Geotechnical lecture presentation · Surcharge loads

Point surcharge load

Lecture: the lateral pressure a point load at the surface produces on a wall; two methods, the resultant, the distribution in plan, and multiple loads, with worked examples.

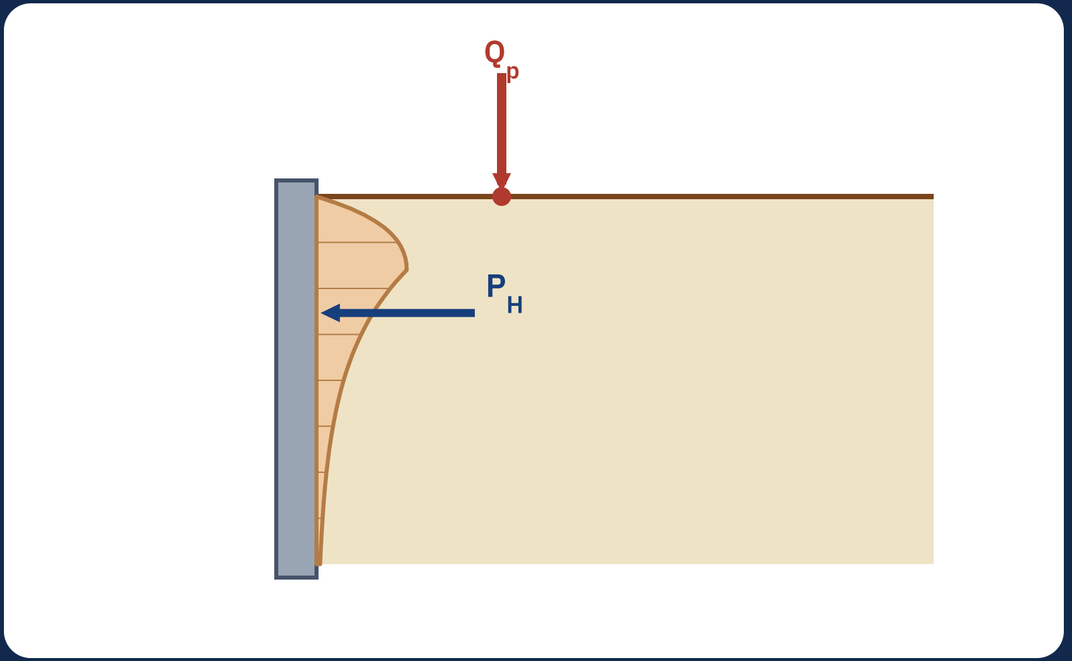
$m = x/H \cdot n = z/H$

$\sigma_H = (3Q_p/\pi)\, x^2 z/R^5$

Terzaghi–NAVFAC chart

P_H and L

Distribution in plan



What will we learn in this lecture?

- The geometry of a point load: **x**, **y**, **z**, **R** and the dimensionless **m = x/H**, **n = z/H**
- **Elastic (rigid wall)** solution: twice the Boussinesq stress, maximum pressure at **z = x/2**; x_{min} **= 0.30 m**
- **Terzaghi–NAVFAC chart**: Case 1 (m ≤ 0.4) and Case 2 (m > 0.4)
- Resultant $P_H = (Q_p/H)\cdot I_0$ and height $L = H\cdot I_1/I_0$; closed-form expressions for I_0, I_1
- Distribution **in plan** along the wall ($\cos^2(1.1\theta)$) and **at most three loads on the same line**

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example wall: H = 15 m, point load Q_p = 38 kN at a distance x = 5 m from the wall. P_H is the force per unit length of wall at the section of the load (kN/m).

Point load acting on the wall

THEORY

The concentrated load **Q_p** at the surface creates a three-dimensional stress field in the soil; if the load lies within the influence zone of a vertical wall, part of the stress increase is transmitted to the wall as **lateral pressure**

The module gives the variation of pressure with depth along the line perpendicular to the wall, its **resultant** and **line of action**, as well as its distribution along the wall in plan

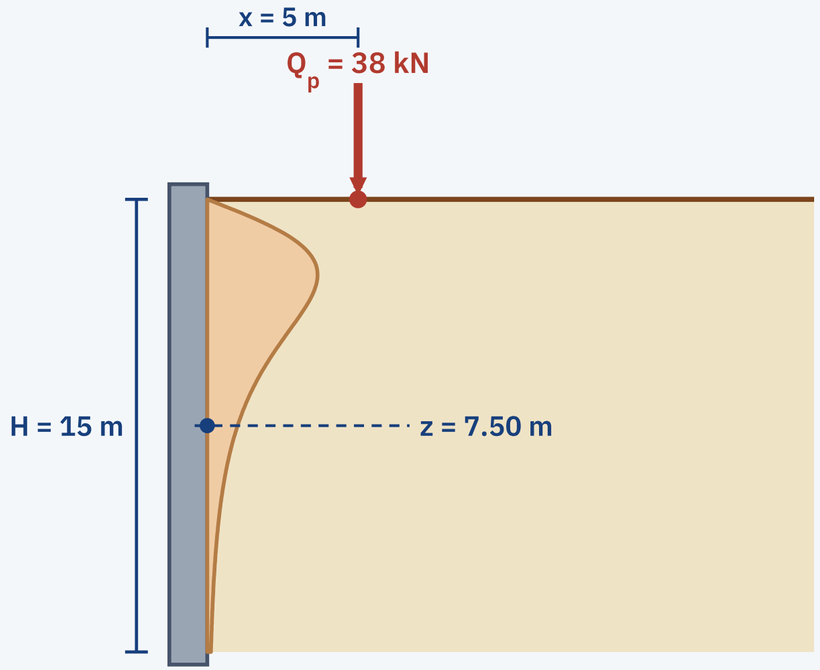
$$m = \frac{x}{H}, \qquad n = \frac{z}{H}$$

H — wall height; **x** — horizontal distance from the wall to the load; **z** — depth from the surface

y — distance along the wall from the line perpendicular to the wall through the load; **R = √(x² + y² + z²)**

σ_H — lateral pressure on the axis line; **σ'_H** — in plan; **P_H** — resultant; **L** — distance from the base to the line of action

θ — plan angle in the chart; **x_{min}** — smallest computation distance in the elastic method (0.30 m)



