Geotechnical lecture presentation · Surcharge loads

Strip surcharge load

Lecture: lateral pressure produced on the wall by a long strip load parallel to the wall; pressure profile, resultant and line of action, worked examples.

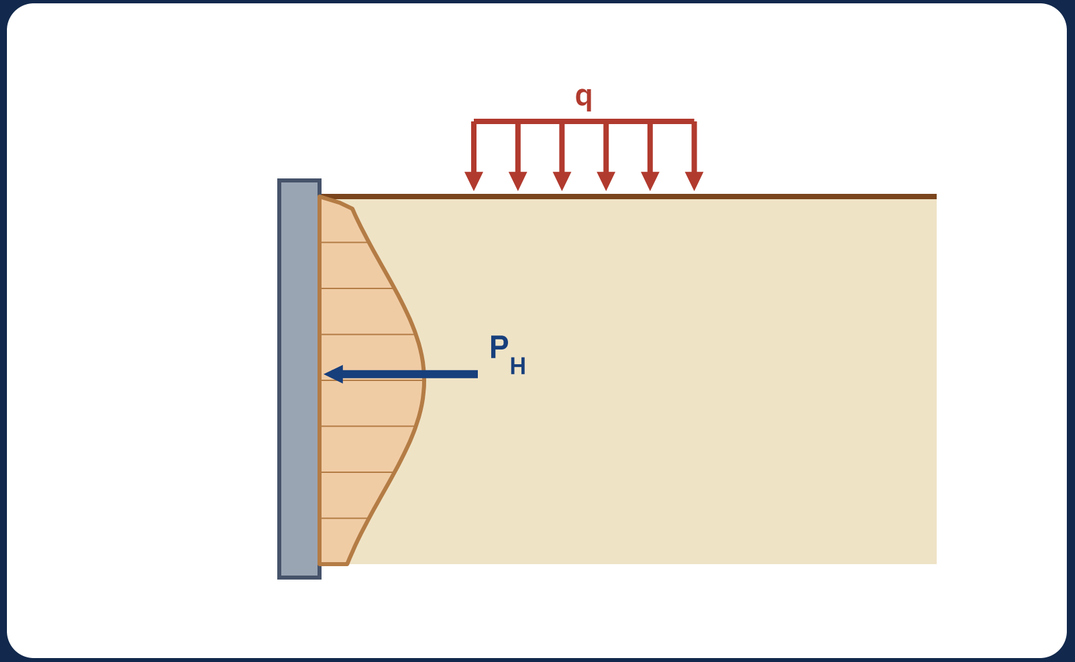
β · α

$\sigma_H = \psi\,(q/\pi)[\beta - \sin\beta\cos 2\alpha]$

Rigid wall ψ = 2

P_H and z_R

Pressure profile



Introduction

What will we learn in this lecture?

- The geometry of the strip load: x_1, x_2, $b = x_2 - x_1$ and the **rays** from a point on the wall to the two edges of the strip
- Determination of the pressure from angles **β** and **α**: $\sigma_H = \psi\,(q/\pi)[\beta - \sin\beta\cos 2\alpha]$
- The form for a **rigid wall (ψ = 2)** versus a **semi-infinite medium (ψ = 1)** and when each is appropriate
- The area of the pressure diagram gives P_H, and from the first moment, z_R; the depth of the maximum pressure
- How the **position and width** of the strip and the **wall height** affect the result

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example wall: H = 25 m, strip between x_1 = 4 m and x_2 = 12 m, q = 20 kPa. Forces are per unit length of wall (kN/m).

