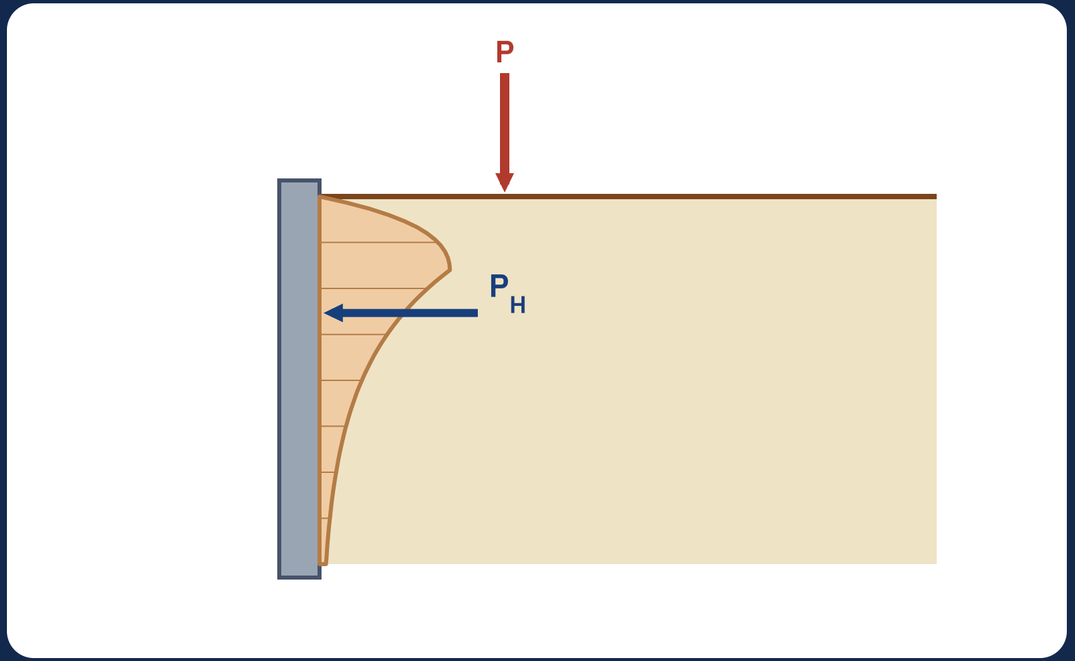
Geotechnical lecture presentation · Surcharge loads

Line surcharge load

Lecture: the lateral pressure exerted on a wall by a line load parallel to the wall; two methods, the resultant and its line of action, worked examples.

m = x/H · n = z/H | Elastic rigid wall | Terzaghi–NAVFAC chart | $I_0 \cdot I_1$ | P_H and z_R



What will we learn in this lecture?

- The **dimensionless** geometry of the line load: load offset **m = x/H** and depth **n = z/H**
- **Elastic (rigid wall)** solution: twice the half-space stress; the x_{min} **= 0.30 m** rule
- The **Terzaghi–NAVFAC chart**: the cases m > 0.4 and m ≤ 0.4; the ineffectiveness of distance within 0.4H
- The resultant with the closed-form expressions $P_H = P \cdot I_0$ and the line of action $z_R = H \cdot I_1 / I_0$
- The comparison of the two methods with respect to distance, wall height and load

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example wall: H = 25 m, line load P = 100 kN/m at a distance x = 5 m from the wall. Forces are per unit length of wall (kN/m).

Line load acting on the wall

THEORY

A surface line load **P** parallel to the wall, given as force per unit length, is at a horizontal distance **x** from the wall

The lateral surcharge pressure depends on the dimensionless load distance **m = x/H** and the dimensionless wall depth **n = z/H**

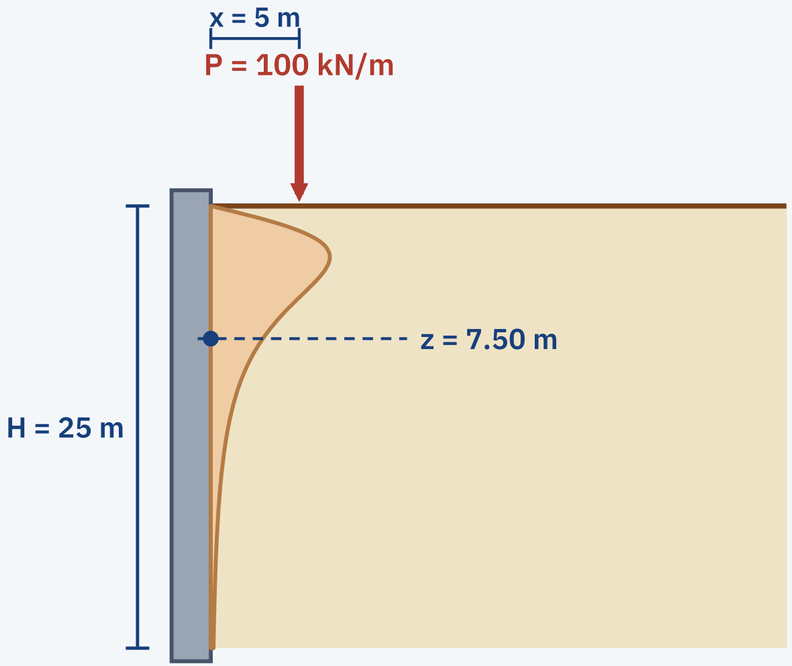
The module offers two methods: the **elastic (rigid wall)** solution and the **Terzaghi–NAVFAC chart**

