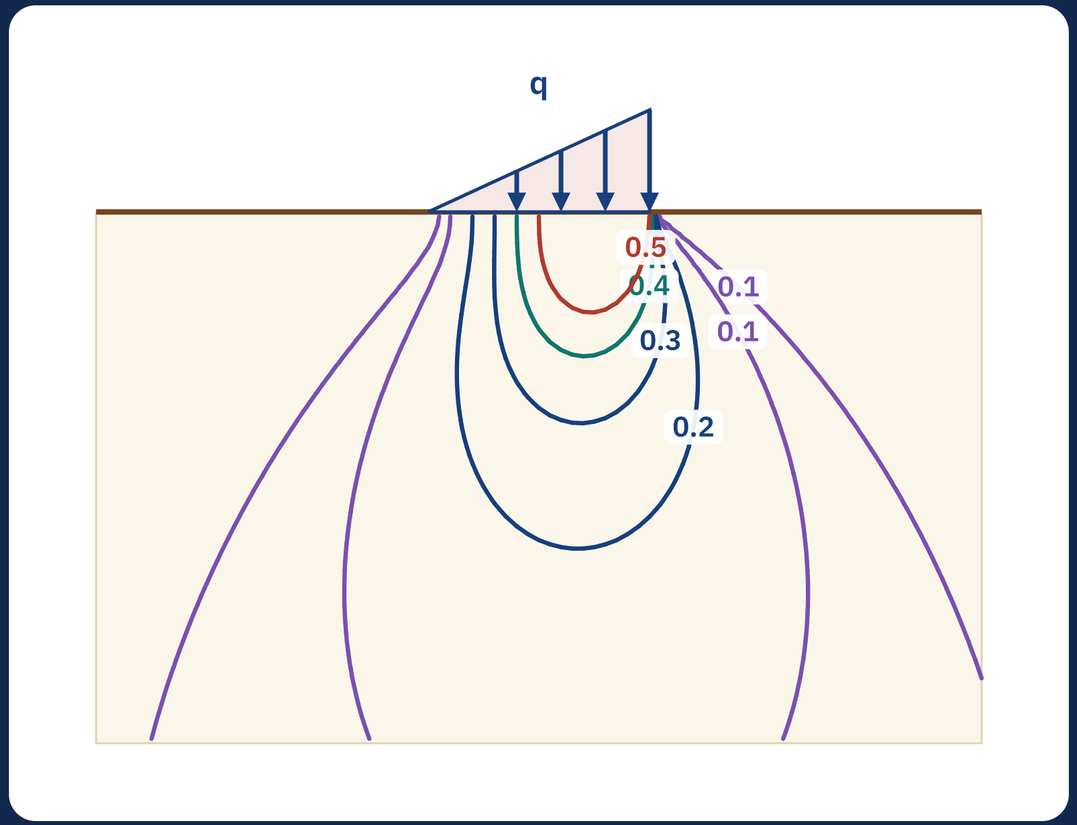
Geotechnical lecture presentation · Stresses in the soil mass

Stress under a triangular strip load

Lecture: triangular strip load increasing from zero to q along one edge; a point manually, profiles, symmetric triangle and embankment.

- Integral of the line load
- Angle form
- Sliding of the crest
- Fill = uniform + triangles
- Solvable by hand



Introduction

What will we learn in this lecture?

Vertical stress increase beneath a strip load **increasing linearly** from zero at one edge to q at the other (embankment slope, stockpile against a wall, toe of a fill): the integral of a line load of intensity (q/B) r dr

The **angle form**: $\Delta\sigma_z = (q/2\pi)[(2x/B)\alpha - \sin 2\delta]$; the definition of α and δ and the **sign** of δ

That the stress at the surface follows the load and at depth spreads out and shifts toward the **toe** into the strip; the **mirror** case with the zero edge on the right

A point load **end-to-end by hand**; with two strips a **symmetrical triangle**; with a uniform strip + two triangles an **embankment** load

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example: q = 100 kN/m², B = 2 m, load zero at x = 0, q at x = B; point A at x = −1 m, z = 1.5 m.

