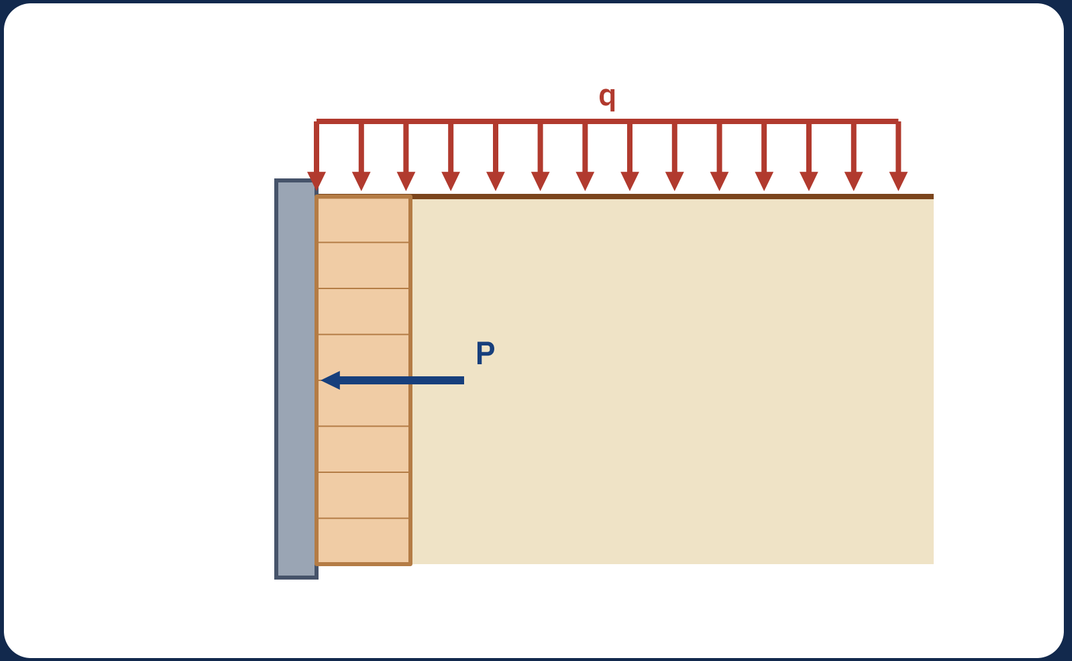
Geotechnical lecture presentation · Surcharge loads

Uniform surcharge load

Lecture: uniform load spread over the entire surface behind the wall; constant lateral pressure with depth, resultant and line of action, worked examples.

$\sigma_H = K \cdot q$ $K_0 \cdot K_a$ $P = K\,q\,H$ $z_R = H/2$

Rectangular diagram



What will we learn in this lecture?

- how a uniformly distributed load q is reflected on the wall as **lateral pressure**: $\sigma_H = K \cdot q$, the same at every depth
- Selection of the **coefficient K** according to wall behaviour: at-rest (K_0, Jaky), active (K_a, Rankine) or an entered value
- The **resultant** of the rectangular diagram P = K q H and the **line of action** z_R = H/2
- How K_0 and K_a vary with φ′; the linear increase of P with q and H
- How the surcharge is **added** to the earth pressure diagram and its share

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example wall: H = 6 m, q = 10 kPa, φ′ = 30°. Forces are per unit length of wall (kN/m); pressures are in kPa.

Uniformly distributed load acting on the wall

THEORY

A uniform **q** load spread over the **entire ground surface** behind the wall increases the vertical stress at every depth by q

The wall feels only a part of this: this part is determined by the lateral earth pressure coefficient **K**

The added pressure is **the same** from the top of the wall to its base: the pressure diagram is a rectangle

