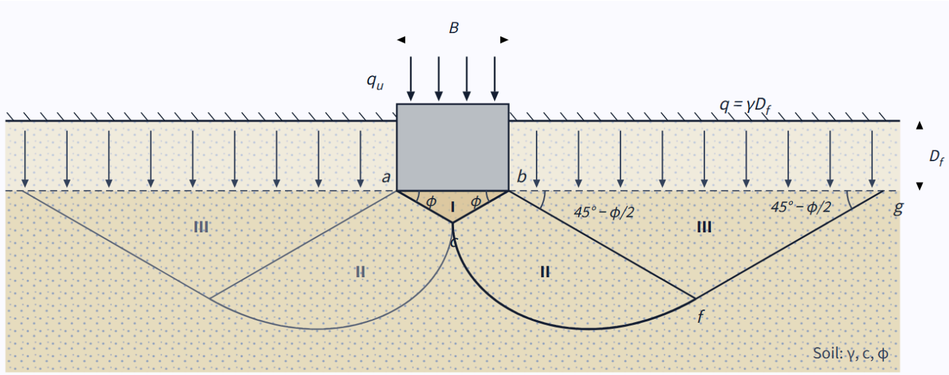
Geotechnical lecture presentation · Foundations

Terzaghi bearing capacity

Lecture: from Terzaghi's failure mechanism and his derivation to the N_q, N_c, N_γ coefficients, from square and circular footings to local shear and groundwater; a worked example in every topic.

Three zones | Log spiral | $N_c \cdot N_q \cdot N_\gamma$ | Square and circular footing | Local shear | Worked examples



Introduction

What will we learn in this lecture?

- the **assumptions** of the Terzaghi equation and how to distinguish **general / local shear** failure
- To follow the failure mechanism (**three zones**, log spiral) and the decomposition of the passive force into three parts
- to determine the N_q **and** N_c factors from closed-form expressions and N_γ from the Kumbhojkar table
- To write the equation for a **strip, square, circular** footing; to treat the undrained ($\varphi = 0$) case
- Applying the same equation with c' and φ' for **local shear**
- To compute the three conditions of the **groundwater** and the gross/net safe pressure and **check** the footing

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example foundation, the model opened by the module: a 1.5 m × 1.5 m square footing in soil with c = 15.2 kPa, φ' = 20°, γ = 17.8 kN/m³, D_f = 1 m, GWT at 2 m.

What is the Terzaghi equation based on?

THEORY

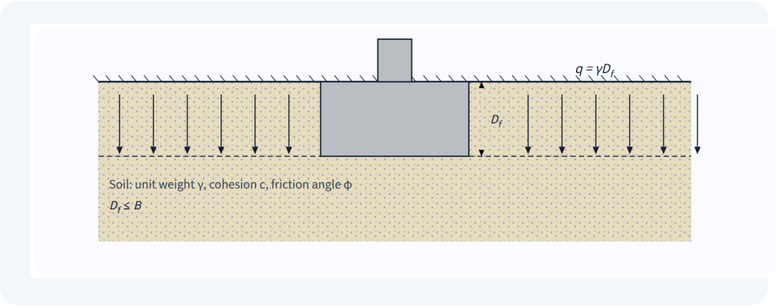
Terzaghi (1943) obtained the bearing capacity q_u, that is, the average pressure at which the soil beneath the footing fails in shear, from an assumed failure mechanism and the equilibrium of its parts. The derivation rests on the following assumptions:

The foundation is a **long strip** (plane strain), rigid and **rough-based**: the soil beneath cannot slide at the base and moves down as a single body together with the foundation.

The soil below the base is **homogeneous** and very deep; the load is **vertical and central**.

The footing is **shallow**, $D_f \leq B$: the soil above the base does not participate in the mechanism and acts only as a **surcharge $q = \gamma D_f$**; shear strength is neglected (a conservative simplification).

The soil fails by **general shear**; local shear is treated afterwards with reduced parameters.



Shear strength follows Mohr–Coulomb:

$$s = c + \sigma' \tan \varphi$$

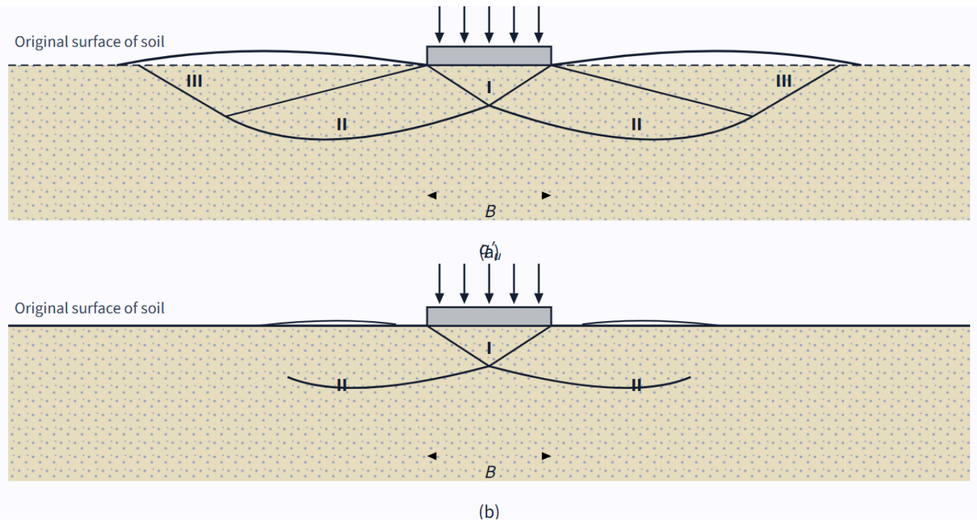
How does a footing fail?

