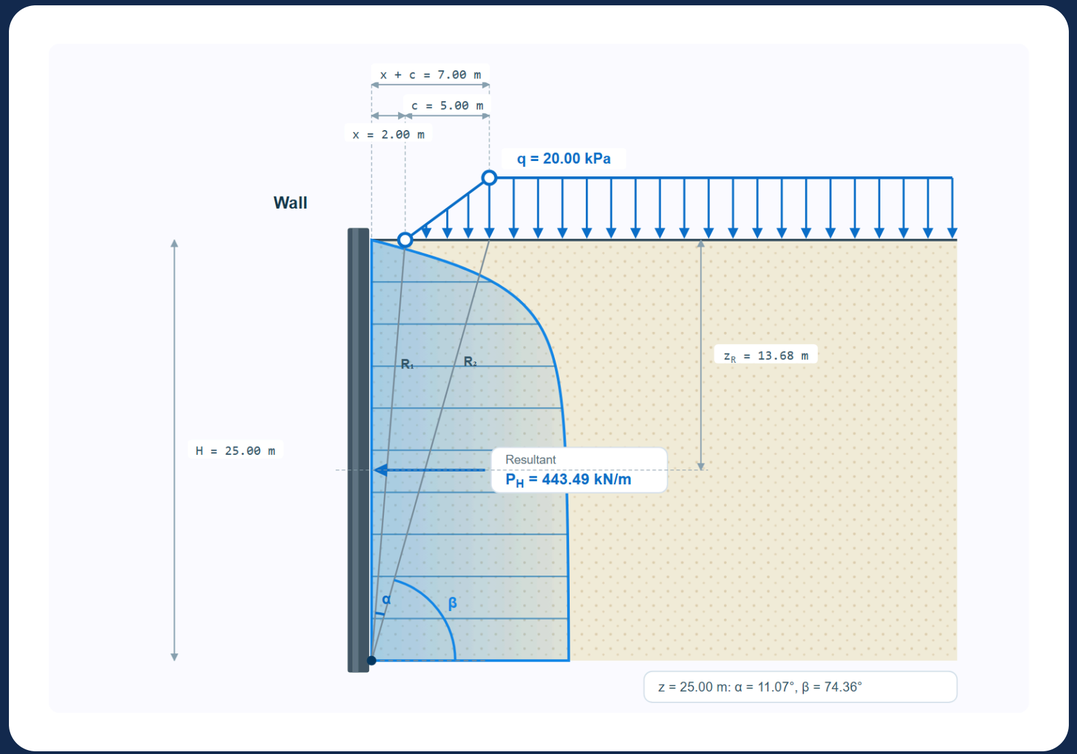
Geotechnical lecture presentation · Surcharge loads

Ramp surcharge load

Lecture: ramp loading starting from zero and reaching a plateau; the ordinate at a given depth is solved by hand, with profiles, the resultant, and variants presented in tables.

- Superposition of line loads
- R_1, R_2, α, β
- Rigid wall ψ = 2
- Resultant and z_R
- Can be solved by hand



Introduction

What will we learn in this lecture?

- The geometry of the **ramp load**: it starts at zero at distance **x** from the wall, rises to the value q over width **c**, and continues at constant q beyond that
- Finding the radii $\mathbf{R_1}$, $\mathbf{R_2}$ and the angles **α, β** at a depth z (radians)
- The closed form of the lateral pressure coming from the **superposition of line loads**: $\sigma_H = \psi\, q/(\pi c)\,[c\beta - x\alpha + 2z\,\ln(R_2/R_1)]$
- The difference between a **rigid wall** (ψ = 2) and a **semi-infinite medium** (ψ = 1)
- **Resultant P_H** and **depth of action z_R**: the area and first moment of the diagram; the effect of x and c

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example: H = 25 m, x = 2 m, c = 5 m, q = 20 kPa; rigid wall. One ordinate at z = 5 m is solved by hand.