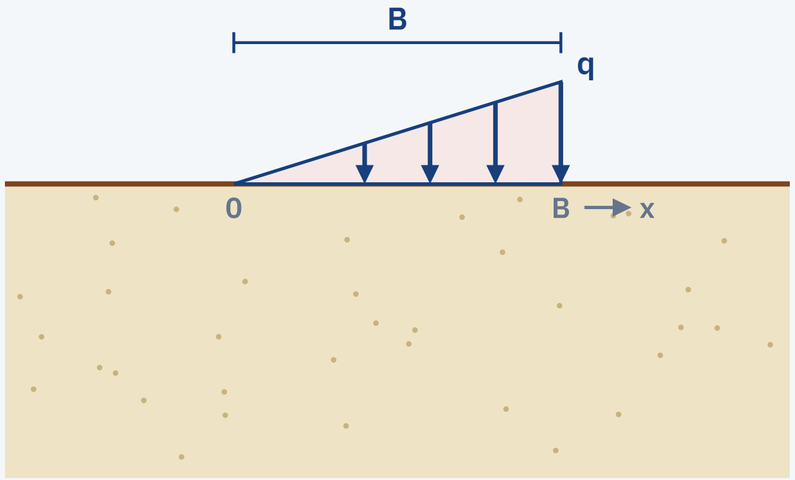
Triangular strip: from zero to q at one edge

THEORY

The slope of a fill, a heap resting against a wall, the toe of an embankment: a strip load whose intensity increases from zero at one edge to the value **q** at the other. Together with a uniform strip load it forms the **fill load**

Flexible strip, width B, infinite length; the load is **q·x/B** in the range $0 \le x \le B$ and zero elsewhere (plane strain)

The soil is a homogeneous, isotropic, linear elastic half-space; $\Delta\sigma_z$ is only the increase caused by the load



Section: the load is zero at x = 0 and q at x = B; the same stress in every section

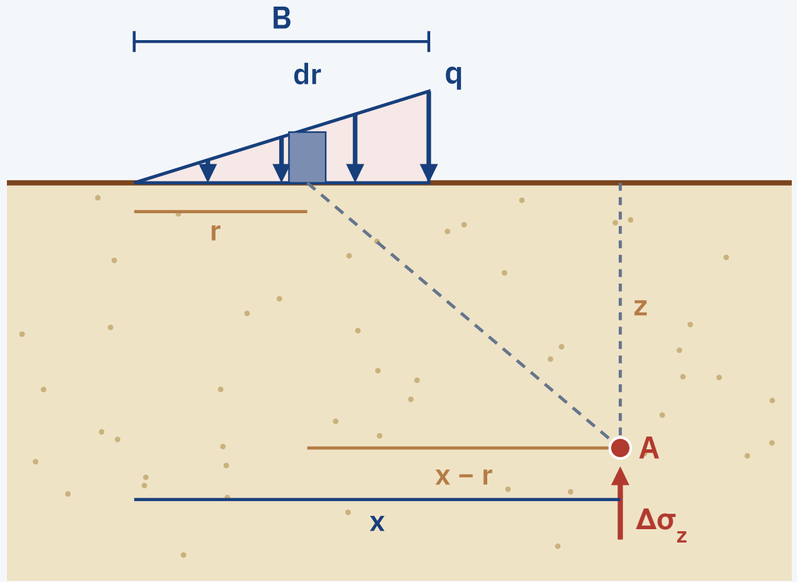
Line load of intensity (q/B) r dr

THEORY

The elementary strip at position r carries a load **(q/B) r dr** per unit length. It is treated as a **line load** at a horizontal distance x − r from point A and summed over the width

$$\Delta\sigma_z = \int_0^B \frac{2\,(q/B)\,r\,dr\,z^3}{\pi\,[(x-r)^2+z^2]^2} = \frac{1}{B}\,\frac{2q}{\pi}\int_0^B \frac{z^3\,r\,dr}{[(x-r)^2+z^2]^2}$$

The integral is **elementary**; written with the angles in the figure, the result is on the next slide. Numerical check (q = 100, B = 2, x = −1, z = 1.5): Simpson value of the integral **4.4699 kPa**, closed form **4.4699 kPa**