Elastic solution and Terzaghi–NAVFAC chart

THEORY

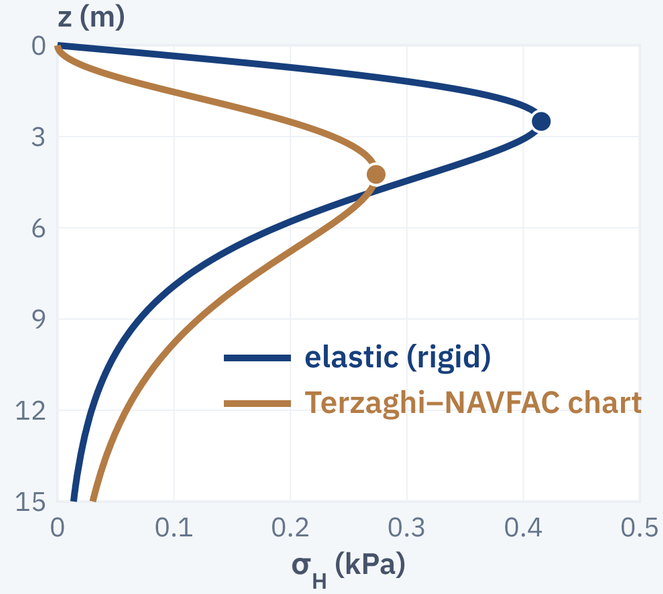
Both describe the **non-yielding wall**; they differ most **under a load close to the wall**

Elastic: **twice** the Boussinesq stress: the mirror-image load on the opposite side of the wall reduces the lateral movement at the face to zero (Mindlin 1936); Poisson's ratio 0.5

Chart (Terzaghi 1954): the distance effect stops at **m = 0.4**; a load within 0.4H is drawn as if at a distance of 0.4H

For a load within 0.4H, the elastic solution concentrates **increasing pressure** near the upper part of the wall, while the chart preserves a single diagram; farther away they give the same order of magnitude

Profile (H = 15 m, x = 5 m, Q_p = 38 kN)



Rigid wall: pressure, peak and closed-form resultant

THEORY

On the vertical through the load (y = 0): **$\sigma_H = (3Q_p/\pi)\, x^2 z/R^5$**; the peak is at **$z = x/2$**, **$\sigma_{H,max} = 0.2733\, Q_p/x^2$**

As the load approaches the wall the solution becomes **singular**: P_H increases as $\approx Q_p/(\pi x)$; therefore, for **$x < x_{min} = 0.30$ m** the load is computed at 0.30 m

Within the critical distance of about **0.4H** the elastic equations cease to be reliable: engineering judgement is required, and the charts provide a comparison

$$\sigma_H = \frac{3}{\pi}\,\frac{Q_p}{H^2}\,\frac{m^2\, n}{\left(m^2+n^2\right)^{5/2}}$$

$$I_0 = \frac{1}{\pi}\left[\frac{1}{m} - \frac{m^2}{\left(1+m^2\right)^{3/2}}\right] \qquad I_1 = I_0 - \frac{1}{\pi\left(1+m^2\right)^{3/2}}$$

Terzaghi–NAVFAC: Case 1 and Case 2

THEORY

At each depth, **n = z/H** is computed, and the equation is selected according to **m**

The Case 1 equation does not contain m: it is the value of Case 2 for m = 0.4 (1.77 × 0.16 = 0.2832, written as **0.28**); the load within 0.4H, wherever it is located, gives **the same diagram, P_H and L**

Due to rounding, P_H makes a small step at the limit m = 0.4 (≈ 1.1%); L remains continuous

$$\sigma_H = 0.28\,\frac{Q_p}{H^2}\,\frac{n^2}{(0.16+n^2)^3} \qquad m \le 0.4\ (\text{Case 1})$$

$$\sigma_H = 1.77\,\frac{Q_p}{H^2}\,\frac{m^2\,n^2}{(m^2+n^2)^3} \qquad m > 0.4\ (\text{Case 2})$$

P_H, L and I_0, I_1

THEORY

P_H is the area of the pressure diagram; **L** is the height of the centroid above the wall base: the resultant acts at a depth of **H – L** from the surface

P_H is a force per unit section of the load itself (y = 0); since the pressure decreases along the wall, it is **not multiplied by the wall length**

When written as $\sigma_H = (Q_p/H^2)\cdot f(n)$, two pure numbers remain; closed-form in elasticity, numerical (or closed-form) integral in the chart. The chart constants are this integral read at a single value of m: for **m = 0.4**, P_H = 0.787 Q_p/H, L = 0.587 H; for **m = 0.6**, 0.457 Q_p/H and 0.476 H

$$P_H = \frac{Q_p}{H} I_0 \qquad L = H \frac{I_1}{I_0}$$

$$I_0 = \int_0^1 f(n)\, dn \qquad I_1 = \int_0^1 f(n)\,(1-n)\, dn$$

Pressure along the wall and multiple loads

THEORY

Elastic: the σ_H formula is valid at every point of the wall ($R^2 = x^2 + y^2 + z^2$); at constant depth the pressure decreases along the wall as $\mathbf{(R_0/R)^5}$, with a peak at $z = \sqrt{(x^2 + y^2)}/2$

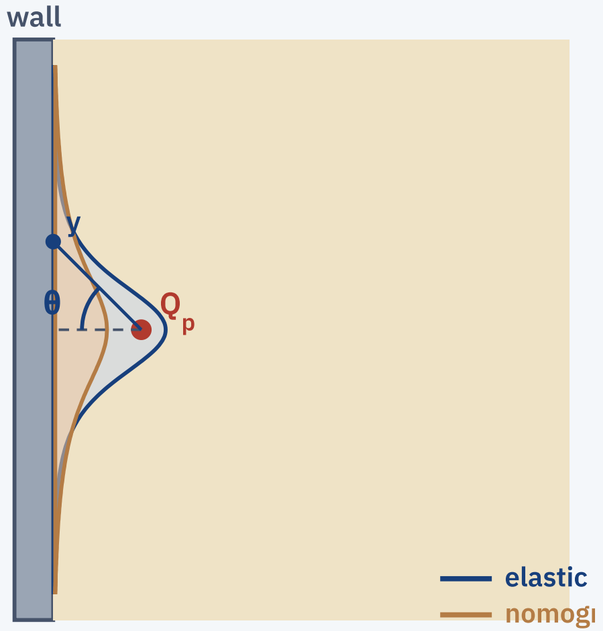
Chart: the axial line pressure is reduced by the plan angle; when the angle 1.1θ reaches 90° ($\boldsymbol{\theta \approx 81.8°}$) the reduction is held at zero