$$m = \frac{x}{H}, \qquad n = \frac{z}{H}$$

P — surface line load per unit length parallel to the wall

x = mH — horizontal distance from the wall to the load; **z = nH** — depth from the surface

σ_H, P_H, z_R — lateral pressure, resultant force, depth of the resultant from the surface



Rigid wall: twice the elastic stress

THEORY

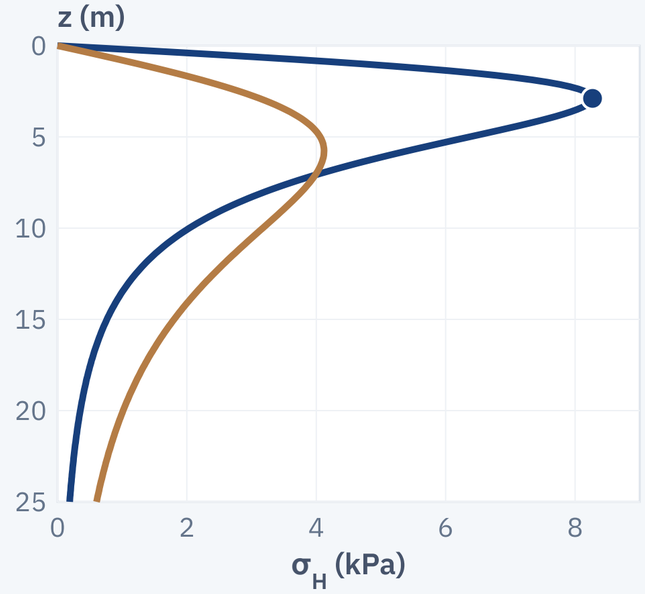
It is **twice** the horizontal stress produced by a line load in an elastic half-space (Flamant, Boussinesq): a wall that does not yield prevents the movement of the soil at its face

It is valid for every distance; the maximum pressure occurs at depth **n = m/√3**

A load closer than $\mathbf{x_{min}}$ **= 0.30 m** is computed at this distance: the maximum pressure increases without bound as the load approaches the wall (the resultant remains finite)

$$\sigma_H = \frac{4P}{\pi H} \cdot \frac{m^2 n}{(m^2 + n^2)^2}$$

Elastic profile (m = 0.2)



Case 1 (m ≤ 0.4) and Case 2 (m > 0.4)

THEORY

For **m > 0.4**, the **same equation** as the elastic solution; for **m ≤ 0.4** the diagram is **held constant** at m = 0.4 (Case 1, which contains no m)

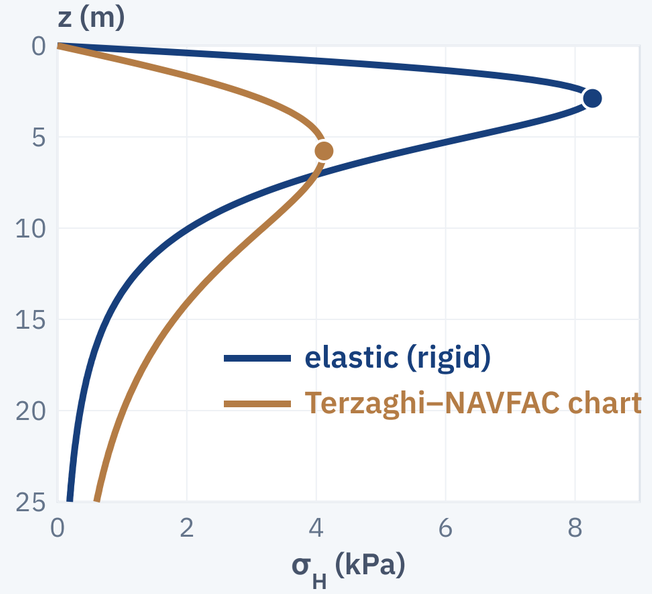
The coefficient 0.203 is the rounded value of 4 × 0.16/π; therefore **within 0.4H the distance has no effect**

m is constant for a given model; n varies with depth

$$\sigma_H = \frac{P}{H} \cdot \frac{0.203\, n}{(0.16 + n^2)^2} \qquad m \le 0.4 \text{ (Case 1)}$$

$$\sigma_H = \frac{4P}{\pi H} \cdot \frac{m^2\, n}{(m^2 + n^2)^2} \qquad m > 0.4 \text{ (Case 2)}$$