THEORY

Let us load a strip footing on dense sand or stiff clay in steps. Settlement first increases slowly; then at a certain pressure the settlement runs away **without any increase in load**: a continuous slip surface has formed extending from the footing edge to the surface, and the ground heaves beside the footing. This is **general shear** failure and that pressure is $\mathbf{q_u}$.

In **loose sand and soft clay** the curve has no peak: after q'_u it passes to a steeper but still rising straight line; the slip surfaces remain within the soil, heave is small: **local shear**.

Three zones beneath the footing: the **wedge (I)** that moves down as a single body, the **radial zone (II)** pushed by the wedge, and the **passive zone (III)** pushed up and sideways. In local shear the wedge forms the same way, but zones II and III dissipate within the soil without reaching the surface.



Three zones and logarithmic spiral

THEORY

Terzaghi divided the soil on both sides into three zones:

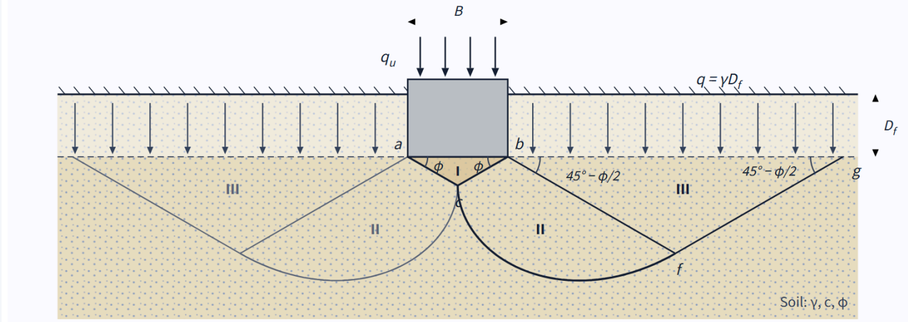
I · wedge abc: the rough base is bonded; it moves down rigidly together with the footing. Its sides ac and bc make **α = φ** with the horizontal.

II · radial zone bcf (Prandtl): every ray emanating from b is a slip line; the boundary cf is a **log spiral** arc with pole b.

III · passive Rankine zone bfg: bf and fg are straight slip lines making 45° − φ/2 with the horizontal.

$$r = r_0\, e^{\theta \tan\varphi} \qquad \theta_1 = \angle cbf = 180^\circ - \varphi - \left(45^\circ - \tfrac{\varphi}{2}\right) = \frac{3\pi}{4} - \frac{\varphi}{2}$$

The property of the spiral: at every point the radius drawn from the pole makes an angle φ with the normal to the curve. Therefore friction along the spiral **produces no moment** about the pole, and equilibrium is written with moments about this pole.



It is vertical and divided into three parts

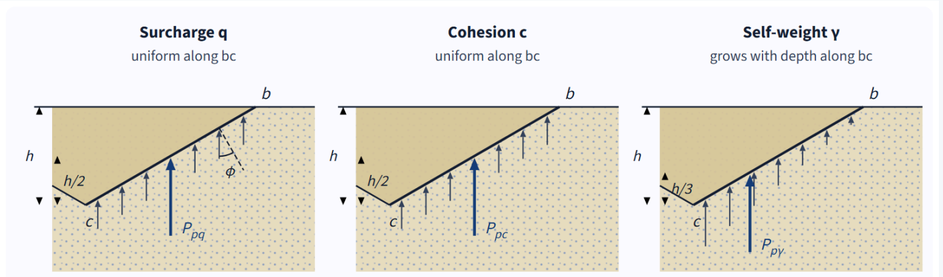
THEORY

The footing pushes the wedge downward; the soil in zones II and III resists on each face with a **passive force P_p**. Since the soil slides against the soil, the full friction angle acts: P_p makes an angle φ with the normal to the face. Since face bc makes an angle φ with the horizontal, its normal makes an angle φ with the vertical; the force deviating by φ from this normal is **exactly vertical**.

P_p depends simultaneously on q, c and γ′, and the three together do not yield a closed-form solution. Terzaghi determined each as if the other two were zero and **added** them (superposition):

$$P_p = P_{pq} + P_{pc} + P_{p\gamma} \qquad h = \frac{B}{2}\tan\varphi$$

- P_{pq}: surcharge only (γ = 0, c = 0) → **N_q**
- P_{pc}: cohesion only (γ = 0, q = 0) → **N_c**
- $P_{p\gamma}$: soil weight only (q = 0, c = 0) → **N_γ**



Moments about pole b

THEORY

With surcharge alone the pole is at b. On the mass bcfj act P_{pq} along bc, q along bj, the Rankine passive force along jf, and the resultant F along the spiral; F passes through the pole and drops out of the moment equation:

$$P_{p(1)} = q\,H_d \tan^2\left(45^\circ + \tfrac{\varphi}{2}\right) \qquad P_{pq}\frac{B}{4} = q\,\overline{bj}\,\frac{\overline{bj}}{2} + P_{p(1)}\frac{H_d}{2}$$

$$r_0 = \frac{B}{2}\sec\varphi \quad r_1 = r_0\, e^{\left(\frac{3\pi}{4}-\frac{\varphi}{2}\right)\tan\varphi} \quad \overline{bj} = r_1\cos\left(45^\circ - \tfrac{\varphi}{2}\right) \quad H_d = r_1\sin\left(45^\circ - \tfrac{\varphi}{2}\right)$$

Since tan(45° + φ/2) = cot(45° − φ/2), the two terms come out equal: $P_{pq} = (4/B)\, q\, r_1^2 \cos^2(45° - \varphi/2)$. With the wedge equilibrium $\mathbf{q_q\, B = 2\, P_{pq}}$:

$$q_q = qN_q \qquad N_q = \frac{e^{2\left(\frac{3\pi}{4}-\frac{\varphi}{2}\right)\tan\varphi}}{2\cos^2\left(45^\circ + \frac{\varphi}{2}\right)}$$

RESULT

For φ = 0, N_q = 1: with no friction the surcharge is transmitted unchanged. For φ = 30°, **N_q = 22.46**.

