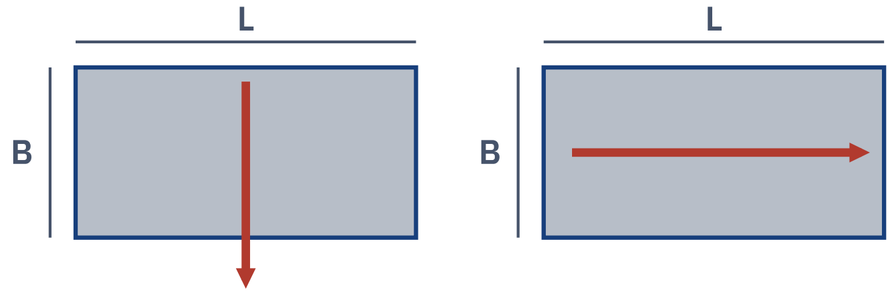
GeoTerva

Geotechnical lecture presentation · Foundations

Vesic bearing capacity equation

Lecture: the coefficients adopted by Vesic and his own contribution: the inclination exponent accounting for the footing shape and the load direction, the base inclination and the ground slope; a worked example on every topic.

Slope exponent m | Inclined load | Base inclination | Ground slope | overlap with Meyerhof | Worked examples



Introduction

What will we learn in this lecture?

To distinguish which coefficients **the Vesic equation** adopts and which are **its own**

Computing the **exponent m** in the slope coefficients according to the foundation B/L ratio and the load direction

Finding the **inclination coefficients** with the actual loads h and v and checking the slip limits

Applying the **base inclination** and **ground slope** factors (Vesic forms); comparing them with those of Hansen

How to handle the undrained (φ = 0) form and groundwater conditions

To see that, with no inclination, base or ground slope, the equation reduces to the **Meyerhof general equation**

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example foundation, the model opened by the module: 1.2 m × 1.2 m square footing in sand, D_f = 1 m, GWT 0.5 m, c = 0, φ′ = 32°.

Adopted coefficients and its own coefficients

THEORY

The Vesic equation has **the same structure** as the Hansen equation: shape, depth, inclination, base inclination and ground inclination corrections in each term:

$$q_u = c\,N_c\,F_{cs}F_{cd}\,i_c b_c g_c + q\,N_q\,F_{qs}F_{qd}\,i_q b_q g_q + \tfrac{1}{2}\gamma B\,N_\gamma\,F_{\gamma s}F_{\gamma d}\,i_\gamma b_\gamma g_\gamma$$

Vesic **did not rederive everything**: he took **De Beer's shape** and **Hansen's depth** coefficients as they were; N_γ is also his closed form: $N_\gamma = 2(N_q + 1)\tan\varphi$.

Its own contribution: the slope coefficients (the exponent depends on the footing shape and the load direction) and the base inclination and ground slope coefficients.

With no slope, base inclination or ground inclination (the module's defaults) i = b = g = 1 and the equation is **exactly the same as the Meyerhof general equation**.

SOURCE OF THE COEFFICIENTS

Coefficient	Source
N_c, N_q	Prandtl, Reissner
N_γ	Vesic
F_s (figure)	De Beer
F_d (depth)	Hansen
i (slope)	**Vesic**
b (base inclination)	**Vesic**
g (ground slope)	**Vesic**

Problem statement

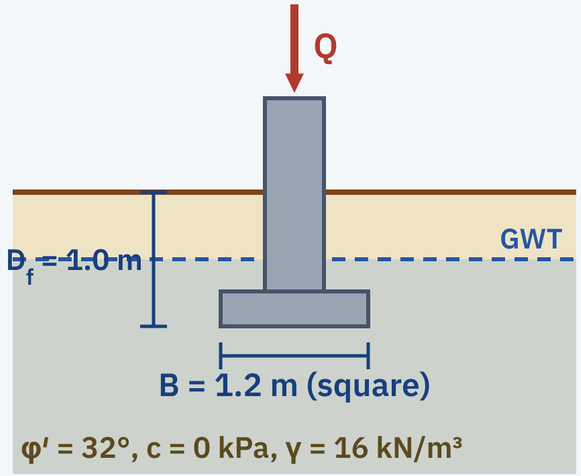
EXAMPLE

Given

Foundation	**square, B = L = 1.2 m, base depth D_f = 1.0 m**
Soil	**sand:** $\varphi' = 32^\circ,\ c = 0$
Unit weight	$\gamma = 16\ \mathrm{kN/m^3}$ **(above),** $\gamma_{sat} = 19.5\ \mathrm{kN/m^3}$ **(below the GWT)**
Load	**vertical, central**

REQUIRED

Determine the values of **N_c, N_q, N_γ** along with **F_{cs}, F_{qs}, $F_{\gamma s}$** and **F_{qd}, F_{cd}**.



Solution

SOLUTION

1 Bearing capacity coefficients (φ′ = 32°, tan 32° = 0.6249)

$$N_q = e^{\pi \tan 32^\circ} \tan^2(61^\circ) = 23.18 \qquad N_c = \frac{23.18 - 1}{0.6249} = 35.49$$

$$N_\gamma = 2(23.18 + 1)(0.6249) = \mathbf{30.21}$$

2 Shape factors (De Beer; B/L = 1)

$$F_{cs} = 1 + \frac{23.18}{35.49} = \mathbf{1.653} \qquad F_{qs} = 1 + \tan 32^\circ = \mathbf{1.625} \qquad F_{\gamma s} = 1 - 0.4 = \mathbf{0.600}$$

3 Depth factors (Hansen; D_f/B = 1.0/1.2 = 0.833 ≤ 1)

$$F_{qd} = 1 + 2(0.6249)(0.2210)(0.833) = \mathbf{1.230} \qquad F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \varphi} = \mathbf{1.241}$$

Identical to the coefficients in Meyerhof's presentation: Vesic did not change them.