Profile of the two methods (m = 0.2)



Area, first moment and I_0, I_1

THEORY

In both methods the **area of the resultant diagram** is the **point at which the first moment balances** from the surface; once σ_H is written as a function of n, the load and the height come outside the integral

Elastic and closed-form expressions for **m > 0.4** in the chart; the chart resultant ≈ 0.64P/(1 + m²). **m ≤ 0.4** chart: I_0 = 0.5469 (the chart's 0.55P value)

$$P_H = \int_0^H \sigma_H \, dz = P \, I_0 \qquad z_R = \frac{\int_0^H z \, \sigma_H \, dz}{P_H} = H \, \frac{I_1}{I_0}$$

$$I_0 = \frac{2/\pi}{1 + m^2} \qquad I_1 = \frac{2}{\pi} \left[m \tan^{-1} \frac{1}{m} - \frac{m^2}{1 + m^2} \right]$$

$$I_0 = 0.1015 \left(\tfrac{1}{0.16} - \tfrac{1}{1.16} \right) = 0.5469 \qquad I_1 = 0.203 \left[\frac{\tan^{-1} 2.5}{0.8} - \frac{1}{2.32} \right] \; (m \le 0.4)$$

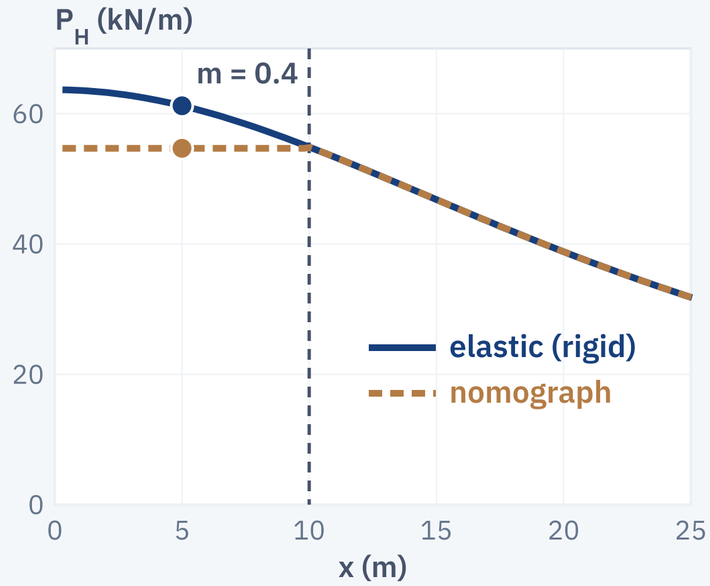
Resultant and line of action, with load distance

THEORY

x (m)	m	P_H elastic	z_R elastic	P_H chart	z_R chart	Condition
0.50	0.020	63.64	0.77	54.69	9.81	Case 1
1.00	0.040	63.56	1.49	54.69	9.81	Case 1
2.00	0.080	63.26	2.84	54.69	9.81	Case 1
5.00	0.200	61.21	6.14	54.69	9.81	Case 1
8.00	0.320	57.75	8.56	54.69	9.81	Case 1
10.00	0.400	54.88	9.81	54.69	9.81	Case 1
12.50	0.500	50.93	11.05	50.93	11.05	Case 2
15.00	0.600	46.81	12.02	46.81	12.02	Case 2
25.00	1.000	31.83	14.27	31.83	14.27	Case 2

m ≤ 0.4: the chart gives the same P_H and z_R; the elastic solution increases P_H as the load approaches and pulls z_R upward

