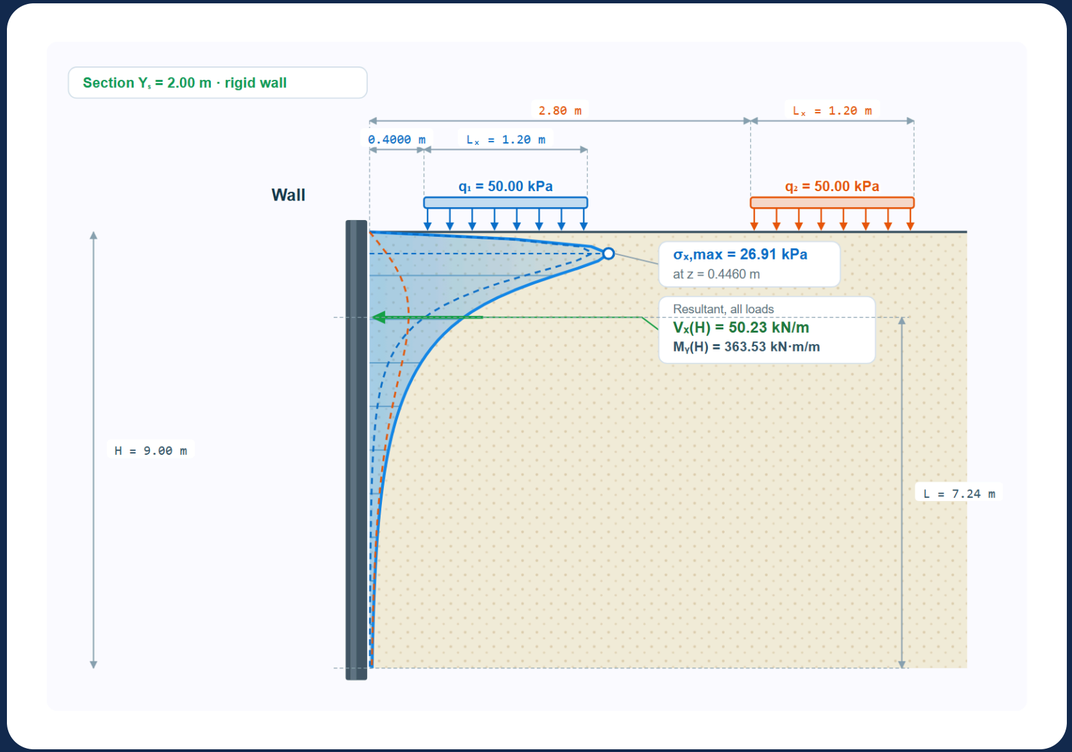
Geotechnical lecture presentation · Surcharge loads

Area surcharge loads

Lecture: rectangular surface loads near a wall; the opening example is solved by hand at the depth of maximum pressure, and the remaining topics are covered with tables.

- Boussinesq σx
- Rectangular integration
- Rigid wall ψ = 2
- Shear force and moment
- Can be solved by hand



What will we learn in this lecture?

The lateral pressure on the wall from a rectangular surface load: the load is the sum of **dP = q·dA** point loads, and the effect of each is found from the Boussinesq solution

The quantities x_1, x_2, y_1, y_2 that give the position and size in plan, and integration that is **exact along the wall and numerical in the direction perpendicular to the wall**

The difference between a **rigid wall** (ψ = 2) and a **semi-infinite medium** (ψ = 1)

Determining the **shear force V_x** and **bending moment M_y** values from the pressure profile

The effect of the section position, the distance of the load from the wall and the **Poisson ratio**

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example: H = 9 m, μ = 0.5, section Y_s = 2 m; two rectangles of 1.2 × 3.6 m, q = 50 kPa, X_0 = 1.0 and 3.4 m; rigid wall.

Rectangular loads acting on the wall

THEORY

A rectangular surface surcharge is a continuous field of vertical point loads **dP = q·dA**

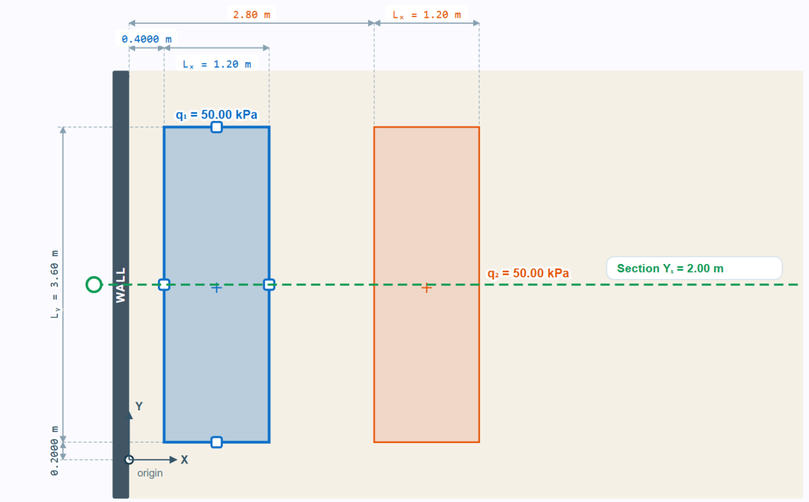
The **horizontal stress** produced by each individual load at the selected vertical section of the wall is found from the Boussinesq solution, and the contributions of each rectangle are summed