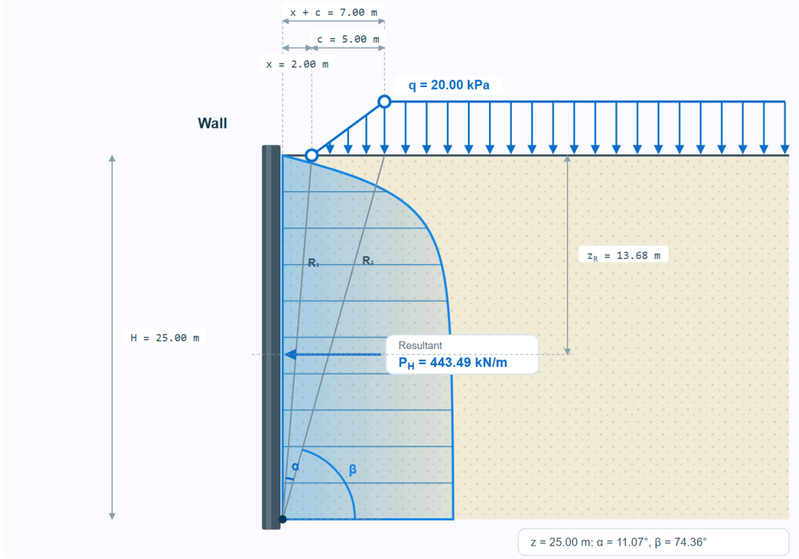
Ramp surcharge acting on the wall

THEORY

The ramp surcharge starts at a horizontal distance **x** from the wall

The surface pressure increases linearly from zero to the full value **q** over the ramp width **c**; beyond **x + c** the surcharge continues at a constant intensity q

The load is long and parallel to the wall; the resultant is given **per unit length** of the wall



H: total wall height
x: distance from the wall to the start of the ramp
c: horizontal width over which the load rises from 0 to q
q: full uniform surcharge intensity beyond the ramp
z: depth from the soil surface at which σ_H is calculated

Geometry at depth z

THEORY

At depth z, the lengths of the two rays from the point on the wall to the **start** and the **end** of the ramp, and the angles between them, are given below

In the pressure equation the angles are taken in **radians**; degree values may be shown in the interface and the calculation report for readability

$$R_1 = \sqrt{x^2 + z^2} \qquad R_2 = \sqrt{(x + c)^2 + z^2}$$

$$\alpha = \tan^{-1}\frac{x + c}{z} - \tan^{-1}\frac{x}{z} \qquad \beta = \frac{\pi}{2} - \tan^{-1}\frac{x + c}{z}$$

Superposition of line loads

THEORY

The ramp load is a continuous series of line loads, each of intensity **w(s)·ds** (increasing on the ramp, constant on the platform); the elastic horizontal stresses (Flamant) are summed

The rising ramp and plateau produce additional algebraic terms; these cancel exactly, leaving **angles and a logarithm**

$$\sigma_H = \psi \frac{2}{\pi} \int w(s) \frac{s^2 z}{(s^2 + z^2)^2} ds$$

$$\sigma_H = \psi \frac{q}{\pi c} \left[c\beta - x\alpha + 2z \ln \frac{R_2}{R_1} \right]$$

ψ = 1 is a semi-infinite medium (no wall); **ψ = 2** is a rigid, non-yielding wall that doubles the horizontal stress: the module default. σ_H is zero at the ground surface

Area and first moment of the diagram

THEORY

The **resultant surcharge force** per unit length of the wall and its depth from the ground surface are the area and the first moment of the diagram; the module computes both integrals numerically with full precision

$$P_H = \int_0^H \sigma_H(z)\, dz$$

$$z_R = \frac{\int_0^H z\, \sigma_H(z)\, dz}{P_H}$$

In the example **P_H = 443.49 kN/m** (rigid wall) or **221.74 kN/m** (semi-infinite medium); in both cases **z_R = 13.68 m**, below the ground surface

