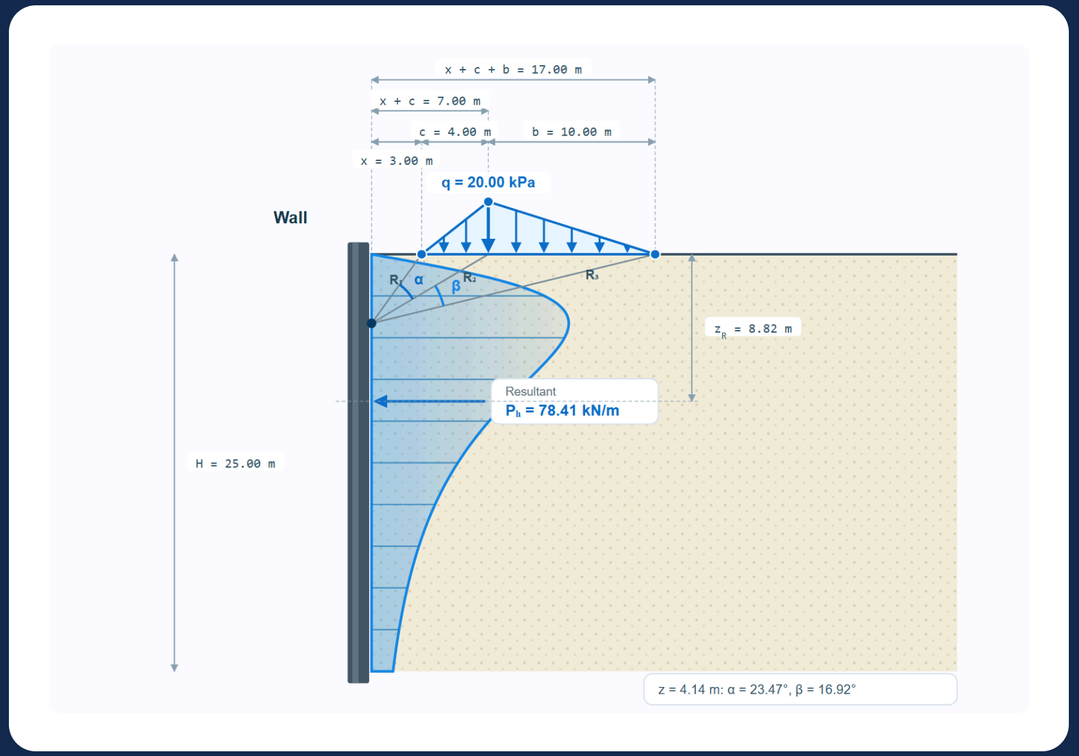
Geotechnical lecture presentation · Surcharge loads

Triangular surcharge load

Lecture: a triangular load starting at x, rising to a peak value and descending to zero; the ordinate at a given depth is solved by hand, and the profile, resultant and variants are presented in tables.

- Superposition of line loads
- A and B functions
- Rigid wall ψ = 2
- Resultant and z_R
- Solvable by hand



What will we learn in this lecture?

- The geometry of the **triangular load**: it starts at zero at distance **x** from the wall, rises to the value q over the width **c**, and decreases back to zero over the width **b**
- At a depth z, finding the radii $\mathbf{R_1}$, $\mathbf{R_2}$, $\mathbf{R_3}$ and the angles **α, β** (in radians)
- The closed form of the lateral pressure obtained from superposition of line loads: with the functions **A(s)** and **B(s)**
- The difference between a **rigid wall** (ψ = 2) and a **semi-infinite medium** (ψ = 1)
- **Resultant** $\mathbf{P_H}$ and **depth of action** $\mathbf{z_R}$: the area and first moment of the diagram; the effect of the load shape via x

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example: H = 25 m, x = 3 m, c = 4 m, b = 10 m, q = 20 kPa; rigid wall. One ordinate is solved by hand at z = 5 m.