Problem statement

EXAMPLE

Given

Soil — $\varphi' = 30^\circ$ **and** $\varphi' = 20^\circ$ **(module model)**

Formula

$$N_q = \frac{e^{2(3\pi/4 - \varphi/2)\tan\varphi}}{2\cos^2(45^\circ + \varphi/2)}$$

REQUIRED

For each φ′, find the value of **N_q** from the closed form.

Solution

SOLUTION

1 φ′ = 30° (3π/4 − φ/2 = 2.0944 rad; tan 30° = 0.5774; 45° + 15° = 60°)

$$N_q = \frac{e^{2(2.0944)(0.5774)}}{2\cos^2 60^\circ} = \frac{e^{2.4184}}{2(0.25)} = \frac{11.228}{0.5} = \mathbf{22.46}$$

2 φ′ = 20° (3π/4 − φ/2 = 2.1817 rad; tan 20° = 0.3640; 45° + 10° = 55°)

$$N_q = \frac{e^{2(2.1817)(0.3640)}}{2\cos^2 55^\circ} = \frac{e^{1.5881}}{2(0.3290)} = \frac{4.895}{0.6580} = \mathbf{7.44}$$

$N_q(30°)$ = **22.46**, $N_q(20°)$ = **7.44**; for φ′ = 0, N_q = 1. These are the values in the module's table.

Check the solution in GeoTerva ↗

cot φ (N_q − 1)

THEORY

With cohesion alone the pole is again at b. In the mass bcfj the cohesion force C = c·bc acts (along a radius, with no moment), on jf the cohesive Rankine passive force acts, and along the spiral c acts. The spiral element is ds = r sec φ dθ and its moment is c r² dθ:

$$P_{p(2)} = 2cH_d \tan\left(45^\circ + \tfrac{\varphi}{2}\right) \qquad M_c = \int_0^{\theta_1} c\, r^2 d\theta = \frac{c}{2\tan\varphi}\left(r_1^2 - r_0^2\right)$$

From the moment equilibrium P_{pc} B/4 = $P_{p(2)}$ H_d/2 + M_c, P_{pc} is found; from the wedge equilibrium **q_c B = 2 C sin φ + 2 P_{pc}** (the vertical component of C). With the two identities the terms in parentheses cancel:

$$q_c = c\cot\varphi\left[\frac{e^{2\left(\frac{3\pi}{4}-\frac{\varphi}{2}\right)\tan\varphi}}{2\cos^2\left(45^\circ+\frac{\varphi}{2}\right)} - 1\right] = c\cot\varphi\,(N_q - 1) = cN_c$$

$$N_c = \cot\varphi\,(N_q - 1)$$

RESULT

As φ → 0, N_c → 3π/2 + 1 = 5.71; Terzaghi's φ = 0 value is **5.7** (a method different from Prandtl's 5.14). For φ = 30°, **N_c = 37.16**.

Problem statement

EXAMPLE

Given

Soil	$\varphi' = 30^\circ$ **and** $\varphi' = 20^\circ$
Previous example	$N_q(30^\circ) = 22.46,\ N_q(20^\circ) = 7.44$

REQUIRED

Find the $\mathbf{N_c}$ values from $N_c = \cot \varphi\,(N_q - 1)$; write the limit as $\varphi \to 0$.

Solution

SOLUTION

1 φ′ = 30° and φ′ = 20°

$$N_c(30^\circ) = \frac{22.46 - 1}{0.5774} = \mathbf{37.16} \qquad N_c(20^\circ) = \frac{7.44 - 1}{0.3640} = \mathbf{17.69}$$

2 φ → 0 limit

$$N_c \to \frac{3\pi}{2} + 1 = \mathbf{5.71} \approx 5.7 \text{ (Terzaghi's value)}$$

$N_c(30°)$ = **37.16**, $N_c(20°)$ = **17.69**. At the same φ′, N_c is much larger than N_q: the cohesion term dominates at low φ′.

Check the solution in GeoTerva ↗

Trial pole and the coefficient with no closed form

THEORY

With the weight, the spiral at pole b is no longer critical: pole O is moved **by trial** along the extension of line fb beyond b; each position gives a new spiral and a new bcfj mass. Moments about O:

$$P_{p\gamma} = \frac{W\, l_w + P_{p(3)}\, l_R}{l_p} \qquad P_{p(3)} = \tfrac{1}{2}\gamma H_d^2 \tan^2\!\left(45^\circ + \tfrac{\varphi}{2}\right)$$

The smallest $P_{p\gamma}$ is the value developed by the soil. The wedge now also carries its own weight: $W_w = ½\ B\ h\ \gamma = (B^2/4)\ \gamma \tan\varphi$. If $P_{p\gamma} = ⅛\ \gamma\ B^2\ K_{p\gamma} \tan^2\varphi$ is written ($K_{p\gamma}$ depends only on φ):

$$q_\gamma B = 2P_{p\gamma} - W_w \;\Rightarrow\; q_\gamma = \tfrac{1}{2}\gamma B\, N_\gamma \qquad N_\gamma = \tfrac{1}{2}K_{p\gamma} \tan^2\varphi - \tfrac{1}{2}\tan\varphi$$

Since $K_{p\gamma}$ comes from a trial-and-error reduction, **there is no closed form for N_γ**; it is computed numerically for each φ. The module uses **Kumbhojkar (1993)** values (Table 1) and interpolates linearly between them: for φ = 30°, N_γ = 19.13.