Six steps and one ordinate

THEORY

1 · H, x, c, q are defined and ψ is selected

2 · Select a depth **z** between the ground surface and the base of the wall

3 · R_1 and R_2 are computed; **4** · α and β are computed (radians)

5 · σ_H is computed: q, c, x, α, β, R_1, R_2 are substituted

6 · Iterate and integrate: pressure profile, then P_H and z_R

$$z = 5\ \text{m}: \quad R_1 = 5.385\ \text{m} \quad R_2 = 8.602\ \text{m} \qquad \alpha = 32.661^\circ \quad \beta = 35.538^\circ$$

$$\sigma_H = \frac{20}{5\pi}\left[5(0.62025) - 2(0.57004) + 10\ln\frac{8.6023}{5.3852}\right] = 8.461\ \text{kPa}\ \ (\psi = 1) \ \rightarrow\ 16.921\ \text{kPa}\ \ (\psi = 2)$$

Problem statement

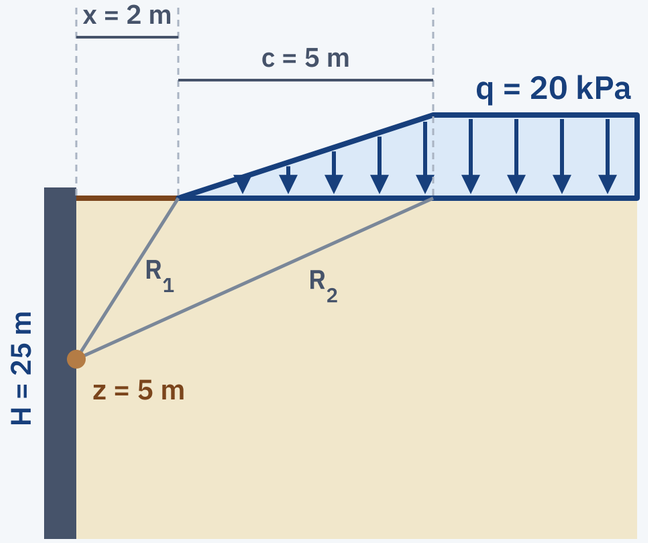
EXAMPLE

REQUIRED

At depth **z = 5 m**, **R_1, R_2, α, β** and the horizontal pressure **σ_H** (for two ψ values); comparison with the module.

Given

Wall and load	$H = 25.0$ **m;** $x = 2.0$ **m;** $c = 5.0$ **m;** $q = 20.0$ **kPa**
Wall	**semi-infinite medium** $(\psi = 1)$ **and rigid wall** $(\psi = 2)$



Solution (1/2)

SOLUTION

1 Rays from the point on the wall to the start (x) and end (x + c) of the ramp

$$R_1 = \sqrt{2^2 + 5^2} = \sqrt{29} = \mathbf{5.3852}\ \mathrm{m}$$

$$R_2 = \sqrt{7^2 + 5^2} = \sqrt{74} = \mathbf{8.6023}\ \mathrm{m}$$

2 Angles (radians)

$$\alpha = \tan^{-1}\frac{7}{5} - \tan^{-1}\frac{2}{5} = 0.95055 - 0.38051 = \mathbf{0.57004}\ \mathrm{rad}\ \ (32.661^\circ)$$

$$\beta = \frac{\pi}{2} - \tan^{-1}\frac{7}{5} = 1.57080 - 0.95055 = \mathbf{0.62025}\ \mathrm{rad}\ \ (35.538^\circ)$$

The start of the ramp is seen from the point on the wall at an angle of **32.66°** and its end at **35.54°**; the rays are 5.385 and 8.602 m long.