m > 0.4: both methods give the same result



Problem statement

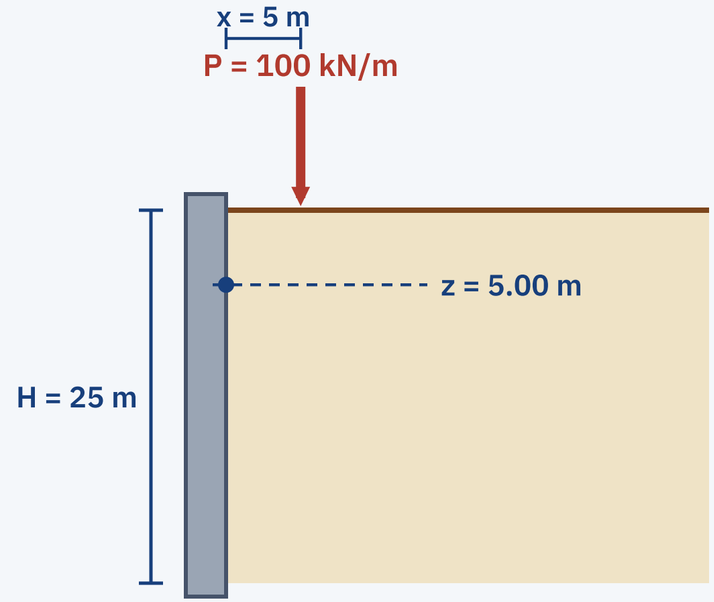
EXAMPLE

Given

Wall and load	$H = 25.00$ **m; line load** $P = 100.0$ **kN/m,** $x = 5.00$ **m**
Depth	$z = 5.00$ **m**

REQUIRED

m, **n** and σ_H: first elastic (rigid wall), then the Terzaghi–NAVFAC chart.



Solution (1/2)

SOLUTION

1 Ratios

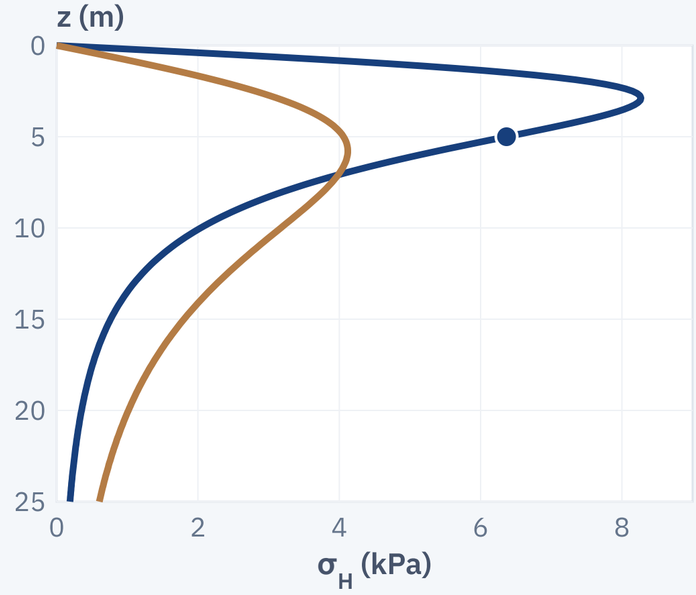
$$m = \frac{x}{H} = \frac{5}{25} = \mathbf{0.20} \qquad n = \frac{z}{H} = \frac{5}{25} = \mathbf{0.20}$$

2 Elastic (rigid wall) pressure

$$\sigma_H = \frac{4 \times 100}{\pi \times 25}\left[\frac{0.20^2 \times 0.20}{(0.20^2 + 0.20^2)^2}\right]$$

$$\sigma_H = 5.0930 \times 1.25 = \mathbf{6.37}\ \text{kPa}$$

At z = 5 m the elastic solution gives **6.37 kPa**. This depth is below the maximum pressure (8.27 kPa, z = 2.89 m); the profile decays rapidly with depth.



Solution (2/2)

SOLUTION

3 Case selection

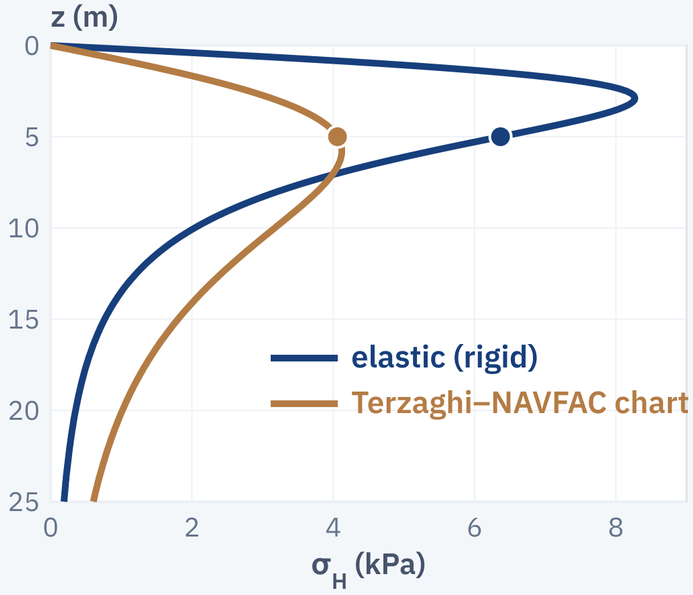
$$m = 0.20 \leq 0.4 \;\Rightarrow\; \text{Case 1 (diagram } m = 0.4 \text{(fixed at)}$$

4 Chart pressure

$$\sigma_H = \frac{100}{25}\left[\frac{0.203 \times 0.20}{(0.16 + 0.20^2)^2}\right]$$

$$\sigma_H = 4.00 \times 1.015 = \mathbf{4.06}\ \text{kPa}$$

The chart gives **4.06 kPa**, the elastic solution **6.37 kPa**: at this depth the chart value is 64% of the elastic value. The chart spreads the pressure with depth for a nearby load.



Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Wall and load	$H = 25.00$ **m,** $P = 100.0$ **kN/m,** $x = 5.00$ **m** $(m = 0.20)$
Depths	$z = 0$ **–** 25 **m, every** 2.5 **m**

REQUIRED

σ_H profile (two methods); **maximum pressure** and its depth.

Solution (1/2)

SOLUTION

z (m)	n = z/H	σ_H elastic (kPa)	σ_H chart (kPa)
0.00	0.00	0.000	0.000
2.50	0.10	8.149	2.810
5.00	**0.20**	**6.366**	**4.060**
7.50	0.30	3.616	3.898
10.00	0.40	2.037	3.172
12.50	0.50	1.211	2.415
15.00	0.60	0.764	1.802
17.50	0.70	0.508	1.345
20.00	0.80	0.352	1.015
22.50	0.90	0.254	0.777
25.00	1.00	0.188	0.603

The elastic pressure reaches its maximum just below the surface and decreases with depth; the chart gives a smoother profile with a peak at greater depth. The row z = 5.00 m is Example 1.

Solution (2/2)

SOLUTION

1 Elastic: peak n = m/√3

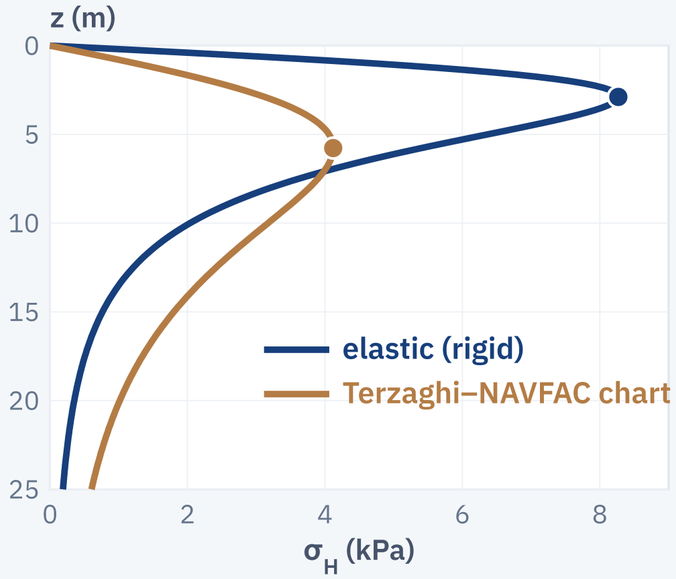
$$n = \frac{m}{\sqrt{3}} = \frac{0.20}{1.7321} = 0.11547 \;\Rightarrow\; z = nH = \mathbf{2.887}\ \mathrm{m}$$

$$\sigma_{H,max} = \frac{9P}{4\sqrt{3}\,\pi\,x} = \frac{9 \times 100}{4(1.7321)\pi(5)} = \mathbf{8.270}\ \mathrm{kPa}$$

2 Chart (Case 1): peak n = 0.4/√3

$$n = \frac{0.4}{\sqrt{3}} = 0.23094 \;\Rightarrow\; z = \mathbf{5.774}\ \mathrm{m}$$

$$\sigma_{H,max} = 4\left[\frac{0.203 \times 0.23094}{(0.16 + 0.05333)^2}\right] = \mathbf{4.120}\ \mathrm{kPa}$$



Elastic peak **8.27 kPa** (z = 2.89 m), chart peak **4.12 kPa** (z = 5.77 m): the elastic solution concentrates toward the upper part of the wall, while the chart gives a peak lower down and half as large.