Check the solution in GeoTerva ↗

An exponent that accounts for the footing shape and load direction

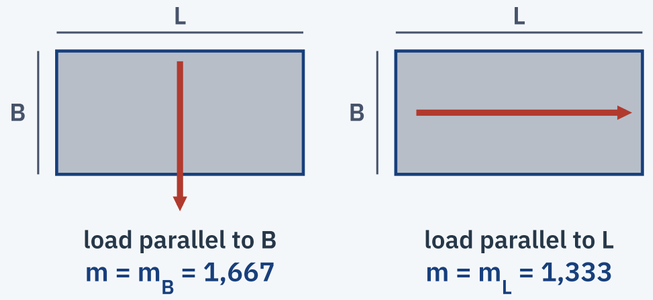
THEORY

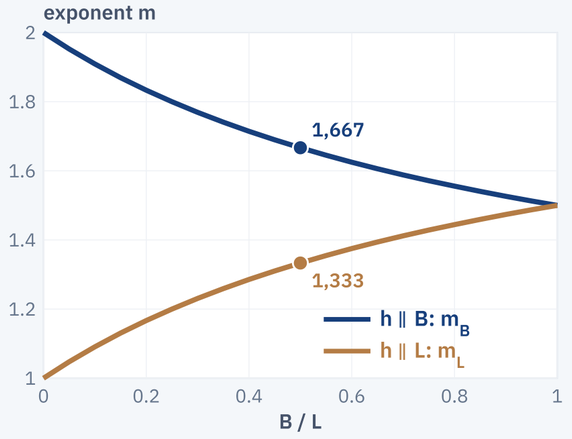
In Meyerhof's inclination factors the exponent is constant (2), and in Hansen's too (5). Vesic's depends on the footing's **B/L ratio** and on **which edge** the horizontal load is **parallel to**:

$$m_B = \frac{2 + B/L}{1 + B/L} \quad (h \parallel B) \qquad m_L = \frac{2 + L/B}{1 + L/B} \quad (h \parallel L)$$

Strip footing (B/L → 0): **m_B = 2**, **m_L = 1**. In a square footing, both directions are the same: m = 1.5.

When the same rectangular footing is loaded in two directions, m differs: **for load parallel to the short side the exponent is larger**, and the coefficients decrease more rapidly. None of the other methods in this section make this distinction.





Problem statement

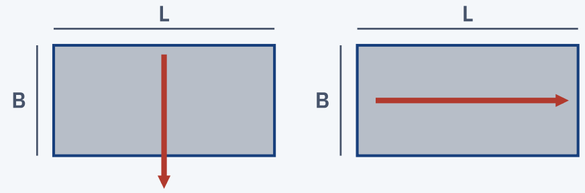
EXAMPLE

Given

Foundations	**(a) square 1.2 × 1.2 m · (b) strip · (c) rectangle 1.2 × 2.4 m (B/L = 0.5)**
Load	**horizontal component parallel to B and parallel to L**

REQUIRED

Find the value of **m** for each footing and each direction.



load parallel to B
$m = m_B = 1{,}667$

load parallel to L
$m = m_L = 1{,}333$

Solution

SOLUTION

1 (a) Square (B/L = 1) and (b) strip (B/L → 0)

$$(\mathrm{a}):\ m = \frac{2+1}{1+1} = \mathbf{1.500}\ \ (\text{in two directions})$$

$$(\mathrm{b}):\ m_B = \frac{2+0}{1+0} = \mathbf{2.000}, \quad m_L = \mathbf{1.000}$$

2 (c) Rectangle 1.2 × 2.4 m (B/L = 0.5; L/B = 2)

$$m_B = \frac{2+0.5}{1+0.5} = \mathbf{1.667} \qquad m_L = \frac{2+2}{1+2} = \mathbf{1.333}$$

For the same rectangular footing, if the load is parallel to the short side m = 1.667, and if parallel to the long side 1.333. If the load is in the short-side direction, the inclination factor $(1 - x)^m$ decreases more rapidly: this direction is **more unfavorable**.

Check the solution in GeoTerva ↗

Vesic: $(1 - h/(v + c \cot \varphi))^m$

THEORY

h = H/A and v = V/A, in the same form as in Hansen, are the pressures of **the same load case** (v = q_{app}; q_{app} is required when h > 0):

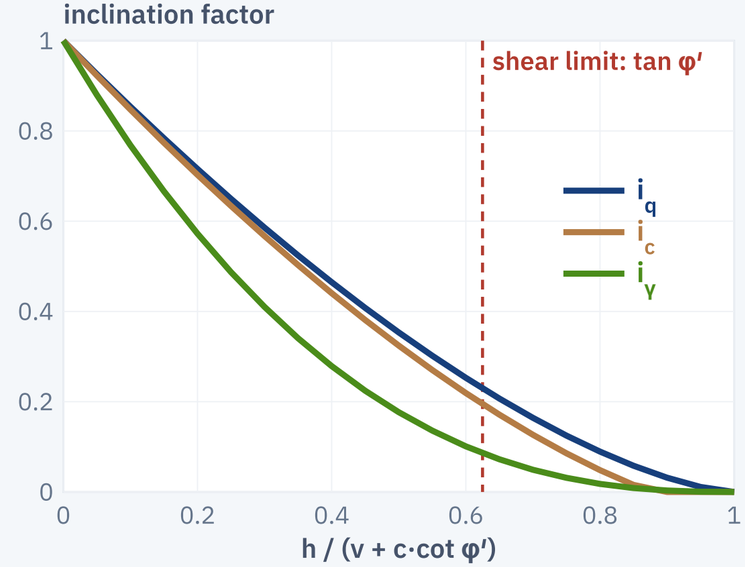
$$i_q = \left(1 - \frac{h}{v + c\cot\varphi}\right)^m \qquad i_\gamma = \left(1 - \frac{h}{v + c\cot\varphi}\right)^{m+1}$$