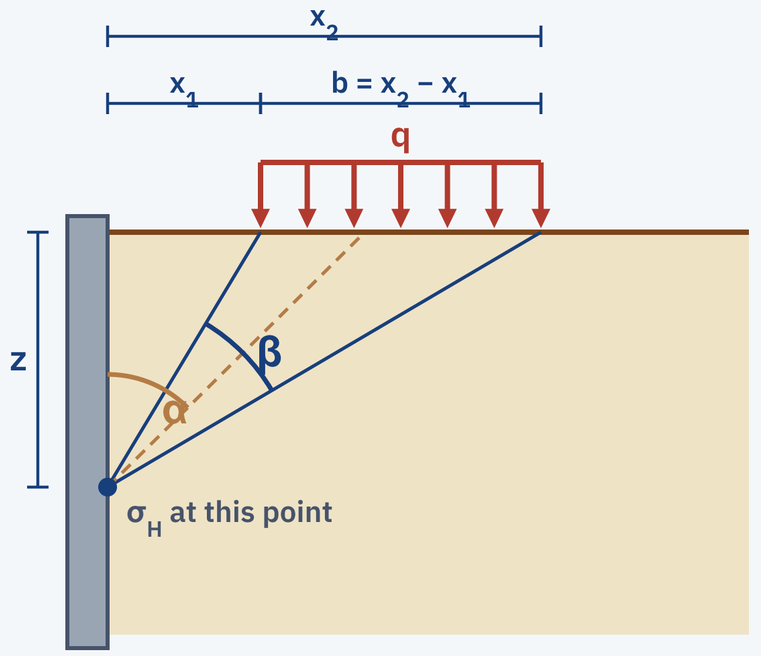
Strip load acting on the wall

THEORY

Between distances x_1 and x_2 from the wall, a uniformly distributed strip load **q** acting on the ground surface adds a horizontal pressure to the wall

The pressure at depth z is obtained from the geometry of the two rays joining that point on the wall to the **near and far edges** of the strip

The strip is long and parallel to the wall: all results are **per unit length of wall**



H — wall height; **z** — depth from the surface

x_1, x_2 — horizontal distances from the wall to the near and far edge of the strip

$b = x_2 - x_1$ — strip width; **q** — uniform load on the strip

β, α and σ_H

THEORY

At depth z, the strip sees angle **β** at the point on the wall; the **bisector** of this angle makes angle **α** with the vertical

Trigonometric functions take angles in **radians**; degrees are only for readability. In the ratios, **z** itself is used

$$\beta = \tan^{-1}\left(\frac{x_2}{z}\right) - \tan^{-1}\left(\frac{x_1}{z}\right) \qquad \alpha = \tan^{-1}\left(\frac{x_1}{z}\right) + \frac{\beta}{2}$$

$$\sigma_H = \psi \frac{q}{\pi} \left[\beta - \sin\beta \cos 2\alpha\right]$$

At the ground surface (z = 0): a strip away from the wall gives no pressure; if x_1 = 0 then β = 90°, α = 45° and $\sigma_H = \psi q/2$

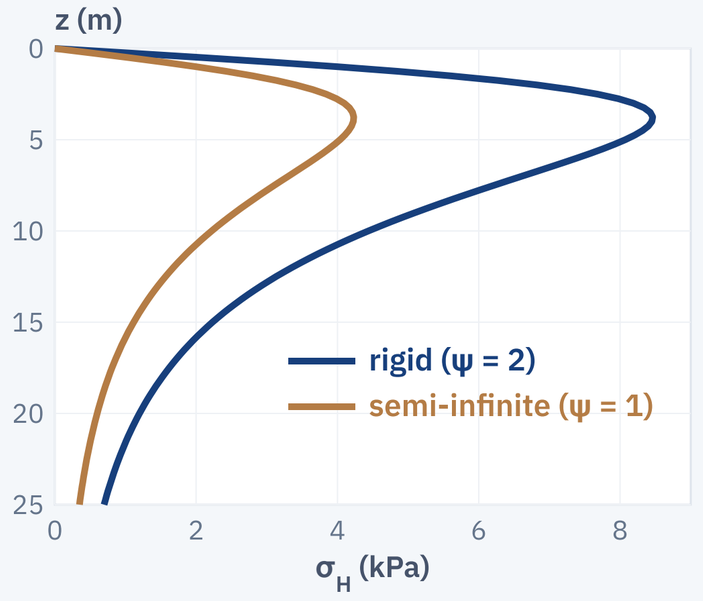
ψ = 2 and ψ = 1: two forms of expression

THEORY

ψ = 1 (semi-infinite medium): the horizontal stress produced by the strip load in an elastic medium as if no wall existed; suitable for a wall that deforms enough to relieve the soil stress

