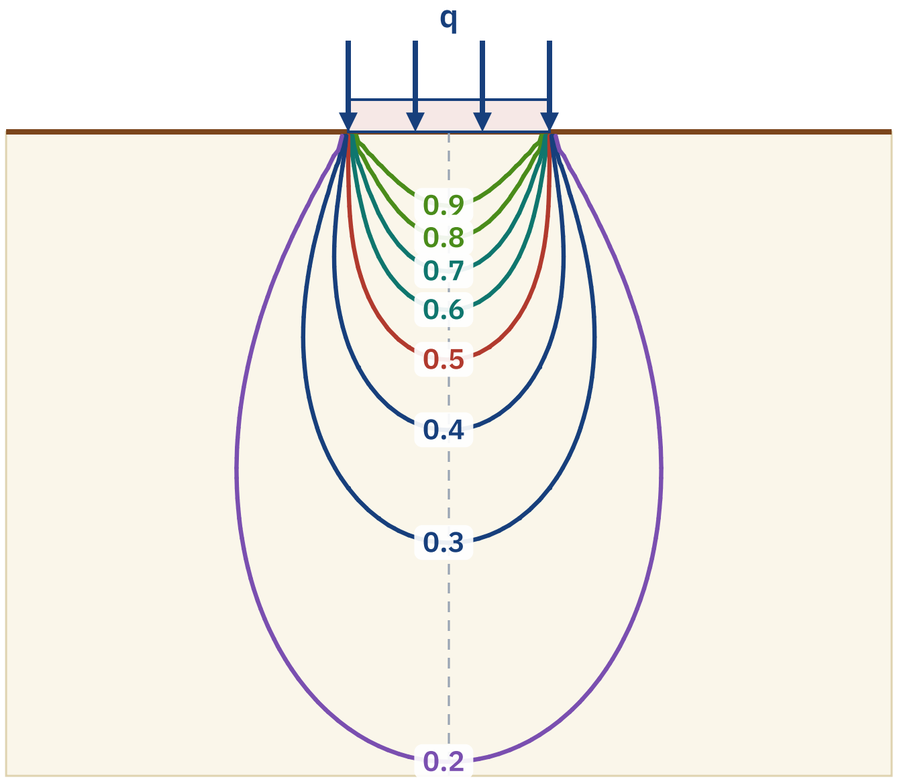
Geotechnical lecture presentation · Stresses in the soil mass

Stress under a vertical strip load

Lecture: flexible, infinitely long vertical strip load; a point by hand, depth and horizontal profiles, superposition.

- Integral of the line load
- Angle form
- Stress bulb
- Superposition
- Solvable by hand



Introduction

What will we learn in this lecture?

The vertical stress increase beneath a **strip load**: an infinitely long flexible strip of width B carrying a uniform pressure q; **the integral of the line load across the width**

The closed **arctangent** form and the **angle form**; the **π correction** added to the first arctangent when x < B/2

That the stress depends only on **x/B** and **z/B** and never exceeds q; the **stress bulb** and the influence factor table

Work one point **by hand from start to finish**; depth and horizontal profiles in tables; multiple strips by **superposition**

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example: q = 100 kN/m², B = 4 m; point A at x = 1 m laterally from the centerline, at depth z = 1 m.

Strip load: finite width, infinite length

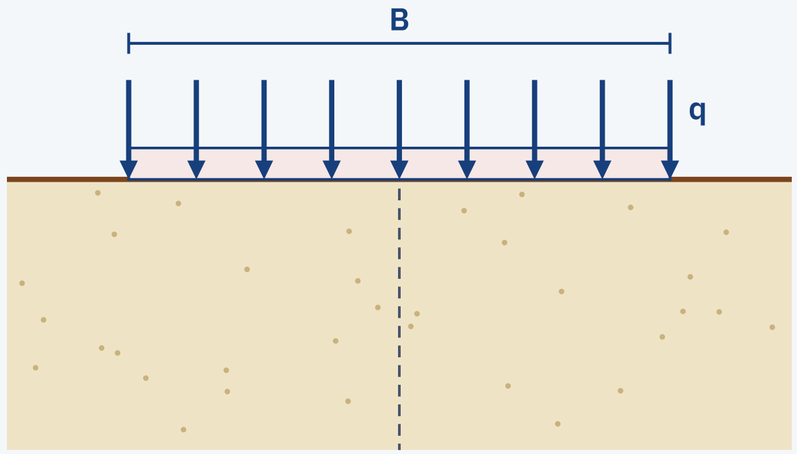
THEORY

A long foundation, the upper surface of a road or embankment, a wall base: many loads are long in one direction and of finite width in the other. A **strip load** models these

Flexible strip: width B, infinite length, uniform vertical pressure q. The stress is the same in every section; the problem is one of **plane strain** (two-dimensional).