$\Delta\sigma_z = (q/2\pi)[(2x/B)\,\alpha - \sin 2\delta]$

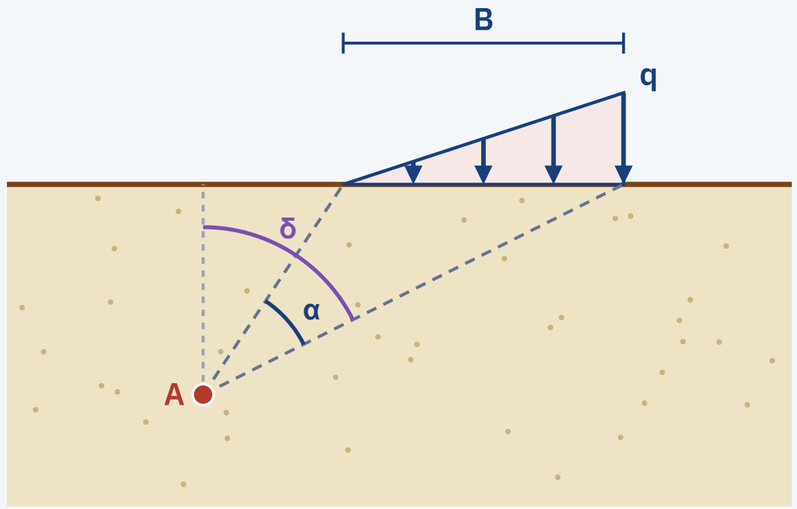
THEORY

$$\Delta\sigma_z = \frac{q}{2\pi}\left(\frac{2x}{B}\,\alpha - \sin 2\delta\right)$$

$$\alpha = \tan^{-1}\frac{x}{z} - \tan^{-1}\frac{x-B}{z} \qquad \delta = \tan^{-1}\frac{x-B}{z}$$

α is in radians. **δ** is the angle from the vertical through A to the line drawn toward the **high edge**. Its sign matters: if the high edge lies on the +x side of A, δ is negative

With arctangents the sign is determined automatically; the result is valid **for every x**: beside, beneath or beyond the strip. x is measured from the **zero edge** toward the loaded edge



A (x = −1, z = 1.5): α and δ (δ negative)

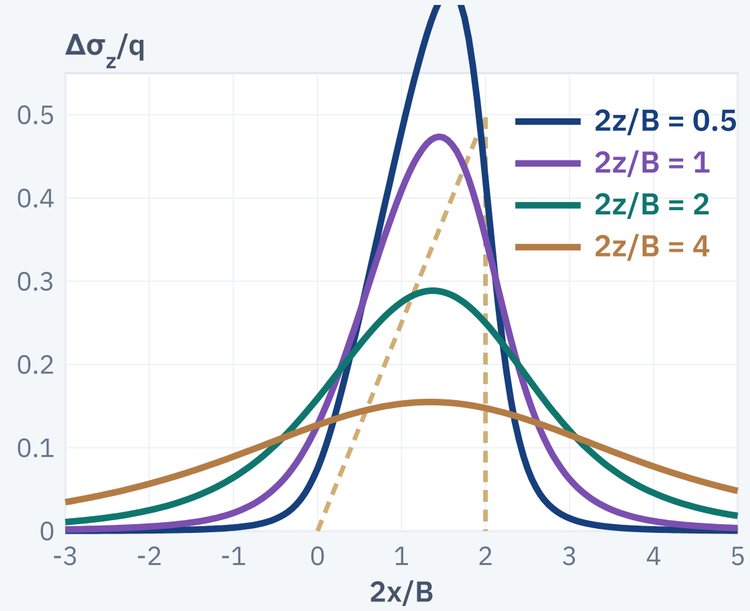
Follows the load at the surface, spreads with depth

THEORY

In the region near the surface, the stress **follows the load itself**; at greater depths the stress **spreads** and its maximum value **shifts toward the interior of the strip**

The peak is at 2x/B = 1.58 for 2z/B = 0.5 (0.65 q); at 2z/B = 4 it is at 2x/B = 1.34 (0.15 q): at great depth it remains beneath the **centroid** of the load (x = 2B/3, i.e. 2x/B = 1.33)

Outside the zero edge (x < 0) the stress is small but not zero; there is no load, but the soil carries stress



Δσz/q — 2x/B (load shown as yellow dashed triangle)