ψ = 2 (rigid wall): the non-yielding wall prevents movement of the soil at its face and takes **twice** this stress; as with the point load, this is the module's default

z (m)	β (°)	α (°)	σ_H ψ = 1 (kPa)	σ_H ψ = 2 (kPa)
0.00	0.000	90.000	0.00	0.00
2.50	20.237	68.113	3.84	7.68
5.00	28.720	53.020	4.04	8.07
7.50	29.922	43.034	3.11	6.21
10.00	28.393	35.998	2.22	4.44
15.00	23.728	26.796	1.12	2.23
20.00	19.654	21.137	0.5994	1.1988
25.00	16.551	17.366	0.3486	0.6971



Area and first moment of the diagram

THEORY

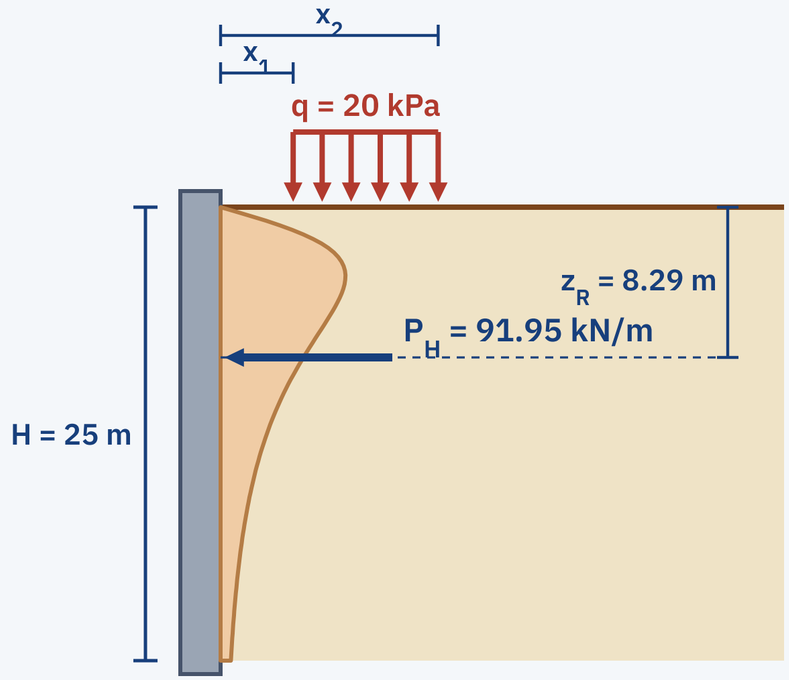
The **resultant** pressure per unit length of wall is the **area** under the pressure diagram; its **depth from the ground surface** is the point where the first moment of the diagram balances

The module computes both integrals numerically at full precision; the **closed-form expression for the area** is given as a check

For a very deep wall, P_H approaches ψ·q·b/π

$$P_H = \int_0^H \sigma_H \, dz = \psi \, \frac{q}{\pi} \, H \, (\theta_2 - \theta_1) \qquad \theta_i = \tan^{-1}\left(\frac{x_i}{H}\right)$$

$$z_R = \frac{\int_0^H z \, \sigma_H \, dz}{P_H}$$



Problem statement

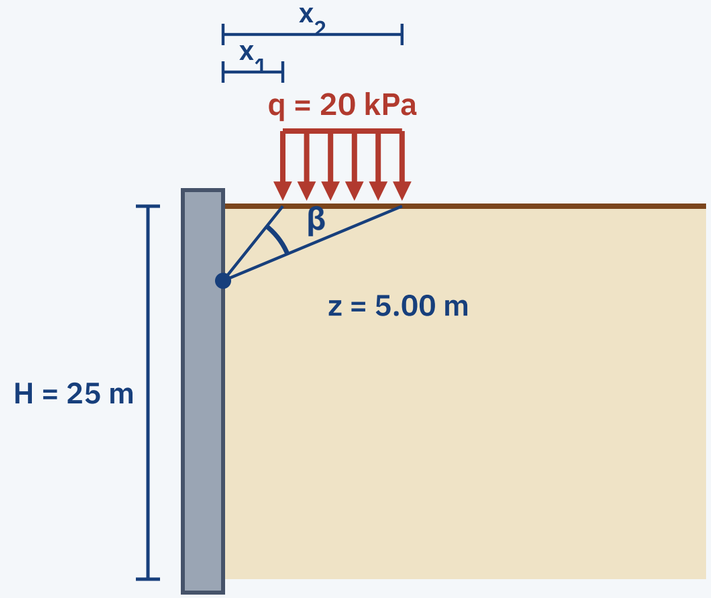
EXAMPLE

REQUIRED

β, **α** and **σ_H**: first semi-infinite medium (ψ = 1), then rigid wall (ψ = 2).