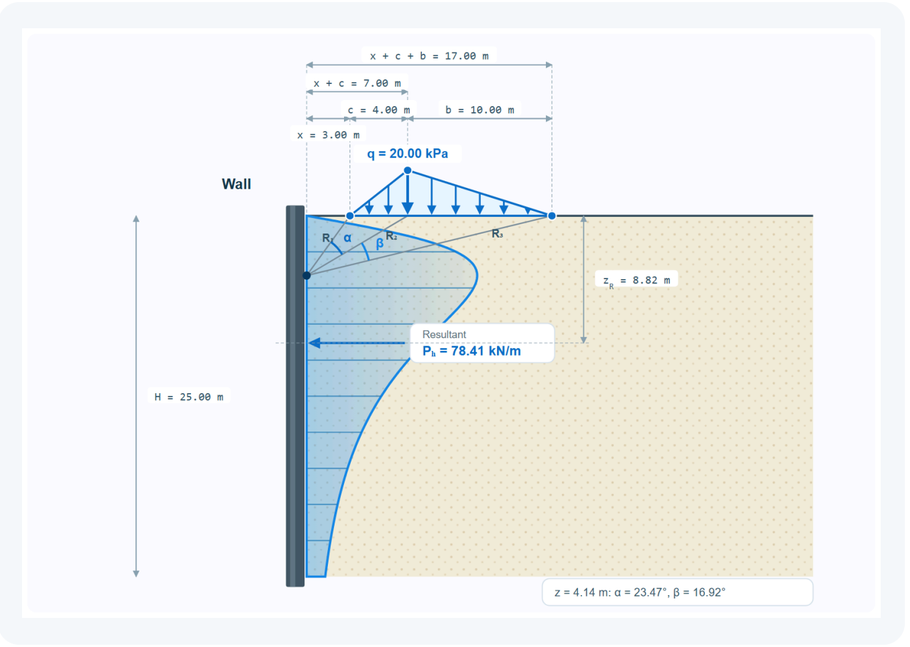
Triangular surcharge acting on the wall

THEORY

The surface surcharge starts at a horizontal distance **x** from the wall

Its intensity increases linearly from zero to the maximum value **q** over the width **c**, then decreases linearly back to zero over the width **b**

At every depth z of the wall, the geometry is defined by the three radii R_1, R_2, R_3 and the two angles **α, β**



H: total wall height; **x**: distance from the wall to the start of the load
c: width of the increasing arm, **b**: width of the decreasing arm
q: maximum surcharge intensity; **z**: depth from the ground surface

Geometry at depth z

THEORY

At depth z, the lengths of the three rays from the point on the wall to the **start**, the **peak**, and the **end** of the load, and the two angles between them, are given below

In the pressure equation the angles are taken in **radians**; degree values are useful only for reporting

$$R_1 = \sqrt{x^2 + z^2} \quad R_2 = \sqrt{(x+c)^2 + z^2} \quad R_3 = \sqrt{(x+c+b)^2 + z^2}$$

$$\alpha = \tan^{-1}\frac{x+c}{z} - \tan^{-1}\frac{x}{z} \qquad \beta = \tan^{-1}\frac{x+c+b}{z} - \tan^{-1}\frac{x+c}{z}$$

Superposition of line loads

THEORY

A triangular load is a series of continuous line loads, each of intensity **w(s)·ds**; the elastic horizontal stresses (Flamant) are summed along the load

With s_1 = x, s_2 = x + c, s_3 = x + c + b, $R^2 = s^2 + z^2$ and the functions A, B below, the integral becomes closed-form

$$\sigma_H = \psi \frac{2}{\pi} \int w(s) \frac{s^2 z}{(s^2 + z^2)^2} ds$$

$$A(s) = \frac{1}{2}\left[\tan^{-1}\frac{s}{z} - \frac{sz}{R^2}\right] \qquad B(s) = \frac{z}{2}\left[\ln R^2 + \frac{z^2}{R^2}\right]$$

$$\sigma_H = \psi \frac{2q}{\pi}\left\{\frac{B_2 - B_1 - x(A_2 - A_1)}{c} + \frac{s_3(A_3 - A_2) - (B_3 - B_2)}{b}\right\}$$

A, B, α, β and ψ

THEORY

The $A_2 - A_1$ and $A_3 - A_2$ terms involve the angles α and β, and the $B_2 - B_1$ and $B_3 - B_2$ terms involve the logarithms $\ln(R_2/R_1)$ and $\ln(R_3/R_2)$

When the same superposition is applied to a uniform strip, the known expression **$(q/\pi)[\beta - \sin\beta\cdot\cos 2\alpha]$** is obtained exactly; the equation is checked in this form