Multiple loads (at most 3): each has its own x_i and $Q_{p,i}$ value; since pressures are stress increases, they are **superposed**; the resultant acts at the point where the moments about the base balance

$$\sigma'_H = \sigma_H \cos^2(1.1\,\theta) \qquad \theta = \tan^{-1}\frac{y}{x}\ \text{(nomograph)}$$

$$\sigma_H = \sum_i \sigma_{H,i} \qquad P_H = \sum_i \frac{Q_{p,i}}{H} I_{0,i} \qquad L = \frac{\sum_i P_{H,i}\, L_i}{P_H}$$

Plan: at depth z = 2.5 m



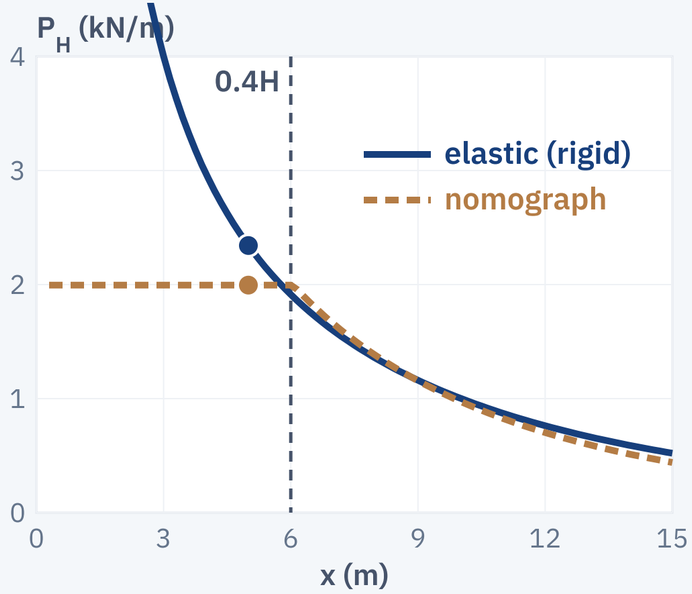
Resultant and its location, with load distance

THEORY

x (m)	m	P_H elastic	Elastic L	P_H chart	L chart	Condition
0.30	0.020	40.32	14.70	1.99	8.81	Case 1
1.00	0.067	12.09	14.01	1.99	8.81	Case 1
2.00	0.133	6.03	13.05	1.99	8.81	Case 1
3.00	0.200	4.00	12.15	1.99	8.81	Case 1
4.00	0.267	2.97	11.33	1.99	8.81	Case 1
5.00	0.333	2.34	10.59	1.99	8.81	Case 1
6.00	0.400	1.91	9.94	1.99	8.81	Case 1
8.00	0.533	1.35	8.87	1.38	7.61	Case 2
10.00	0.667	1.00	8.05	0.9755	6.74	Case 2
15.00	1.000	0.5213	6.80	0.4402	5.45	Case 2

m ≤ 0.4 (x ≤ 6 m): the chart gives the same P_H and L; the elastic P_H increases rapidly as the load approaches

m > 0.4: the elastic resultant acts somewhat higher (L is larger)



Problem statement

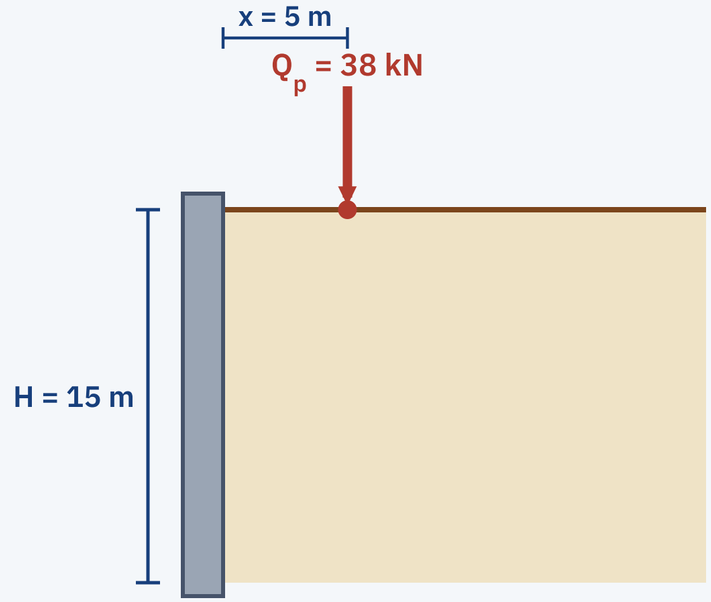
EXAMPLE

REQUIRED

m, the maximum pressure $\sigma_{H,max}$ (z = x/2), I_0, I_1, P_H, **L** and the depth of the resultant below the surface.

Given

Wall	$H = 15.00$ **m**
Load	$Q_p = 38.0$ **kN,** $x = 5.00$ **m from the wall**
Method	**elastic (rigid wall);** $x \geq x_{min}$



Solution (1/3)

SOLUTION

1 Distance ratio

$$m = \frac{x}{H} = \frac{5}{15} = \frac{1}{3} = \mathbf{0.333} \quad (x \geq x_{min} = 0.30\ \text{m})$$

2 Peak depth

$$z = \frac{x}{2} = \mathbf{2.50}\ \text{m}$$

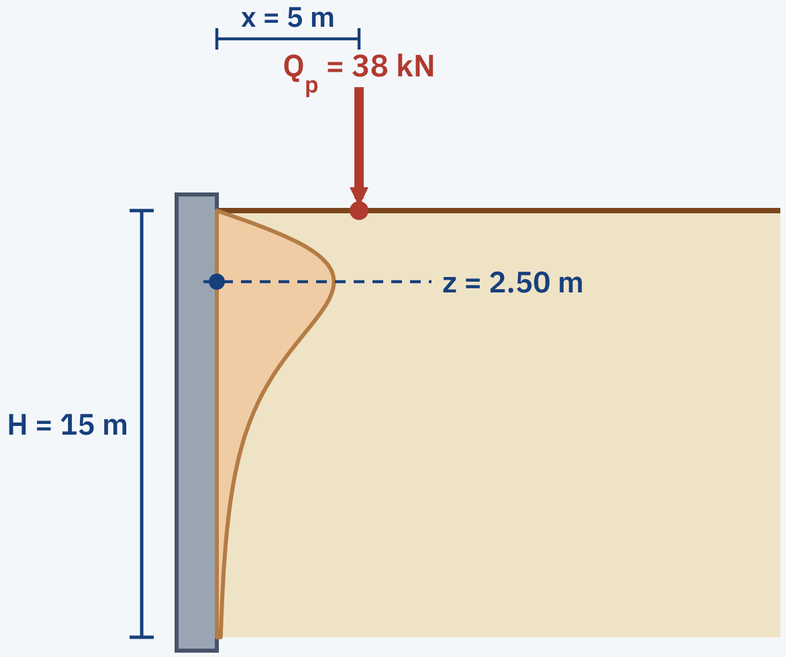
$$R = \sqrt{x^2 + z^2} = \sqrt{25.00 + 6.25} = \mathbf{5.5902}\ \text{m}$$