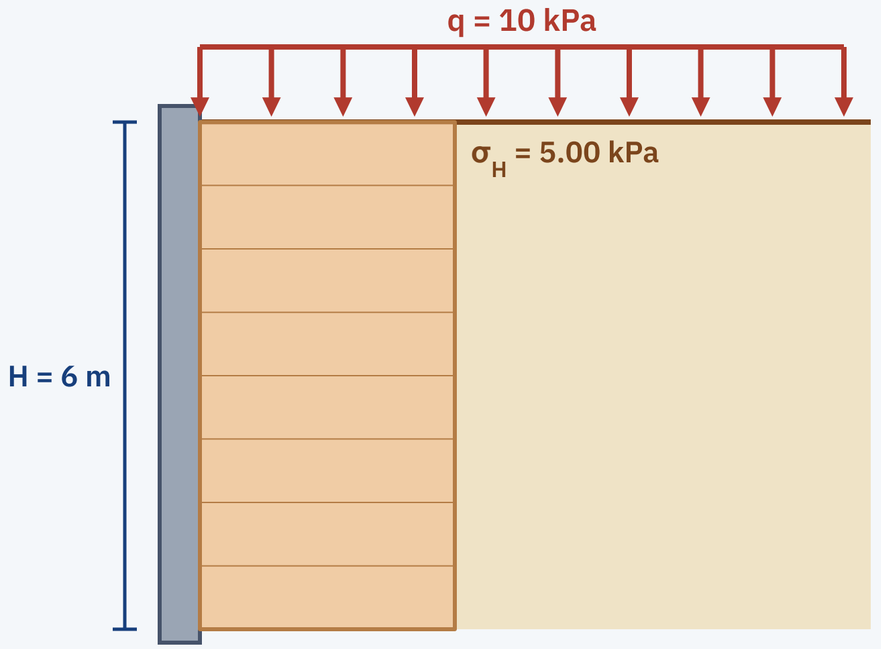
$$\sigma_H = K\,q$$

q — uniform surcharge on the retained soil surface

φ′ — effective angle of internal friction of the retained soil

K — lateral earth pressure coefficient (K_0, K_a, or user-entered)

σ_H — added lateral pressure, constant with depth



Wall behaviour governs K

THEORY

A **non-yielding** (supported) wall keeps the soil in the at-rest state: in normally consolidated soil this is the default **Jaky** coefficient

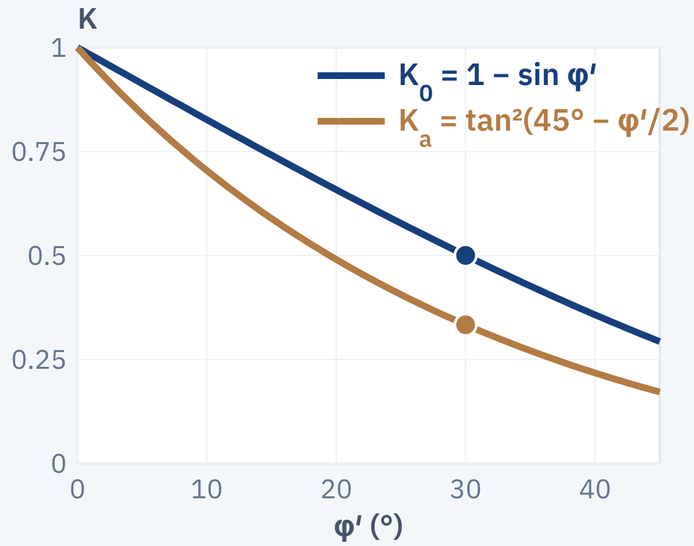
A **yielding** wall mobilises the active state: on a vertical wall and horizontal surface the **Rankine** coefficient is used

A coefficient from another source (overconsolidated K_0, Coulomb K_a for an inclined or rough wall) is **entered directly**

$$K_0 = 1 - \sin\varphi' \qquad \text{(at rest, Jaky)}$$

$$K_a = \tan^2\left(45^\circ - \frac{\varphi'}{2}\right) \qquad \text{(active, Rankine)}$$

K_0 and K_a – φ′



Variation of the two coefficients with the angle of internal friction

THEORY

φ′ (°)	0	20	25	30	35	40
K_a	1.0000	0.4903	0.4059	0.3333	0.2710	0.2174
K_0	1.0000	0.6580	0.5774	0.5000	0.4264	0.3572

As φ′ increases, **both coefficients decrease**: in stronger soil the wall feels a smaller share

At the same φ′, **$K_0 > K_a$**: a wall that cannot move takes more load than a yielding wall

For $\varphi' = 0$, $K_0 = K_a = 1$: the load is transmitted fully to the wall, as in a fluid