Given

Wall	$H = 25.00$ **m**
Strip	$x_1 = 4.00$ **m,** $x_2 = 12.00$ **m,** $q = 20.00$ **kPa**
Depth	$z = 5.00$ **m**



Solution (1/2)

SOLUTION

1 Strip width

$$b = x_2 - x_1 = 12.00 - 4.00 = \mathbf{8.00}\ \text{m}$$

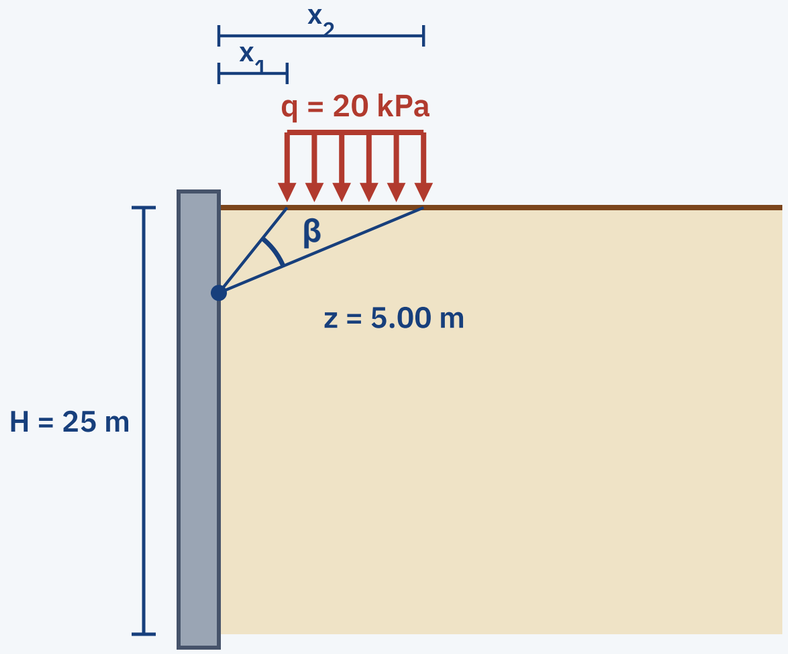
2 β: angle between the two rays

$$\beta = \tan^{-1}\tfrac{12}{5} - \tan^{-1}\tfrac{4}{5} = 67.380^\circ - 38.660^\circ = \mathbf{28.720}^\circ$$

3 α: angle of the bisector with the vertical

$$\alpha = \tan^{-1}\tfrac{4}{5} + \tfrac{\beta}{2} = 38.660^\circ + 14.3602^\circ = \mathbf{53.020}^\circ$$

In radians β = **0.50126** and 2α = **1.85075** (used for σ_H on the next page).



Solution (2/2)

SOLUTION

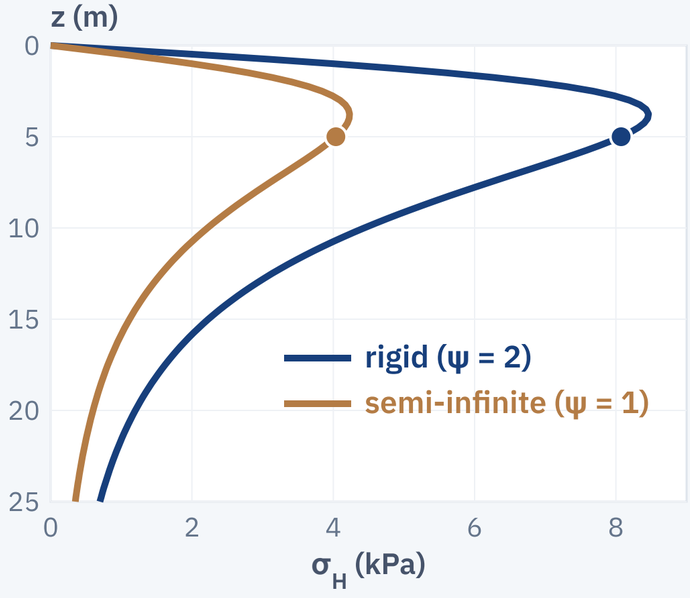
4 Semi-infinite medium (ψ = 1)

$$\sigma_H = \frac{20}{\pi}\left[0.50126 - \sin(0.50126)\cos(1.85075)\right] = \mathbf{4.0364}\ \text{kPa}$$