$$i_c = i_q - \frac{1 - i_q}{N_q - 1} \; (\geq 0)$$

$$\varphi = 0: \quad i_c = 1 - \frac{m\,h}{c\,N_c}, \qquad i_q = i_\gamma = 1$$

h > v tan φ + c: the base slides; the module warns (sliding is also checked separately). **h ≥ v + c cot φ** (at φ = 0, **h ≥ c**): the module stops.

i_c does not become negative, it is held at zero. In the chart the slip limit is the line h/(v + c cot φ) = tan φ.



Problem statement

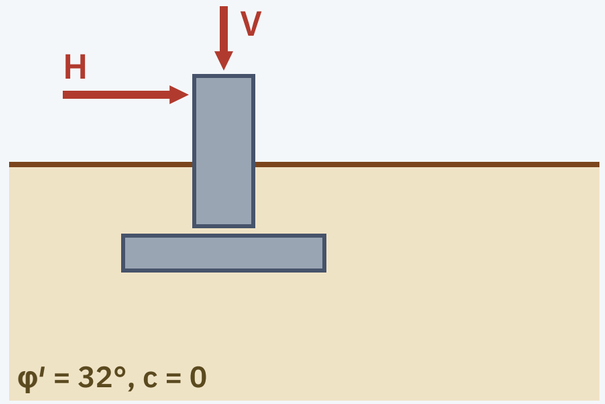
EXAMPLE

Given

Foundation, soil	**square 1.2 × 1.2 m,** $\varphi' = 32^\circ,\ c = 0$ **(module model);** $N_q = 23.18$
Load case	$v = q_{apply} = 150\ \mathrm{kPa},\ h = 30\ \mathrm{kPa}$ **(ek: 60, 90, 120)**
Load direction	**square footing: m = 1.5**

REQUIRED

Determine the i_q, i_c, i_γ coefficients and check the shear limit.



Solution (1/2)

SOLUTION

1 Ratio (c = 0: v + c cot φ = v = 150 kPa)

$$x = \frac{h}{v + c\cot\varphi} = \frac{30}{150} = \mathbf{0.200} \qquad m = 1.5$$

2 Coefficients

$$i_q = (1 - 0.2)^{1.5} = \mathbf{0.716} \qquad i_\gamma = (1 - 0.2)^{2.5} = \mathbf{0.572}$$

$$i_c = i_q - \frac{1 - i_q}{N_q - 1} = 0.716 - \frac{0.284}{22.18} = \mathbf{0.703}$$

3 Slip boundary

$$v\tan\varphi + c = 150(0.6249) = \mathbf{93.7}\ \text{kPa} \geq h = 30 \ \Rightarrow\ \text{no slip}$$

$$(h \geq v = 150 \text{ it stands if})$$

Solution (2/2)

SOLUTION

h (kPa)	h/(v + c cot φ)	i_c	i_q	i_γ	sliding check
0	0.00	1.000	1.000	1.000	passes
30	0.20	0.703	0.716	0.572	passes
60	0.40	0.441	0.465	0.279	passes
90	0.60	0.219	0.253	0.101	passes
120	0.80	0.048	0.089	0.018	**warning: h > 93.7**

h = 30 kPa (20% of v) reduces the q term by **28%** and the self-weight term by **43%**. Since the exponent is m + 1, the **self-weight term is always affected more**. If c = 5 kPa, then v + c cot φ = 158.0 kPa and i_q = 0.729.

Check the solution in GeoTerva ↗

Problem statement

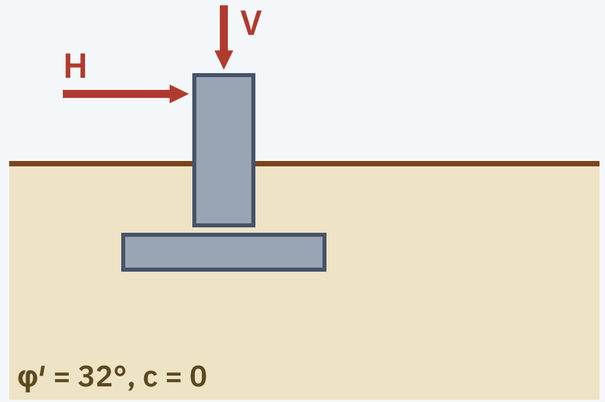
EXAMPLE

Given

Foundation, soil	**square 1.2 m,** $\varphi' = 32°,\ c = 0$
Load case	$v = 150\ \text{kPa},\ h = 30\ \text{kPa}$**: to the vertical** $\beta = \tan^{-1}(30/150) = 11.31°$

REQUIRED

Determine the i_q and i_γ values using the Meyerhof, Hansen and Vesic methods and compare them.