WHY A TABLE?

N_c and N_q are obtained from closed-form expressions. N_γ, however, is the result of a minimization: although written by the same method, there is no formula universally used.

Problem statement

EXAMPLE

Given

Soil	$\varphi' = 20^\circ$ **(module model) and** 30°**; also** 22.5°
Source	**Kumbhojkar (1993) Table 1: $N_\gamma(22°) = 5.09$, $N_\gamma(23°) = 6.00$**

REQUIRED

$N_\gamma(20°)$, $N_\gamma(30°)$; $K_{p\gamma}$ from these; and $N_\gamma(22.5°)$ (linear interpolation).

Solution

SOLUTION

1 Table 1 ($N_\gamma = ½\, K_{p\gamma} \tan^2\varphi - ½ \tan\varphi \Rightarrow K_{p\gamma} = (2N_\gamma + \tan\varphi)/\tan^2\varphi$)

$$N_\gamma(20^\circ) = 3.64 \;\Rightarrow\; K_{p\gamma} = \frac{2(3.64) + 0.3640}{0.3640^2} = 57.7$$

$$N_\gamma(30^\circ) = 19.13 \;\Rightarrow\; K_{p\gamma} = \frac{2(19.13) + 0.5774}{0.5774^2} = 116.5$$

2 $\varphi' = 22.5°$: halfway between 22° and 23° (t = 0.5)

$$N_\gamma = 5.09 + 0.5(6.00 - 5.09) = \mathbf{5.545}$$

$N_\gamma(20°) = \mathbf{3.64}$, $N_\gamma(30°) = \mathbf{19.13}$; $K_{p\gamma}$ 57.7 → 116.5. The module also linearly interpolates N_c and N_q from the same table: at 22.5°, $N_c = 21.01$, $N_q = 9.71$.

Check the solution in GeoTerva ↗

Sum of the three parts and Table 1

THEORY

Summing the three parts gives the bearing capacity of the **strip footing** on a cohesive-frictional soil with weight:

$$q_u = q_c + q_q + q_\gamma = c\,N_c + q\,N_q + \tfrac{1}{2}\gamma B\,N_\gamma$$

N_q, N_c, N_γ depend **only on φ**; Table 1 gives the values read by the module.

Superposition is not exact: N_q and N_c come from spirals with pole at the foundation edge, N_γ from a spiral with pole at O. The sum of separately minimized parts cannot exceed the minimum of the combined problem; the error is **on the safe side**.

Tests show that the wedge edges are not at φ but at approximately **45° + φ/2**. Terzaghi's coefficients (and this module) keep α = φ; the 45° + φ/2 wedge is the basis of the Prandtl–Reissner coefficients (Meyerhof, Hansen, Vesic modules), and their N_c, N_q values therefore differ.

TABLE 1 (Terzaghi)

φ′ (°)	N_c	N_q	N_γ
0	5.70	1.00	0.00
5	7.34	1.64	0.14
10	9.60	2.69	0.56
15	12.86	4.45	1.52
20	17.69	7.44	3.64
25	25.13	12.72	8.34
30	37.16	22.46	19.13
35	57.75	41.44	45.41
40	95.66	81.27	116.31
45	172.29	173.29	325.34
50	347.51	415.15	1,072.80

Problem statement

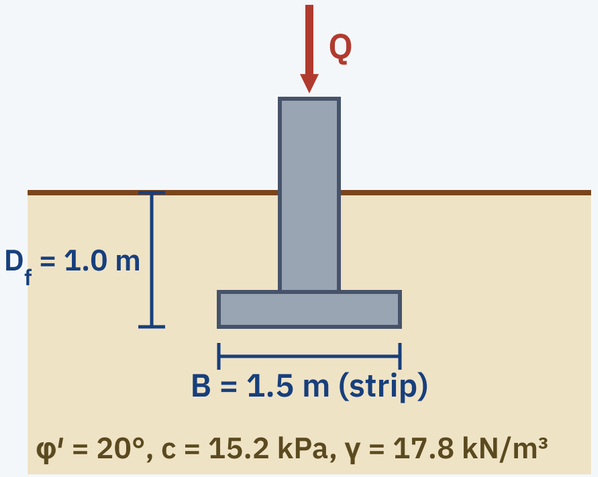
EXAMPLE

Given

Foundation	**strip (continuous), B = 1.5 m, D_f = 1.0 m**
Soil	$c = 15.2\ \mathrm{kPa},\ \varphi' = 20^\circ,\ \gamma = 17.8\ \mathrm{kN/m^3}$
Groundwater	**none (at depth)**
Table 1 (20°)	**N_c = 17.69, N_q = 7.44, N_γ = 3.64**

REQUIRED

Find the values of **q_u** and **Q_u** per unit length.