$\psi = 1$ is the semi-infinite medium (no wall); **$\psi = 2$** is the rigid non-yielding wall that doubles the horizontal stress: the module default

Repeating the calculation over the wall height gives **$\sigma_H(z)$**; the pressure is zero at the ground surface

Area and first moment of the diagram

THEORY

The **resultant surcharge force** per unit length of the wall and its depth from the ground surface are the area and the first moment of the diagram; the module computes both integrals numerically with full precision

$$P_H = \int_0^H \sigma_H(z)\, dz$$

$$z_R = \frac{\int_0^H z\, \sigma_H(z)\, dz}{P_H}$$

In the example, **P_H = 78.41 kN/m** (rigid wall) or **39.21 kN/m** (semi-infinite medium); in both, **z_R = 8.82 m**, below the ground surface

Six steps and one ordinate

THEORY

1 · H, x, c, b, q are defined and ψ is selected; **2** · a depth z is selected

3 · R_1, R_2, R_3 are computed; **4** · α and β are computed (radians)

5 · σ_H is calculated: the functions A and B are found at the points s_1, s_2, s_3 and substituted

6 · Iterate and integrate: pressure profile, then P_H and z_R

$$z = 5\ \text{m}: \quad R_1 = 5.831\ \text{m} \quad R_2 = 8.602\ \text{m} \quad R_3 = 17.720\ \text{m} \qquad \alpha = 23.499^\circ \quad \beta = 19.148^\circ$$

$$\sigma_H = 3.247\ \text{kPa}\ \ (\psi = 1) \ \rightarrow\ 2 \times 3.247 = 6.494\ \text{kPa}\ \ (\psi = 2) \qquad P_H = 78.41\ \text{kN/m} \quad z_R = 8.82\ \text{m}$$

Problem statement

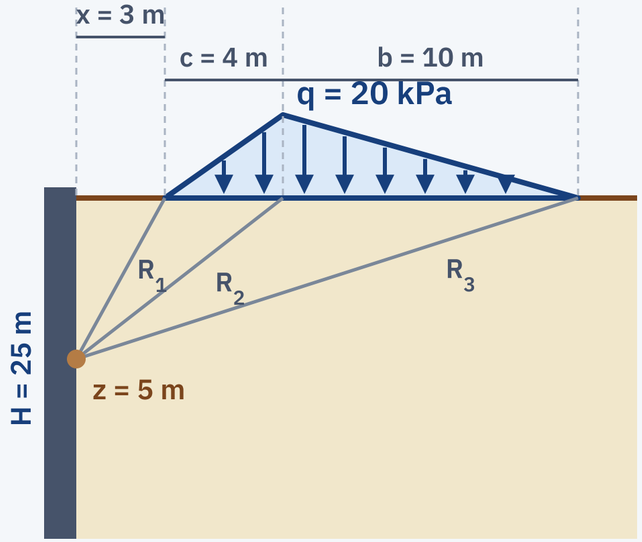
EXAMPLE

Given

Wall and load	$H = 25.0$ **m;** $x = 3.0$ **m;** $c = 4.0$ **m;** $b = 10.0$ **m;** $q = 20.0$ **kPa**
Wall	**semi-infinite medium ($\psi = 1$) and rigid wall ($\psi = 2$)**

REQUIRED

At depth **z = 5 m**, the values of **R_1, R_2, R_3, α, β**, A and B, and the horizontal pressure **σ_H** (for two ψ values); comparison with the module.



Solution (1/3)

SOLUTION

1 The points s_1 = x, s_2 = x + c, s_3 = x + c + b and the rays

$$s_1 = 3\ \text{m} \quad s_2 = 3 + 4 = 7\ \text{m} \quad s_3 = 7 + 10 = 17\ \text{m}$$

$$R_1 = \sqrt{3^2 + 5^2} = \mathbf{5.831} \quad R_2 = \sqrt{7^2 + 5^2} = \mathbf{8.602} \quad R_3 = \sqrt{17^2 + 5^2} = \mathbf{17.720}\ \text{m}$$

2 Angles (radians)

$$\alpha = \tan^{-1}\frac{7}{5} - \tan^{-1}\frac{3}{5} = 0.95055 - 0.54042 = \mathbf{0.41013}\ \text{rad}\ \ (23.499^\circ)$$

$$\beta = \tan^{-1}\frac{17}{5} - \tan^{-1}\frac{7}{5} = 1.28474 - 0.95055 = \mathbf{0.33420}\ \text{rad}\ \ (19.148^\circ)$$

The angle between the start of the load and its peak, measured from the point on the wall, is **23.50°**; the angle between the peak and the end is **19.15°**.