Whatever the settlement, the pressure remains q across the width; a rigid foundation would redistribute this pressure

The soil is a **homogeneous, isotropic, linearly elastic** half-space. $\Delta\sigma_z$ is only the increase due to the load; the overburden is added separately



Section: the strip extends infinitely in the y direction; the same stress in every section (plane strain)

From line load to strip

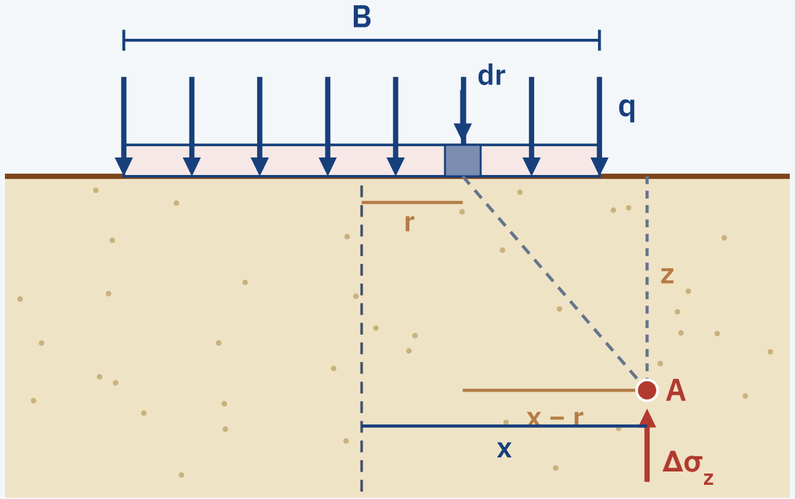
THEORY

An elementary strip of width dr carries a load **q dr** per unit length and acts like a **line load**; its horizontal distance to point A is x – r

$$d\sigma_z = \frac{2\,(q\,dr)\,z^3}{\pi\,[(x-r)^2+z^2]^2}$$

The entire strip is the sum of these line loads from r = –B/2 … +B/2

$$\Delta\sigma_z = \int_{-B/2}^{+B/2} \frac{2q}{\pi}\,\frac{z^3}{[(x-r)^2+z^2]^2}\,dr$$



Closed-form of the integral

THEORY

u = x − r is taken; the **antiderivative** for 1/(u² + z²)² is (q/π)[tan^{-1}(u/z) + z u/(u² + z²)]. Evaluating the integral at the limits u = x − B/2 and u = x + B/2 gives the **arctangent form**

$$\Delta\sigma_z = \frac{q}{\pi}\left\{\tan^{-1}\frac{z}{x-B/2} - \tan^{-1}\frac{z}{x+B/2} - \frac{B\,z\,[x^2 - z^2 - B^2/4]}{[x^2 + z^2 - B^2/4]^2 + B^2 z^2}\right\}$$

The arctangents are in **radians**; $\Delta\sigma_z$ is the same at the +x and −x positions (the strip is symmetric about its centerline)

As written, the equation is valid for x ≥ B/2. For 0 ≤ x < B/2 the first arctangent becomes negative and is replaced by **π + tan^{-1}[z/(x − B/2)]**; exactly at x = B/2 it is π/2

Numerical verification (q = 100, B = 4, x = 1, z = 1): the numerical integral of the line-load solution over the strip width is **90.2232 kPa**, the closed form is **90.2232 kPa**

Stress depends only on the angles

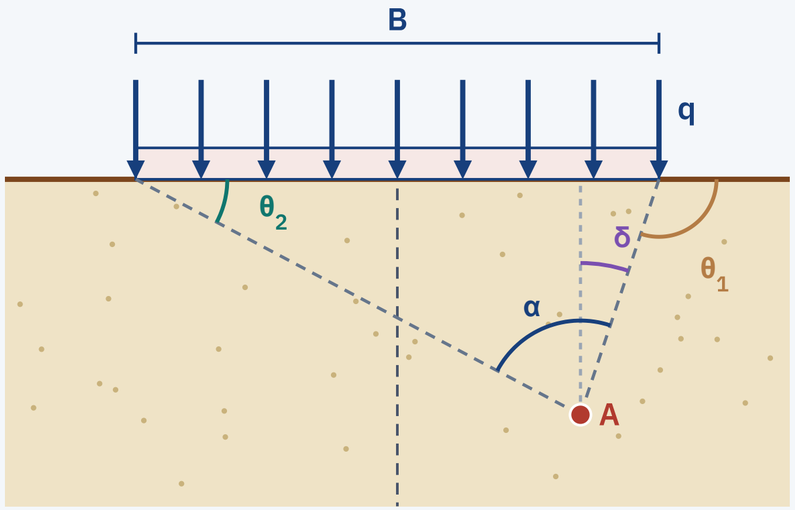
THEORY

The same result is written with the angles in the figure. θ is measured from the surface; α = $\theta_1 - \theta_2$ is the angle at which the strip is seen from A. **δ** is the angle between the vertical and the line drawn from A to the +B/2 edge: δ = 90° − θ_1 (negative below the strip)

$$\Delta\sigma_z = \frac{q}{\pi}\left[\alpha + \sin\alpha\,\cos(\alpha + 2\delta)\right]$$

$$F(u) = \tan^{-1}\frac{u}{z} + \frac{zu}{u^2+z^2} = \varphi + \tfrac{1}{2}\sin 2\varphi \quad (u = z\tan\varphi)$$

The stress depends only on **x/B and z/B** and **does not exceed q**: at the surface beneath the strip α = π and $\Delta\sigma_z$ = q; beside the strip α = 0



Below the axis and below the edge

THEORY

Under the axis (x = 0; b = B/2) the equation reduces to a single term; **under the edge** (x = b) the first arctangent is π/2 and at the surface $\Delta\sigma_z = q/2$

$$\left.\frac{\Delta\sigma_z}{q}\right|_{x=0} = \frac{2}{\pi}\left[\tan^{-1}\frac{b}{z} + \frac{bz}{b^2+z^2}\right]$$

$$\left.\frac{\Delta\sigma_z}{q}\right|_{x=b} = \frac{1}{\pi}\left[\frac{\pi}{2} - \tan^{-1}\frac{z}{B} + \frac{Bz}{B^2+z^2}\right]$$