By integrating the pressure profile along the wall, the **shear force** and **bending moment** are obtained

$$A = L_x L_y \qquad P = qA$$

$$x_1 = X_0 - \frac{L_x}{2} \quad x_2 = X_0 + \frac{L_x}{2}$$

$$y_1 = Y_0 - \frac{L_y}{2} - Y_s \quad y_2 = Y_0 + \frac{L_y}{2} - Y_s$$



X_0, Y_0: coordinates of the centroid perpendicular to and along the wall

L_x, L_y: sides; **Y_s**: coordinate of the section

The rectangle is behind the wall face: $x_1 \geq 0$

Boussinesq σ_x on the wall plane

THEORY

For an element dP located at a point of the section at depth z and at plan position **(x, y)**, the **horizontal normal stress** perpendicular to the wall is given below

$r^2 = x^2 + y^2$ and $R^2 = r^2 + z^2$; $dP = q \cdot dx \cdot dy$

The horizontal stress depends on **Poisson's ratio μ**; for μ = 0.5 the term with the factor (1 – 2μ) is zero

$$\sigma_x = \frac{dP}{2\pi}\left[\frac{3x^2 z}{R^5} - (1-2\mu)\left(\frac{x^2-y^2}{r^2 R(R+z)} + \frac{y^2 z}{r^2 R^3}\right)\right]$$

$$r^2 = x^2 + y^2 \qquad R^2 = r^2 + z^2 \qquad dP = q\,dx\,dy$$

Closed-form solution along the wall: G

THEORY

Replacing the rectangle with a finite mesh of point loads **converges slowly near the wall**, where the stress varies most rapidly

Along the wall (y direction) the integral has a **closed form**; the remaining integral in the direction perpendicular to the wall (x) is computed **adaptively** with a tight tolerance

$$G(x,y,z) = \frac{1}{2\pi}\left[\frac{x^2 z\, y\,(2y^2+3a^2)}{a^4 R^3} - (1-2\mu)\frac{y\,(1-z/R)}{r^2}\right] \qquad a^2 = x^2+z^2$$

$$\sigma_x(z) = \psi \sum_{\text{loads}} q \int_{x_1}^{x_2} \left[G(x,y_2,z) - G(x,y_1,z)\right] dx$$

A very long rectangle, whatever μ may be, gives the **plane strain strip load** solution; the engine is tested with this check

Wall effect: ψ

THEORY

A smooth wall that does not move is represented by **reflecting** every load with respect to the wall face: the reflection sets the displacement normal to the wall and the shear stress on that plane to zero

Therefore the pressure acting on the wall is, regardless of μ, **exactly twice** the semi-infinite medium stress

A wall that deflects enough to loosen the soil is closer to the semi-infinite medium value

$$\sigma_{wall} = \psi\,\sigma_x \qquad \psi = 2\ (\text{rigid wall, default}) \qquad \psi = 1\ (\text{semi-infinite medium})$$

Integration of the profile along the wall

THEORY

The pressure profile is integrated along the wall **per unit length** of the section in the cross-section, once for shear force and twice for bending moment

Both are integrated **adaptively** between the rows of the depth table; the number of rows n_z only determines how much of the profile is listed

$$V_x(z) = \int_0^z \sigma_x \, dt$$

$$M_y(z) = \int_0^z V_x \, dt = \int_0^z (z - t) \, \sigma_x \, dt$$

Six steps and an opening example

THEORY

1 · Wall and section: H, μ and Y_s are entered

2 · Each rectangle: q, X_0, Y_0, L_x, L_y; compute x_1, x_2, y_1, y_2

3 · Point load stress: r, R and σ_x for dP = q·dx·dy

4 · Integration of the area: exact with G in the y direction, numerical in the x direction

5 · Superposition and ψ: each rectangle is summed at depth z and multiplied by two for a rigid wall

6 · Integration along the wall: $\sigma_x(z)$, then $V_x(z)$ and $M_y(z)$

$$A = 2 \times 1.2 \times 3.6 = 8.64\ \mathrm{m}^2 \qquad P = 432\ \mathrm{kN} \qquad \sigma_x = 24.98 + 1.93 = 26.91\ \mathrm{kPa}$$