Solution (2/3)

SOLUTION

3 At three points A(s) = ½[tan⁻¹(s/z) − sz/R²] and B(s) = (z/2)[ln R² + z²/R²]

$$A_1 = \frac{1}{2}\left[0.54042 - \frac{3(5)}{34}\right] = 0.04962 \qquad A_2 = \frac{1}{2}\left[0.95055 - \frac{7(5)}{74}\right] = 0.23879$$

$$A_3 = \frac{1}{2}\left[1.28474 - \frac{17(5)}{314}\right] = 0.50702$$

s (m)	R² (m²)	tan⁻¹(s/z)	A(s)	ln R²	B(s)
3	34	0.54042	**0.04962**	3.52636	**10.6541**
7	74	0.95055	**0.23879**	4.30407	**11.6048**
17	314	1.28474	**0.50702**	5.74939	**14.5725**

The A differences carry the angles, the B differences carry the logarithms: **$A_2 - A_1$** = 0.18917, **$A_3 - A_2$** = 0.26824; **$B_2 - B_1$** = 0.9506, **$B_3 - B_2$** = 2.9678.

Solution (3/3)

SOLUTION

4 Contribution of increasing and decreasing arms

$$\text{increasing} = \frac{B_2 - B_1 - x(A_2 - A_1)}{c} = \frac{0.9506 - 3(0.18917)}{4} = 0.09578$$

$$\text{decreasing} = \frac{s_3(A_3 - A_2) - (B_3 - B_2)}{b} = \frac{17(0.26824) - 2.9678}{10} = 0.15922$$

5 Semi-infinite medium (ψ = 1) and rigid wall (ψ = 2)

$$\sigma_H = \frac{2q}{\pi}\,(\text{increasing} + \text{decreasing}) = \frac{2(20)}{\pi}(0.09578 + 0.15922) = \mathbf{3.247}\ \text{kPa}\ \ (\psi = 1)$$

$$\sigma_H = 2 \times 3.247 = \mathbf{6.494}\ \text{kPa}\ \ (\psi = 2)$$

At z = 5 m, **σ_H = 6.494 kPa** (rigid wall; 3.247 in the semi-infinite medium). The module gives the same value.

Check the solution in GeoTerva ↗

Problem statement

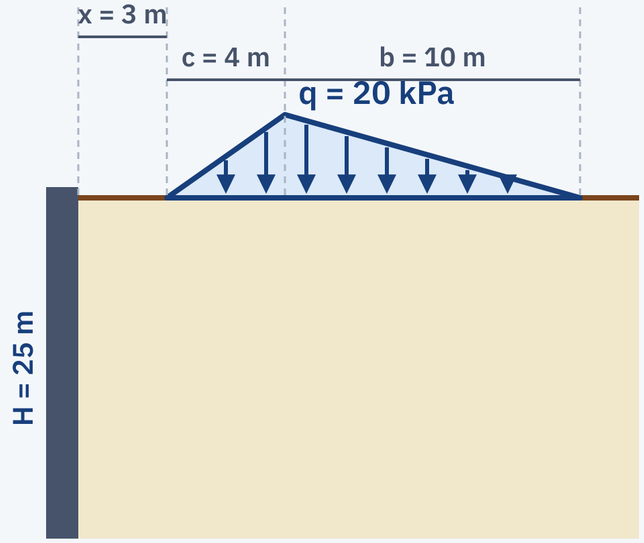
EXAMPLE

Given

Model	**Triangular load in Example 1; rigid wall; table at every $H/30 = 0.833$ m**
On each row	**R_1, R_2, R_3, α, β and σ_H; the semi-infinite medium values are half those of the rigid wall**

REQUIRED

Pressure profile; the maximum pressure and its depth; the angles at the peak depth.