5 Rigid wall (ψ = 2): twice

$$\sigma_H = 2 \times 4.0364 = \mathbf{8.0728}\ \text{kPa}$$

At z = 5 m, **4.0364 kPa** for a semi-infinite medium and **8.0728 kPa** for a rigid wall. The same β and α are used in both forms; ψ is only a scale factor.



Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Wall and strip	$H = 25.00$ **m,** $x_1 = 4.00$ **m,** $x_2 = 12.00$ **m,** $q = 20.00$ **kPa**
Depths	$z = 0$ **–** 25 **m, every** 2.5 **m**

REQUIRED

The **β**, **α**, **σ_H** profile (two forms); the **maximum pressure** and its depth.

Solution (1/2)

SOLUTION

z (m)	β (°)	α (°)	σ_H ψ = 1 (kPa)	σ_H ψ = 2 (kPa)
0.00	0.000	90.000	0.0000	0.0000
2.50	20.237	68.113	3.8387	7.6773
5.00	**28.720**	**53.020**	**4.0364**	**8.0728**
7.50	29.922	43.034	3.1069	6.2137
10.00	28.393	35.998	2.2191	4.4382
12.50	26.086	30.788	1.5660	3.1319
15.00	23.728	26.796	1.1160	2.2319
17.50	21.564	23.657	0.8096	1.6193
20.00	19.654	21.137	0.5994	1.1988
22.50	17.992	19.077	0.4528	0.9056
25.00	16.551	17.366	0.3486	0.6971

At z = 0 there is no pressure (the strip is far from the wall); the pressure increases with depth, peaks at **z ≈ 3.81 m**, and then decreases. The z = 5.00 m row is Example 1. At the base (z = 25 m) 0.6971 kPa remains for ψ = 2.

Solution (2/2)

SOLUTION

1 Crest depth: condition $d\sigma_H/dz = 0$

$$\frac{x_1^3}{(x_1^2+z^2)^2} = \frac{x_2^3}{(x_2^2+z^2)^2} \;\Rightarrow\; z^2 = \frac{(x_1x_2)^{3/2}\left(\sqrt{x_2}-\sqrt{x_1}\right)}{x_2^{3/2}-x_1^{3/2}}$$

$$z^2 = 14.504 \;\Rightarrow\; z = \mathbf{3.8084}\ \mathrm{m}$$

2 Angles at this depth

$$\beta = \tan^{-1}\frac{12}{3.8084} - \tan^{-1}\frac{4}{3.8084} = \mathbf{25.99}^\circ \qquad \alpha = \tan^{-1}\frac{4}{3.8084} + \frac{\beta}{2} = \mathbf{59.40}^\circ$$

3 Maximum pressure

$$\sigma_{H,max} = \frac{20}{\pi}[0.45355 - \sin(0.45355)\cos(2.07341)] = \mathbf{4.23}\ (\psi = 1);\quad \mathbf{8.46}\ \mathrm{kPa}\ (\psi = 2)$$

At the peak z = **3.81 m**: **8.46 kPa** (ψ = 2), **4.23 kPa** (ψ = 1); the depth depends only on x_1, x_2.