Resultant P = K q H, at mid-height

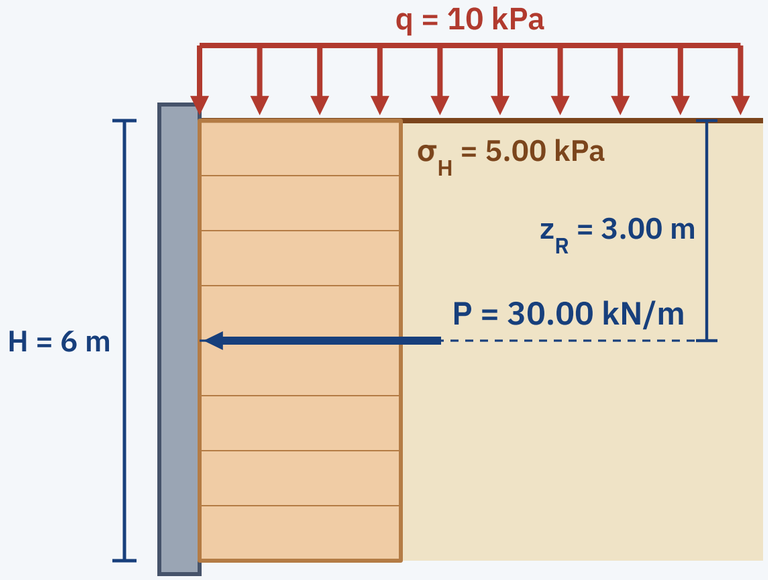
THEORY

The diagram is a rectangle: the **resultant per unit length** of the wall is the area, and the **line of action** passes through the centre of the rectangle

The resultant acts at the **middle** of the height: H/2 below the ground surface (also H/2 above the base)

This is only the **surcharge component**: it is added to the soil and water pressure diagrams

$$P = \sigma_H H = K q H \qquad z_R = \frac{H}{2}$$



P — resultant force per unit length of wall (kN/m)

z_R — depth of the resultant below the ground surface

Problem statement

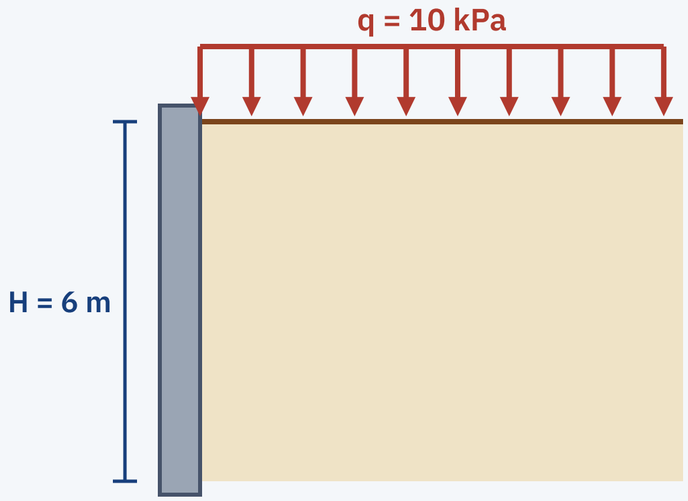
EXAMPLE

Given

Wall	$H = 6.00$ **m, rigid (non-yielding) wall**
Load	$q = 10.00$ **kPa over the entire surface**
Soil	$\varphi' = 30^\circ$

REQUIRED

K, the added lateral pressure σ_H, the resultant **P** and its depth z_R.



Solution

SOLUTION

1 Coefficient: wall cannot move, at-rest condition (Jaky)

$$K_0 = 1 - \sin\varphi' = 1 - \sin 30^\circ = 1 - 0.5 = \mathbf{0.5000}$$

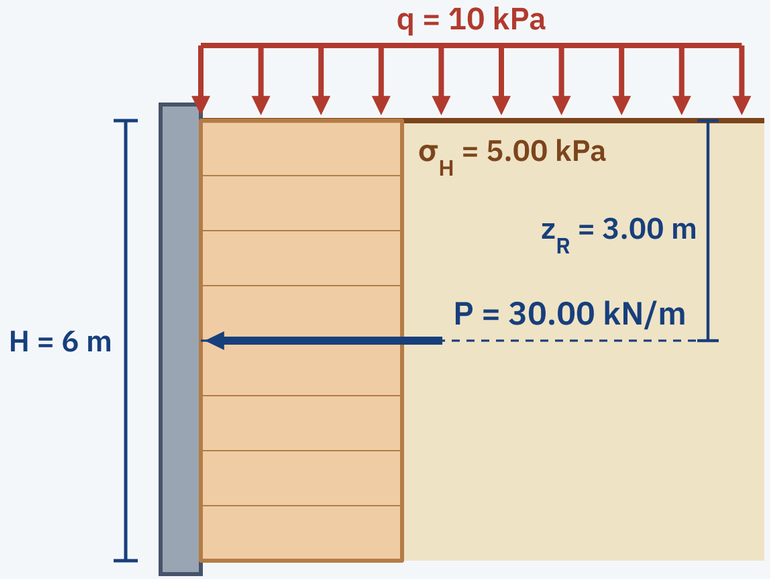
2 Added lateral pressure: same at every depth