Solution (2/2)

SOLUTION

3 Semi-infinite medium (ψ = 1): the three terms in the bracket

$$\sigma_H = \frac{q}{\pi c}\left[c\beta - x\alpha + 2z\ln\frac{R_2}{R_1}\right] = \frac{20}{5\pi}\left[5(0.62025) - 2(0.57004) + 10\ln\frac{8.6023}{5.3852}\right]$$

$$= 1.27324\,[3.1012 - 1.1401 + 4.6838] = 1.27324(6.6450) = \mathbf{8.461}\ \text{kPa}$$

4 Rigid wall (ψ = 2): twice the semi-infinite medium value

$$\sigma_H = 2 \times 8.461 = \mathbf{16.921}\ \text{kPa}$$

At z = 5 m, **σ_H = 16.921 kPa** (rigid wall; 8.461 in a semi-infinite medium). The module gives the same value. The x·α term enters with a minus sign: a rising ramp reduces the load near the wall relative to a full strip starting at x.

Check the solution in GeoTerva ↗

Problem statement

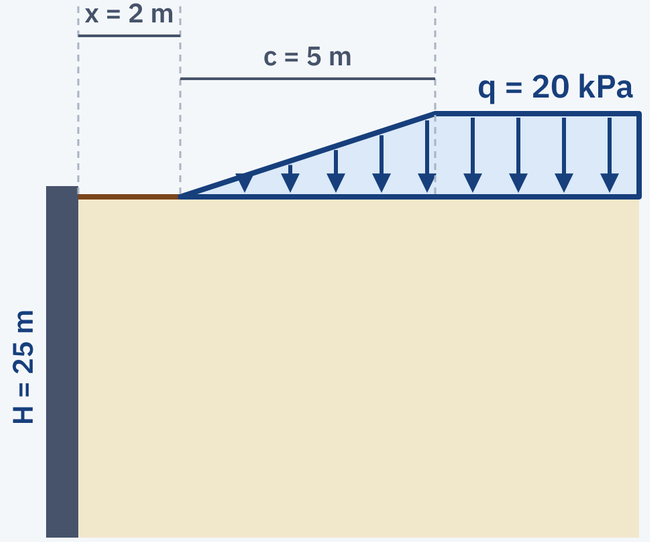
EXAMPLE

Given

Model	**Ramp from Example 1; rigid wall; table at every $H/30 = 0.833$ m**
On each row	**R_1, R_2, α, β and σ_H; the semi-infinite medium values are half those of the rigid wall**

REQUIRED

Pressure profile and the value at the wall base; behavior of pressure at depth (the z → ∞ limit of the formula).



Solution (1/2)

SOLUTION

z (m)	R_1 (m)	R_2 (m)	α (°)	β (°)	σ_H (kPa)
0.00	2.000	7.000	0.00	0.00	0.000
0.83	2.167	7.049	15.83	6.79	5.109
1.67	2.603	7.196	26.41	13.39	9.258
2.50	3.202	7.433	31.69	19.65	12.275
3.33	3.887	7.753	33.57	25.46	14.395
4.17	4.622	8.146	33.60	30.76	15.877
5.00	**5.385**	**8.602**	**32.66**	**35.54**	**16.921**
5.83	6.167	9.112	31.27	39.81	17.665
6.67	6.960	9.667	29.70	43.60	18.202
7.50	7.762	10.259	28.09	46.97	18.596
8.33	8.570	10.883	26.53	49.97	18.887
9.17	9.382	11.534	25.06	52.63	19.107
10.00	10.198	12.207	23.68	55.01	19.275
10.83	11.016	12.898	22.41	57.13	19.405
11.67	11.837	13.606	21.24	59.04	19.506
12.50	12.659	14.327	20.16	60.75	19.586