3 Pressure at the top

$$\sigma_{H,max} = \frac{3 \times 38}{\pi} \cdot \frac{5.00^2 \times 2.50}{5.5902^5}$$

$$\sigma_{H,max} = 36.287 \times 0.011449 = \mathbf{0.4154}\ \text{kPa}$$

The elastic peak is **0.4154 kPa**, at depth z = 2.50 m: 0.2733 Q_p/x^2 = 0.4154 kPa gives the same value.



Solution (2/3)

SOLUTION

4 Dimensionless value of the area I_0

$$I_0 = \frac{1}{\pi}\left[\frac{1}{m} - \frac{m^2}{(1+m^2)^{3/2}}\right] = \frac{1}{\pi}\left[3 - \frac{1/9}{(10/9)^{3/2}}\right] = \frac{1}{\pi}[3 - 0.09487] = \mathbf{0.92473}$$

5 Dimensionless value of the moment I_1

$$I_1 = I_0 - \frac{1}{\pi(1+m^2)^{3/2}} = 0.92473 - \frac{1}{\pi(10/9)^{3/2}} = 0.92473 - 0.27178 = \mathbf{0.65295}$$

I_0 = **0.92473**, I_1 = **0.65295**: two numbers depending only on m; the load and the height lie outside them.

Solution (3/3)

SOLUTION

6 Resultant

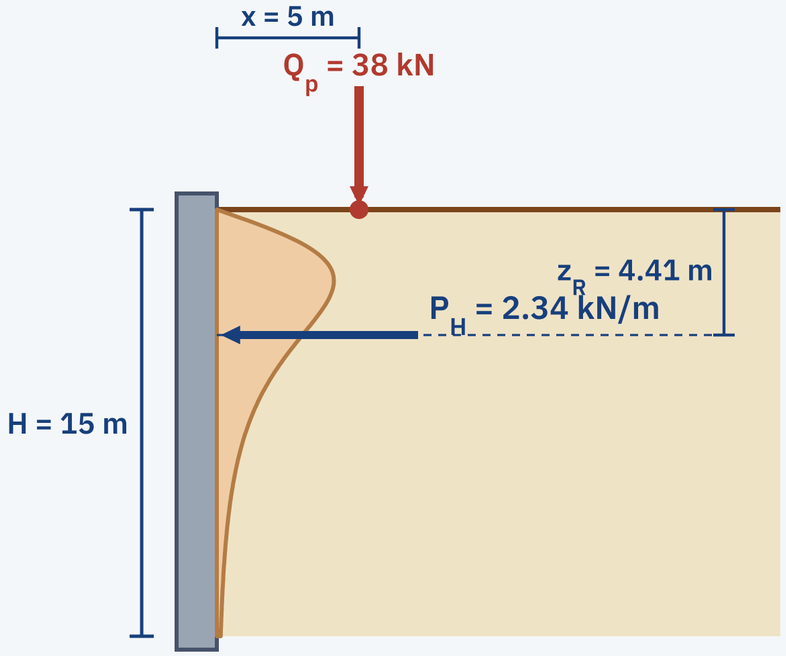
$$P_H = \frac{Q_p}{H} I_0 = \frac{38}{15} \times 0.92473 = \mathbf{2.34}\ \mathrm{kN/m}$$

7 Influence line (from base)

$$L = H \frac{I_1}{I_0} = 15 \times \frac{0.65295}{0.92473} = \mathbf{10.59}\ \mathrm{m}$$

8 Depth from surface

$$H - L = 15.00 - 10.59 = \mathbf{4.41}\ \mathrm{m}$$



P_H = 2.34 kN/m, at **4.41 m** below the surface (10.59 m from the base). The force acts at the section of the load: it is not multiplied by the wall length.

Check the solution in GeoTerva ↗

Problem statement

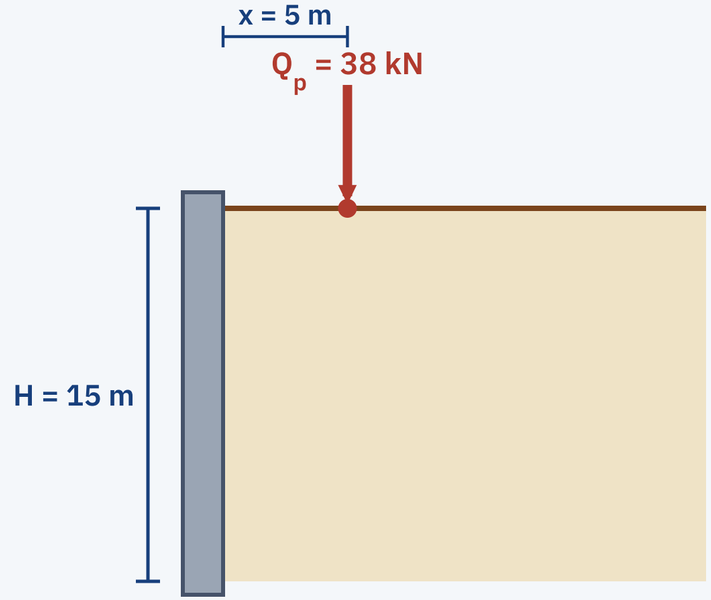
EXAMPLE

Given

Wall and load	$H = 15.00$ **m,** $Q_p = 38.0$ **kN,** $x = 5.00$ **m**
Method	**Terzaghi–NAVFAC chart; pressure at** $z = 2.50$ **m**

REQUIRED

Case selection, σ_H (z = 2.5 m), I_0, I_1, P_H and **L**.