$$\sigma_H = K\,q = 0.5000 \times 10.00 = \mathbf{5.00}\ \mathrm{kPa}$$

3 Resultant and line of action

$$P = \sigma_H\,H = 5.00 \times 6.00 = \mathbf{30.00}\ \mathrm{kN/m}$$

$$z_R = \frac{H}{2} = \frac{6.00}{2} = \mathbf{3.00}\ \mathrm{m}$$



The wall receives a lateral pressure of **5.00 kPa** over its full height; the resultant is **30.00 kN/m**, located **3.00 m** below the ground surface (3.00 m above the base).

Check the solution in GeoTerva ↗

Problem statement

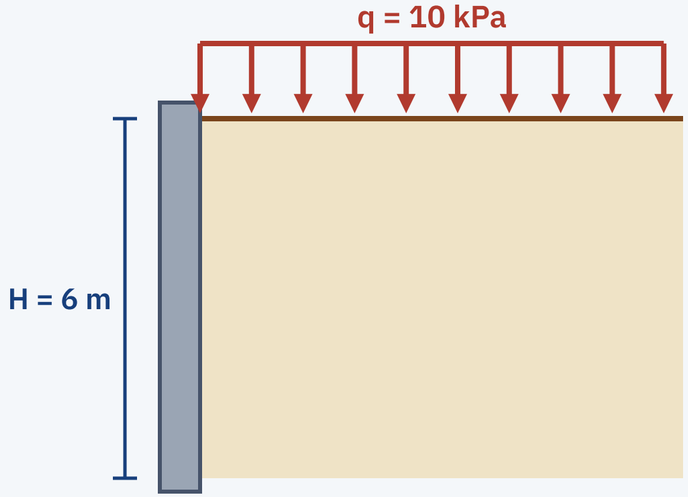
EXAMPLE

Given

Wall and load	**same wall:** $H = 6.00$ **m,** $q = 10.00$ **kPa,** $\varphi' = 30^\circ$
Difference	**the wall deforms enough to mobilize the active state (vertical wall, horizontal surface)**

REQUIRED

K_a, σ_H, P, z_R; comparison with the at-rest condition.



Solution (1/2)

SOLUTION

1 Coefficient: active state (Rankine)

$$K_a = \tan^2\left(45^\circ - \frac{\varphi'}{2}\right) = \tan^2 30^\circ = 0.5774^2 = \mathbf{0.3333}$$

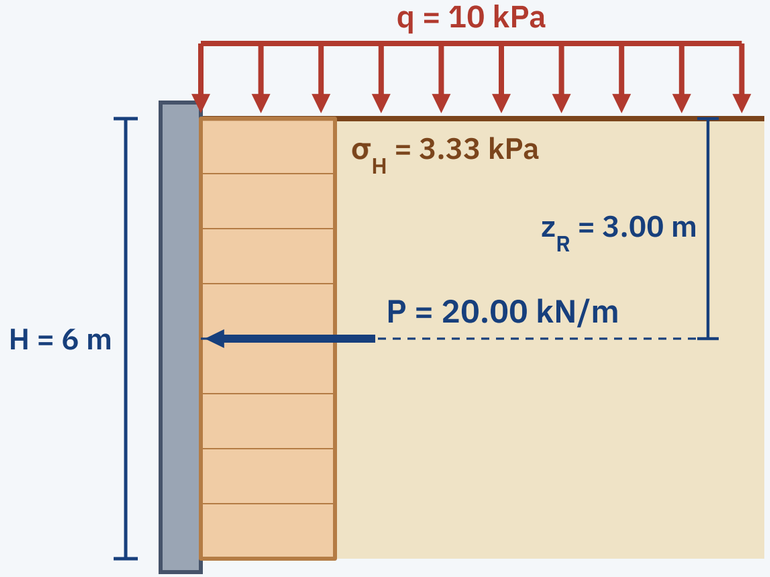
2 Superimposed lateral pressure

$$\sigma_H = K_a\, q = 0.3333 \times 10.00 = \mathbf{3.33}\ \mathrm{kPa}$$

3 Resultant and line of action

$$P = \sigma_H\, H = 3.333 \times 6.00 = \mathbf{20.00}\ \mathrm{kN/m}$$

$$z_R = \frac{H}{2} = \mathbf{3.00}\ \mathrm{m}$$



In the active state the pressure is **3.333 kPa**, the resultant is **20.00 kN/m**; the line of action is at the same height (z_R = 3.00 m).