Solution

SOLUTION

1 Surcharge q = γ D_f = 17.8 × 1.0 = 17.8 kPa; no shape factor for the strip

$$q_u = 15.2(17.69) + 17.8(7.44) + \tfrac{1}{2}(17.8)(1.5)(3.64)$$
$$= 268.89 + 132.43 + 48.59 = \mathbf{449.91}\ \mathrm{kPa}$$

2 Load per unit length

$$Q_u = q_u B = 449.91(1.5) = \mathbf{674.9}\ \mathrm{kN/m}$$

Bearing capacity of the strip footing is **449.9 kPa**: cohesion term 60%, surcharge 29%, self-weight 11%.

Check the solution in GeoTerva ↗

Semi-empirical shape factors

THEORY

A square or circular footing does not put the soil beneath it into a plane-strain condition; the derivation cannot be applied directly. Terzaghi proposed **semi-empirical coefficients** for the cohesion and unit-weight terms:

$$q_u = 1.3\,c\,N_c + q\,N_q + 0.4\,\gamma B\,N_\gamma \qquad (\text{square, B} \times \text{B})$$

$$q_u = 1.3\,c\,N_c + q\,N_q + 0.3\,\gamma B\,N_\gamma \qquad (\text{circle, diameter B})$$

Undrained loading in saturated clay (φ = 0, c = c_u; N_c = 5.7, N_q = 1, N_γ = 0):

$$q_u = 5.7\,c_u + q\ (\text{strip}) \qquad q_u = 1.3 \times 5.7\,c_u + q = 7.41\,c_u + q\ (\text{square, circle})$$

The module's **Shape** option selects these coefficients. Terzaghi did not provide coefficients for a rectangular footing; that case is covered by the general bearing capacity equation with shape factors (Meyerhof, Hansen, Vesic).

COEFFICIENTS

Foundation	c N_c	γ B N_γ	B
strip	1	0.5	width
square	1.3	0.4	edge
circle	1.3	0.3	diameter

Problem statement

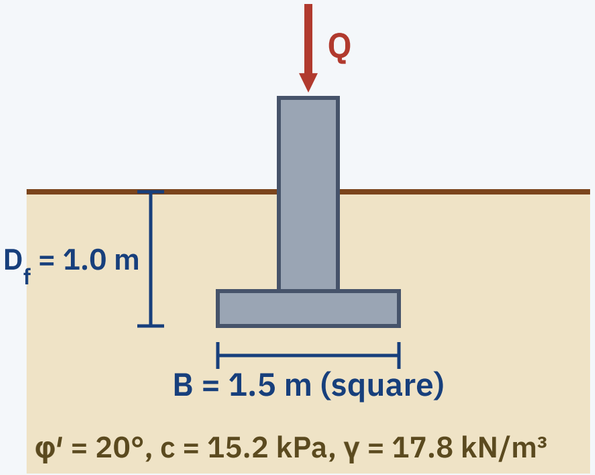
EXAMPLE

Given

Foundation	**square, B = L = 1.5 m, base depth D_f = 1.0 m**
Soil	$c = 15.2\ \mathrm{kPa},\ \varphi' = 20^\circ$
Unit weight	$\gamma = 17.8\ \mathrm{kN/m^3}$ **(top)**
Groundwater	**none (at depth)**
Comparison	**circle, diameter B = 1.5 m; also clay: φ = 0, c_u = 40 kPa, γ = 18 kN/m³, D_f = 1 m**

REQUIRED

q_u and **Q_u** for the square and the circle; **q_u** for the strip and the square in clay.



Solution (1/2)

SOLUTION

1 Square (0.4) and circle (0.3); $N_c = 17.69$, $N_q = 7.44$, $N_\gamma = 3.64$

$$\text{square}: \quad q_u = 1.3(15.2)(17.69) + 17.8(7.44) + 0.4(17.8)(1.5)(3.64)$$
$$= 349.55 + 132.43 + 38.88 = \mathbf{520.86}\ \text{kPa}$$
$$\text{circle}: \quad q_u = 349.55 + 132.43 + 0.3(17.8)(1.5)(3.64) = \mathbf{511.14}\ \text{kPa}$$

2 Loads

$$Q_u^{square} = q_u B^2 = 520.86(2.25) = \mathbf{1{,}171.9}\ \text{kN}$$
$$Q_u^{circle} = q_u \frac{\pi B^2}{4} = 511.14(1.767) = \mathbf{903.3}\ \text{kN}$$

The q_u of the square footing is only 1.9% higher than that of the circle (only the γ term differs).

Solution (2/2)

SOLUTION

3 Undrained clay (q = 18 × 1 = 18 kPa; N_c = 5.7)

$$\text{strip}: \quad q_u = 5.7(40) + 18 = \mathbf{246.0}\ \text{kPa}$$

$$\text{square, circular}: \quad q_u = 1.3(5.7)(40) + 18 = 7.41(40) + 18 = \mathbf{314.4}\ \text{kPa}$$

Foundation	c N_c term	q	q_u (kPa)
strip	228.0	18.0	**246.0**
square / circle	296.4	18.0	**314.4**

In clay the bearing capacity comes almost solely from cohesion; square and circular footings carry **28%** more than a strip footing (factor 1.3).