Solution (1/3)

SOLUTION

1 Case selection

$$m = \frac{1}{3} = 0.333 \leq 0.4 \;\Rightarrow\; \text{Case 1: diagram } m = 0.4\text{constant at }'$$

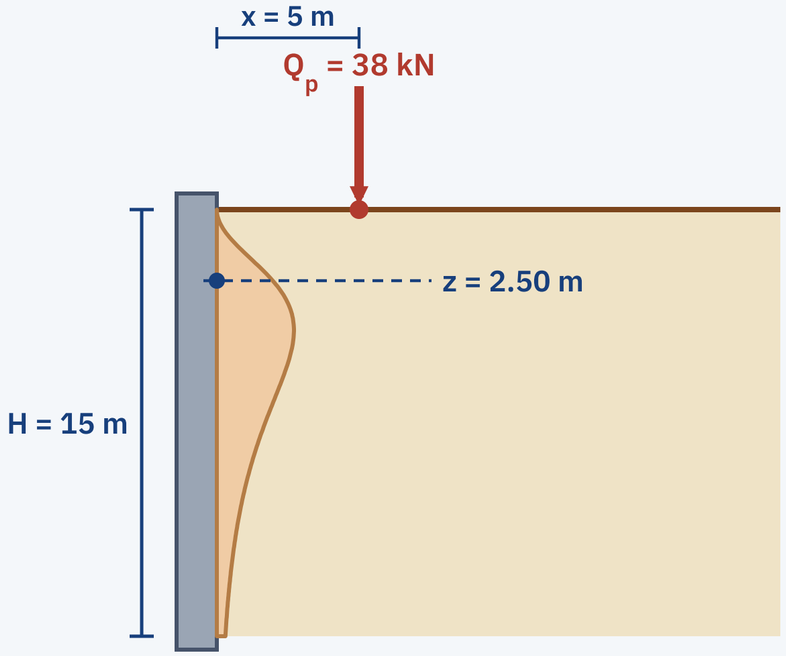
2 Depth ratio

$$n = \frac{z}{H} = \frac{2.50}{15} = \mathbf{0.1667}$$

3 Pressure

$$\sigma_H = 0.28\,\frac{38}{15^2}\cdot\frac{n^2}{(0.16+n^2)^3} = 0.04729 \times \frac{0.02778}{0.006621}$$

$$\sigma_H = 0.04729 \times 4.1953 = \mathbf{0.1984}\ \text{kPa}$$



At z = 2.5 m the chart pressure is **0.1984 kPa**; at the same depth the elastic solution gives 0.4154 kPa (peak).

Solution (2/3)

SOLUTION

4 Integral (closed form, a = 0.16)

$$I_0 = \int_0^1 0.28\,\frac{n^2}{(a+n^2)^3}\,dn = 0.28\left[\frac{1}{8a(1+a)} - \frac{1}{4(1+a)^2} + \frac{\tan^{-1}(1/\sqrt{a})}{8a^{3/2}}\right]$$

5 Numbers

$$\frac{1}{8(0.16)(1.16)} = 0.6735 \qquad \frac{1}{4(1.16)^2} = 0.1858 \qquad \frac{\tan^{-1} 2.5}{8(0.16)^{3/2}} = 2.3248$$

$$I_0 = 0.28\,[0.6735 - 0.1858 + 2.3248] = 0.28 \times 2.8125 = \mathbf{0.7875}$$

I_0 = **0.7875**: it agrees with the constant 0.787 Q_p/H that the chart gives for m = 0.4. The module computes this integral numerically (Simpson).

Solution (3/3)

SOLUTION

6 $I_1 = I_0 - \int n\,f\,dn$ (closed form, a = 0.16)

$$\int_0^1 nf\,dn = 0.28 \cdot \tfrac{1}{2}\left[\frac{1}{2a} - \frac{1}{1+a} + \frac{a}{2(1+a)^2}\right]$$

$$= 0.28 \times 1.1612 = 0.3251 \;\Rightarrow\; I_1 = 0.7875 - 0.3251 = \mathbf{0.4624}$$

7 Resultant and line of action

$$P_H = \frac{38}{15} \times 0.7875 = \mathbf{1.99}\ \mathrm{kN/m}$$

$$L = 15 \times \frac{0.4624}{0.7875} = \mathbf{8.81}\ \mathrm{m}$$

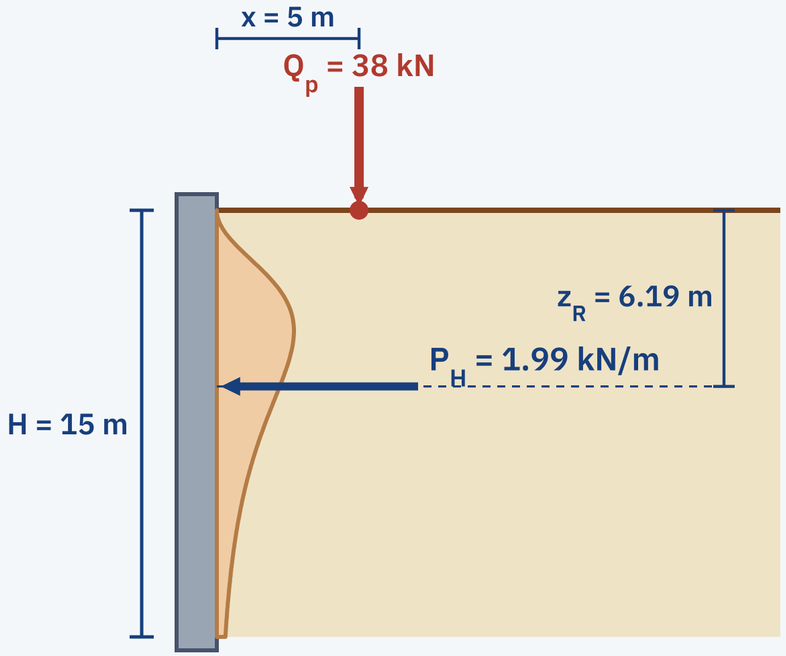


Chart: **P_H = 1.99 kN/m**, L = **8.81 m** (6.19 m from the surface). Compared with the elastic solution the resultant is 15% smaller and the line of action is 1.78 m lower.