$$V_x(H) = 50.23\ \mathrm{kN/m} \qquad M_y(H) = 363.53\ \mathrm{kN \cdot m/m}$$

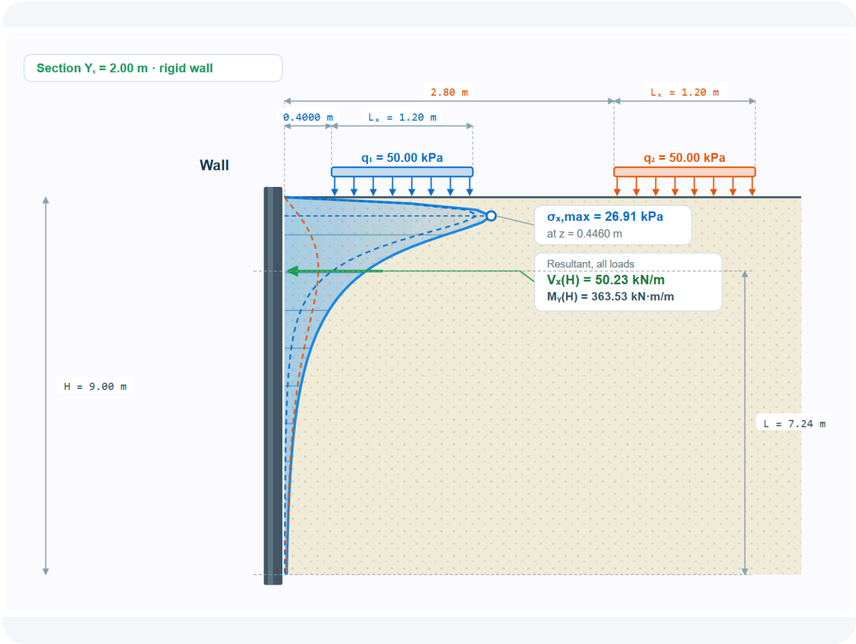
Location, distance and Poisson's ratio

THEORY

The method preserves the **three-dimensional position and size** of each load: moving a rectangle along the wall or away from the wall, changing its size, shifting the section or changing μ alters the pressure profile

The pressure reaches its maximum value **in sections passing in front of** a load and decreases rapidly with distance in plan; when loads are short relative to their distance from the wall, **several sections** are checked

When μ is below 0.5, **tension ordinates** may appear near the surface; these are given with their signs



Problem statement

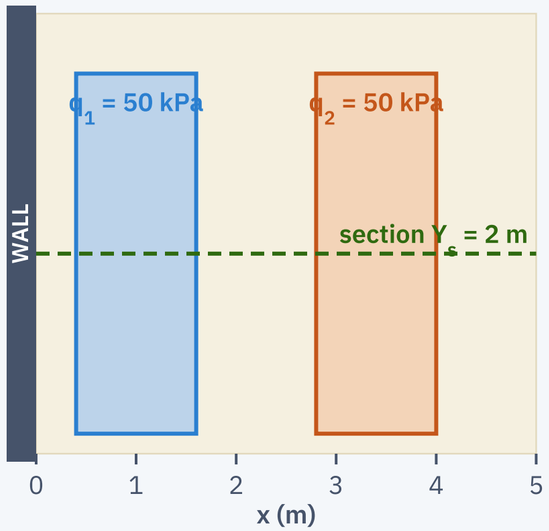
EXAMPLE

Given

Wall and section	$H = 9.0$ **m;** $\mu = 0.5$**; section** $Y_s = 2.0$ **m; rigid wall (**$\psi = 2$**)**
Rectangle 1	$q = 50$ **kPa;** $L_x = 1.2$ **m,** $L_y = 3.6$ **m;** $X_0 = 1.0$ **m,** $Y_0 = 2.0$ **m**
Rectangle 2	$q = 50$ **kPa;** 1.2×3.6 **m;** $X_0 = 3.4$ **m,** $Y_0 = 2.0$ **m**

REQUIRED

The σ_x contribution of each rectangle at depth **z = 0.446 m** (by hand with **Simpson**) and the total **σ_x**; comparison with the module result. This depth is the depth at which the pressure is greatest.



Solution (1/4)

SOLUTION

1 Position of the rectangles relative to the section (x measured from the wall, y from the section)

$$\text{Rect. 1: } x_1 = 1.0 - 0.6 = 0.4\ \text{m} \quad x_2 = 1.0 + 0.6 = 1.6\ \text{m}$$

$$\text{Rect. 2: } x_1 = 3.4 - 0.6 = 2.8\ \text{m} \quad x_2 = 3.4 + 0.6 = 4.0\ \text{m}$$

$$y_1 = 2.0 - 1.8 - 2.0 = -1.8\ \text{m} \quad y_2 = 2.0 + 1.8 - 2.0 = +1.8\ \text{m}$$

2 Area and total vertical load (for each)

$$A = L_x L_y = 1.2 \times 3.6 = 4.32\ \text{m}^2 \qquad P = qA = 50 \times 4.32 = 216\ \text{kN} \qquad \Sigma P = 432\ \text{kN}$$

The section passes exactly through the center of both rectangles: $\mathbf{y_1 = -y_2}$. Rectangle 1 starts 0.4 m from the wall and rectangle 2 starts 2.8 m away.

Solution (2/4)

SOLUTION

3 μ = 0.5: (1 − 2μ) = 0, the second term of G drops out; since $y_1 = -y_2$, $G(x, y_2) - G(x, y_1) = 2G(x, y_2)$

$$f(x) = G(x, y_2, z) - G(x, y_1, z) = \frac{x^2\, z\, y_2\, (2y_2^2 + 3a^2)}{\pi\, a^4 R^3}$$

$$z\, y_2 = 0.8028 \qquad 2y_2^2 = 6.48 \qquad a^2 = x^2 + 0.1989 \qquad R^2 = x^2 + 3.4389$$

4 Simpson's rule in the direction perpendicular to the wall (n even intervals, $h = (x_2 - x_1)/n$); $\sigma_x = \psi\, q \int f\, dx$

$$\int_{x_1}^{x_2} f\, dx \approx \frac{h}{3}\left[f_0 + 4f_1 + 2f_2 + 4f_3 + \cdots + 4f_{n-1} + f_n\right]$$

$$\sigma_x = \psi\, q\, \frac{h}{3} \sum c_i\, f_i \qquad (c_i = 1, 4, 2, 4, \ldots, 4, 1)$$

Rectangle 1 (x = 0.4 – 1.6 m) is near the wall where the integrand changes rapidly: **n = 8** intervals; for rectangle 2 (x = 2.8 – 4.0 m) **n = 4** suffices.