2z/B	0.5	1.0	2.0	3.0	4.0	6.0
Under center (2x/B = 0)	0.959	0.818	0.550	0.396	0.306	0.208
Under edge (2x/B = 1)	0.497	0.480	0.409	0.334	0.275	0.198

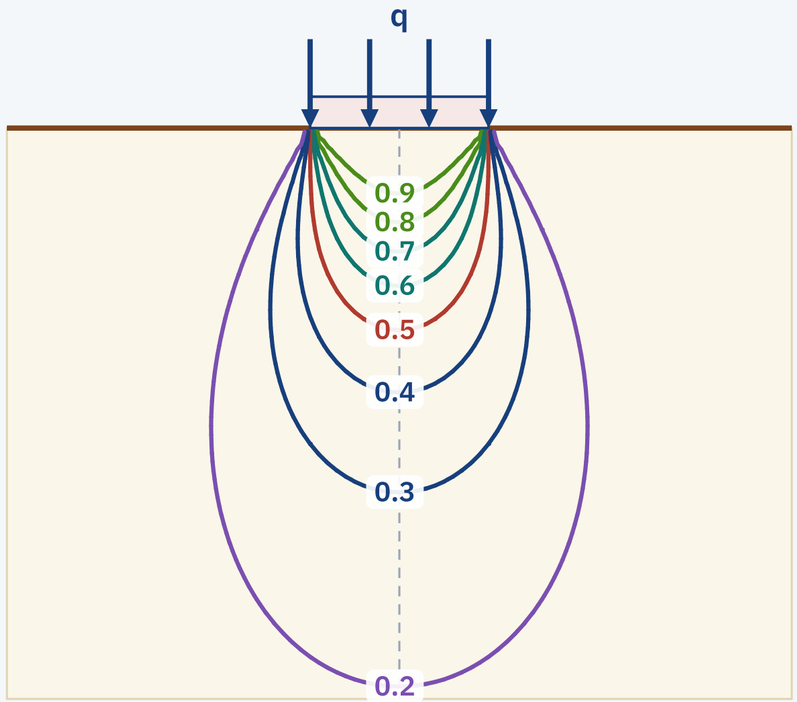
Contour lines: $\Delta\sigma_z/q$

THEORY

Below the axis, the stress drops to half of q at a depth of about **z = 1.1B**, to one fifth at **3.1B**, and only to one tenth at **6.3B**

The 0.6 – 0.9 contours become denser below the strip in the near-surface zone: the familiar **pressure bulb**

When distances are divided by B, the curves are **the same for every strip**: a single diagram plotted against x/B and z/B is valid for every q and B



$\Delta\sigma_z/q$ values

THEORY

2z/B \ 2x/B	0.0	0.5	1.0	1.5	2.0	2.5	3.0
0.0	1.000	1.000	0.500	0.000	0.000	0.000	0.000
0.5	0.959	0.902	0.497	0.089	0.019	0.006	0.003
1.0	0.818	0.735	0.480	0.214	0.084	0.036	0.017
1.5	0.668	0.607	0.448	0.271	0.146	0.077	0.042
2.0	0.550	0.510	0.409	0.288	0.185	0.114	0.071
2.5	0.462	0.436	0.370	0.285	0.205	0.141	0.095
3.0	0.396	0.379	0.334	0.274	0.211	0.157	0.114
3.5	0.345	0.334	0.302	0.258	0.210	0.165	0.127
4.0	0.306	0.298	0.275	0.242	0.205	0.168	0.134
5.0	0.248	0.244	0.231	0.212	0.188	0.164	0.139
6.0	0.208	0.206	0.198	0.186	0.171	0.154	0.136

$\Delta\sigma_z = q \cdot$ (table value). Every value in the table comes from the same equation; the z = 0 row is the surface (1 under the strip, 0.5 at the edge, 0 outside).

Superposition

THEORY

Since the soil is **elastic**, the stress of each strip is found separately and summed. Each strip is defined by its own q, B and axis line; the module accepts at most eight strips

The strips **cannot overlap**: two may meet at an edge. The stress of each strip is found with **its own x value**, i.e. the distance of point A to that strip's centreline

$$\Delta\sigma_z = \sum_k \Delta\sigma_{z(k)} \qquad x_k = x_A - x_{c,k}$$

$\Delta\sigma_{z(k)}$ is calculated from the equation above (q_k, B_k, x_k, z); due to symmetry only $|x_k|$ is required

Problem statement

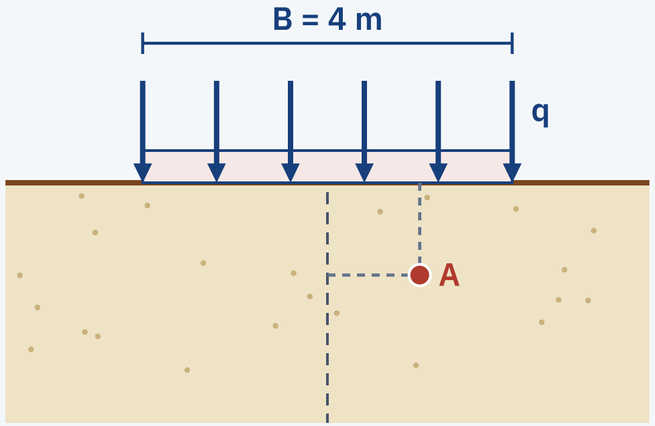
EXAMPLE

Given

Strip load	**Flexible, vertical: $q = 100$ kN/m², $B = 4$ m ($b = B/2 = 2$ m); axis line $x = 0$**
Point A	**$x = 1$ m to the side of the centerline, at a depth of $z = 1$ m (beneath the strip)**

REQUIRED

$\Delta\sigma_z$ and **$\Delta\sigma_z/q$**: first with the closed (arctangent) form, then verification with the **angle form**.