Check the solution in GeoTerva ↗

Problem statement

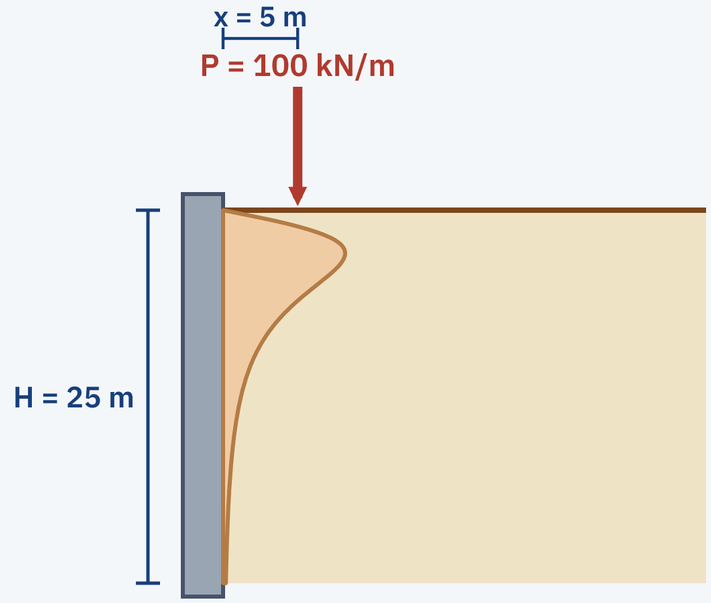
EXAMPLE

Given

Wall and load	$H = 25.00$ **m**, $P = 100.0$ **kN/m**, $x = 5.00$ **m**
Methods	**elastic (rigid wall) and Terzaghi–NAVFAC chart**

REQUIRED

$\mathbf{I_0}$, $\mathbf{I_1}$, resultant $\mathbf{P_H}$, $\mathbf{z_R}$ and its height above the base (for the two methods).



Solution (1/2)

SOLUTION

1 I_0 (area)

$$I_0 = \frac{2/\pi}{1+m^2} = \frac{0.63662}{1+0.20^2} = \mathbf{0.61213}$$

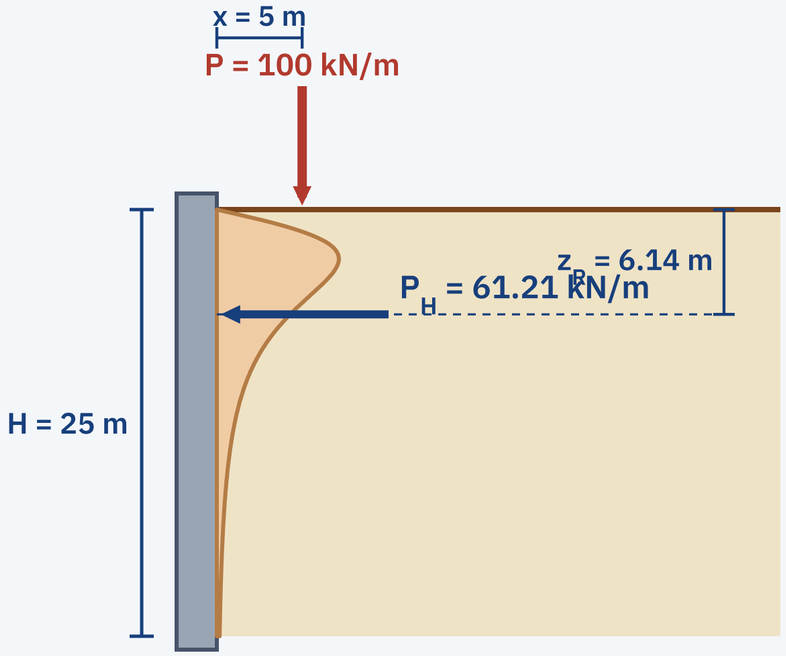
2 I_1 (moment): $(2/\pi)[m \tan^{-1}(1/m) - m^2/(1+m^2)]$

$$I_1 = 0.63662\,[0.20(1.37340) - 0.03846] = \mathbf{0.15038}$$

3 Resultant and line of action

$$P_H = P\,I_0 = 100 \times 0.61213 = \mathbf{61.21}\ \mathrm{kN/m}$$

$$z_R = H\frac{I_1}{I_0} = 25 \times \frac{0.15038}{0.61213} = \mathbf{6.14}\ \mathrm{m}$$



Elastic: **P_H = 61.21 kN/m**, at a depth of **6.14 m** below the surface (18.86 m above the base).

Solution (2/2)