Check the solution in GeoTerva ↗

Problem statement

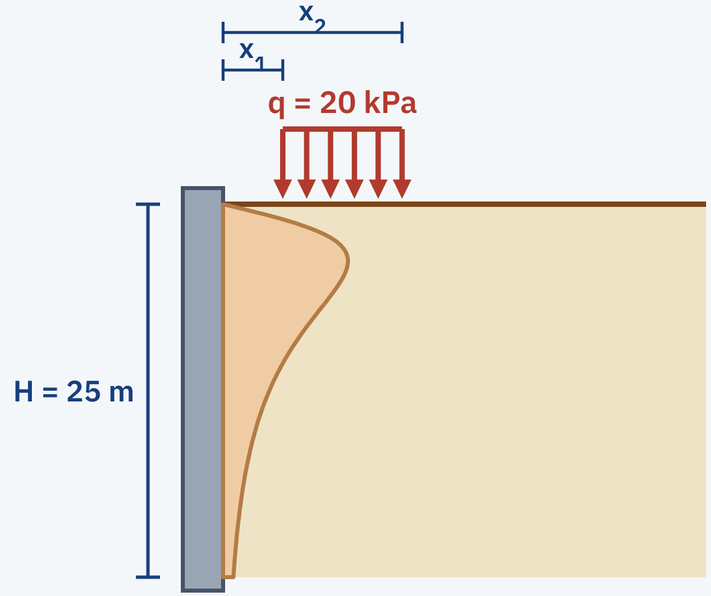
EXAMPLE

Given

Wall and strip	$H = 25.00$ **m,** $x_1 = 4.00$ **m,** $x_2 = 12.00$ **m,** $q = 20.00$ **kPa**
Shapes	**semi-infinite medium ($\psi = 1$) and rigid wall ($\psi = 2$)**

REQUIRED

P_H (with the closed-form expression for the area), z_R and its height above the base.



Solution (1/2)

SOLUTION

1 Angles (at depth H = 25 m)

$$\theta_1 = \tan^{-1}\frac{4}{25} = \mathbf{0.158655}\ \text{rad}$$

$$\theta_2 = \tan^{-1}\frac{12}{25} = \mathbf{0.447520}\ \text{rad}$$

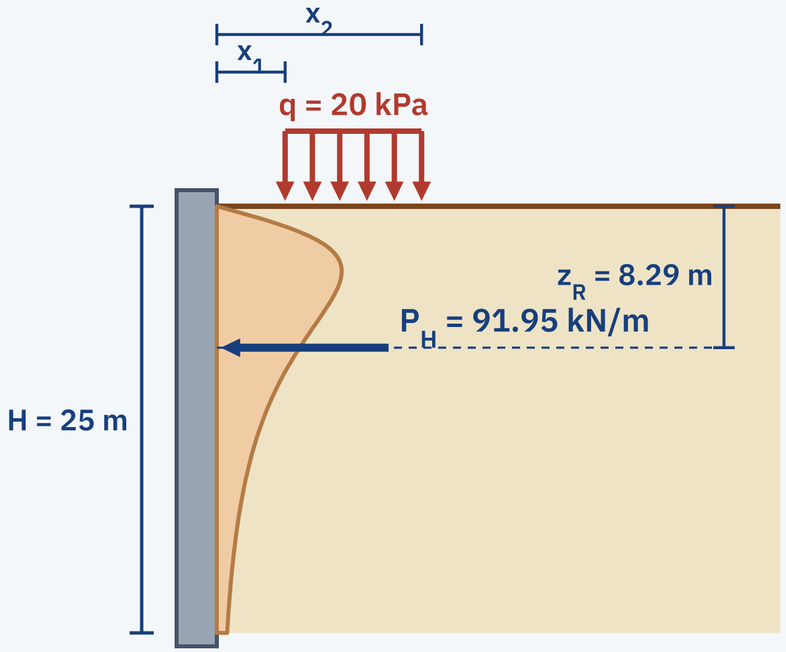
2 Semi-infinite medium (ψ = 1)

$$P_H = \frac{20}{\pi} \times 25 \times (0.447520 - 0.158655) = \mathbf{45.97}\ \text{kN/m}$$

3 Rigid wall (ψ = 2): twice

$$P_H = 2 \times 45.974 = \mathbf{91.95}\ \text{kN/m}$$

The resultant is **45.97 kN/m** for a semi-infinite medium and **91.95 kN/m** for a rigid wall; the module's numerical integration gives the same value.