Solution (1/3)

SOLUTION

1 Half-width and the distances from point A to the edges

$$b = \frac{B}{2} = 2\ \text{m} \qquad x - b = 1 - 2 = -1\ \text{m} \qquad x + b = 1 + 2 = 3\ \text{m}$$

$$\frac{2x}{B} = 0.5 \qquad \frac{2z}{B} = 0.5$$

2 Since x < B/2 the first arctangent is negative; π is added

$$\tan^{-1}\frac{z}{x-b} = \tan^{-1}\frac{1}{-1} = -0.7854 \;\Rightarrow\; A_1 = \pi + (-0.7854) = \mathbf{2.3562}$$

$$A_2 = \tan^{-1}\frac{z}{x+b} = \tan^{-1}\frac{1}{3} = \mathbf{0.3218}$$

Point A lies **below** the strip (|x| = 1 m < B/2 = 2 m). The π correction of the first arctangent is therefore required; A_1 = 2.3562 and A_2 = 0.3218 rad.

Solution (2/3)

SOLUTION

3 Third (algebraic) term

$$T = \frac{B\,z\,(x^2 - z^2 - b^2)}{(x^2 + z^2 - b^2)^2 + B^2 z^2} = \frac{4(1)(1 - 1 - 4)}{(1 + 1 - 4)^2 + 16(1)} = \frac{-16}{20} = \mathbf{-0.8000}$$

4 Influence value and stress

$$\frac{\Delta\sigma_z}{q} = \frac{1}{\pi}\left[A_1 - A_2 - T\right] = \frac{1}{\pi}\left[2.3562 - 0.3218 - (-0.8)\right] = \mathbf{0.9022}$$

$$\Delta\sigma_z = 0.9022\,(100) = \mathbf{90.22}\ \mathrm{kPa}$$

$\Delta\sigma_z$ = 90.22 kPa: The stress at point A is **90.2**% of the surface pressure ($\Delta\sigma_z$/q = 0.9022). The stress does not exceed q = 100 kPa.

Solution (3/3)

SOLUTION

5 Angles: θ from the surface, δ from the vertical

$$\theta_1 = 135.00^\circ \quad (x - b < 0) \qquad \theta_2 = \tan^{-1}\frac{z}{x+b} = 18.43^\circ$$

$$\alpha = \theta_1 - \theta_2 = 116.57^\circ = 2.0344 \text{ rad}$$

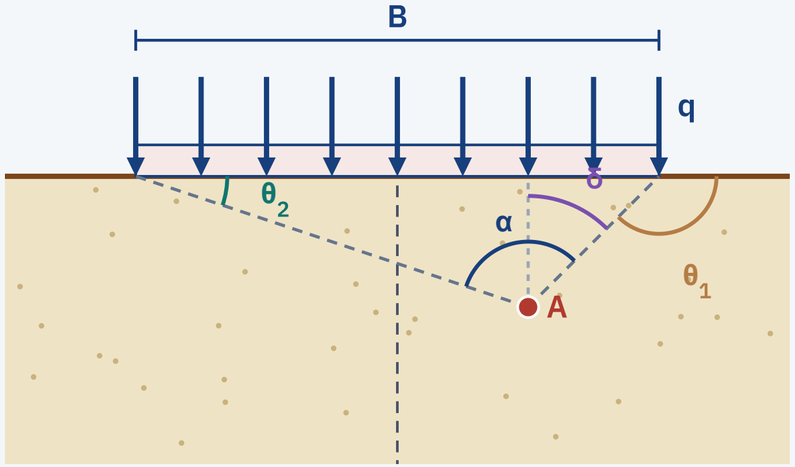
$$\delta = 90^\circ - \theta_1 = -\mathbf{45}^\circ$$

6 Angle form

$$\Delta\sigma_z = \frac{q}{\pi}\left[\alpha + \sin\alpha\cos(\alpha + 2\delta)\right]$$

$$= \frac{100}{\pi}\left[2.0344 + (0.8944)\cos(26.57^\circ)\right]$$

$$= \frac{100}{\pi}(2.8344) = \mathbf{90.22}\text{ kPa}$$



The two forms give the **same result**: $\Delta\sigma_z$ = 90.22 kPa. The module gives the same value; the angular form shows that the stress depends only on the angles, i.e. on x/B and z/B.