Solution

SOLUTION

1 Meyerhof (with angle β), Hansen (x = h/v = 0.2) and Vesic (m = 1.5)

$$\text{Meyerhof}: F_{;qi} = \left(1 - \frac{11.31}{90}\right)^2 = \mathbf{0.764}, \quad F_{\gamma i} = \left(1 - \frac{11.31}{32}\right)^2 = \mathbf{0.418}$$

$$\text{Hansen}: i_{;q} = (1 - 0.5x)^5 = \mathbf{0.590}, \quad i_{\gamma} = (1 - 0.7x)^5 = \mathbf{0.470}$$

$$\text{Vesic}: i_{;q} = (1 - 0.2)^{1.5} = \mathbf{0.716}, \quad i_{\gamma} = (1 - 0.2)^{2.5} = \mathbf{0.572}$$

	i_q	i_γ
Meyerhof	0.764	0.418
Hansen	0.590	0.470
Vesic	**0.716**	**0.572**

Each coefficient must be used together with its own N_γ and shape factors. Vesic's i_q is close to Meyerhof's, and its i_γ is the highest of the three.

Check the solution in GeoTerva ↗

Vesic's own coefficients

THEORY

Vesic (1975) also proposed his own coefficients for base inclination and ground inclination. For φ > 0 the cohesion coefficient is derived from the surcharge coefficient (η and ω in the exponent in radians):

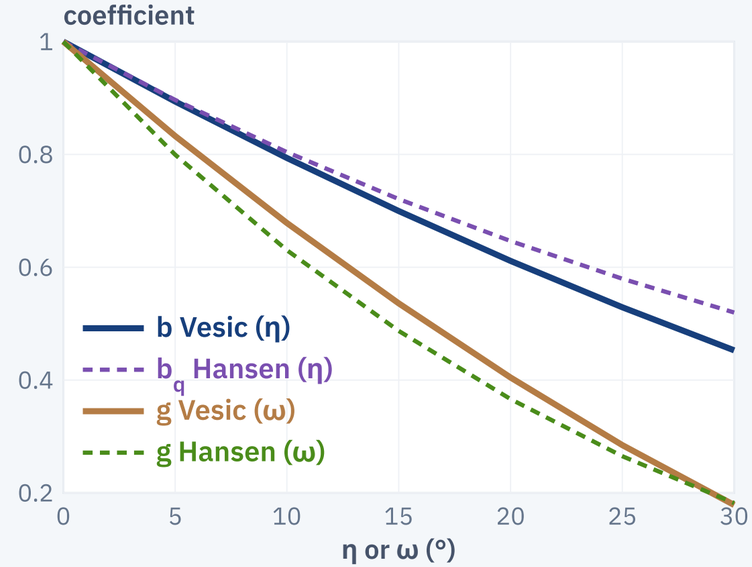
$$b_q = b_\gamma = (1 - \eta \tan\varphi)^2 \qquad g_q = g_\gamma = (1 - \tan\omega)^2$$

$$\varphi > 0: \quad b_c = b_q - \frac{1 - b_q}{N_q - 1} \qquad g_c = g_q - \frac{1 - g_q}{N_q - 1}$$

$$\varphi = 0: \quad b_c = 1 - \frac{2\eta}{\pi + 2} \quad g_c = 1 - \frac{2\omega}{\pi + 2} \quad b_q = b_\gamma = 1$$

Hansen's b_q, b_γ coefficients are exponential ($e^{-2\eta \tan\varphi}$) and $g_q = (1 - 0.5 \tan\omega)^5$; at small angles they are close, and they diverge as ω increases (figure).

If ω > φ the slope is not in equilibrium on its own: the module stops. η and ω are entered at most 45° ($(1 - \tan\omega)^2$ goes to zero there).



Problem statement

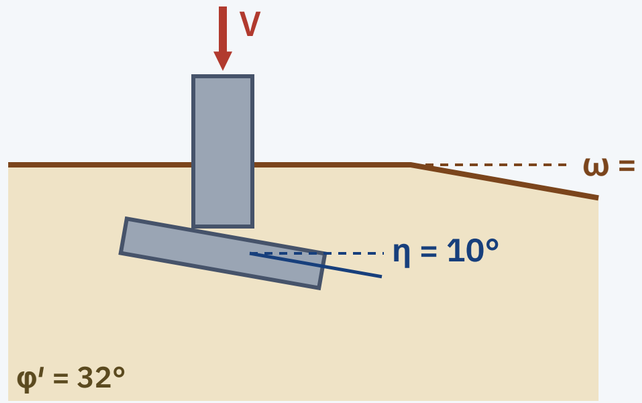
EXAMPLE

Given

Soil	$\varphi' = 32^\circ,\ \tan\varphi = 0.6249,\ N_q = 23.18$
Base slope	**η = 10° (0.1745 rad)**
Ground slope	**ω = 10°**
Comparison	**Hansen coefficients**

REQUIRED

Find Vesic's b_c, b_q, b_γ and g_c, g_q, g_γ values; compare them with Hansen's; write the φ = 0 forms.