SOLUTION

4 I_0 and I_1 (m ≤ 0.4, Case 1)

$$I_0 = 0.1015\left(\frac{1}{0.16} - \frac{1}{1.16}\right) = \mathbf{0.5469}$$

$$I_1 = 0.203\left[\frac{\tan^{-1} 2.5}{0.8} - \frac{1}{2.32}\right]$$

$$I_1 = 0.203\,[1.48786 - 0.43103] = \mathbf{0.21454}$$

5 Resultant and line of action

$$P_H = 100 \times 0.5469 = \mathbf{54.69}\ \mathrm{kN/m}$$

$$z_R = 25 \times \frac{0.21454}{0.5469} = \mathbf{9.81}\ \mathrm{m}$$

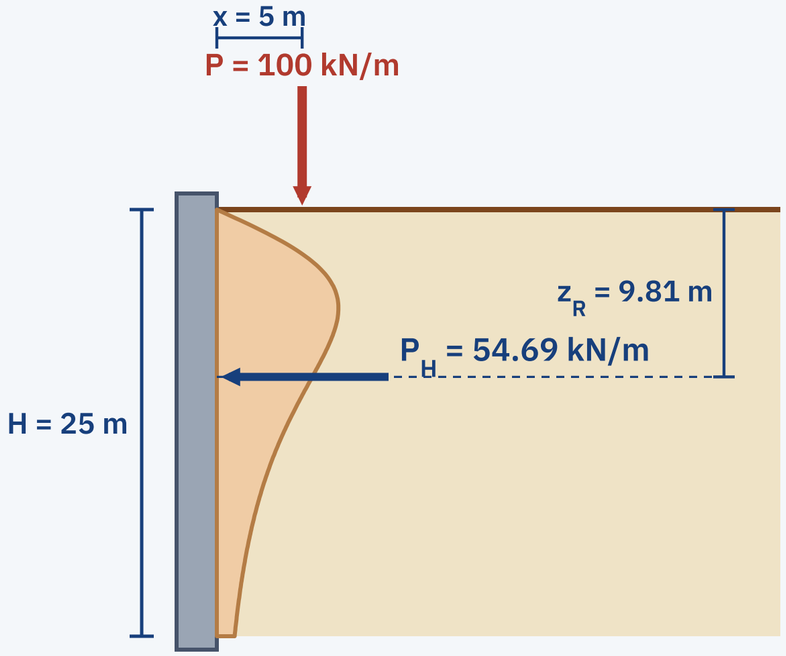


Chart: **P_H = 54.69 kN/m**, **z_R = 9.81 m**. Compared with the elastic solution, the resultant is 10.7% smaller and the line of action is 3.67 m lower.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Wall and load	$H = 25.00$ **m,** $P = 100.0$ **kN/m**
Distances	$x = 0.5;\ 1;\ 2;\ 5;\ 8;\ 10;\ 12.5;\ 15;\ 25$ **m**

REQUIRED

P_H and z_R for the two methods; hand check for **x = 12.5 m**.

Solution (1/2)

SOLUTION

x (m)	m	P_H elastic (kN/m)	z_R elastic (m)	P_H chart (kN/m)	z_R chart (m)	Condition
0.50	0.020	63.64	0.77	54.69	9.81	Case 1
1.00	0.040	63.56	1.49	54.69	9.81	Case 1
2.00	0.080	63.26	2.84	54.69	9.81	Case 1
5.00	**0.200**	**61.21**	**6.14**	**54.69**	**9.81**	**Case 1**
8.00	0.320	57.75	8.56	54.69	9.81	Case 1
10.00	0.400	54.88	9.81	54.69	9.81	Case 1
12.50	0.500	50.93	11.05	50.93	11.05	Case 2
15.00	0.600	46.81	12.02	46.81	12.02	Case 2
25.00	1.000	31.83	14.27	31.83	14.27	Case 2

x ≤ 10 m (m ≤ 0.4): the chart gives 54.69 kN/m and 9.81 m, the distance has no effect; the elastic resultant load decreases slowly as it moves away (63.64 → 54.88 kN/m) and its line of action deepens. **m > 0.4**: the two methods are the same.

Solution (2/2)

SOLUTION

1 Ratio and condition

$$m = \frac{12.5}{25} = \mathbf{0.50} > 0.4 \;\Rightarrow\; \text{chart Case 2: same equation as the elastic solution}$$

2 I_0 and I_1

$$I_0 = \frac{2/\pi}{1 + 0.50^2} = \frac{0.63662}{1.25} = \mathbf{0.50930}$$

$$I_1 = 0.63662\,[0.50(1.10715) - 0.2] = \mathbf{0.22509}$$

3 Resultant and line of action

$$P_H = 100 \times 0.50930 = \mathbf{50.93}\ \text{kN/m} \qquad z_R = 25 \times \frac{0.22509}{0.50930} = \mathbf{11.05}\ \text{m}$$

For x = 12.5 m both methods give **50.93 kN/m** and **11.05 m**.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Wall and load	$H = 25.00$ **m,** $P = 100.0$ **kN/m**
Adjacent load	$x = 0.05;\ 0.1;\ 0.2;\ 0.3;\ 0.5;\ 1$ **m**
Limit	**chart:** $x = 9.99;\ 10;\ 10.01$ **m (**$0.4H = 10$ **m)**

REQUIRED

The effect of the $\mathbf{x_{min}}$ rule; the elastic maximum pressure; the chart's behaviour at the m = 0.4 limit.