z (m)	R_1 (m)	R_2 (m)	α (°)	β (°)	σ_H (kPa)
13.33	13.482	15.059	19.17	62.30	19.651
14.17	14.307	15.802	18.26	63.71	19.703
15.00	15.133	16.553	17.42	64.98	19.745
15.83	15.959	17.312	16.65	66.15	19.780
16.67	16.786	18.077	15.94	67.22	19.809
17.50	17.614	18.848	15.28	68.20	19.833
18.33	18.442	19.624	14.67	69.10	19.853
19.17	19.271	20.405	14.11	69.94	19.870
20.00	20.100	21.190	13.58	70.71	19.885
20.83	20.929	21.978	13.09	71.43	19.897
21.67	21.759	22.769	12.63	72.10	19.908
22.50	22.589	23.564	12.20	72.72	19.918
23.33	23.419	24.361	11.80	73.30	19.926
24.17	24.249	25.160	11.42	73.85	19.933
25.00	25.080	25.962	11.07	74.36	19.939

Solution (2/2)

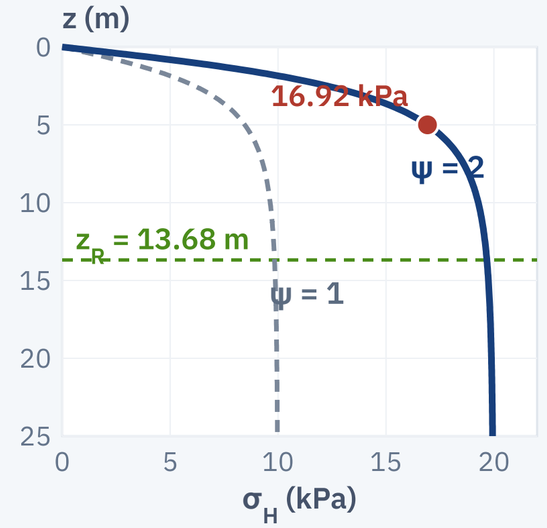
SOLUTION

1 At the surface and at depth

$$z = 0: \quad \sigma_H = 0 \qquad z = H = 25\ \text{m}: \quad \sigma_H = \mathbf{19.94}\ \text{kPa}$$

$$z \to \infty: \quad \alpha \to 0,\ \beta \to \frac{\pi}{2},\ 2z \ln \frac{R_2}{R_1} \to 0 \ \Rightarrow\ \sigma_H \to \psi \frac{q}{2}$$

The pressure is zero at the surface and **increases with depth**: the maximum value is at the wall base (19.94 kPa). At depth $\sigma_H \to \psi q/2$ = **q**: 19.99 at z = 50 m, 20.00 kPa at z = 1000 m (rigid wall). The ordinate at z = 5 m is 16.921 kPa (bold row, Example 1).



Check the solution in GeoTerva ↗

Problem statement

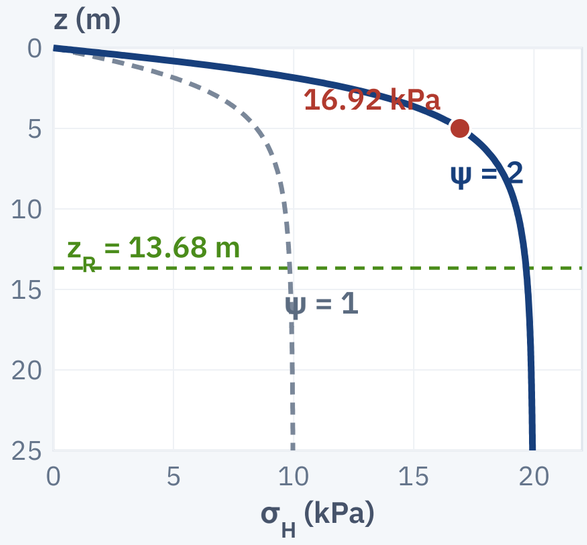
EXAMPLE

Given

Model	**The ramp in Example 1; rigid wall; the $H/30$ table of Example 2 ($n = 30$, $h = H/30 = 0.8333$ m)**
Method	**Simpson's rule; $z_R = \int z\,\sigma\,dz \,/\, P_H$**

REQUIRED

P_H and z_R (by hand, from the table); semi-infinite medium values; comparison with the module result.