Check the solution in GeoTerva ↗

Problem statement

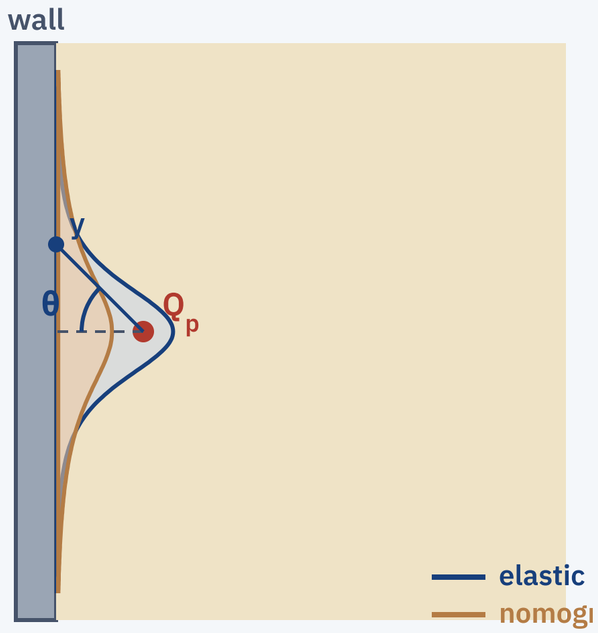
EXAMPLE

Given

Wall and load	$H = 15.00$ **m,** $Q_p = 38.0$ **kN,** $x = 5.00$ **m**
Depth	$z = 2.50$ **m**
Location	**at** $y = 5.00$ **m from the perpendicular through the load (**$\theta = 45°$**)**

REQUIRED

In both methods **σ′H** (y = 5 m); distribution for y = 0 – 15 m.



Solution (1/2)

SOLUTION

1 Elastic: attenuation with $(R_0/R)^5$

$$R_0^2 = x^2 + z^2 = 31.25 \qquad R^2 = x^2 + y^2 + z^2 = 56.25$$

$$\sigma'_H = \sigma_H \left(\frac{R_0}{R}\right)^5 = 0.4154 \times \left(\frac{31.25}{56.25}\right)^{5/2} = 0.4154 \times 0.2300 = \mathbf{0.0956}\ \text{kPa}$$

2 Chart: attenuation with $\cos^2(1.1\theta)$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{5}{5} = 45^\circ$$

$$\sigma'_H = \sigma_H \cos^2(1.1\theta) = 0.1984 \times \cos^2(49.5^\circ) = 0.1984 \times 0.4218 = \mathbf{0.0837}\ \text{kPa}$$

At y = 5 m the elastic pressure is 0.23 of the axis value (0.0956 kPa), while the chart gives 0.42 of it (0.0837 kPa): the chart decreases more slowly along the wall, but from a smaller initial value.

Solution (2/2)

SOLUTION

y (m)	θ (°)	σ'_H elastic (kPa)	ratio	σ'_H chart (kPa)	ratio	elastic peak z (m)	peak pressure (kPa)
0.0	0.0	0.4154	1.00	0.1984	1.00	2.50	0.4154
2.5	26.6	0.2634	0.63	0.1511	0.76	2.80	0.2659
5.0	45.0	0.0956	0.23	0.0837	0.42	3.54	0.1039
7.5	56.3	0.0317	0.08	0.0439	0.22	4.51	0.0393
10.0	63.4	0.0115	0.03	0.0237	0.12	5.59	0.0166
15.0	71.6	0.0022	0.01	0.0076	0.04	7.91	0.0042

The pressure is largest along the normal to the load and decays rapidly along the wall; in the elastic solution the depth of the peak decreases to $\sqrt{(x^2 + y^2)}/2$ as y increases. In the chart the reduction is held at zero after $\theta \approx 81.8°$.

Check the solution in GeoTerva ↗

Problem statement

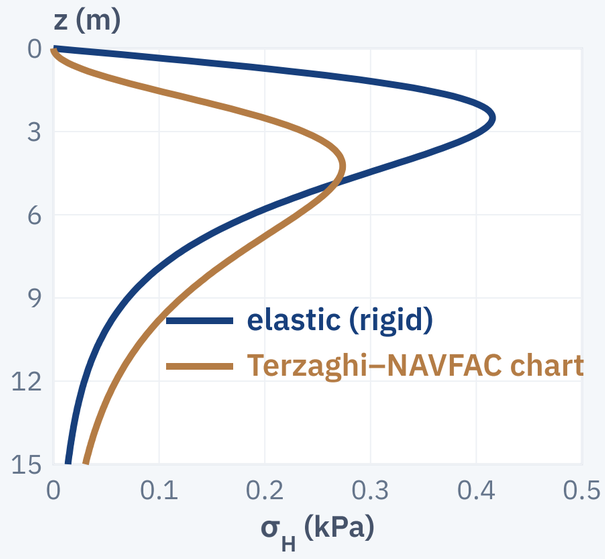
EXAMPLE

Given

Wall and load	$H = 15.00$ **m,** $Q_p = 38.0$ **kN,** $x = 5.00$ **m**
Depths	$z = 0$ **–** 15 **m, every** 1.5 **m**

REQUIRED

σ_H profile for the two methods; the **top** of the chart (Case 1) by hand.



Solution (1/2)

SOLUTION

z (m)	n = z/H	σ_H elastic (kPa)	σ_H chart (kPa)
0.00	0.00	0.0000	0.0000
1.50	0.10	0.3511	0.0963
3.00	0.20	0.4038	0.2364
4.50	0.30	0.2964	0.2724
6.00	0.40	0.1873	0.2309
7.50	0.50	0.1143	0.1715
9.00	0.60	0.0706	0.1211
10.50	0.70	0.0448	0.0844
12.00	0.80	0.0293	0.0591
13.50	0.90	0.0198	0.0420
15.00	1.00	0.0138	0.0303

The elastic pressure peaks at 0.4154 kPa at a depth near the surface (2.5 m) and decays rapidly; the chart gives a smoother profile, with its peak deeper and smaller.

Solution (2/2)

SOLUTION

1 Derivative zero: d/dn [n²/(0.16 + n²)³] = 0

$$2n(0.16+n^2)-6n^3=0 \;\Rightarrow\; n^2=\frac{0.16}{3}=0.0533$$

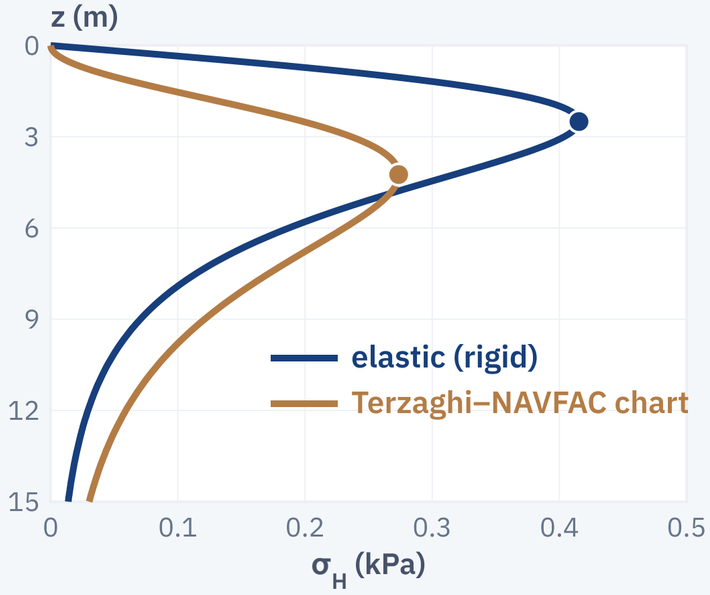
$$n=\mathbf{0.2828} \;\Rightarrow\; z=nH=\mathbf{4.243}\ \mathrm{m}$$