Solution (1/2)

SOLUTION

z (m)	R_1 (m)	R_2 (m)	R_3 (m)	α (°)	β (°)	σ_H (kPa)
0.00	3.000	7.000	17.000	0.00	0.00	0.000
0.83	3.114	7.049	17.020	8.74	3.98	2.513
1.67	3.432	7.196	17.082	15.66	7.79	4.520
2.50	3.905	7.433	17.183	20.15	11.29	5.815
3.33	4.485	7.753	17.324	22.55	14.37	6.463
4.17	5.134	8.146	17.503	23.48	16.99	6.635
5.00	**5.831**	**8.602**	**17.720**	**23.50**	**19.15**	**6.494**
5.83	6.560	9.112	17.973	22.98	20.87	6.168
6.67	7.311	9.667	18.260	22.17	22.19	5.746
7.50	8.078	10.259	18.581	21.22	23.17	5.286
8.33	8.857	10.883	18.933	20.23	23.86	4.821
9.17	9.645	11.534	19.314	19.24	24.30	4.374
10.00	10.440	12.207	19.723	18.29	24.54	3.955
10.83	11.241	12.898	20.158	17.39	24.62	3.569
11.67	12.046	13.606	20.618	16.54	24.58	3.218
12.50	12.855	14.327	21.101	15.75	24.42	2.901
13.33	13.667	15.059	21.605	15.02	24.19	2.617
14.17	14.481	15.802	22.129	14.34	23.90	2.363
15.00	15.297	16.553	22.672	13.71	23.56	2.136
15.83	16.115	17.312	23.231	13.12	23.18	1.933
16.67	16.935	18.077	23.807	12.58	22.78	1.752
17.50	17.755	18.848	24.398	12.07	22.37	1.591
18.33	18.577	19.624	25.002	11.60	21.94	1.447
19.17	19.400	20.405	25.620	11.17	21.51	1.319
20.00	20.224	21.190	26.249	10.76	21.07	1.204
20.83	21.048	21.978	26.889	10.38	20.64	1.101
21.67	21.873	22.769	27.540	10.02	20.21	1.009
22.50	22.699	23.564	28.200	9.69	19.79	0.926
23.33	23.525	24.361	28.869	9.37	19.38	0.851
24.17	24.352	25.160	29.547	9.08	18.97	0.784
25.00	25.179	25.962	30.232	8.80	18.57	0.723

Solution (2/2)

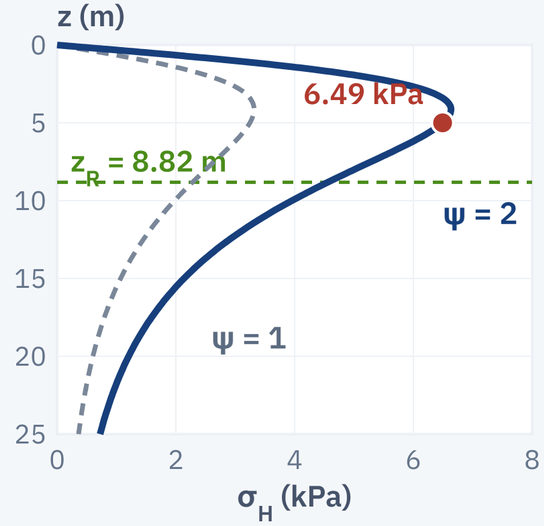
SOLUTION

1 The pressure is zero at the surface, reaches its maximum at the peak depth, then decreases

$$z = 0: \quad \sigma_H = 0 \qquad z = 4.14\ \text{m}: \quad \sigma_H = \mathbf{6.635}\ \text{kPa}$$

$$\alpha = 23.47^\circ \quad \beta = 16.92^\circ \qquad z = H: \quad \sigma_H = 0.723\ \text{kPa}$$

The maximum pressure is **6.63 kPa** at **z = 4.14 m** (4.14 m below the top of the load); it decreases to 0.72 kPa at the base. These angles are shown in the module's figure. The ordinate at z = 5 m is 6.494 kPa (thick line, Example 1).