$\Delta\sigma_z/q$ values

THEORY

2z/B \ 2x/B	-2.0	-1.0	0.0	0.5	1.0	1.5	2.0	3.0	4.0
0.25	0.0001	0.0006	0.0392	0.2516	0.4969	0.7272	0.4604	0.0024	0.0003
0.50	0.0008	0.0041	0.0749	0.2574	0.4797	0.6448	0.4220	0.0152	0.0019
1.00	0.0053	0.0217	0.1273	0.2620	0.4092	0.4726	0.3524	0.0622	0.0119
1.50	0.0140	0.0447	0.1528	0.2473	0.3341	0.3598	0.2952	0.1010	0.0285
2.00	0.0249	0.0643	0.1592	0.2235	0.2749	0.2870	0.2500	0.1206	0.0457
3.00	0.0448	0.0854	0.1469	0.1774	0.1979	0.2017	0.1872	0.1259	0.0691
4.00	0.0567	0.0894	0.1273	0.1431	0.1529	0.1545	0.1476	0.1154	0.0775
6.00	0.0622	0.0798	0.0955	0.1010	0.1042	0.1047	0.1024	0.0909	0.0740

$\Delta\sigma_z = q \cdot$ (table value). The $2x/B = -1$, $2z/B = 1.5$ cell is 0.0447: the module's opening example. The load spans $0 \le 2x/B \le 2$; the loaded edge is $2x/B = 2$.

If the zero edge is on the right; multiple strips

THEORY

A strip may also be zero at its **right edge**: this is the **mirror image** of the strip in the example; the same equation is used and x is measured from the **zero edge** toward the higher edge (if the right edge is zero, $x = x_L + B - x_A$)

Since the soil is elastic, the stress of each strip is found separately and summed. Strips cannot overlap (they may meet at an edge); the module accepts at most eight strips, each with its own q, B, left edge and direction

Placing the two triangles side by side with mirrored orientations gives a **symmetric triangular** load; a uniform strip plus two triangles forms the **embankment** load

$$\Delta\sigma_z = \sum_k \Delta\sigma_{z(k)}$$

Problem statement

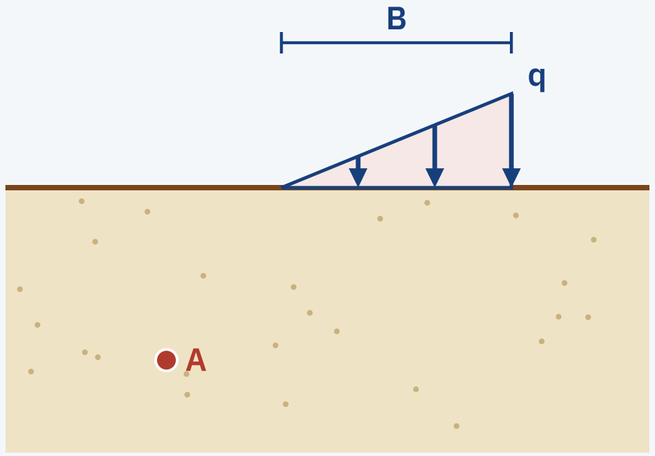
EXAMPLE

Given

Strip load	**Flexible, vertical; zero at $x = 0$, increases linearly to q at $x = B$: $q = 100$ kN/m², $B = 2$ m**
Point A	**$x = -1$ m (1 m outside the zero edge), $z = 1.5$ m**

REQUIRED

The angles **α** and **δ**, **$\Delta\sigma_z/q$** and **$\Delta\sigma_z$**; comparison with the mirror strip for which the zero edge is on the right.



Solution (1/2)

SOLUTION

1 Angles measured from the surface (from A to the high edge and to the zero edge)

$$\alpha_1 = \tan^{-1}\frac{1.5}{3} = 26.57^\circ \qquad \alpha_2 = \tan^{-1}\frac{1.5}{1} = 56.31^\circ$$