2 Peak pressure (n² = 0.08)

$$\sigma_{H,max}=0.28\,\frac{38}{15^2}\cdot\frac{0.08}{(0.16+0.08)^3}$$

$$\sigma_{H,max}=0.04729\times 5.787=\mathbf{0.2737}\ \mathrm{kPa}$$

Chart peak **0.2737 kPa** (z = 4.24 m); elastic peak **0.4154 kPa** (z = 2.50 m): the chart shifts the peak downward and reduces it to two-thirds.



Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Wall and load	$H = 15.00$ **m,** $Q_p = 38.0$ **kN**
Distances	$x = 0.3;\ 1;\ 2;\ 3;\ 4;\ 5;\ 6;\ 8;\ 10;\ 15$ **m**
Ek	$x < 0.30$ **m and the** $0.4H = 6$ **m limit of the chart**

REQUIRED

P_H and **L** for the two methods; hand check for **x = 10 m**; x_{min} and the limit.

Solution (1/3)

SOLUTION

x (m)	m	P_H elastic (kN/m)	L elastic (m)	P_H chart (kN/m)	L chart (m)	Condition
0.30	0.020	40.32	14.70	1.99	8.81	Case 1
1.00	0.067	12.09	14.01	1.99	8.81	Case 1
2.00	0.133	6.03	13.05	1.99	8.81	Case 1
3.00	0.200	4.00	12.15	1.99	8.81	Case 1
4.00	0.267	2.97	11.33	1.99	8.81	Case 1
5.00	**0.333**	**2.34**	**10.59**	**1.99**	**8.81**	**Case 1**
6.00	0.400	1.91	9.94	1.99	8.81	Case 1
8.00	0.533	1.35	8.87	1.38	7.61	Case 2
10.00	0.667	1.00	8.05	0.9755	6.74	Case 2
15.00	1.000	0.5213	6.80	0.4402	5.45	Case 2

x ≤ 6 m: the chart gives 1.99 kN/m and L = 8.81 m; the distance has no effect; the elastic P_H load increases rapidly as the load approaches (40.32 kN/m for x = 0.3 m). **x > 6 m**: the two methods approach each other, and the elastic resultant acts higher up.

Solution (2/3)

SOLUTION

1 m = 10/15 = 0.667 > 0.4: chart Case 2. Elastic: I_0, I_1, P_H, L

$$I_0 = \frac{1}{\pi}\left[\frac{1}{m} - \frac{m^2}{(1+m^2)^{3/2}}\right] = 0.3960 \qquad I_1 = I_0 - \frac{1}{\pi(1+m^2)^{3/2}} = 0.2126$$

$$P_H = \frac{38}{15} \times 0.3960 = \mathbf{1.00}\ \mathrm{kN/m} \qquad L = 15 \times \frac{0.2126}{0.3960} = \mathbf{8.05}\ \mathrm{m}$$

2 Chart (Case 2): same closed-form expressions as Example 2, a = m² and coefficient 1.77 m²

$$I_0 = \int_0^1 f\, dn = 0.3851 \qquad I_1 = \int_0^1 f\,(1-n)\, dn = 0.1730$$

$$P_H = \frac{38}{15} \times 0.3851 = \mathbf{0.9755}\ \mathrm{kN/m} \qquad L = 15 \times \frac{0.1730}{0.3851} = \mathbf{6.74}\ \mathrm{m}$$

x = 10 m: elastic **1.00 kN/m**, L = 8.05 m; chart **0.9755 kN/m**, L = 6.74 m (row in table).

Solution (3/3)

SOLUTION

x (m)	calculation x (m)	P_H elastic (kN/m)	L (m)	$\sigma_{H,max}$ elastic (kPa)	P_H chart (kN/m)
0.05	**0.30**	40.32	14.70	115.4	1.995
0.10	**0.30**	40.32	14.70	115.4	1.995
0.20	**0.30**	40.32	14.70	115.4	1.995
0.30	0.30	40.32	14.70	115.4	1.995
0.50	0.50	24.19	14.50	41.5	1.995

x < 0.30 m the load is computed at 0.30 m; even at this distance the peak is 115.4 kPa.

x (m)	m	Condition	P_H chart (kN/m)	L chart (m)
5.99	0.3993	Case 1	1.9950	8.8069
6.00	0.4000	Case 1	1.9950	8.8069
6.000001	0.400000	Case 2	2.0178	8.8069
6.01	0.4007	Case 2	2.0137	8.8000

The chart increases P_H by 1.14% when crossing the **m = 0.4** limit; L remains continuous.

Check the solution in GeoTerva ↗

Problem statement

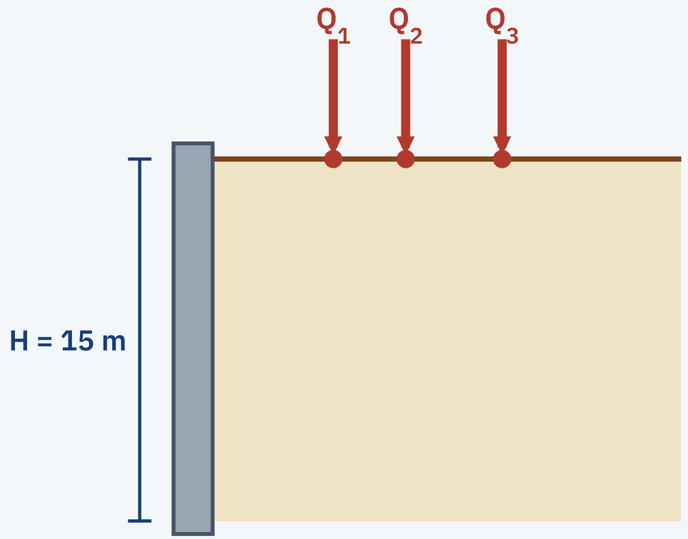
EXAMPLE

REQUIRED

The $P_{H,i}$ and L_i value of each load; the total P_H, L and the peak of the total pressure.