Check the solution in GeoTerva ↗

Problem statement

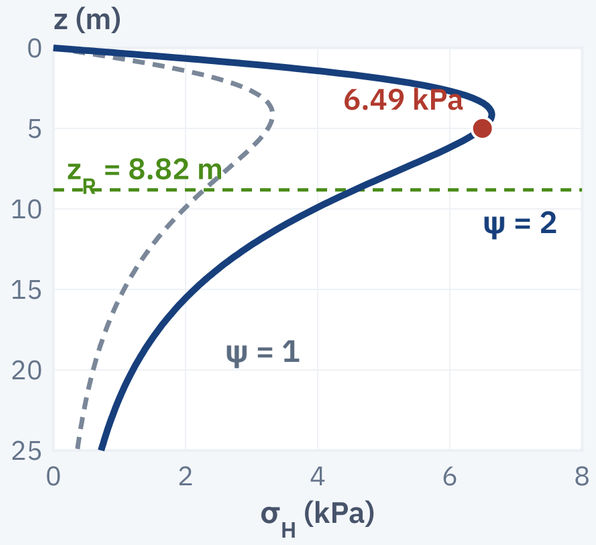
EXAMPLE

Given

Model	**The triangular load in Example 1; rigid wall; the $H/30$ table of Example 2 ($n = 30$, $h = H/30 = 0.8333$ m)**
Method	**Simpson's rule; $z_R = \int z\,\sigma\,dz \,/\, P_H$**

REQUIRED

P_H and z_R (by hand, from the table); semi-infinite medium values; comparison with the module result.



Solution (1/2)

SOLUTION

	σ_0	Σ odd (i = 1, 3, …, 29)	Σ even (i = 2, 4, …, 28)	σ_{30}
σ_H (kPa)	0.000	47.277	46.234	0.723

1 Simpson sum and area

$$\Sigma = \sigma_0 + 4(47.277) + 2(46.234) + \sigma_{30} = 282.30$$

$$P_H \approx \frac{h}{3}\,\Sigma = \frac{0.8333}{3}(282.30) = \mathbf{78.42}\ \mathrm{kN/m}$$

By hand **78.42 kN/m**; the module's precise integral gives **78.41 kN/m** (difference 0.004%). In a semi-infinite medium, half of it: **39.21 kN/m**.

Solution (2/2)

SOLUTION

	$z_0\sigma_0$	Σ single	Σ double	$z_{30}\sigma_{30}$
$z\sigma_H$ (kPa·m)	0.000	414.83	405.58	18.07

2 First moment and depth

$$\int z\,\sigma\,dz \approx \frac{0.8333}{3}\left[0 + 4(414.83) + 2(405.58) + 18.07\right] = 691.3\ \text{kN}$$

$$z_R = \frac{691.3}{78.42} = \mathbf{8.82}\ \text{m}\ \ (\text{from the ground}) \qquad H - z_R = 16.18\ \text{m}\ (\text{from the base})$$

By hand **z_R = 8.82 m**; the module gives 8.82 m. The resultant 78.41 kN/m acts **8.82 m below the ground surface** (16.18 m above the base); z_R does not change when ψ changes.

Check the solution in GeoTerva ↗

Problem statement

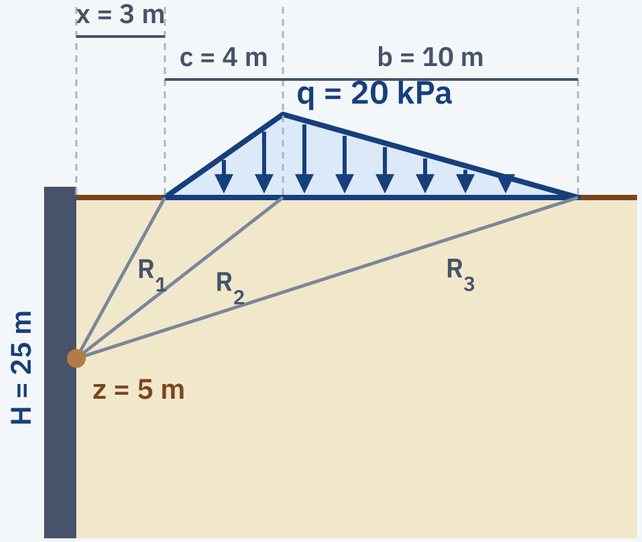
EXAMPLE

Given

Model	**Example 1:** $z = 5$ **m; semi-infinite medium (** $\psi = 1$**)**
Method	$\sigma_H = \frac{2}{\pi} \int w(s) \, \frac{s^2 z}{(s^2 + z^2)^2} \, ds$**;** w **triangle:** 0 **at** $s = 3$ **m,** q **at** 7 **m,** 0 **at** 17 **m**
Calculation	**Simpson,** $n = 14$**,** $h = 1.0$ **m (**$s = 3, 4, \ldots, 17$**)**

REQUIRED

σ_H from the sum of the line loads; comparison with the closed form of Example 1.