Solution (3/4)

SOLUTION

i	x (m)	a² (m²)	R (m)	f(x) (1/m)	c	c·f
0	0.400	0.3589	1.8971	0.35129	1	0.35129
1	0.550	0.5014	1.9343	0.33921	4	1.35683
2	0.700	0.6889	1.9821	0.28954	2	0.57909
3	0.850	0.9214	2.0400	0.23681	4	0.94723
4	1.000	1.1989	2.1069	0.19155	2	0.38310
5	1.150	1.5214	2.1821	0.15520	4	0.62080
6	1.300	1.8889	2.2647	0.12657	2	0.25315
7	1.450	2.3014	2.3540	0.10408	4	0.41632
8	1.600	2.7589	2.4493	0.08632	1	0.08632
				Σ c·f =		**4.99412**

5 σ_x for rectangle 1 (h = 0.15 m)

$$\sigma_{x,1} = \psi\, q\, \frac{h}{3} \sum c_i f_i = 2(50)\frac{0.15}{3}(4.99412) = \mathbf{24.97}\ \text{kPa}$$

Simpson (8 intervals): **24.97 kPa**. The module's adaptive integration gives 24.98 kPa; difference 0.03%.

Solution (4/4)

SOLUTION

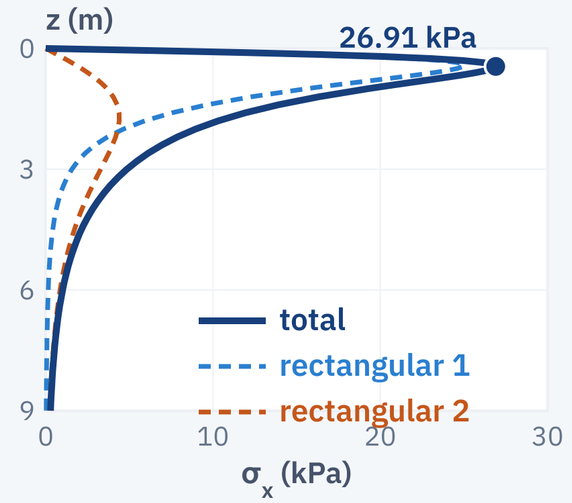
i	x (m)	a² (m²)	R (m)	f(x) (1/m)	c	c·f
0	2.800	8.0389	3.3584	0.02504	1	0.02504
1	3.100	9.8089	3.6123	0.01944	4	0.07777
2	3.400	11.7589	3.8728	0.01536	2	0.03071
3	3.700	13.8889	4.1387	0.01232	4	0.04927
4	4.000	16.1989	4.4090	0.01001	1	0.01001
				Σ c·f =		**0.19281**

6 σ_x for rectangle 2 (h = 0.30 m) and superposition

$$\sigma_{x,2} = 2(50)\frac{0.30}{3}(0.19281) = 1.93\ \text{kPa}$$

$$\sigma_x = \sigma_{x,1} + \sigma_{x,2} = 24.97 + 1.93 = \mathbf{26.90}\ \text{kPa}$$

By hand: **24.97 + 1.93 = 26.90 kPa**. Module: 24.98 + 1.93 = **26.91 kPa** (z = 0.446 m); identical to the example on the page.



Check the solution in GeoTerva ↗

Problem statement

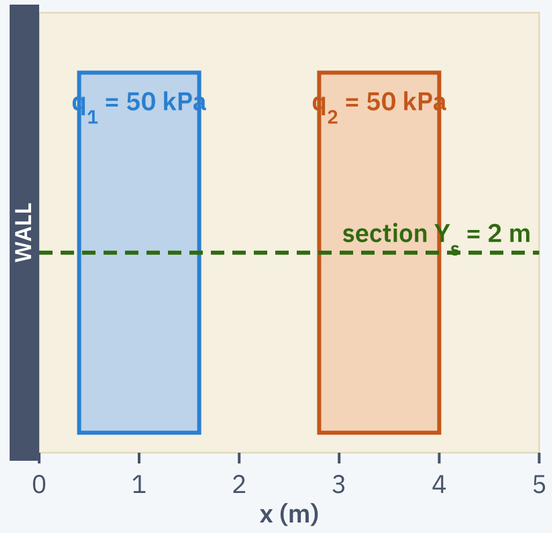
EXAMPLE

Given

Model	**The two rectangles in Example 1; $H = 9.0$ m; $n_z = 20$ (row spacing $H/20 = 0.45$ m)**
Method	**rigid wall ($\psi = 2$); $V_x(z) = \int_0^z \sigma_x \, dt$, $M_y(z) = \int_0^z (z - t)\, \sigma_x \, dt$**

REQUIRED

For each row, **σ_x** (the separate contributions of the two rectangles and their sum), **$V_x(z)$** and **$M_y(z)$**; at the base, **$V_x(H)$**, **$M_y(H)$** and the depth of the resultant.