Solution (2/2)

SOLUTION

1 First moment (closed form; ψ = 1, in [ ] minus for x_2, plus for x_1)

$$M = \int_0^H z\,\sigma_H\,dz = \frac{q}{\pi}\left[\frac{H^2}{2}\tan^{-1}\frac{x}{H} - \frac{x}{2}\left(H - x\tan^{-1}\frac{H}{x}\right)\right]_{x_1}^{x_2}$$

2 Numbers (ψ = 1)

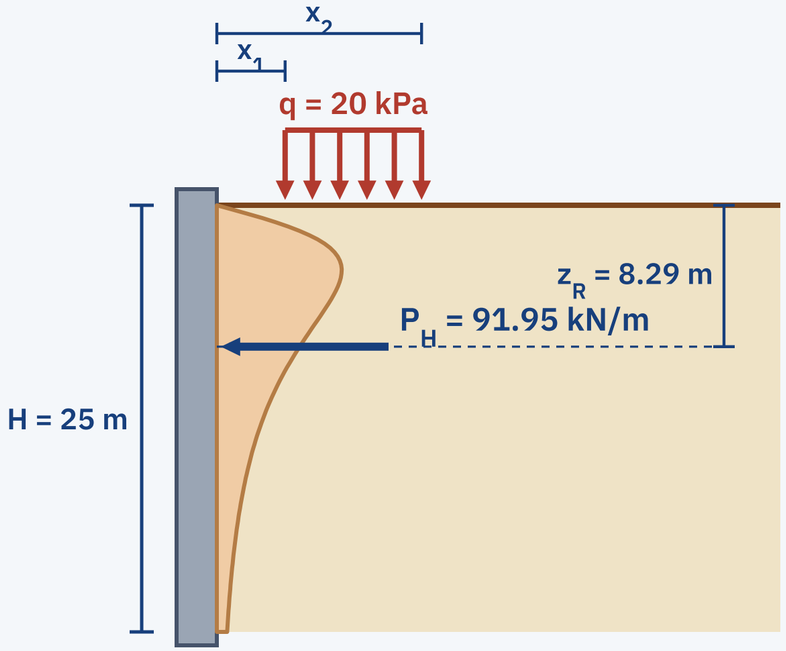
$$x_2: \quad 312.5(0.447520) - 6(25 - 12\cdot 1.12328) = 70.7259$$

$$x_1: \quad 312.5(0.158655) - 2(25 - 4\cdot 1.41214) = 10.8769$$

$$M = \tfrac{20}{\pi}(70.7259 - 10.8769) = \mathbf{381.011}\ \text{kN}$$

3 Depth and height above the base

$$z_R = \frac{M}{P_H} = \frac{381.011}{45.974} = \mathbf{8.287}\ \text{m} \qquad H - z_R = \mathbf{16.71}\ \text{m}$$



The resultant is **8.29 m** below the surface and **16.71 m** above the base; the depth is the same either way.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Wall and load	$H = 25.00$ **m,** $q = 20.00$ **kPa, rigid wall (** $\psi = 2$**)**
Strip	**width** $b = 8.00$ **m constant;** $x_1 = 0,\ 2,\ 4,\ 8,\ 12,\ 20$ **m**

REQUIRED

P_H, **z_R**, the maximum pressure and depth; manual check for **$x_1 = 0$**.

Solution (1/2)

SOLUTION

x_1 (m)	x_2 (m)	P_H (kN/m)	z_R (m)	$\sigma_{H,max}$ (kPa)	z (m)
0	8	98.58	4.80	20.00	0.00
2	10	95.71	6.72	11.53	2.33
4	**12**	**91.95**	**8.29**	**8.46**	**3.81**
8	16	82.64	10.63	5.57	6.40
12	20	72.33	12.21	4.16	8.85
20	28	53.22	14.06	2.76	13.60

As the strip moves away from the wall **the resultant decreases** (98.58 → 53.22 kN/m), **the line of action deepens** (4.80 → 14.06 m) and the pressure peak moves downward; the x_1 = 4 m row is Examples 1 and 3.