Solution (1/2)

SOLUTION

x (m)	calculation x (m)	P_H elastic (kN/m)	z_R elastic (m)	$\sigma_{H,max}$ elastic (kPa)	z (m)	P_H chart (kN/m)
0.05	**0.30**	63.65	0.46	137.8	0.17	54.69
0.10	**0.30**	63.65	0.46	137.8	0.17	54.69
0.20	**0.30**	63.65	0.46	137.8	0.17	54.69
0.30	0.30	63.65	0.46	137.8	0.17	54.69
0.50	0.50	63.64	0.77	82.7	0.29	54.69
1.00	1.00	63.56	1.49	41.3	0.58	54.69

A load with **x < 0.30 m** is computed at 0.30 m: the resultant, the line of action and the maximum pressure are the same as the values at 0.30 m; the elastic peak rises to **137.8 kPa** (z = 0.17 m) at this distance. The chart gives 54.69 kN/m at every distance.

Solution (2/2)

SOLUTION

1 0.203 and 4 × 0.16/π

$$\frac{4 \times 0.16}{\pi} = 0.2037 \approx 0.203 \Rightarrow \frac{I_0(m = 0.4^+)}{I_0(\text{Case 1})} = \frac{0.63662/1.16}{0.5469} = \mathbf{1.0033}$$

2 Nomogram resultant on both sides of the limit

x (m)	m	Condition	P_H chart (kN/m)	z_R chart (m)
9.99	0.3996	Case 1	54.6875	9.8074
10.00	0.4000	Case 1	54.6875	9.8074
10.01	0.4004	Case 2	54.8659	9.8130

Due to rounding, the resultant jumps by about **0.33%** as it crosses the m = 0.4 limit (54.69 → 54.87 kN/m); the line of action goes from 9.807 → 9.813 m. The jump is small; the published coefficient of the nomograph is retained.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Load	$P = 100.0$ **kN/m,** $x = 5.00$ **m**
Height	$H = 5,\ 10,\ 25,\ 50,\ 100$ **m**
Load intensity	$P = 50,\ 100,\ 200$ **kN/m** ($H = 25$ **m**)

REQUIRED

$\mathbf{P_H}$ and $\mathbf{z_R}$ for both methods; the effect of P and H.

Solution

SOLUTION

H (m)	m	P_H elastic	z_R elastic	P_H chart	z_R chart	Condition
5	1.000	31.83	2.85	31.83	2.85	Case 2
10	0.500	50.93	4.42	50.93	4.42	Case 2
25	0.200	61.21	6.14	54.69	9.81	Case 1
50	0.100	63.03	6.93	54.69	19.61	Case 1
100	0.050	63.50	7.37	54.69	39.23	Case 1

For a high wall m decreases: the elastic P_H approaches 0.64P; the chart's P_H does not change (0.5469P), while z_R grows with H.

P (kN/m)	P_H elastic	P_H chart	z_R elastic	z_R chart
50	30.61	27.34	6.14	9.81
100	61.21	54.69	6.14	9.81
200	122.43	109.38	6.14	9.81

The resultant increases **linearly** with **P**; z_R is independent of P (elastic · chart).

Check the solution in GeoTerva ↗

Summary

Key points

m = x/H, **n = z/H**; elastic (rigid wall): $\boldsymbol{\sigma_H = (4P/\pi H)\, m^2 n/(m^2 + n^2)^2}$, peak at n = m/√3; $\boldsymbol{x_{min} = 0.30\ m}$

Nomograph: for m > 0.4 the same equation; for **m ≤ 0.4**, $\sigma_H = (P/H)\ 0.203\ n/(0.16 + n^2)^2$, distance has no effect

Example (H = 25 m, x = 5 m, P = 100 kN/m): at z = 5 m, elastic 6.37, chart 4.06 kPa; maximum pressure 8.27 kPa (z = 2.89 m) and 4.12 kPa (z = 5.77 m)

Resultant $P_H = P \cdot I_0$, $\boldsymbol{z_R = H \cdot I_1/I_0}$: elastic 61.21 kN/m and 6.14 m; chart 54.69 kN/m and 9.81 m

Comparison: beyond 0.4H the two methods agree; inside it, the elastic resultant load increases as the load approaches, while the chart value is constant

References

Das, B.M. (2016). Principles of Foundation Engineering, 8th ed. Cengage Learning; this page follows it.

Terzaghi, K. (1954). Anchored bulkheads. Transactions of the ASCE, 119, 1243–1280.

NAVFAC DM-7.02 (1986). Foundations and Earth Structures. Naval Facilities Engineering Command.

NOTE

All numerical examples are produced with the module's own calculation engine; the module gives the same values. The module's opening wall is H = 25 m, x = 5 m and P = 100 kN/m (elastic method).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own wall, the intensity and distance of your line load in GeoTerva; the module gives the pressure profile, the resultant and the line of action together with a step-by-step solution and report using two methods.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com