Solution (1/2)

SOLUTION

z (m)	$\sigma_{x,1}$ (kPa)	$\sigma_{x,2}$ (kPa)	σ_x (kPa)	V_x (kN/m)	M_y (kN·m/m)
0.00	0.00	0.00	0.00	0.00	0.00
0.45	**24.96**	**1.94**	**26.91**	**8.45**	**1.45**
0.90	17.46	3.43	20.89	19.44	7.84
1.35	10.24	4.23	14.47	27.30	18.46
1.80	5.91	4.39	10.31	32.80	32.06
2.25	3.49	4.13	7.62	36.80	47.76
2.70	2.13	3.65	5.78	39.79	65.02
3.15	1.35	3.10	4.45	42.08	83.47
3.60	0.89	2.58	3.46	43.85	102.82
4.05	0.60	2.11	2.71	45.23	122.87
4.50	0.42	1.71	2.13	46.31	143.48
4.95	0.30	1.39	1.69	47.17	164.52
5.40	0.22	1.13	1.35	47.85	185.91
5.85	0.16	0.92	1.08	48.40	207.57
6.30	0.13	0.75	0.88	48.84	229.45
6.75	0.10	0.62	0.72	49.19	251.51
7.20	0.08	0.51	0.59	49.48	273.71
7.65	0.06	0.42	0.49	49.72	296.03
8.10	0.05	0.36	0.40	49.92	318.46
8.55	0.04	0.30	0.34	50.09	340.96
9.00	0.03	0.25	0.28	50.23	363.53

Solution (2/2)

SOLUTION

1 Shear force and moment at the base (last row of the table)

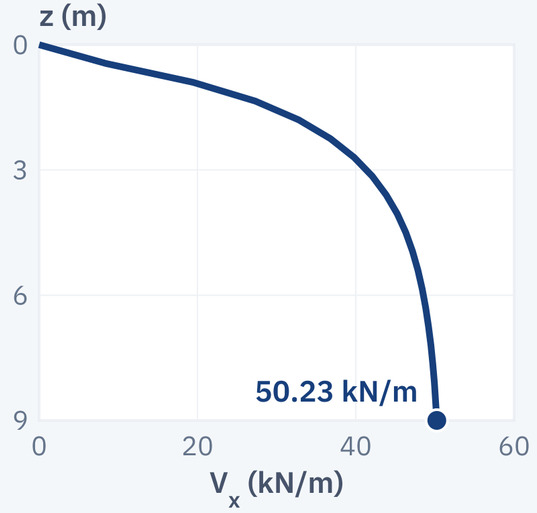
$$V_x(H) = \int_0^H \sigma_x \, dz = \mathbf{50.23}\ \mathrm{kN/m}$$

$$M_y(H) = \int_0^H (H - t)\, \sigma_x \, dt = \mathbf{363.53}\ \mathrm{kN \cdot m/m}$$

2 Line of action of the resultant: moment arm and its depth

$$L = \frac{M_y(H)}{V_x(H)} = \frac{363.53}{50.23} = \mathbf{7.237}\ \mathrm{m}\ \ (\text{from the base})$$

$$z_R = H - L = 9.0 - 7.237 = \mathbf{1.763}\ \mathrm{m}\ \ (\text{from the ground})$$



The resultant is **50.23 kN/m** and acts **1.76 m** below the ground surface. **92%** of the force comes from the upper 4.5 m (half the wall height): $V_X(4.5)$ = 46.31 kN/m.

Check the solution in GeoTerva ↗

Problem statement

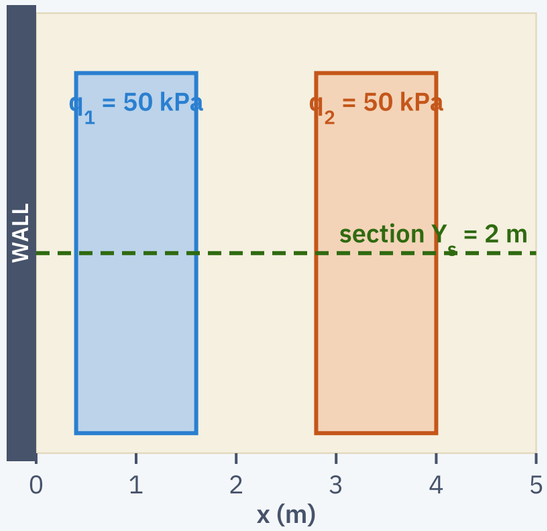
EXAMPLE

Given

Model	**The rectangles in Example 1; each alone, then both together**
Methods	**rigid wall ($\psi = 2$) and semi-infinite medium ($\psi = 1$)**

REQUIRED

For each rectangle **individually** and together **$\sigma_{x,max}$**, **$V_x(H)$**, **$M_y(H)$** and **z_R**; the same quantities in a semi-infinite medium.