Solution (1/2)

SOLUTION

1 Base slope η = 10° (Vesic)

$$b_q = b_\gamma = (1 - 0.1745 \cdot 0.6249)^2 = \mathbf{0.7938} \qquad b_c = 0.7938 - \frac{1 - 0.7938}{23.18 - 1} = \mathbf{0.7845}$$

2 Ground slope ω = 10° (tan 10° = 0.1763)

$$g_q = g_\gamma = (1 - 0.1763)^2 = \mathbf{0.6784} \qquad g_c = 0.6784 - \frac{1 - 0.6784}{23.18 - 1} = \mathbf{0.6639}$$

3 φ = 0 forms (for 10°)

$$b_c = 1 - \frac{2(0.1745)}{\pi + 2} = \mathbf{0.932} \qquad g_c = 1 - \frac{2(0.1745)}{\pi + 2} = \mathbf{0.932} \qquad b_q = b_\gamma = 1$$

Solution (2/2)

SOLUTION

	b_c	b_q	b_γ	g_c	$g_q = g_\gamma$
Vesic	0.784	0.794	0.794	0.664	0.678
Hansen	0.932	0.804	0.745	0.932	0.630

b_q and g_q are **close** in the two methods (0.794 and 0.804; 0.678 and 0.630) but **the cohesion coefficients differ greatly**: Vesic's b_c, g_c values are derived from the surcharge coefficient (0.784 and 0.664), Hansen's are 1 – angle/147° (0.932). In sand with c = 0 this difference does not affect the result.

Check the solution in GeoTerva ↗

Combining the equation and checking

THEORY

Although the six corrections may look daunting, **all of them equal 1 in the ordinary case**: with the module defaults (h = 0, η = 0, ω = 0), i = b = g = 1 and the equation is identical to the general Meyerhof equation. q_u is the vertical pressure at failure; the module keeps gross and net pressure separate:

$$q_{u,net} = q_u - q \qquad q_{all,g} = \frac{q_u}{FS} \qquad q_{all,n} = \frac{q_{u,net}}{FS}$$

The applied pressure $q_{applied}$ is the **vertical gross pressure at the base** for the same loading case (V/A). The footing is acceptable under the following conditions:

$$q_{apply} \le q_{all,g} \qquad FS_{\text{actual}} = \frac{q_u}{q_{apply}} \qquad q_{apply} - q \le q_{all,n}$$

Allowable vertical load $Q_{all} = q_{all,g} \cdot A$. When h > 0, $q_{applied}$ must be entered.

WHEN IS THE COEFFICIENT 1?

Coefficient	case where it is 1
F_s	strip footing (B/L = 0)
F_d	D_f = 0 (base at the surface)
i	h = 0 (vertical load)
b	η = 0 (smooth base)
g	ω = 0 (level ground)

Coefficients unchanged; q and γ change

THEORY

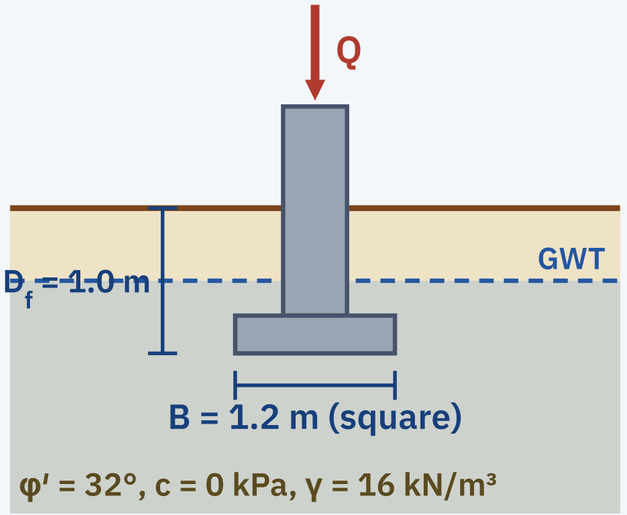
Groundwater is treated with the **same two cases** as in the other modules in this section. Below the GWT the effective unit weight is $\gamma' = \gamma_{sat} - \gamma_w$ ($\gamma_w = 9.81$). D_w is the depth of the groundwater from the surface:

$$\text{I } (D_w \le D_f):\ q = \gamma D_w + \gamma'(D_f - D_w),\ \ \gamma \to \gamma' \text{ (third term)}$$

$$\text{III } (D_f < D_w \le D_f + B):\ q = \gamma D_f,\ \ \bar{\gamma} = \frac{1}{B}\left[\gamma D + \gamma'(B - D)\right],\ D = D_w - D_f$$

For $D_w = D_f$, $q = \gamma D_f$ and γ' is used in the third term.

Water deeper than B below the base is **ineffective**. Water enters only through **q and γ**; F, i, b, g remain the same.



Problem statement

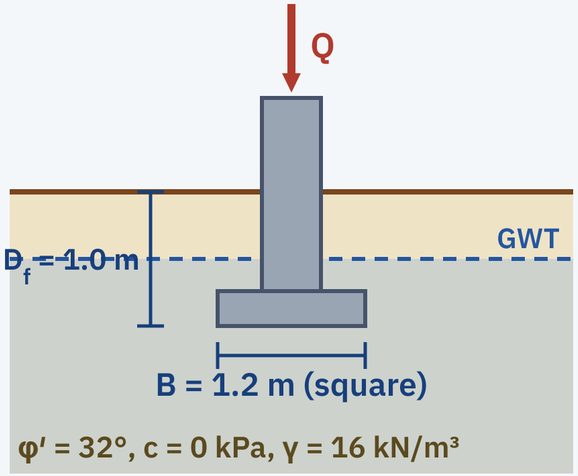
EXAMPLE

Given

Foundation	**square, B = L = 1.2 m, base depth D_f = 1.0 m**
Soil	**sand:** $\varphi' = 32^\circ,\ c = 0$
Unit weight	$\gamma = 16\ \mathrm{kN/m^3}$ **(above),** $\gamma_{sat} = 19.5\ \mathrm{kN/m^3}$ **(below the GWT)**
Groundwater	**D_W = 0.5 m (case I)**
Load	**vertical, central**
Safety	**FS = 3**

REQUIRED

q_u, gross and net **q_{all}** and the allowable load **Q_{all}**; then q_u for c = 5 kPa.