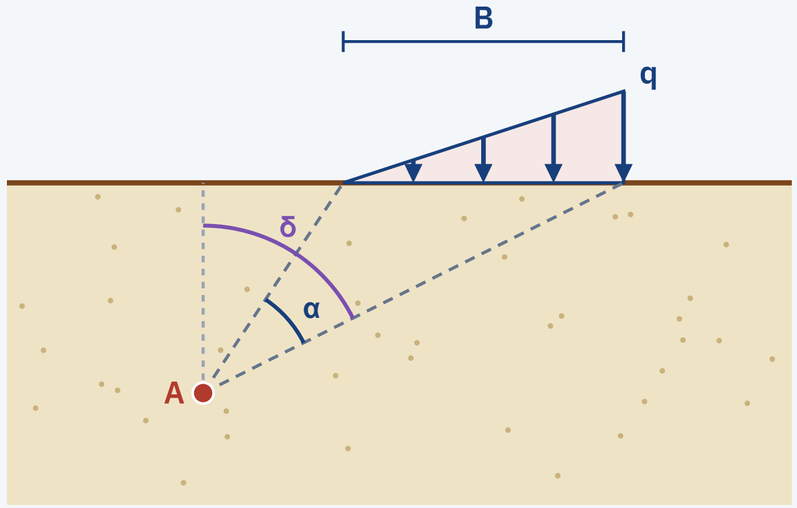
$$\alpha = \alpha_2 - \alpha_1 = 29.74^\circ \qquad \delta = -(90^\circ - \alpha_2 + \alpha) = \mathbf{-63.43^\circ}$$

2 Same angles from the equation (x = −1 m, B = 2 m, z = 1.5 m)

$$\alpha = \tan^{-1}\frac{x}{z} - \tan^{-1}\frac{x-B}{z}$$

$$= \tan^{-1}(-0.6667) - \tan^{-1}(-2) = 29.74^\circ = \mathbf{0.5191}\ \text{rad}$$

$$\delta = \tan^{-1}\frac{x-B}{z} = \tan^{-1}\frac{-3}{1.5} = \mathbf{-63.43^\circ}$$



The two routes give the **same** angles: α = 29.74° (0.5191 rad), δ = -63.43°. δ is **negative** because the higher edge (x = B = 2 m) is on the +x side of A.

Solution (2/2)

SOLUTION

3 Influence value and stress

$$\frac{\Delta\sigma_z}{q} = \frac{1}{2\pi}\left[\frac{2x}{B}\,\alpha - \sin 2\delta\right] = \frac{1}{2\pi}\left[(-1)(0.5191) - \sin(-126.87^\circ)\right]$$

$$= \frac{1}{2\pi}\left[-0.5191 + 0.8000\right] = \mathbf{0.0447} \qquad \Delta\sigma_z = 0.0447\,(100) = \mathbf{4.47}\ \text{kPa}$$

4 Mirror strip: zero edge on the right (left edge 0, B = 2 m), point A at x = 3 m

$$x = x_L + B - x_A = 0 + 2 - 3 = -1\ \text{m} \;\Rightarrow\; \text{same value: } \Delta\sigma_z = \mathbf{4.47}\ \text{kPa}$$

There is no load **1 m outside** the zero edge, yet the stress is **4.47 kPa** (4.5% of q). In the mirror strip the counterpart of A is x = 3 m; the module gives the same result in both cases.

Check the solution in GeoTerva ↗

Problem statement

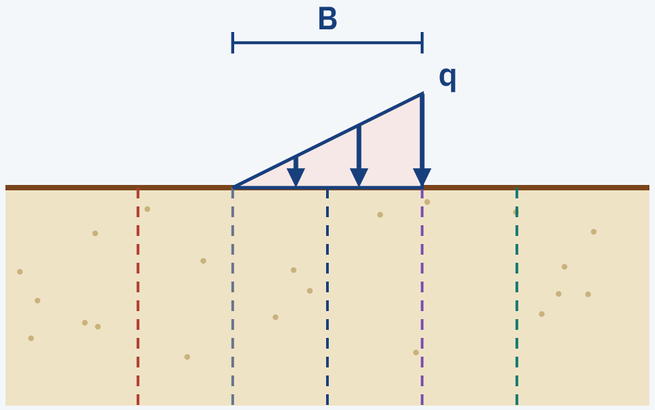
EXAMPLE

Given

Strip load	**Strip in Example 1: $q = 100$ kN/m², $B = 2$ m ($x = 0$ zero, $x = 2$ high edge)**
Hand calculation	**At $z = 1.5$ m, below the high edge ($x = B$) and below the zero edge ($x = 0$)**
Profile	**$x = -1,\ 0,\ 1,\ 2,\ 3$ m lines; $z = 0.5$ … 8 m**

REQUIRED

$\Delta\sigma_z$ by hand at the two edge points; the variation of stress with depth along five lines.