Solution (1/2)

SOLUTION

	σ_0	Σ odd (i = 1, 3, …, 29)	Σ even (i = 2, 4, …, 28)	σ_{30}
σ_H (kPa)	0.000	266.553	255.220	19.939

1 Simpson sum and area

$$\Sigma = \sigma_0 + 4(266.553) + 2(255.220) + \sigma_{30} = 1{,}596.59$$

$$P_H \approx \frac{h}{3}\,\Sigma = \frac{0.8333}{3}(1{,}596.59) = \mathbf{443.50}\ \mathrm{kN/m}$$

By hand **443.50 kN/m**; the module's precise integral **443.49 kN/m** (difference 0.002%). In a semi-infinite medium, half of it: **221.74 kN/m**.

Solution (2/2)

SOLUTION

	$z_0\sigma_0$	Σ single	Σ double	$z_{30}\sigma_{30}$
$z\sigma_H$ (kPa·m)	0.000	3,638.41	3,393.35	498.48

2 First moment and depth

$$\int z\,\sigma\,dz \approx \frac{0.8333}{3}\left[0 + 4(3{,}638.41) + 2(3{,}393.35) + 498.48\right] = 6{,}066.3\ \text{kN}$$

$$z_R = \frac{6{,}066.3}{443.50} = \mathbf{13.68}\ \text{m}\ \ (\text{from the ground}) \qquad H - z_R = 11.32\ \text{m}\ (\text{from the base})$$

By hand **z_R = 13.68 m**; the module 13.68 m. The resultant 443.49 kN/m acts **13.68 m below the ground surface** (11.32 m above the base); when ψ changes, z_R does not change.

Check the solution in GeoTerva ↗

Problem statement

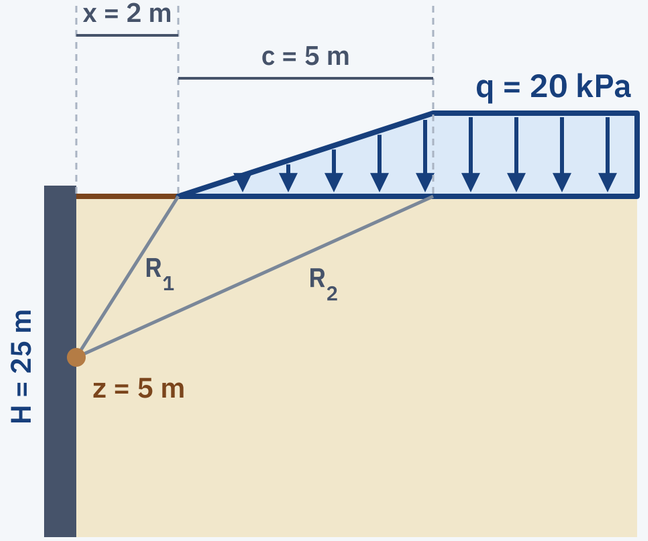
EXAMPLE

Given

Model	**Example 1: $z = 5$ m; semi-infinite medium ($\psi = 1$)**
Method	$\sigma_H = \frac{2}{\pi} \int w(s) \frac{s^2 z}{(s^2 + z^2)^2} ds$; **$w = q(s - x)/c$ on the ramp, $w = q$ on the plateau**
Calculation	**ramp: Simpson ($n = 10$, $h = 0.5$ m); plateau: closed-form integral**

REQUIRED

σ_H by finding and summing the ramp and plateau contributions separately; comparison with the closed form of Example 1.