Solution (1/2)

SOLUTION

1 Coefficients (Example 1: φ′ = 32°; B/L = 1; D_f/B = 0.833; h = η = ω = 0)

$$N_q = 23.18,\ N_\gamma = 30.21 \qquad F_{qs} = 1.625,\ F_{\gamma s} = 0.6,\ F_{qd} = 1.230 \qquad i = b = g = 1$$

2 Case I: q = 16(0.5) + (19.5 – 9.81)(0.5); γ′ = 9.69 kN/m³ (in the third term)

$$q = \mathbf{12.845}\ \mathrm{kPa} \qquad \gamma' = \mathbf{9.69}\ \mathrm{kN/m^3}$$

Solution (2/2)

SOLUTION

3 Bearing capacity (c = 0: no cohesion term)

$$q_u = (12.845)(23.18)(1.625)(1.230) + \tfrac{1}{2}(9.69)(1.2)(30.21)(0.6)$$
$$= 595.1 + 105.4 = \mathbf{700.46}\ \text{kPa}$$

4 Allowable pressure and load (A = 1.2² = 1.44 m²)

$$q_{all,g} = \frac{700.46}{3} = \mathbf{233.49}\ \text{kPa} \qquad q_{all,n} = \frac{700.46 - 12.845}{3} = 229.20\ \text{kPa}$$

$$Q_{all} = 233.49(1.44) = \mathbf{336.2}\ \text{kN}$$

q_u = **700.46 kPa** found with the same inputs (β = 0) using the general Meyerhof equation: the two equations are **exactly identical**. If c = 5 kPa, the cohesion term would add 363.9 kPa and q_u = 1,064.3 kPa.

Check the solution in GeoTerva ↗

Problem statement

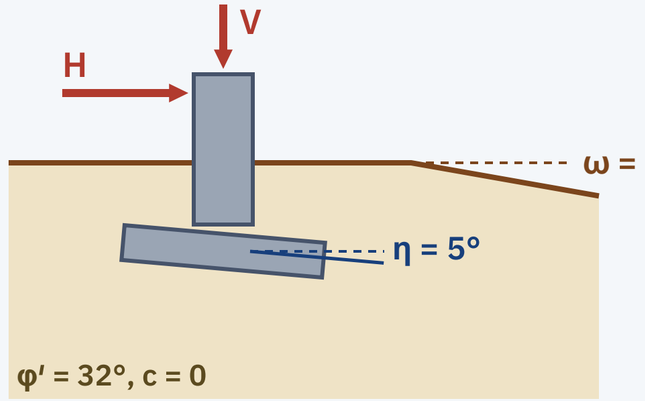
EXAMPLE

Given

Foundation	**square, B = L = 1.2 m, base depth D_f = 1.0 m**
Soil	**sand:** $\varphi' = 32^\circ,\ c = 0$
Unit weight	$\gamma = 16\ \mathrm{kN/m^3}$ **(above),** $\gamma_{sat} = 19.5\ \mathrm{kN/m^3}$ **(below the GWT)**
Groundwater	**D_w = 0.5 m**
Load case	$v = q_{apply} = 90\ \mathrm{kPa},\ h = 18\ \mathrm{kPa}$ **(h ∥ B)**
Base, soil	**η = 5°, ω = 10°**
Safety	**FS = 3**

REQUIRED

Find the **q_u** and **q_{all}** values; check whether the footing passes for q_{app} = 90 kPa and check sliding.



Solution (1/3)

SOLUTION

1 Slope coefficients (x = h/v = 18/90 = 0.2; m = 1.5)

$$i_q = (1 - 0.2)^{1.5} = \mathbf{0.716} \qquad i_\gamma = (1 - 0.2)^{2.5} = \mathbf{0.572} \qquad i_c = 0.703$$

2 Base and ground slope factors (η = 5° = 0.0873 rad; ω = 10°)

$$b_q = b_\gamma = (1 - 0.0873 \cdot 0.6249)^2 = \mathbf{0.894} \qquad b_c = 0.889$$

$$g_q = g_\gamma = (1 - \tan 10^\circ)^2 = \mathbf{0.678} \qquad g_c = 0.664$$

Solution (2/3)

SOLUTION

3 Three terms (c = 0; F coefficients as in Example 6)

$$q\text{ term} = (12.845)(23.18)(1.625)(1.230)(0.716)(0.894)(0.678) = 258.2$$

$$\gamma\text{ term} = \tfrac{1}{2}(9.69)(1.2)(30.21)(0.6)(0.572)(0.894)(0.678) = 36.6$$

4 Bearing capacity and allowable pressure

$$q_u = 258.2 + 36.6 = \mathbf{294.8}\ \text{kPa} \qquad q_{all,g} = \frac{294.8}{3} = \mathbf{98.3}\ \text{kPa}$$

Solution (3/3)