Given

Wall	$H = 15.00$ m
Loads	$Q_1 = 38$ kN ($x_1 = 5$ m), $Q_2 = 60$ kN ($x_2 = 8$ m), $Q_3 = 25$ kN ($x_3 = 12$ m); all on a single line perpendicular to the wall
Method	elastic (rigid wall); chart for comparison



Solution (1/3)

SOLUTION

Load	x (m)	Q_p (kN)	m	I_0	I_1	$P_{H,i}$ (kN/m)	L_i (m)
Q_1	5	38	0.3333	0.92473	0.65295	2.3427	10.592
Q_2	8	60	0.5333	0.53463	0.31597	2.1385	8.865
Q_3	12	25	0.8000	0.30089	0.14933	0.5015	7.444

1 Each load at its own m value (elastic)

$$P_{H,i} = \frac{Q_{p,i}}{H}\, I_{0,i} \qquad L_i = H\,\frac{I_{1,i}}{I_{0,i}} \qquad \text{e.g. } P_{H,2} = \frac{60}{15} \times 0.53463 = \mathbf{2.1385}\ \text{kN/m}$$

Each load acts at its own distance x_i; all act on the same section of the wall, so the resultants can be summed.

Solution (2/3)

SOLUTION

2 Total resultant (elastic)

$$P_H = \sum P_{H,i} = 2.3427 + 2.1385 + 0.5015 = \mathbf{4.9827}\ \mathrm{kN/m}$$

3 Influence line: equilibrium of moments about the base

$$L = \frac{\sum P_{H,i} L_i}{P_H} = \frac{2.343(10.592) + 2.139(8.865) + 0.501(7.444)}{4.983} = \mathbf{9.534}\ \mathrm{m}$$

Method	$P_{H,1}$	$P_{H,2}$	$P_{H,3}$	P_H total (kN/m)	L total (m)	from surface (m)
Elastic	2.3427	2.1385	0.5015	**4.9827**	**9.534**	5.466
Nomogram	1.9950	2.1774	0.4624	**4.6347**	**7.975**	7.025

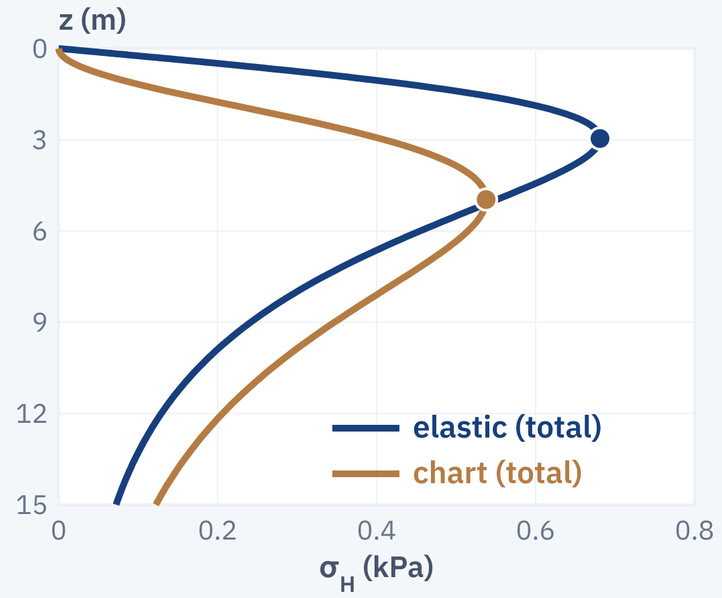
The resultant of the three loads is **4.98 kN/m** elastically and **4.63 kN/m** from the chart; the line of action is 9.53 m and 7.97 m above the base.

Solution (3/3)

SOLUTION

z (m)	$\sigma_{H,1}$ (kPa)	$\sigma_{H,2}$ (kPa)	$\sigma_{H,3}$ (kPa)	total σ_H (kPa)
0.0	0.0000	0.0000	0.0000	**0.0000**
1.5	0.3511	0.1540	0.0199	**0.5250**
3.0	0.4038	0.2416	0.0356	**0.6810**
4.5	0.2964	0.2533	0.0447	**0.5944**
6.0	0.1873	0.2200	0.0475	**0.4548**
9.0	0.0706	0.1304	0.0407	**0.2417**
12.0	0.0293	0.0705	0.0293	**0.1291**
15.0	0.0138	0.0387	0.0197	**0.0722**

The pressures are summed (stress increase): the elastic total peak is **0.6811 kPa**, at z = 2.96 m; on the chart 0.5377 kPa, at z = 4.97 m. The peak occurs at the depth where the share of the largest load (Q_2) becomes dominant.



Check the solution in GeoTerva ↗

Summary

Key points

m = x/H, **n = z/H**; elastic (rigid wall): **$\sigma_H = (3Q_p/\pi)\, x^2z/R^5$**, peak at z = x/2, $\sigma_{H,max} = 0.2733\ Q_p/x^2$; **$x_{min} = 0.30$ m**

Chart: Case 1 ($m \le 0.4$): $\sigma_H = 0.28\ (Q_p/H^2)\ n^2/(0.16 + n^2)^3$; Case 2: $1.77\ (Q_p/H^2)\ m^2n^2/(m^2 + n^2)^3$; within 0.4H the distance has no effect

Example (H = 15 m, x = 5 m, Q_p = 38 kN): elastic peak 0.4154 kPa (z = 2.5 m), $I_0 = 0.9247$, $I_1 = 0.6530$, **$P_H = 2.34$ kN/m**, L = 10.59 m; chart $P_H = 1.99$ kN/m, L = 8.81 m

In plan: elastic $(R_0/R)^5$, chart $\cos^2(1.1\theta)$ (up to $\theta \approx 81.8°$); **multiple loads** (at most 3): the pressures and $P_{H,i}$ are summed, L from moment equilibrium

Comparison: the chart is constant for $m \le 0.4$; the elastic P_H increases as the load approaches; P_H is at the section of the load and is not multiplied by the wall length

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NOTE

All numerical examples have been produced with the module's own calculation engine; the module gives the same values. The module's opening wall is H = 15 m, x = 5 m and Q_p = 38 kN (elastic method).

GeoTerva

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