Solution (2/2)

SOLUTION

Condition	K	σ_H (kPa)	P (kN/m)	z_R (m)
At-rest (K_0)	0.5000	5.00	30.00	3.00
Active (K_a)	0.3333	3.33	20.00	3.00
Ratio K_a / K_0	**0.6667**	**0.6667**	**0.6667**	1

For $\varphi' = 30°$, $K_a / K_0 = 2/3$: a wall that yields takes **two-thirds** of the surcharge pressure taken by a wall that cannot move. In both the diagram is rectangular and the resultant is at mid-height.

Whether the wall actually moves is the sole basis for this choice: a supported or anchored wall does not reach the active state, so K_0 is taken.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Wall and load	$H = 6.00$ **m,** $q = 10.00$ **kPa**
Soil	$\varphi' = 0°,\ 20°,\ 25°,\ 30°,\ 35°,\ 40°$

REQUIRED

For each φ′: **K_0**, **K_a**, **σ_H** and **P** (for two cases); hand check for φ′ = 40°.

Solution (1/2)

SOLUTION

φ′ (°)	K_0	K_a	σ_H at-rest (kPa)	σ_H active (kPa)	P at-rest (kN/m)	P active (kN/m)
0	1.0000	1.0000	10.00	10.00	60.00	60.00
20	0.6580	0.4903	6.58	4.90	39.48	29.42
25	0.5774	0.4059	5.77	4.06	34.64	24.35
30	**0.5000**	**0.3333**	**5.00**	**3.33**	**30.00**	**20.00**
35	0.4264	0.2710	4.26	2.71	25.59	16.26
40	0.3572	0.2174	3.57	2.17	21.43	13.05

The φ′ = 30° row corresponds to Examples 1 and 2. As φ′ increases the resultant decreases: at rest from 60.00 kN/m (φ′ = 0°) to 21.43 kN/m (φ′ = 40°), and in the active case to 13.05 kN/m.

Solution (2/2)

SOLUTION

1 Coefficients

$$K_0 = 1 - \sin 40^\circ = 1 - 0.6428 = \mathbf{0.3572}$$

$$K_a = \tan^2(45^\circ - 20^\circ) = \tan^2 25^\circ = 0.4663^2 = \mathbf{0.2174}$$

2 Pressure

$$\sigma_{H,0} = 0.3572 \times 10.00 = \mathbf{3.57}\ \mathrm{kPa}$$

$$\sigma_{H,a} = 0.2174 \times 10.00 = \mathbf{2.17}\ \mathrm{kPa}$$

3 Resultant (z_R = 3.00 m, in both cases)

$$P_0 = 3.572 \times 6.00 = \mathbf{21.43} \qquad P_a = 2.174 \times 6.00 = \mathbf{13.05}\ \mathrm{kN/m}$$

The φ′ = 40° row in the table is also obtained by hand: **21.43** and **13.05 kN/m**.

Check the solution in GeoTerva ↗

Example 4 · Coefficient entered: K = 0.30

Problem statement

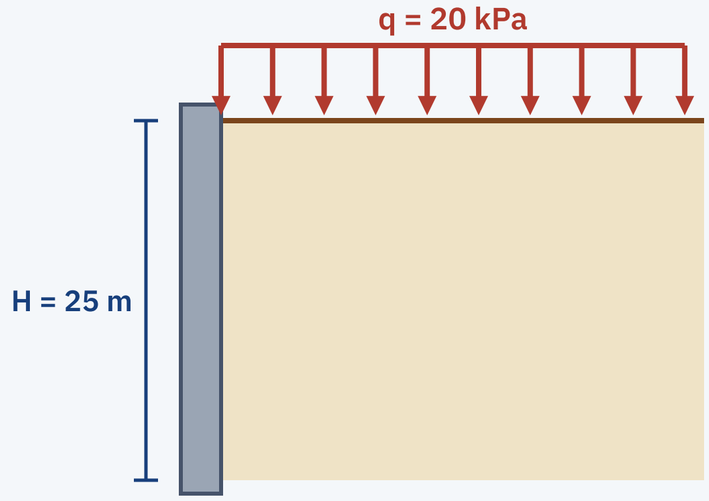
EXAMPLE

Given

Wall	$H = 25.00$ **m**
Load	$q = 20.00$ **kPa over the entire surface**
Coefficient	$K = 0.30$ **from another source (φ′ is not used)**

REQUIRED

σ_H, P, z_R and the variation of pressure with depth.