SOLUTION

5 Check (q_{appl} = 90 kPa)

$$q_{apply} = 90 \leq q_{all,g} = 98.3 \;\Rightarrow\; \textbf{valid}$$

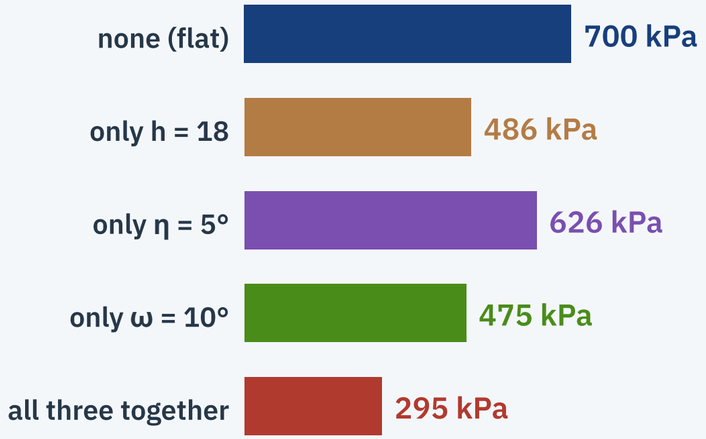
$$FS_{\text{actual}} = \frac{294.8}{90} = \mathbf{3.28} \geq 3$$

$$q_{apply} - q = 77.2 \leq q_{all,n} = 94.0 \;\Rightarrow\; \textbf{valid}$$

6 Slip boundary

$$h = 18 \leq v \tan\varphi = 56.2\ \text{kPa} \;\Rightarrow\; \text{no slip}$$

The q_u of 700 kPa in the level model drops to **295 kPa** (58%) under this loading condition. Individually: ground slope −32%, horizontal load −31%, base inclination −11%. The footing is safe in all three conditions (FS = 3.28).



Check the solution in GeoTerva ↗

φ = 0: the forms of the cohesion coefficients themselves

THEORY

At φ = 0, N_c = 5.14, N_q = 1, N_γ = 0; the self-weight term drops out and the surcharge term becomes only q g_q. In this module the cohesion term remains **multiplicative** (unlike Hansen's additive form):

$$q_u = c\,N_c\,F_{cs}F_{cd}\,i_c\,b_c\,g_c + q\,g_q$$

The φ = 0 forms of the cohesion coefficients are given in the table: for inclination, **i_c = 1 − m h/(c N_c)**; for base and ground inclination, **1 − 2·angle/(π + 2)** (angle in radians).

If **h ≥ c**, the base slides and the module stops. At φ = 0, F_{cs} = 1 + (B/L)/5.14 (N_q/N_c = 1/5.14) and F_{cd} = 1 + 0.4 k.

φ = 0 CASES

Coefficient	Value
N_c	5.14
F_{cs}	1 + (B/L)/5.14
F_{cd}	1 + 0.4 k
i_c	1 − m h/(c N_c)
b_c	1 − 2η/(π + 2)
g_c	1 − 2ω/(π + 2)
g_q	(1 − tan ω)²

Problem statement

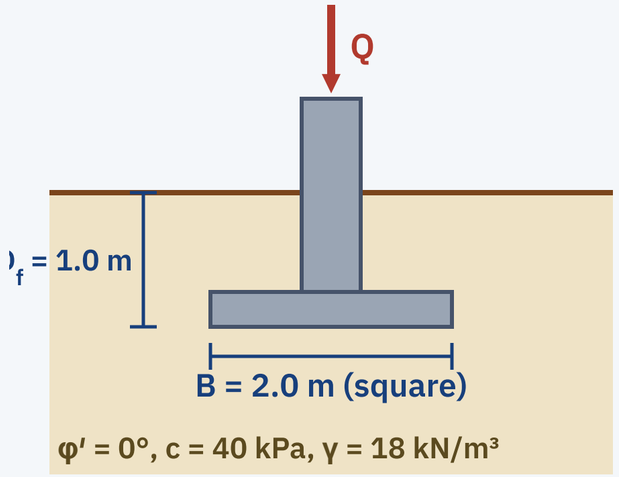
EXAMPLE

Given

Foundation	**square B = L = 2 m, D_f = 1 m**
Soil	**clay:** $\varphi = 0,\ c_u = 40\ \mathrm{kPa},\ \gamma = 18\ \mathrm{kN/m^3}$**; no groundwater**
Cases	**(a) vertical load · (b)** $v = 100,\ h = 10\ \mathrm{kPa}$ **· (c)** $h = 10$ **and** $\eta = \omega = 10^\circ$

REQUIRED

Find the q_u value for each case.



Solution (1/2)

SOLUTION

1. (a) Vertical load: F_{cs} = 1 + 1/5.14; F_{cd} = 1 + 0.4(0.5); q = 18 kPa

$$q_u = 40(5.14)(1.195)(1.200) + 18 = 294.72 + 18 = \mathbf{312.72}\ \text{kPa}$$

2. (b) h = 10 kPa, c = 40 (m = 1.5)

$$i_c = 1 - \frac{1.5(10)}{40(5.14)} = \mathbf{0.927} \Rightarrow q_u = 273.22 + 18 = \mathbf{291.22}\ \text{kPa}$$

Solution (2/2)

SOLUTION

3 (c) h = 10, η = ω = 10°: $b_c = g_c = 1 - 2(0.1745)/(\pi + 2) = 0.932$; $g_q = (1 - \tan 10°)^2$

$$q_u = 40(5.14)(1.195)(1.200)(0.927)(0.932)(0.932) + 18(0.678) = 237.4 + 12.2 = \mathbf{249.6}\ \text{kPa}$$

Condition	$c\,N_c F_{cs} F_{cd} \cdot (i\,b\,g)$	$q\,g_q$	q_u (kPa)
(a) vertical	294.7	18.0	**312.7**
(b) h = 10	273.2	18.0	**291.2**
(c) h, η, ω	237.4	12.2	**249.6**

A load of only h = 10 kPa reduces the bearing capacity by 7%, and together with h the base and ground inclination reduce it by 20%. Hansen's additive form gives 305.84 kPa for the same vertical case (Hansen presentation): the two methods agree within 2%.