Check the solution in GeoTerva ↗

Problem statement

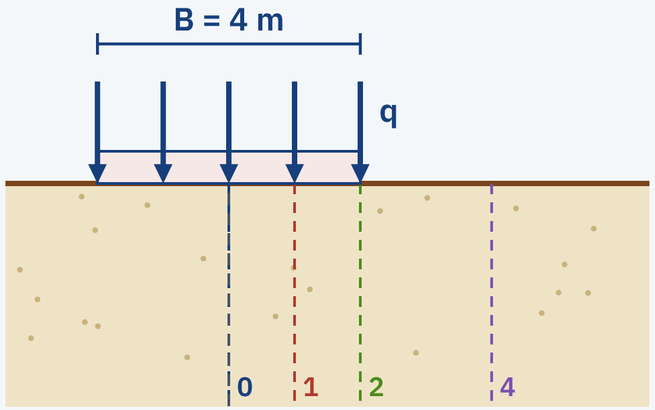
EXAMPLE

Given

Strip load	$q = 100$ **kN/m²,** $B = 4$ **m, centreline** $x = 0$ **(the strip in Example 1)**
Lines	$x = 0$ **(axis),** $x = 1$ **m (line of A),** $x = 2$ **m (edge),** $x = 4$ **m (2 m beyond the edge)**
Depth	$z = 0.5$ **…** 12 **m**

REQUIRED

The variation of $\Delta\sigma_z$ with depth along each line; the depths under the axis at which the stress is **q/2, q/5, q/10**.

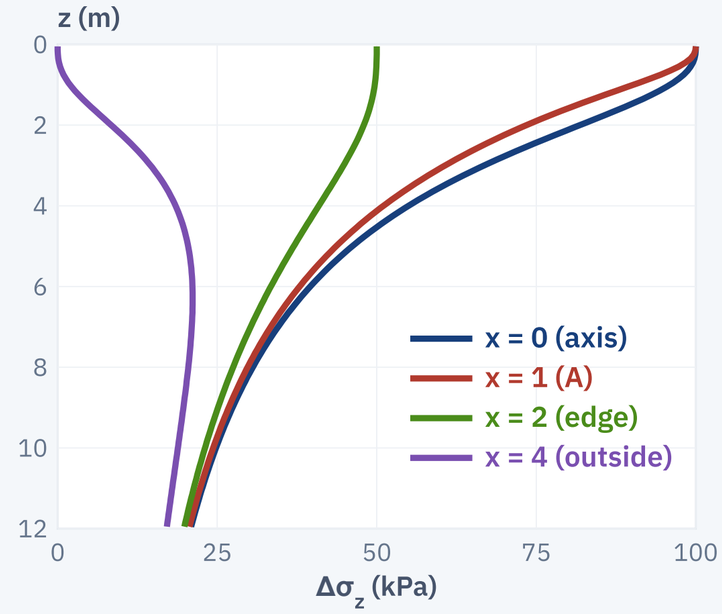


Solution (1/2)

SOLUTION

z (m)	2z/B	x = 0 (axis)	x = 1 m (A)	x = 2 m (edge)	x = 4 m (outside)
0.5	0.250	99.38	97.88	49.96	0.30
1.0	0.500	95.95	**90.22**	49.69	1.93
1.5	0.750	89.59	81.38	49.04	4.90
2.0	1.000	81.83	73.47	47.97	8.39
3.0	1.500	66.82	60.71	44.80	14.57
4.0	2.000	54.98	51.05	40.92	18.48
5.0	2.500	46.18	43.65	37.00	20.45
6.0	3.000	39.58	37.91	33.41	21.12
8.0	4.000	30.58	29.76	27.49	20.47
10.0	5.000	24.81	24.36	23.09	18.84
12.0	6.000	20.84	20.57	19.79	17.07

Under the centre, the stress decreases slowly from the surface downward (20.8 kPa at z = 12 m); **under the edge**, it is very close to q/2 at the surface (50.0 kPa), then decreases; **outside** on the straight line, the stress first increases, reaches its maximum at z ≈ 6 m (21.1 kPa) and then decreases.



Solution (2/2)

SOLUTION

1 Under the axis, the equation reduces to a single term (x = 0, b = B/2 = 2 m)

$$\frac{\Delta\sigma_z}{q} = \frac{2}{\pi}\left[\tan^{-1}\frac{b}{z} + \frac{bz}{b^2+z^2}\right] \qquad z = 1.132B = 4.53\ \text{m} \Rightarrow \frac{2}{\pi}[0.4159 + 0.3695] = \mathbf{0.500}$$

2 Values of the equation at three depths

Influence value	z/B	z (m)	$\tan^{-1}(b/z)$	$bz/(b^2+z^2)$	$\Delta\sigma_z/q$	$\Delta\sigma_z$ (kPa)
q/2	1.132	4.53	0.4159	0.3695	**0.500**	50.00
q/5	3.130	12.52	0.1584	0.1558	**0.200**	20.00
q/10	6.340	25.36	0.0787	0.0784	**0.100**	10.00

Below the axis, the stress falls to q/2 at **z = 1.13B = 4.5 m**, to q/5 at **3.13B = 12.5 m**, and to q/10 at **6.34B = 25.4 m**: the soil remains under load down to a depth of several times the strip width.