Solution

SOLUTION

	$\sigma_{x,max}$ (kPa)	z (m)	V_x(H) (kN/m)	M_y(H) (kN·m/m)	z_R (m)	V_x(H), ψ = 1	M_y(H), ψ = 1
Rectangle 1	25.03	0.419	33.14	260.49	1.140	16.57	130.25
Rectangle 2	4.40	1.717	17.09	103.04	2.971	8.55	51.52
Both together	26.91	0.446	50.23	363.53	1.763	25.12	181.77

1 V and M are summed; the depth of the resultant is the force-weighted average

$$V_x = 33.14 + 17.09 = \mathbf{50.23}\ \mathrm{kN/m} \qquad M_y = 260.49 + 103.04 = \mathbf{363.53}\ \mathrm{kN \cdot m/m}$$

$$z_R = \frac{V_1 z_{R,1} + V_2 z_{R,2}}{V_1 + V_2} = \frac{33.14(1.140) + 17.09(2.971)}{50.23} = \mathbf{1.763}\ \mathrm{m}$$

The half-space values are **exactly half** those of the rigid wall (V_x 25.12, M_y 181.77), while z_R is unchanged. The overall maximum pressure (26.91 kPa) is smaller than the sum of the two separate maxima (29.44): the peaks occur at different depths (0.42 and 1.72 m).

Check the solution in GeoTerva ↗

Problem statement

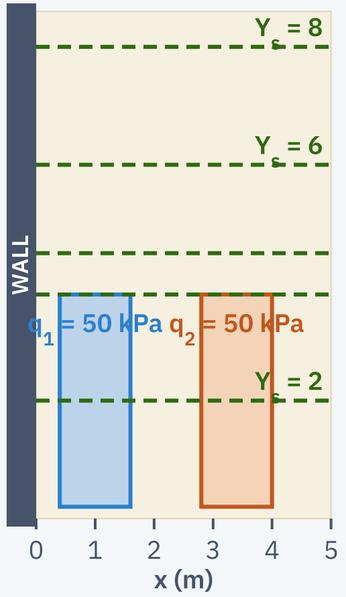
EXAMPLE

REQUIRED

For each section, $\sigma_{x,max}$ and its depth, $V_x(H)$, $M_y(H)$ and z_R; the position of the section relative to the load.

Given

Model	**The two rectangles of Example 1 ($Y_0 = 2.0$ m, $L_y = 3.6$ m: the loads are between $y = 0.2 – 3.8$ m)**
Sections	$Y_s = 2.0; 3.0; 3.8; 4.5; 6.0; 8.0$ m



Solution

SOLUTION

Section Y_s (m)	By load	$\sigma_{x,max}$ (kPa)	z (m)	$V_x(H)$ (kN/m)	$M_y(H)$ (kN·m/m)	z_R (m)
2.0	at the center of the load	26.91	0.446	50.23	363.53	1.763
3.0	within the load	25.04	0.428	45.78	330.75	1.775
3.8	at the edge of the load	14.13	0.467	31.31	217.96	2.038
4.5	outside, 0.7 m	5.00	0.998	17.39	110.14	2.667
6.0	beyond, 2.2 m	1.39	2.112	6.77	36.89	3.549
8.0	beyond, 4.2 m	0.41	3.094	2.44	11.53	4.270

The pressure is largest in the section **at the centre of the load** (26.91 kPa) and decreases rapidly with distance in plan: at Y_s = 6 m, V_x drops to about one-seventh, and at 8 m to about one-twentieth. The Y_s = 3.8 m section is **symmetric** with the y = 0.2 m section (Y_0 ± 1.8 m): the results are the same (V_x = 31.31 kN/m). If the loads are short relative to the wall, several sections are checked.

Check the solution in GeoTerva ↗

Problem statement

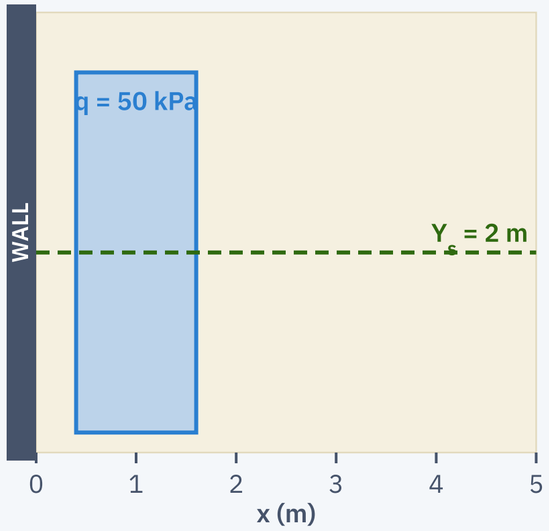
EXAMPLE

REQUIRED

For each location, $\sigma_{x,max}$ and its depth, $V_x(H)$, $M_y(H)$ and z_R.