Solution (1/2)

SOLUTION

i	s (m)	w(s) (kPa)	f(s) = s²z/(s²+z²)² (1/m)	c	c·w·f
0	3	0.00	0.03893	1	0.00000
1	4	5.00	0.04759	4	0.95181
2	5	10.00	0.05000	2	1.00000
3	6	15.00	0.04837	4	2.90245
4	7	20.00	0.04474	2	1.78963
5	8	18.00	0.04040	4	2.90872
6	9	16.00	0.03604	2	1.15344
7	10	14.00	0.03200	4	1.79200
8	11	12.00	0.02838	2	0.68118
9	12	10.00	0.02521	4	1.00837
10	13	8.00	0.02245	2	0.35923
11	14	6.00	0.02007	4	0.48156
12	15	4.00	0.01800	2	0.14400
13	16	2.00	0.01621	4	0.12968
14	17	0.00	0.01466	1	0.00000
			Σ c·w·f =		**15.30207**

Solution (2/2)

SOLUTION

1 Simpson and ψ

$$\sigma_H = \frac{2}{\pi}\frac{h}{3}\sum c_i\, w_i f_i = \frac{2}{\pi}\frac{1}{3}(15.30207) = \mathbf{3.2472}\ \mathrm{kPa}\ \ (\psi = 1)$$

$$\psi = 2:\ \ \sigma_H = \mathbf{6.494}\ \mathrm{kPa}$$

	Sum of line loads (Simpson)	Closed form (A, B; Example 1)	Difference
σ_H, ψ = 1 (kPa)	3.2472	3.2468	%0.012
σ_H, ψ = 2 (kPa)	6.4944	6.4936	%0.012

Both approaches give the same result: the closed-form A and B terms are the sum of the line-load integrals of the increasing and decreasing arms. As the Simpson interval decreases, the difference tends to zero (0.0006% for h = 0.5 m).

Check the solution in GeoTerva ↗

Problem statement

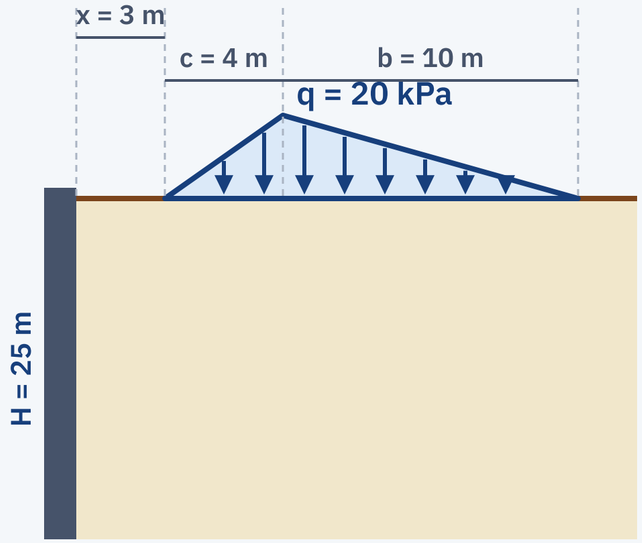
EXAMPLE

Given

Model	$H = 25.0$ **m;** $q = 20$ **kPa; rigid wall; only one changes at a time**
Variables	**starting distance** $x = 0$**;** 3**;** 6**;** 10 **m (**$c = 4$**,** $b = 10$ **m) and figure** $(c, b) = (2, 12)$**;** $(4, 10)$**;** $(7, 7)$**;** $(10, 4)$**;** $(12, 2)$ **m (**$x = 3$ **m)**
Comparison	**the weight of the triangle** $Q = q(c + b)/2$**, the line load with its centroid at position** $x + (2c + b)/3$

REQUIRED

For each case, compare P_H, z_R, $\sigma_{H,max}$ and its depth with the resultant of the equivalent line load.