Check the solution in GeoTerva ↗

Problem statement

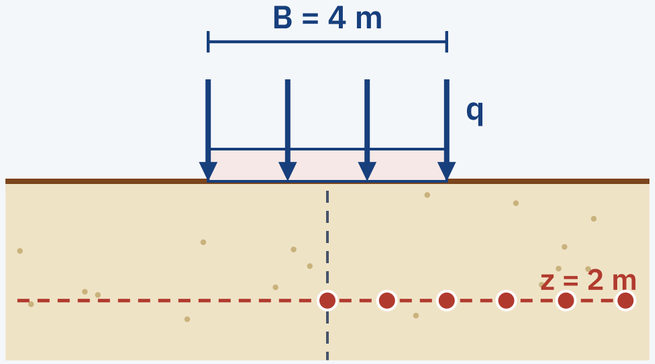
EXAMPLE

Given

Strip load	$q = 100$ **kN/m²,** $B = 4$ **m,** $x = 0$ **(**$b = 2$ **m)**
Line	**horizontal line at a depth of** $z = 2$ **m; points** $x = 0, 1, 2, 3, 4, 5, 6, 8$ **m**
Hand calculation	**below the edge** $x = b = 2$ **m and outside the strip** $x = 3$ **m**

REQUIRED

Below the edge (x = B/2) and **outside the strip** (x = 3 m), compute $\Delta\sigma_z$ by hand; the entire horizontal profile at z = 2 m; comparison of x = −3 m with x = +3 m.



Solution (1/3)

SOLUTION

1 x = b = 2 m, z = 2 m: the first arctangent is exactly π/2

$$A_1 = \tan^{-1}\frac{z}{x-b} = \tan^{-1}\frac{2}{0} = \frac{\pi}{2} = 1.5708 \qquad A_2 = \tan^{-1}\frac{2}{4} = 0.4636$$

$$T = \frac{4(2)(4-4-4)}{(4+4-4)^2+16(4)} = \frac{-32}{80} = -0.4$$

2 Influence value and stress

$$\frac{\Delta\sigma_z}{q} = \frac{1}{\pi}\left[1.5708 - 0.4636 - (-0.4)\right] = \mathbf{0.4797}$$

$$\Delta\sigma_z = 0.4797\,(100) = \mathbf{47.97}\ \mathrm{kPa}$$

Below the edge the stress drops from q/2 at the surface to **47.97 kPa** (48.0% of q) at a depth of z = 2 m. Exactly at the edge the first arctangent is π/2; no π correction is needed.

Solution (2/3)

SOLUTION

3 x = 3 m > b: both arctangents are taken directly (x − b = 1, x + b = 5)

$$A_1 = \tan^{-1}\frac{2}{1} = 1.1071 \qquad A_2 = \tan^{-1}\frac{2}{5} = 0.3805$$

$$T = \frac{4(2)(9-4-4)}{(9+4-4)^2+16(4)} = \frac{8}{145} = 0.0552$$

4 Influence value and stress

$$\frac{\Delta\sigma_z}{q} = \frac{1}{\pi}\left[1.1071 - 0.3805 - 0.0552\right] = \mathbf{0.2137}$$

$$\Delta\sigma_z = 0.2137\,(100) = \mathbf{21.37}\ \text{kPa}$$

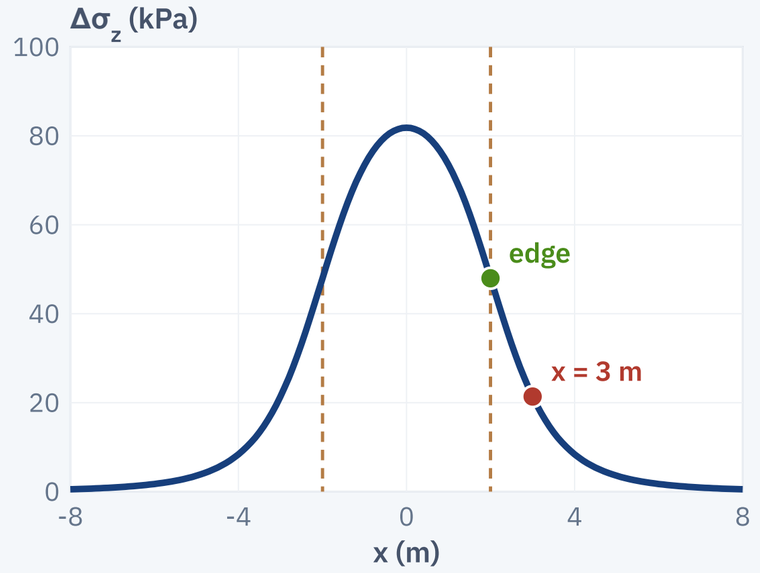
1 m outside the strip (x = 3 m) the stress is 21.37 kPa; there is no load at this point, yet the soil carries stress. By symmetry, **21.37 kPa** is also obtained for x = −3 m (the module gives the same value).

Solution (3/3)

SOLUTION

x (m)	2x/B	$\Delta\sigma_z$ (kPa)	$\Delta\sigma_z/q$	Location
0	0.00	81.83	0.8183	axis
1	0.50	73.47	0.7347	beneath the strip
2	1.00	47.97	0.4797	edge
3	1.50	21.37	0.2137	outside
4	2.00	8.39	0.0839	outside
5	2.50	3.58	0.0358	outside
6	3.00	1.72	0.0172	outside
8	4.00	0.53	0.0053	outside

The stress is largest at the axis (81.8 kPa), drops to 48.0 kPa at the edge (about half of q) and decays rapidly outside the strip: 0.53 kPa at x = 8 m. The profile is **symmetric** about the x = 0 axis.