Solution (1/2)

SOLUTION

i	s (m)	w(s) (kPa)	f(s) = s²z/(s²+z²)² (1/m)	c	c·w·f
0	2.0	0.00	0.02378	1	0.00000
1	2.5	2.00	0.03200	4	0.25600
2	3.0	4.00	0.03893	2	0.31142
3	3.5	6.00	0.04414	4	1.05941
4	4.0	8.00	0.04759	2	0.76145
5	4.5	10.00	0.04945	4	1.97796
6	5.0	12.00	0.05000	2	1.20000
7	5.5	14.00	0.04955	4	2.77472
8	6.0	16.00	0.04837	2	1.54797
9	6.5	18.00	0.04671	4	3.36314
10	7.0	20.00	0.04474	1	0.89481
			Σ c·w·f =		**14.14688**

1 Integral of the line loads on the ramp

$$I_{ramp} = \frac{h}{3}\sum c_i\, w_i f_i = \frac{0.5}{3}(14.14688) = \mathbf{2.3578}$$

Solution (2/2)

SOLUTION

2 Plateau (s ≥ x + c = 7 m): closed-form integral with A(s) = ½[tan⁻¹(s/z) − sz/(s² + z²)]

$$A(7) = \frac{1}{2}\left[\tan^{-1}\frac{7}{5} - \frac{7(5)}{74}\right] = 0.23879 \qquad A(\infty) = \frac{\pi}{4} = 0.78540$$

$$I_{plateau} = q\,[A(\infty) - A(7)] = 20(0.78540 - 0.23879) = 10.9322$$

3 Total and ψ

$$\sigma_H = \frac{2}{\pi}\,(I_{ramp} + I_{plateau}) = \frac{2}{\pi}(2.3578 + 10.9322) = \mathbf{8.4607}\ \text{kPa}\ \ (\psi = 1)$$

$$\psi = 2: \quad \sigma_H = \mathbf{16.921}\ \text{kPa}$$

Superposition gives **8.4607 kPa**; the closed form (Example 1) gives **8.4607 kPa**. The two approaches are identical: the terms in the brackets are the sum of the ramp and plateau integrals.

Check the solution in GeoTerva ↗

Problem statement

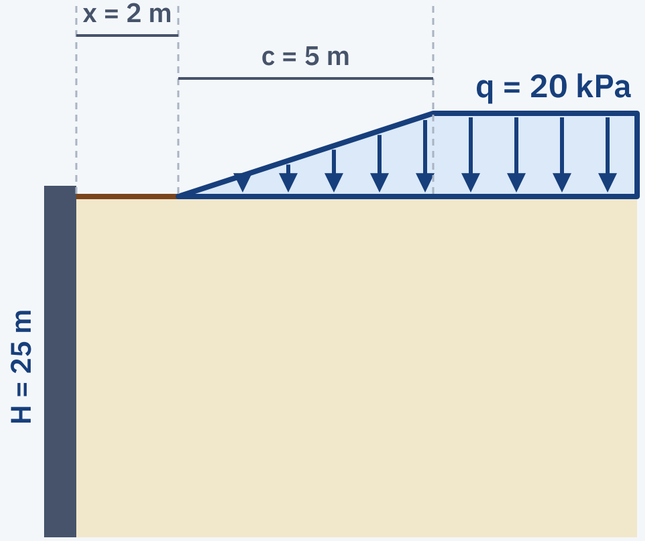
EXAMPLE

Given

Model	$H = 25.0$ **m;** $q = 20$ **kPa; rigid wall; only one changes at a time**
Variables	**start offset** $x = 0; 2; 5; 10$ **m (**$c = 5$ **m) and width** $c = 1; 5; 10; 20$ **m (**$x = 2$ **m)**

REQUIRED

For each case P_H, z_R, the pressure at the base; semi-infinite medium values; upper bound (uniform load).