Solution (2/2)

SOLUTION

1 At the surface the angles take their limiting value (z = 0)

$$\beta = 90^\circ, \quad \alpha = 45^\circ$$

$$\sigma_H = \psi \frac{q}{\pi} \left[\frac{\pi}{2} - \sin 90^\circ \cos 90^\circ \right] = \psi \frac{q}{2} = \frac{2 \times 20}{2} = \mathbf{20.00}\ \mathrm{kPa}$$

2 Resultant ($\theta_1 = 0$, $\theta_2 = \tan^{-1}(8/25)$)

$$P_H = \frac{2 \times 20}{\pi} \times 25 \times (0.309703 - 0) = \mathbf{98.58}\ \mathrm{kN/m}$$

A strip starting at the wall face **directly loads** the upper part of the wall: the pressure at the ground surface is ψq/2 = **20.00 kPa** (10.00 kPa in a semi-infinite medium), the resultant is **98.58 kN/m** and z_R = 4.80 m.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Wall and load	$q = 20.00$ **kPa, rigid wall (**$\psi = 2$**)**
Width	$x_1 = 4.00$ **m constant,** $b = 2,\ 4,\ 8,\ 16,\ 36$ **m (**$H = 25$ **m)**
Height	$x_1 = 4$**,** $x_2 = 12$ **m;** $H = 5,\ 10,\ 25,\ 50,\ 100,\ 250$ **m**

REQUIRED

P_H and z_R; comparison with the deep wall limit $\psi\, q\, b / \pi$.

Solution

SOLUTION

b (m)	x_2 (m)	P_H (kN/m)	z_R (m)	$\sigma_{H,max}$ (kPa)
2	6	**24.47**	6.12	3.32
4	8	**48.08**	6.95	5.57
8	12	**91.95**	8.29	8.46
16	20	**164.28**	10.01	11.53
36	40	**271.69**	11.82	14.63

A **wider strip** carries more load: P_H increases, z_R and the crest deepen.

H (m)	P_H (kN/m)	P_H / (ψqb/π)	z_R (m)	from base (m)
5	**31.91**	0.3133	3.00	2.00
10	**63.10**	0.6194	5.10	4.90
25	**91.95**	0.9027	8.29	16.71
50	**99.13**	0.9732	10.13	39.87
100	**101.16**	0.9931	11.27	88.73
250	**101.75**	0.9989	12.03	237.97

In a deep wall, P_H approaches the value ψqb/π = 101.86 kN/m.

The pressure profile does not depend on wall height ($\sigma_{H,max}$ = 8.46 kPa, z = 3.81 m for every H): the height only changes the upper limit of the integral.

Check the solution in GeoTerva ↗

Summary

Key points

$\beta = \tan^{-1}(x_2/z) - \tan^{-1}(x_1/z)$, $\alpha = \tan^{-1}(x_1/z) + \beta/2$; $\sigma_H = \psi\,(q/\pi)[\beta - \sin\beta\cos 2\alpha]$; $\psi = 2$ rigid wall, $\psi = 1$ semi-infinite medium

Example ($H = 25$ m, $x_1 = 4$, $x_2 = 12$, $q = 20$): at $z = 5$ m, $\beta = 28.720°$, $\alpha = 53.020°$, $\sigma_H = 4.0364$ kPa ($\psi = 1$) and 8.0728 kPa ($\psi = 2$)

Maximum pressure 8.46 kPa ($\psi = 2$) at depth $z = 3.81$ m; zero at the surface, 0.6971 kPa at the base

Resultant $P_H = \psi\,(q/\pi)\,H\,(\theta_2 - \theta_1)$ = **91.95 kN/m** ($\psi = 2$), **$z_R = 8.29$ m** below the surface (16.71 m from the base)

Effects: as the strip moves away P_H decreases and z_R becomes deeper; if $x_1 = 0$ it is $\psi q/2$ at the surface; for a very deep wall $P_H \to \psi q b/\pi$

References

Das, B.M. (2016). Principles of Foundation Engineering, 8th ed. Cengage Learning; this page follows it.

California Department of Transportation. Trenching and Shoring Manual — the rigid-wall (doubled) form of the strip-load pressure.

NOTE

All numerical examples were generated with the module's own calculation engine; the module gives the same values. The module's opening wall has H = 25 m, x_1 = 4 m, x_2 = 12 m and q = 20 kPa.

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own wall, the position and width of your strip load in GeoTerva; the module provides β, α, σ_H values at every depth, the pressure profile, the resultant and the line of action, the step-by-step solution and the report together.

Open the module → **Theory page →**

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com