Check the solution in GeoTerva ↗

Problem statement

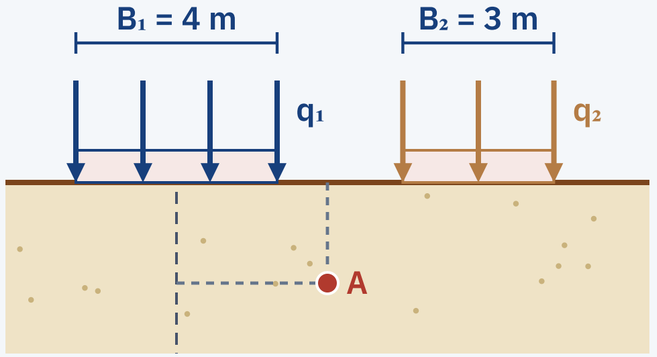
EXAMPLE

Given

1st strip	$q_1 = 100$ **kN/m²,** $B_1 = 4$ **m, centerline** $x_{c1} = 0$ **(–2 … 2 m)**
2nd strip	$q_2 = 60$ **kN/m²,** $B_2 = 3$ **m, centerline** $x_{c2} = 6$ **m (4.5 … 7.5 m)**
Point A	$x = 3$ **m,** $z = 2$ **m (between the two strips)**

REQUIRED

The stress at A from each strip and the **total $\Delta\sigma_z$**; the variation of stress with depth below A.



Solution (1/2)

SOLUTION

1 slice 1: $x_1 = 3 - 0 = 3$ m, $b = 2$ m ($x > b$)

$$A_1 = \tan^{-1}\frac{2}{1} = 1.1071 \quad A_2 = \tan^{-1}\frac{2}{5} = 0.3805 \quad T = \frac{4(2)(1)}{9^2+64} = 0.0552$$

$$\Delta\sigma_{z(1)} = \frac{100}{\pi}\left[A_1 - A_2 - T\right] = 0.2137\,(100) = \mathbf{21.37}\ \text{kPa}$$

2 2nd strip: $x_2 = 3 - 6 = -3$ m ($|x_2| = 3$ m), $b = 1.5$ m

$$A_1 = \tan^{-1}\frac{2}{1.5} = 0.9273 \quad A_2 = \tan^{-1}\frac{2}{4.5} = 0.4182 \quad T = \frac{3(2)(2.75)}{(10.75)^2+36} = 0.1089$$

$$\Delta\sigma_{z(2)} = \frac{60}{\pi}\left[A_1 - A_2 - T\right] = 0.1274\,(60) = \mathbf{7.64}\ \text{kPa}$$

3 Superposition

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} = 21.37 + 7.64 = \mathbf{29.02}\ \text{kPa}$$

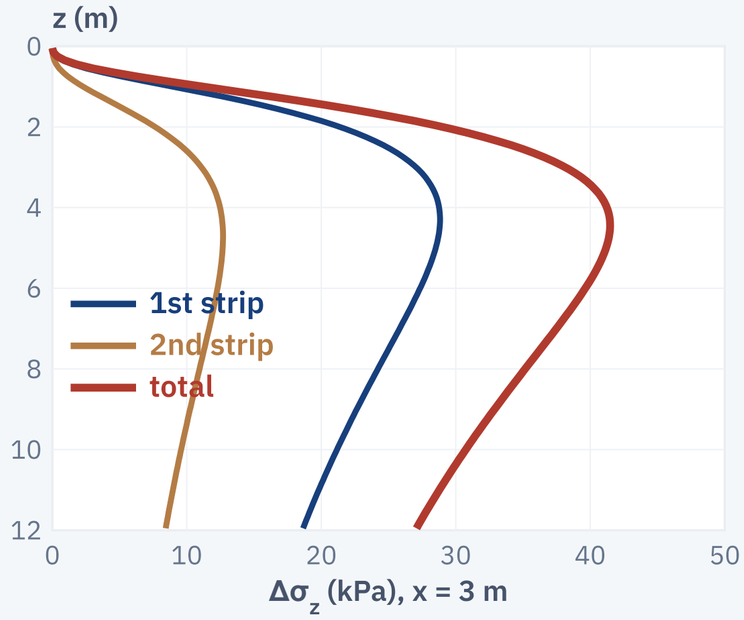
Total **$\Delta\sigma_z$ = 29.02 kPa**; the module gives the same value. The share of the 2nd strip is 26% of the total.

Solution (2/2)

SOLUTION

z (m)	1st strip (kPa)	2nd strip (kPa)	Total (kPa)	share of strip 2
1	8.92	2.28	**11.21**	20 %
2	21.37	7.64	**29.02**	26 %
3	27.05	11.09	**38.14**	29 %
4	28.76	12.47	**41.24**	30 %
6	27.35	12.28	**39.63**	31 %
8	24.21	10.95	**35.16**	31 %
12	18.59	8.41	**26.99**	31 %

The total stress beneath A is greatest at z ≈ 4.5 m (41.5 kPa) and then decreases. In the layer near the surface, strip 1 dominates; at depth, the contributions of the two strips approach each other (at z = 12 m the contribution of strip 2 is 31%).