Check the solution in GeoTerva ↗

Problem statement

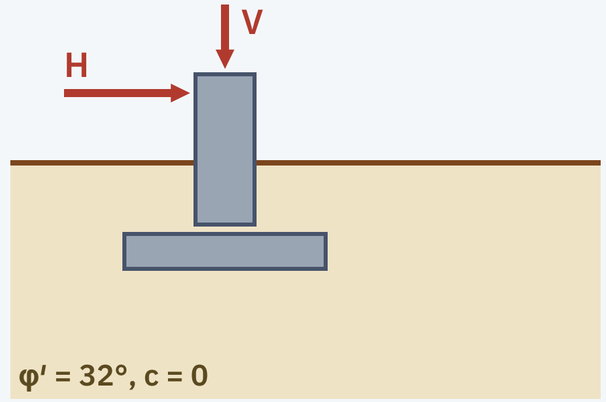
EXAMPLE

Given

Foundation	**square, B = L = 1.2 m, base depth D_f = 1.0 m**
Soil	**sand:** $\varphi' = 32°,\ c = 0$
Unit weight	$\gamma = 16\ \mathrm{kN/m^3}$ **(above),** $\gamma_{sat} = 19.5\ \mathrm{kN/m^3}$ **(below the GWT)**
Groundwater	**D_w = 0.5 m**
Load case	$v = 150\ \mathrm{kPa},\ h = 30\ \mathrm{kPa}$**;** $\beta = 11.31°$ **for Meyerhof**

REQUIRED

Compare the **q_u** values of the two equations term by term for the vertical load and for this inclined load.



Solution (1/2)

SOLUTION

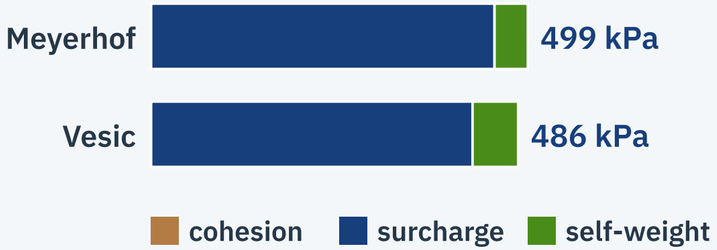
Vertical load	Vesic	Meyerhof
q term	595.1	595.1
γ term	105.4	105.4
q_u (kPa)	**700.46**	**700.46**

When there is no slope, base or ground inclination, the Vesic equation is **exactly the same** as the Meyerhof general equation (700.46 kPa): the divergence of Vesic begins only in inclined cases. This is the module's own verification path.

Solution (2/2)

SOLUTION

Inclined load	Vesic (m = 1.5)	Meyerhof (β = 11.31°)
i_q / F_{qi}	0.716	0.764
i_γ / $F_{\gamma i}$	0.572	0.418
q term	425.8	454.9
γ term	60.3	44.1
q_u (kPa)	**486.1**	**499.0**



Under inclined loading the difference is small: **2.6%** (Vesic is lower). Meyerhof's $F_{\gamma i}$ reduces more and his F_{qi} less; since the q term dominates, the results are close.

Check the solution in GeoTerva ↗

Summary

Key points

The **Vesic equation** has the same structure as the Hansen equation; it takes the **De Beer shape** and **Hansen depth** coefficients as they are, and $N_\gamma = 2(N_q + 1)\tan\varphi$ as it is

His own contribution: the inclination exponent m, base inclination b and ground inclination g coefficients

Inclination: $i_q = (1 - h/(v + c\cot\varphi))^m$, $i_\gamma = (\ldots)^{m+1}$; $m = (2 + B/L)/(1 + B/L)$ ($h \parallel B$) or $(2 + L/B)/(1 + L/B)$ ($h \parallel L$); for a strip, $m_B = 2$, $m_L = 1$

Base inclination $b_q = b_\gamma = (1 - \eta\tan\varphi)^2$; **ground inclination** $g_q = g_\gamma = (1 - \tan\omega)^2$; for $\varphi > 0$, b_c, g_c from the surcharge factor, for $\varphi = 0$, $1 - 2\cdot\text{angle}/(\pi + 2)$

Limits: $h > v\tan\varphi + c$ shear warning; $h \geq v + c\cot\varphi$ (for $\varphi = 0$, $h \geq c$) rest; $\omega > \varphi$ rest

With $i = b = g = 1$ the equation is the **general Meyerhof equation**; the **groundwater table** changes only q and γ

Check: $q_{app} \leq q_{all,g}$, FS_{actual} and net; the coefficients of each method are used together with its own N_γ

References

Vesic's complete bearing-capacity equation — the B/L-dependent inclination exponent and his own base-tilt factors — follows Vesic, A.S., "Analysis of Ultimate Loads of Shallow Foundations," Journal of the Soil Mechanics and Foundations Division, ASCE, 1973, as restated consistently across standard geotechnical references (alongside Hansen's closely related equation — see the Hansen tool in this section). The Nγ closed form used across this whole section is Vesic's, as set out in Das, B.M., Principles of Geotechnical Engineering, 9th ed., Cengage Learning, 2018 (Chapter 16); the depth factors are Hansen's, as restated in Das, B.M., Principles of Foundation Engineering, 8th ed., Cengage Learning, 2016 (Table 4.3). In the text of this page “the source” is the first of these two books and “the second source” the second.

NOTE

All numerical examples have been produced with the module's own computation library; the module gives the same values.

GeoTerva

Online geotechnical analysis

From theory to practice

Compute the example foundation in this lecture step by step in GeoTerva: the module opens with this model.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com