Solution (1/2)

SOLUTION

x (m)	P_H (kN/m), ψ = 2	P_H (kN/m), ψ = 1	z_R (m)	$\sigma_{H,max}$ (kPa)	z (m)
0	83.46	41.73	6.53	10.12	1.93
3	78.41	39.21	8.82	6.63	4.14
6	72.25	36.13	10.51	4.91	6.14
10	63.33	31.67	12.13	3.65	8.64

(c, b) (m)	P_H (kN/m), ψ = 2	P_H (kN/m), ψ = 1	z_R (m)	$\sigma_{H,max}$ (kPa)	z (m)
(2; 12)	79.61	39.81	8.33	7.30	3.63
(4; 10)	78.41	39.21	8.82	6.63	4.14
(7; 7)	76.44	38.22	9.45	5.87	4.78
(10; 4)	74.32	37.16	9.97	5.29	5.31
(12; 2)	72.85	36.43	10.28	4.97	5.62

As the load moves away from the wall (x ↑) **the resultant decreases, and z_R and the depth of the peak increase**. For a constant total width (c + b = 14 m), as the peak moves away from the wall (c ↑) the resultant also decreases and z_R deepens. The ψ = 1 values are half those for a rigid wall.

Solution (2/2)

SOLUTION

1 Weight and centroid of the triangle

$$Q = \frac{q(c+b)}{2} = \frac{20(4+10)}{2} = \mathbf{140}\ \mathrm{kN/m}$$

$$\bar{x} = x + \frac{2c+b}{3} = 3 + \frac{2(4)+10}{3} = \mathbf{9}\ \mathrm{m} \qquad m = \frac{\bar{x}}{H} = 0.36$$

2 Resultant of the line load (ψ = 2): the integral of the line load integrand over depth

$$P_H = \psi \frac{Q}{\pi} \frac{H^2}{\bar{x}^2 + H^2} = \frac{2Q}{\pi} \frac{1}{1+m^2} = \frac{2(140)}{\pi} \frac{1}{1+0.1296} = \mathbf{78.90}\ \mathrm{kN/m}$$

The equivalent line load gives **78.90 kN/m**; the resultant of the triangular load is **78.41 kN/m** (0.6% difference). The maximum pressures are 6.43 and 6.63 kPa; z_R is 9.22 and 8.82 m. In terms of the resultant the difference is small; the pressure diagram, however (maximum pressure and its depth, z_R), departs from that of the line load.

Check the solution in GeoTerva ↗

Summary

Key points

$\sigma_H = \psi\,(2q/\pi)\,\{[B_2 - B_1 - x(A_2 - A_1)]/c + [s_3(A_3 - A_2) - (B_3 - B_2)]/b\}$; $A(s) = \tfrac{1}{2}[\tan^{-1}(s/z) - sz/R^2]$, $B(s) = (z/2)[\ln R^2 + z^2/R^2]$; $\psi = 2$ rigid wall, $\psi = 1$ semi-infinite medium

Example (z = 5 m): R_1 5.831, R_2 8.602, R_3 17.720 m; α 23.499°, β 19.148°; σ_H = 3.247 kPa ($\psi = 1$), **6.494 kPa** ($\psi = 2$)

Profile: 0 at the surface; maximum pressure **6.63 kPa** at z = 4.14 m; then it decreases

Resultant P_H = **78.41 kN/m**, z_R = **8.82 m** (from the ground surface); for $\psi = 1$, P_H = 39.21 kN/m, z_R the same

Comparison: the resultant decreases as the load moves away from the wall; the triangular load gives a resultant close to the equivalent line load at its centroid (78.41 and 78.90 kN/m)

References

Das, B.M. (2016). Principles of Foundation Engineering, 8th ed. Cengage Learning; this page follows it.

NOTE

All numerical examples have been produced with the module's own calculation engine; the module gives the same values. The module's opening example is the wall with H = 25 m, x = 3 m, c = 4 m, b = 10 m, q = 20 kPa.

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own triangular load, start distance, arm widths and wall height in GeoTerva; the module provides the pressure diagram, the resultant and its depth of action, the three views, the step-by-step solution and the report together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com