Solution

SOLUTION

1 Pressure: entered K is multiplied directly

$$\sigma_H = K\,q = 0.30 \times 20.00 = \mathbf{6.00}\ \mathrm{kPa}$$

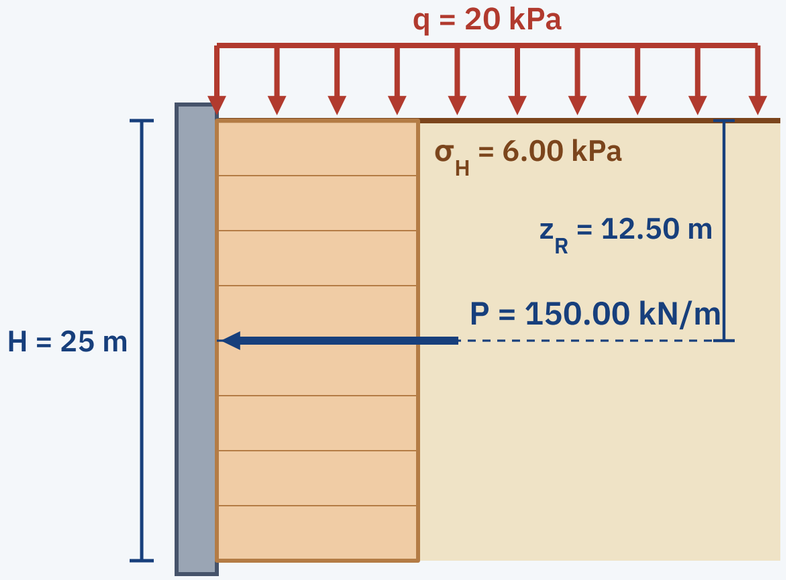
2 Resultant and line of action

$$P = \sigma_H\,H = 6.00 \times 25.00 = \mathbf{150.00}\ \mathrm{kN/m}$$

$$z_R = \frac{H}{2} = \frac{25.00}{2} = \mathbf{12.50}\ \mathrm{m}$$

3 Variation of pressure with depth

z (m)	0.0	5.0	10.0	15.0	20.0	25.0
σ_H (kPa)	6.00	6.00	6.00	6.00	6.00	6.00



Pressure is **6.00 kPa at every depth**; resultant **150.00 kN/m**, 12.50 m below the ground surface.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Wall and soil	$\varphi' = 30^\circ$; **at-rest and active state**
Variable 1	$q = 0,\ 5,\ 10,\ 20,\ 30,\ 50$ **kPa** ($H = 6.00$ **m**)
Variable 2	$H = 2,\ 4,\ 6,\ 8,\ 10$ **m** ($q = 10.00$ **kPa**)

REQUIRED

Variation of the resultant with **q** and **H**; pressure and line of action.

Solution

SOLUTION

q (kPa)	σ_H at-rest (kPa)	P at-rest (kN/m)	P active (kN/m)
0	0.00	**0.00**	**0.00**
5	2.50	**15.00**	**10.00**
10	5.00	**30.00**	**20.00**
20	10.00	**60.00**	**40.00**
30	15.00	**90.00**	**60.00**
50	25.00	**150.00**	**100.00**

When **q** doubles, P also doubles; at q = 0 there is no surcharge.

H (m)	σ_H at-rest (kPa)	P at-rest (kN/m)	P active (kN/m)	z_R (m)
2	5.00	**10.00**	**6.67**	1.00
4	5.00	**20.00**	**13.33**	2.00
6	5.00	**30.00**	**20.00**	3.00
8	5.00	**40.00**	**26.67**	4.00
10	5.00	**50.00**	**33.33**	5.00

As **H** increases, P increases linearly; the pressure stays the same, and the resultant moves down to a depth of H/2.

P = K q H: it is directly proportional to the **product** of q and H; the effect of K is a separate multiplier (active / at-rest = 2/3, in every row).