Given

Model	**single rectangle:** $q = 50$ **kPa,** 1.2×3.6 **m,** $Y_0 = 2.0$ **m;** $\mu = 0.5$**;** $H = 9.0$ **m; rigid wall**
Location	$X_0 = 0.6$**;** 1.0**;** 1.6**;** 2.6**;** 4.6 **m, i.e. the near edge** $x_1 = 0$**;** 0.4**;** 1.0**;** 2.0**;** 4.0 **m**



Solution

SOLUTION

Near edge x_1 (m)	X_0 (m)	$\sigma_{x,max}$ (kPa)	z (m)	$V_x(H)$ (kN/m)	$M_y(H)$ (kN·m/m)	z_R (m)
0.0	0.6	50.00	0.000	35.77	295.98	0.726
0.4	1.0	25.03	0.419	33.14	260.49	1.140
1.0	1.6	14.01	0.791	28.40	208.03	1.675
2.0	2.6	6.89	1.315	21.32	139.72	2.445
4.0	4.6	2.56	2.315	12.59	67.76	3.617

As the load moves away from the wall, the maximum pressure **decreases and deepens**: 50.00 kPa (at the surface) → 2.56 kPa (2.32 m). For a load adjacent to the wall ($x_1 = 0$), the maximum pressure is at the surface and equals **q = 50 kPa**. V_x, however, decreases more slowly (35.77 → 12.59 kN/m); z_R deepens.

Check the solution in GeoTerva ↗

Problem statement

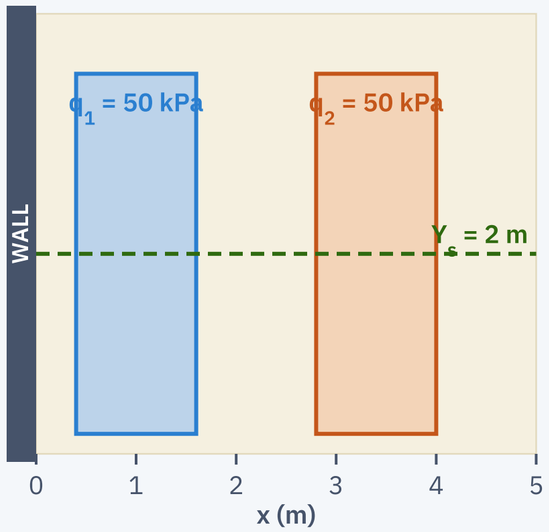
EXAMPLE

Given

Model	**The two rectangles in Example 1; $H = 9.0$ m; $Y_s = 2.0$ m; rigid wall**
Varying	$\mu = 0.5; 0.4; 0.3; 0.2$

REQUIRED

For each μ, **$\sigma_{x,max}$**, **$V_x(H)$**, **$M_y(H)$**; the ordinate at the ground surface $\sigma_x(0)$ and the **manual** determination of this ordinate.



Solution (1/2)

SOLUTION

μ	1 – 2μ	$\sigma_x(0)$ (kPa)	$\sigma_{x,max}$ (kPa)	z (m)	$V_x(H)$ (kN/m)	$M_y(H)$ (kN·m/m)
0.5	0.0	0.00	26.91	0.446	50.23	363.53
0.4	0.2	−4.18	23.54	0.459	41.47	303.91
0.3	0.4	−8.36	20.19	0.473	32.72	244.28
0.2	0.6	−12.54	16.88	0.489	23.96	184.66

As μ decreases, the pressure and force **decrease markedly**: for μ = 0.3, V_x = 32.72 kN/m, 35% less than for μ = 0.5. A **negative (tensile) ordinate** develops at the surface; V_x and M_y are integrated from the signed profile.

Solution (2/2)

SOLUTION

1 For z = 0, R = r, (1 – z/R) = 1: only the second term of G remains and ∫ y/(x² + y²) dx = tan⁻¹(x/y)

$$\sigma_x(0) = -\psi\, q\, \frac{(1-2\mu)}{\pi}\left[\tan^{-1}\frac{x_2}{y_2} - \tan^{-1}\frac{x_1}{y_2}\right] \qquad (y_1 = -y_2)$$

2 Rectangle 1 (x = 0.4 – 1.6 m), rectangle 2 (x = 2.8 – 4.0 m); for μ = 0.4, 1 – 2μ = 0.2

$$\sigma_{x,1}(0) = -2(50)\frac{0.2}{\pi}\left[\tan^{-1}\frac{1.6}{1.8} - \tan^{-1}\frac{0.4}{1.8}\right] = -6.366(0.5080) = -3.234\ \text{kPa}$$

$$\sigma_{x,2}(0) = -6.366\left[\tan^{-1}\frac{4.0}{1.8} - \tan^{-1}\frac{2.8}{1.8}\right] = -6.366(0.1485) = -0.945\ \text{kPa}$$

$$\sigma_x(0) = -3.234 + -0.945 = \mathbf{-4.18}\ \text{kPa}$$

By hand −4.18 kPa; module −4.18 kPa. The surface ordinate is proportional to **(1 – 2μ)**: zero at μ = 0.5, −12.54 kPa at μ = 0.2.