Solution (1/2)

SOLUTION

x (m), c = 5	P_H (kN/m), ψ = 2	P_H (kN/m), ψ = 1	z_R (m)	H − z_R (m)	σ_H(H) (kPa)
0	468.38	234.19	13.19	11.81	19.98
2	443.49	221.74	13.68	11.32	19.94
5	407.49	203.75	14.28	10.72	19.78
10	352.75	176.38	15.00	10.00	19.17

c (m), x = 2	P_H (kN/m), ψ = 2	P_H (kN/m), ψ = 1	z_R (m)	H − z_R (m)	σ_H(H) (kPa)
1	468.28	234.14	13.22	11.78	19.99
5	443.49	221.74	13.68	11.32	19.94
10	414.10	207.05	14.11	10.89	19.76
20	362.68	181.34	14.67	10.33	19.02

As the ramp moves away from the wall (x ↑) or rises slowly (c ↑) **the resultant decreases and z_R deepens**; the pressure at the base is always close to q = 20 kPa. The ψ = 1 values are half of those for a rigid wall.

Solution (2/2)

SOLUTION

1. Upper bound: the load starts at the wall face (x = 0) and immediately becomes q (c → 0): uniform surcharge

$$\sigma_H = \psi \frac{q}{2} = q \quad (\text{at any z}) \;\Rightarrow\; P_H = qH = 20 \times 25 = \mathbf{500}\ \text{kN/m}, \quad z_R = \frac{H}{2} = 12.5\ \text{m}$$

2. Ramp width c → 0: step load starting at x = 2 m

$$c = 0.001\ \text{m}: \quad P_H = 474.58\ \text{kN/m}, \quad z_R = 13.09\ \text{m}$$

$$\text{strip formula}\ (x_2 \to \infty): \quad P_H = 474.59\ \text{kN/m}, \quad z_R = 13.09\ \text{m}$$

The ramp of Example 1 lies between these two limits: **443.49 kN/m**, which is 88.7% of the upper limit of 500 kN/m. As the ramp becomes steeper, the result of the step load approaches the strip solution: the same superposition. (The module gives 499.99 kN/m for x = 0, c = 0.001 m.)

Check the solution in GeoTerva ↗

Summary

Key points

$\sigma_H = \psi\, q/(\pi c)\,[c\beta - x\alpha + 2z \ln(R_2/R_1)]$; $R_1 = \sqrt{(x^2 + z^2)}$, $R_2 = \sqrt{((x + c)^2 + z^2)}$, $\alpha = \tan^{-1}((x + c)/z) - \tan^{-1}(x/z)$, $\beta = \pi/2 - \tan^{-1}((x + c)/z)$ (radians); $\psi = 2$ for a rigid wall, $\psi = 1$ for a semi-infinite medium

Example (z = 5 m): R_1 5.385 m, R_2 8.602 m; α 32.661°, β 35.538°; σ_H = 8.461 kPa ($\psi = 1$), **16.921 kPa** ($\psi = 2$)

Profile: 0 at the surface, increases with depth, approaches $\psi q/2 = q$ at depth (19.94 kPa at the base)

Resultant $P_H = \int \sigma_H dz$ = **443.49 kN/m**, $z_R = \int z\sigma_H dz / P_H$ = **13.68 m** (from the ground surface); for $\psi = 1$, P_H = 221.74 kN/m, z_R the same

Comparison: the resultant decreases as the ramp moves away from the wall and widens; the upper bound is the uniform load (q H = 500 kN/m), and the lower extreme is the step load as the ramp becomes steeper

References

Das, B.M. (2016). Principles of Foundation Engineering, 8th ed. Cengage Learning; this page follows it.

NOTE

All numerical examples were produced with the module's own calculation engine; the module gives the same values. The module's opening example is the wall with H = 25 m, x = 2 m, c = 5 m, q = 20 kPa.

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own ramp load, starting distance, width and wall height in GeoTerva; the module gives the pressure diagram, the resultant and its depth of action, three views, the step-by-step solution and the report together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com