Check the solution in GeoTerva ↗

Same equation, reduced c and φ

THEORY

When the soil fails by local shear, the mechanism does not fully develop and only a part of the strength is mobilized. Terzaghi kept **the same equations** and reduced the strength parameters:

$$c' = \frac{2}{3}\,c \qquad \varphi' = \tan^{-1}\left(\frac{2}{3}\tan\varphi\right)$$

The corrected N'_c, N'_q, N'_γ are the same functions evaluated at φ′ (Table 3):

$$q_u = c' N'_c + q N'_q + \tfrac{1}{2}\gamma B N'_\gamma \ \text{(strip)} \qquad q_u = 1.3\,c' N'_c + q N'_q + 0.4\,\gamma B N'_\gamma \ \text{(square)}$$

Since c′ = ⅔ c, the cohesion term for square and circular footings becomes 1.3 × ⅔ = **0.867 c N′c**.

For sands, Vesic (1973) proposed a reduction depending on relative density instead of a fixed ⅔: **φ′ = tan⁻¹(k tan φ)**, k = 0.67 + D_r − 0.75 D_r^2 (0 ≤ D_r ≤ 0.67).

TABLE 3 (local shear)

φ (°)	φ′ (°)	N′c	N′q	N′γ
10	6.70	8.03	1.94	0.25
15	10.13	9.67	2.73	0.58
20	13.64	11.86	3.88	1.18
25	17.27	14.82	5.61	2.29
30	21.05	18.99	8.31	4.35
35	25.02	25.18	12.75	8.38
40	29.22	34.89	20.53	16.84

Problem statement

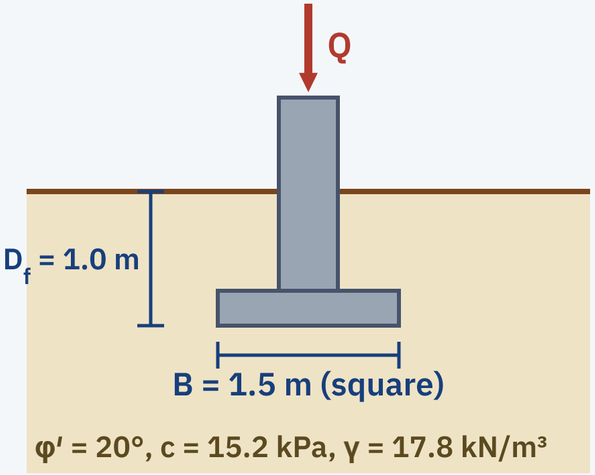
EXAMPLE

Given

Foundation	**square, B = L = 1.5 m, base depth D_f = 1.0 m**
Soil	$c = 15.2\ \mathrm{kPa},\ \varphi' = 20^\circ$
Unit weight	$\gamma = 17.8\ \mathrm{kN/m^3}$ **(top)**
Groundwater	**none (at depth)**
Failure mode	**local shear**

REQUIRED

c′, φ′, corrected coefficients and **q_u**; its ratio to general shear.



Solution (1/2)

SOLUTION

1 Reduced strength (tan 20° = 0.3640)

$$\varphi' = \tan^{-1}\left(\tfrac{2}{3}\cdot 0.3640\right) = \mathbf{13.64}^\circ \qquad c' = \tfrac{2}{3}(15.2) = \mathbf{10.13}\ \mathrm{kPa}$$

2 Table 3 (φ′ = 13.64°, by interpolation)

$$N_c' = \mathbf{11.86} \qquad N_q' = \mathbf{3.88} \qquad N_\gamma' = \mathbf{1.18}$$

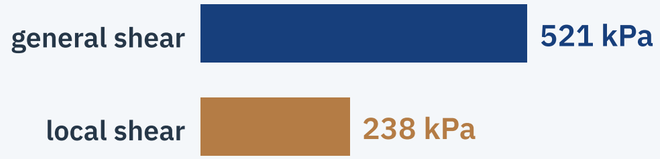
Solution (2/2)

SOLUTION

3 Square footing (q = 17.8 kPa)

$$q_u = 1.3(10.13)(11.86) + 17.8(3.88) + 0.4(17.8)(1.5)(1.18)$$
$$= 156.20 + 69.05 + 12.61 = \mathbf{237.86}\ \text{kPa}$$

For local shear, q_u = **237.86 kPa**, only **46%** of that for general shear (520.86 kPa). In loose or soft soil, this reduction removes **more than half**.



Check the solution in GeoTerva ↗

Water changes only q and γ

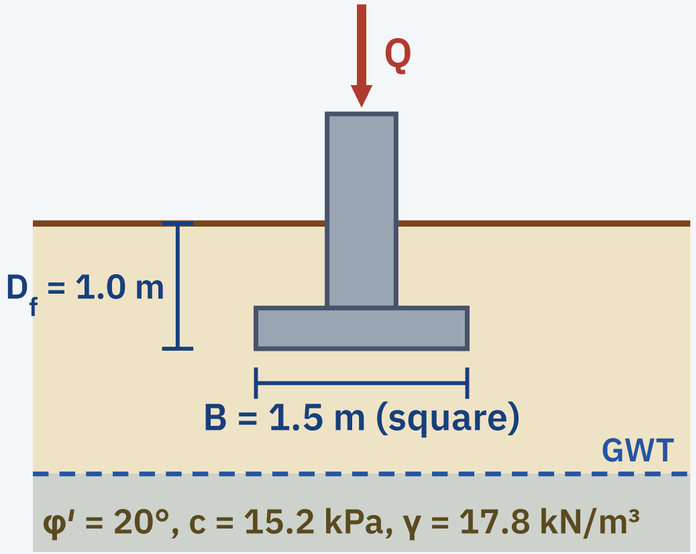
THEORY

The derivation assumes the groundwater table very deep. Water close to the base changes two things: the **surcharge q** and the **unit weight** in the ½γBN$_\gamma$ term; never the coefficients. Below the GWT the effective unit weight is γ′ = γ$_{sat}$ − γ$_w$ (γ$_w$ = 9.81). The module distinguishes three cases (D$_w$ is the depth from the surface):

$$\text{I } (D_w \le D_f):\ q = \gamma D_w + \gamma'(D_f - D_w),\ \ \gamma \to \gamma' \text{ (third term)}$$

$$\text{III } (D_w > D_f):\ q = \gamma D_f,\ \ \bar{\gamma} = \frac{1}{B}\left[\gamma D + \gamma'(B - D)\right],\ D = D_w - D_f \le B$$