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Wall	$H = 6.00$ **m,** $\gamma = 18.0$ **kN/m³,** $\varphi' = 30^\circ$ **(dry fill)**
Surcharge	$q = 10.00$ **kPa over the entire surface**
Cases	**at-rest (K_0) and active (K_a)**

REQUIRED

The resultant of the soil triangle, the resultant of the surcharge, the **total force** and the height of the total force above the base; the contribution of the surcharge.

Solution (1/2)

SOLUTION

1 Earth pressure (triangular diagram), at-rest condition

$$P_z = \tfrac{1}{2} K_0 \gamma H^2 = \tfrac{1}{2}(0.500)(18.0)(6.00)^2 = \mathbf{162.00}\ \mathrm{kN/m} \quad (\text{from the base } 2.00\ \mathrm{m})$$

2 Surcharge component (Example 1)

$$P_q = K_0\, q\, H = 0.500 \times 10.00 \times 6.00 = \mathbf{30.00}\ \mathrm{kN/m} \quad (\text{from the base } 3.00\ \mathrm{m})$$

3 Total and line of action (base moment)

$$P = 162.00 + 30.00 = \mathbf{192.00}\ \mathrm{kN/m}$$

$$L = \frac{162.00(2.00) + 30.00(3.00)}{192.00} = \frac{414.00}{192.00} = \mathbf{2.156}\ \mathrm{m}$$

At rest the total is **192.00 kN/m**, acting **2.156 m** above the base; the surcharge share is 30.00 kN/m (15.6%).

Solution (2/2)

SOLUTION

1 Same steps for the active case (K_a = 0.3333)

$$P_z = \tfrac{1}{2}(0.3333)(18.0)(6.00)^2 = \mathbf{108.00}\ \mathrm{kN/m}$$

$$P_q = 0.3333(10.00)(6.00) = \mathbf{20.00}\ \mathrm{kN/m}$$

$$L = \frac{108.00(2.00) + 20.00(3.00)}{128.00} = \mathbf{2.156}\ \mathrm{m}$$

Condition	K	P_{soil} (kN/m)	$P_{surcharge}$ (kN/m)	P total (kN/m)	Surcharge share	L (m, from the base)
At-rest	0.5000	162.00	30.00	**192.00**	%15.62	2.156
Active	0.3333	108.00	20.00	**128.00**	%15.62	2.156

Since K is a factor in both terms, **the surcharge share and its line of action are independent of K**: share 2q / (γH + 2q) = 15.62%, L = 2.156 m. Only the total force scales with K.

Check the solution in GeoTerva ↗

Summary

Key points

$\sigma_H = K \cdot q$: a uniform load spread over the entire surface adds **the same** lateral pressure to the wall **at every depth**; the diagram is rectangular

K according to the wall behaviour: at-rest **$K_0 = 1 - \sin \varphi'$** (Jaky), active **$K_a = \tan^2(45° - \varphi'/2)$** (Rankine) or the entered value; for $\varphi' = 30°$, 0.5000 and 0.3333

$P = K\,q\,H$ and **$z_R = H/2$**: for the opening wall (H = 6 m, q = 10 kPa), at rest σ_H = 5.00 kPa, P = 30.00 kN/m; in the active case 3.33 kPa and 20.00 kN/m

Linear: P is proportional to q and H; $K_a / K_0 = 2/3$ ($\varphi' = 30°$)

Summation: the surcharge component is added to the soil and water pressure diagrams; the surcharge share is $2q / (\gamma H + 2q)$ and its line of action is independent of K

References

Das, B.M. (2016). Principles of Foundation Engineering, 8th ed. Cengage Learning; this page follows it.

Jaky, J. (1944). The coefficient of earth pressure at rest. Journal of the Society of Hungarian Architects and Engineers, 7, 355–358.

Rankine, W.M.J. (1857). On the stability of loose earth. Philosophical Transactions of the Royal Society of London, 147, 9–27.

NOTE

All numerical examples have been produced with the module's own calculation engine; the module gives the same values. The module's opening wall is H = 6 m, q = 10 kPa, φ′ = 30° (at rest) and its second example is H = 25 m, q = 20 kPa, K = 0.30.

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own wall, your surcharge and your choice of coefficient in GeoTerva; the module gives the K, σ_H, P and z_R values, the pressure diagram, a step-by-step solution and report together.

Open the module → | **Theory page →**

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com