Check the solution in GeoTerva ↗

Problem statement

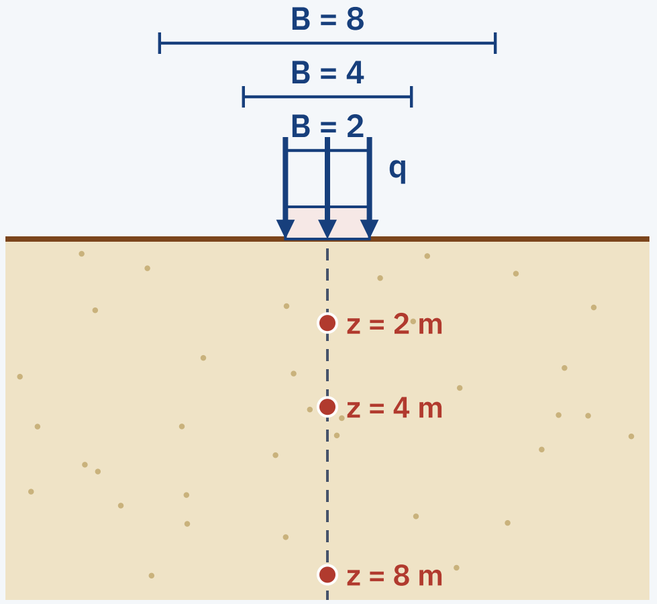
EXAMPLE

Given

Width effect	**same** $q = 100$ **kN/m²;** $B = 2,\ 4,\ 8$ **m; under the axis** $z = 2,\ 4,\ 8$ **m**
Narrow strip	**load intensity** $P = qB = 100$ **kN/m constant;** $B = 2,\ 1,\ 0.5,\ 0.1$ **m; point** $x = 2$ **m,** $z = 2$ **m**
Comparison	**line load:** $\Delta\sigma_z = 2Pz^3/[\pi(x^2 + z^2)^2]$

REQUIRED

To show that the stress is the same when **z/B** is the same; to see that as the strip narrows the result approaches the **line load** value.



Solution (1/2)

SOLUTION

1 Under the axis, B = 2 m, z = 2 m (z/B = 1; b = 1 m)

$$\frac{\Delta\sigma_z}{q} = \frac{2}{\pi}\left[\tan^{-1}\frac{1}{2} + \frac{1(2)}{1+4}\right] = \frac{2}{\pi}\left[0.4636 + 0.4\right] = 0.5498 \;\Rightarrow\; \Delta\sigma_z = \mathbf{54.98}\ \text{kPa}$$

2 Three widths, three depths (below axis, kPa)

B (m)	z = 2 m	z = 4 m	z = 8 m
2	54.98 (z/B = 1.00)	30.58 (z/B = 2.00)	15.75 (z/B = 4.00)
4	81.83 (z/B = 0.50)	54.98 (z/B = 1.00)	30.58 (z/B = 2.00)
8	95.95 (z/B = 0.25)	81.83 (z/B = 0.50)	54.98 (z/B = 1.00)

The stress depends **only on z/B**: B = 2 m, z = 2 m; B = 4 m, z = 4 m; and B = 8 m, z = 8 m always give **54.98 kPa**. With the same q, a **wider strip acts deeper**: at z = 8 m, 15.8 kPa for B = 2 m and 55.0 kPa for B = 8 m.

Solution (2/2)

SOLUTION

3 Line load solution (P = 100 kN/m; x = 2 m, z = 2 m)

$$\Delta\sigma_z = \frac{2Pz^3}{\pi(x^2+z^2)^2} = \frac{2(100)(2)^3}{\pi(4+4)^2} = \frac{1{,}600}{201.06} = \mathbf{7.958}\ \text{kPa}$$

4 Progressively narrower strips with the same P = qB (closed form, same point)

B (m)	q = P/B (kPa)	$\Delta\sigma_z$ (strip, kPa)	Difference from line load
2.0	50	9.242	%16.1
1.0	100	8.287	%4.1
0.5	200	8.041	%1.0
0.1	1,000	7.961	%0.0
→ **0**	→ ∞	**7.958** (line load)	0

As B decreases, the strip solution approaches the line load value of **7.96 kPa** from **9.24 kPa**; for B = 0.1 m the difference is only 0.04%. The line load is the B → 0 limit of the strip load.

Check the solution in GeoTerva ↗

Summary

Key points

Closed form: $\Delta\sigma_z = (q/\pi)\{\tan^{-1}[z/(x - B/2)] - \tan^{-1}[z/(x + B/2)] - Bz[x^2 - z^2 - B^2/4] / ([x^2 + z^2 - B^2/4]^2 + B^2z^2)\}$; for $x < B/2$, π is added to the first arctangent

Angle form: $\Delta\sigma_z = (q/\pi)[\alpha + \sin\alpha\cos(\alpha + 2\delta)]$; $\alpha = \theta_1 - \theta_2$, $\delta = 90° - \theta_1$. The stress depends only on **x/B and z/B** and never exceeds q; at the surface it is q beneath the strip, q/2 at the edge, and 0 outside

Example ($q = 100$ kN/m², $B = 4$ m; A: $x = 1$, $z = 1$ m): A_1 2.3562, A_2 0.3218, T -0.8 → $\Delta\sigma_z/q$ = **0.9022**; **$\Delta\sigma_z$ = 90.22 kPa**; the angle form is the same

Stress bulb: below the centreline the depths of q/2, q/5, q/10 are 1.13B, 3.13B, 6.34B. **Superposition**: each strip with its own x value; in the two-strip example 21.37 + 7.64 = **29.02 kPa**

Limits: the same z/B gives the same stress; as $B \to 0$ with $P = qB$ constant, it approaches the strip, line load solution

References

Das, B.M. (2018). Principles of Geotechnical Engineering, 9th ed. Cengage Learning.

NOTE

All numerical examples are produced with the module's own calculation engine; the module gives the same values. The module opens with q = 100 kN/m², B = 4 m and A (x = 1 m, z = 1 m).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own strips (q, B, centerline) and point A in GeoTerva; the module provides the stress, isostress contours, depth profile, step-by-step solution and report together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com