D$_w$ = D$_f$ (case II) is a special case of I: q = γ D$_f$, with γ′ in the third term. When the base is deeper than one B below the base (D > B), the water is below the failure zone and the equation is used as if there were no water.



Problem statement

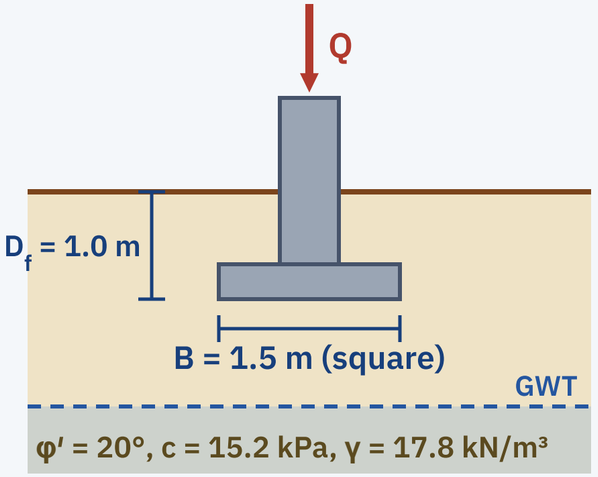
EXAMPLE

Given

Foundation	**square, B = L = 1.5 m, base depth D_f = 1.0 m**
Soil	$c = 15.2\ \text{kPa},\ \varphi' = 20^\circ$
Unit weight	$\gamma = 17.8\ \text{kN/m}^3$ **(top)**
Groundwater	$\gamma_{sat} = 19\ \text{kN/m}^3$**;** $D_w = 0.5;\ 1.0;\ 2.0$ **and** $3.5\ \text{m}$

REQUIRED

For each case, find **q**, the unit weight in the third term and $\mathbf{q_u}$ of the square footing.



Solution (1/3)

SOLUTION

1 γ′ = 19 − 9.81 = 9.19 kN/m³. Case I (D_W = 0.5) and II (D_W = D_f = 1.0)

$$D_w = 0.5: \;\; q = 17.8(0.5) + 9.19(0.5) = \mathbf{13.495}\ \mathrm{kPa},\ \gamma' = 9.19$$

$$D_w = 1.0: \;\; q = 17.8(1.0) = \mathbf{17.8}\ \mathrm{kPa},\ \gamma' = 9.19$$

2 Case III (D_W = 2.0 ⇒ D = 1.0 ≤ B = 1.5) and water ineffective (D_W = 3.5 ⇒ D = 2.5 > B)

$$D_w = 2.0: \;\; \bar{\gamma} = \frac{1}{1.5}\left[17.8(1.0) + 9.19(0.5)\right] = \mathbf{14.93}\ \mathrm{kN/m^3},\ q = 17.8\ \mathrm{kPa}$$

$$D_w = 3.5: \;\; q = 17.8,\ \gamma = 17.8$$

Solution (2/3)

SOLUTION

3 q_u for D_w = 2.0 m (square; first and second terms unchanged)

$$q_u = 1.3(15.2)(17.69) + 17.8(7.44) + 0.4(14.93)(1.5)(3.64)$$
$$= 349.55 + 132.43 + 32.61 = \mathbf{514.59}\ \text{kPa}$$

The model with which the module opens (D_w = 2 m, case III): q_u = **514.59 kPa**; without water it would be 520.86 kPa.

Solution (3/3)

SOLUTION

D_w (m)	Condition	q (kPa)	γ third term	q_u (kPa)
0.5	I	13.50	9.19	**470.0**
1.0	II	17.80	9.19	**502.1**
2.0	III	17.80	14.93	**514.6**
3.5	negligible	17.80	17.80	**520.9**

With water at 0.5 m, q_u = 470; when deep, 521 kPa (10% difference). The cohesion term (349.6 kPa) never changes: **water enters only through q and γ**.

Check the solution in GeoTerva ↗

Gross and net, applied pressure

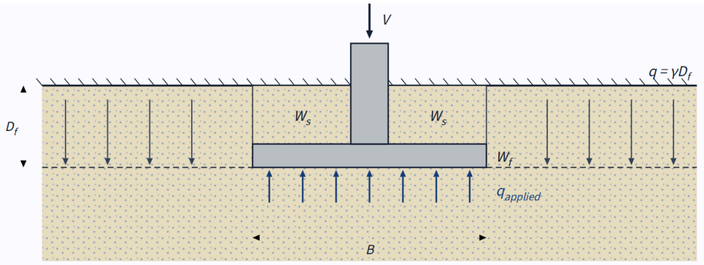
THEORY

q_u is the pressure at failure. The module divides it by the factor of safety FS and keeps the **gross** and **net** pressures separate. The net pressure is the pressure the footing **adds** above the surcharge q that existed at the base before excavation:

$$q_{u,net} = q_u - q \qquad q_{all,g} = \frac{q_u}{FS} \qquad q_{all,n} = \frac{q_{u,net}}{FS}$$