Check the solution in GeoTerva ↗

Problem statement

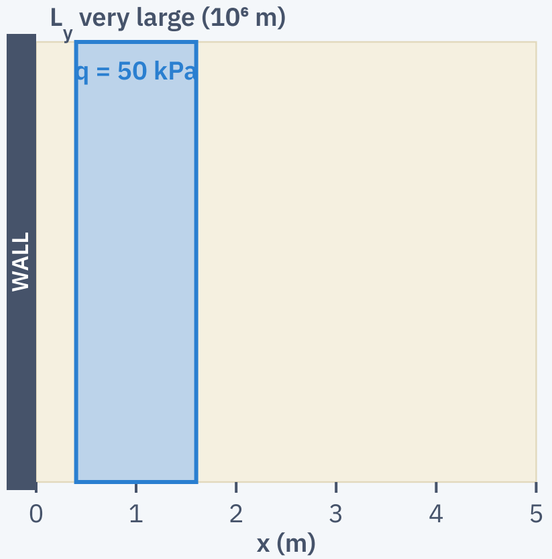
EXAMPLE

Given

Model	**single rectangle: $q = 50$ kPa, $x_1 = 0.4$ m, $x_2 = 1.6$ m ($L_x = 1.2$ m), $L_y = 10^6$ m; $H = 9.0$ m; rigid wall**
Poisson's ratio	$\mu = 0.5$ **and** $\mu = 0.2$

REQUIRED

With the **strip load** formula (by hand) **σ_x** at z = 0.5; 1; 2; 4 m; comparison with the area module; **$V_x(H)$**.



Solution (1/2)

SOLUTION

1 Strip load (plane strain), rigid wall: β = tan⁻¹(x₂/z) − tan⁻¹(x₁/z), α = tan⁻¹(x₁/z) + β/2

$$\sigma_x = \psi \frac{q}{\pi} \left(\beta - \sin\beta \cos 2\alpha\right) \qquad z = 1.0 \text{ m}:$$

$$\beta = \tan^{-1} 1.6 - \tan^{-1} 0.4 = 0.6317 \quad \alpha = 0.3805 + \frac{0.6317}{2} = 0.6964$$

$$\sigma_x = 2\frac{50}{\pi} \left(0.6317 - \sin 0.6317 \cos 1.3927\right) = \mathbf{16.777}\ \text{kPa}$$

The strip formula gives **16.777 kPa**; the area module ($L_y = 10^6$ m) finds the same value: a very long rectangular area turns into a strip load. The comparison for four depths is on the next page.

Solution (2/2)

SOLUTION

z (m)	β (rad)	α (rad)	σ_x, strip formula (kPa)	Area modulus, μ = 0.5	Area modulus, μ = 0.2
0.5	0.5932	0.9713	**25.346**	25.346	25.346
1.0	0.6317	0.6964	**16.777**	16.777	16.777
2.0	0.4773	0.4361	**5.788**	5.788	5.788
4.0	0.2808	0.2401	**1.115**	1.115	1.115

2 Shear force: closed form for the strip; the area module gives the same value

$$V_x(H) = \psi \frac{q}{\pi} H \left[\tan^{-1} \frac{x_2}{H} - \tan^{-1} \frac{x_1}{H} \right] = \mathbf{37.68}\ \mathrm{kN/m}$$

With the strip formula, the area module at μ = 0.5 is **identical**, and at μ = 0.2 it matches with a difference of at most 10^{-4} kPa: for a very long rectangle the result is independent of μ. $V_X(H)$ = 37.68 kN/m.

Check the solution in GeoTerva ↗

Summary

Key points

$\sigma_x(z) = \psi \Sigma q \int [G(x,y_2,z) - G(x,y_1,z)]\, dx$: closed-form solution (G) along the wall, adaptive numerical integration in the direction perpendicular to the wall; **$\psi = 2$** rigid wall, **$\psi = 1$** semi-infinite medium

$V_x(z) = \int \sigma_x\, dt$, **$M_y(z) = \int (z - t)\, \sigma_x\, dt$**; the depth of the resultant $z_R = H - M_y/V_x$

Opening example (z = 0.446 m): σ_x = 24.98 + 1.93 = **26.91 kPa** (largest pressure); $V_x(H)$ = **50.23 kN/m**, $M_y(H)$ = **363.53 kN·m/m**, z_R = 1.76 m

The pressure is greatest when the **section** passes through the center of the load; distance and **the distance of the load from the wall** reduce the pressure rapidly; for a load adjacent to the wall, $\sigma_x = q$ at the surface

Poisson's ratio changes the result considerably ($\mu = 0.3$: V_x is about 35% short); for $\mu < 0.5$ there is a tensile ordinate at the surface; a very long rectangular strip load gives

References

Das, B.M. (2016). Principles of Foundation Engineering, 8th ed. Cengage Learning; this page follows it.

Boussinesq, J. (1883). Application des Potentiels à l’Étude de l’Équilibre et du Mouvement des Solides Élastiques. Gauthier-Villars, Paris.

Mindlin, R.D. (1936). Discussion: Pressure distribution on retaining walls. Proc. 1st Int. Conf. on Soil Mechanics and Foundation Engineering, Harvard, Vol. III, 155–156.

NOTE

All numerical examples have been generated with the module's own calculation engine; the module yields the same values. The module's opening example is a wall with two rectangles (H = 9 m, μ = 0.5, Y_s = 2 m).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own rectangular loads, the section position, Poisson's ratio and the wall height in GeoTerva; the module provides the pressure profile, the shear force and moment, the separate contribution of each load, the three views, the step-by-step solution and the report together.

Open the module → **Theory page →**

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com