The applied pressure $q_{applied}$ is the **vertical gross pressure** at the base under service loads: the column load V, the footing weight W_f and the soil above the footing W_s, divided by the base area (square B^2, circle $\pi B^2/4$, strip B). The footing is acceptable under the following conditions:

$$q_{apply} = \frac{V + W_f + W_s}{A} \le q_{all,g} \qquad FS_{\text{actual}} = \frac{q_u}{q_{apply}} \qquad q_{apply} - q \le q_{all,n}$$



If q ≥ 0 and FS ≥ 1, the **gross check governs**: if it passes, the net check also passes. If q_{app} is not entered, the module gives only the capacities. FS ≈ 3 is customary; it accounts for the uncertainty of the strength parameters more than of the theory.

Problem statement

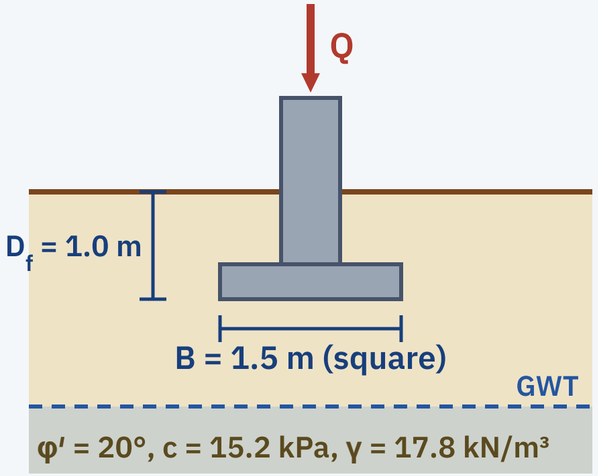
EXAMPLE

Given

Foundation	**square, B = L = 1.5 m, base depth D_f = 1.0 m**
Soil	$c = 15.2\ \mathrm{kPa},\ \varphi' = 20^\circ$
Unit weight	$\gamma = 17.8\ \mathrm{kN/m^3}$ **(top)**
Groundwater	$\gamma_{sat} = 19\ \mathrm{kN/m^3},\ D_w = 2\ \mathrm{m}$ **(Example 7: $q_u = 514.59$ kPa)**
Safety	**FS = 3**
Applied pressure	**(a) q_{app} = 150 kPa · (b) q_{app} = 200 kPa**

REQUIRED

$q_{u,net}$, $q_{all,g}$, $q_{all,n}$, Q_{all} and the check for each $q_{applied}$.



Solution

SOLUTION

1 Allowable pressures (q = 17.8 kPa)

$$q_{u,net} = 514.59 - 17.8 = 496.79 \text{ kPa}$$

$$q_{all,g} = \frac{514.59}{3} = \mathbf{171.53} \text{ kPa} \qquad q_{all,n} = \frac{496.79}{3} = \mathbf{165.60} \text{ kPa}$$

$$Q_{all} = 171.53(1.5^2) = \mathbf{385.9} \text{ kN}$$

2 Check

$$\text{(a) } 150 \leq 171.53 \Rightarrow \textbf{valid}, \; FS_{\text{actual}} = \frac{514.59}{150} = 3.43 \geq 3$$

$$\text{(b) } 200 > 171.53 \Rightarrow \textbf{does not pass}, \; FS_{\text{actual}} = \frac{514.59}{200} = 2.57 < 3$$

The net condition is also satisfied for q_{app} = 150 kPa (132.2 ≤ 165.6 kPa); at 200 kPa neither the gross nor the net condition is satisfied. When the gross condition is satisfied, the net condition is always satisfied.

Check the solution in GeoTerva ↗

Summary

Key points

Terzaghi (1943): strip, rough, shallow ($D_f \le B$) footing; general shear; wedge (I), radial (II) and passive (III) zone; wedge edges at $\alpha = \varphi$

Superposition: $P_p = P_{pq} + P_{pc} + P_{p\gamma}$; $q_u = cN_c + qN_q + \frac{1}{2}\gamma B N_\gamma$

$N_q = e^{2(3\pi/4 - \varphi/2)\tan\varphi}/[2\cos^2(45° + \varphi/2)]$, $N_c = \cot\varphi\,(N_q - 1)$ (closed), **N_γ** from the Kumbhojkar table ($\varphi = 20°$: 17.69; 7.44; 3.64)

Square: 1.3 $cN_c + qN_q + 0.4\,\gamma B N_\gamma$; **circle**: 0.3; **undrained**: $N_c = 5.7$, strip 5.7 $c_u + q$, square 7.41 $c_u + q$

Local shear: the same equation with $c' = \frac{2}{3}\,c$, $\varphi' = \tan^{-1}(\frac{2}{3}\tan\varphi)$ (example: 520.86 → 237.86 kPa)

Groundwater changes only q and γ (3 cases): example 514.59 kPa

Check: $q_{app} \le q_{all,g}$, FS_{actual} and net; $Q_{all} = q_{all,g} \cdot A$

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The figures of this page are drawn from the mechanism itself; the equation, figure and table numbers are its own.

NOTE

Some of the figures are taken from the GeoTerva Theory page; all of the numerical examples were produced with the module's own calculation library and table, and the module gives the same values.

GeoTerva

Online geotechnical analysis

From theory to practice

Compute the example foundation in this lecture step by step in GeoTerva: the module opens with this model.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com