Solution (1/2)

SOLUTION

1 Under the higher edge (x = B = 2 m, z = 1.5 m): δ = 0

$$\alpha = \tan^{-1}\frac{2}{1.5} - \tan^{-1}(0) = \mathbf{0.9273} \qquad \delta = 0 \;\Rightarrow\; \sin 2\delta = 0$$

$$\frac{\Delta\sigma_z}{q} = \frac{1}{2\pi}\left[(2)(0.9273) - 0\right] = 0.2952 \;\Rightarrow\; \Delta\sigma_z = \mathbf{29.52}\ \text{kPa}$$

2 Under the zero edge (x = 0, z = 1.5 m): 2x/B = 0

$$\alpha = \tan^{-1}(0) - \tan^{-1}\frac{-2}{1.5} = 0.9273 \qquad \delta = -0.9273 \;\Rightarrow\; \sin 2\delta = -0.9600$$

$$\frac{\Delta\sigma_z}{q} = \frac{1}{2\pi}\left[(0)(0.9273) - (-0.9600)\right] = 0.1528 \;\Rightarrow\; \Delta\sigma_z = \mathbf{15.28}\ \text{kPa}$$

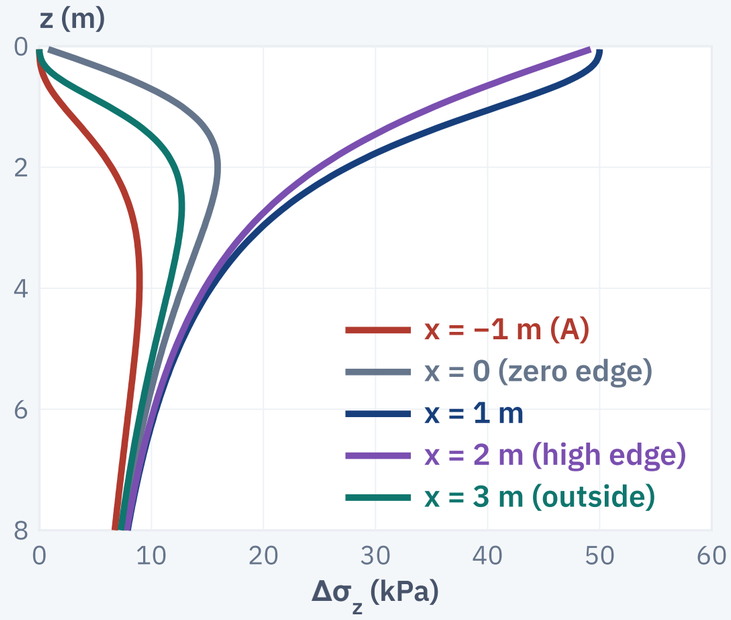
Under the high edge at depth z = 1.5 m, **29.52 kPa** (30% of q); under the zero edge, although the load is zero, there is a stress of **15.28 kPa**: stress exists not only under the load but also around it.

Solution (2/2)

SOLUTION

z (m)	x = −1 m (A)	x = 0 (zero)	x = 1 m	x = 2 m (high)	x = 3 m (outside)
0.5	0.41	7.49	47.97	42.20	1.52
1.0	2.17	12.73	40.92	35.24	6.22
1.5	**4.47**	**15.28**	**33.41**	**29.52**	**10.10**
2.0	6.43	15.92	27.49	25.00	12.06
3.0	8.54	14.69	19.79	18.72	12.59
4.0	8.94	12.73	15.29	14.76	11.54
6.0	7.98	9.55	10.42	10.24	9.09
8.0	6.73	7.49	7.88	7.80	7.27

Beneath the higher edge, the stress starts near **q/2 = 50 kPa** at the surface (42.2 kPa at z = 0.5 m) and decreases with depth; **beneath the zero edge** it first increases and peaks at 15.9 kPa at z ≈ 2 m. Outside the strip (x = −1 m), the value is largest around z ≈ 4 m (8.9 kPa at z = 4 m).



Check the solution in GeoTerva ↗

Problem statement

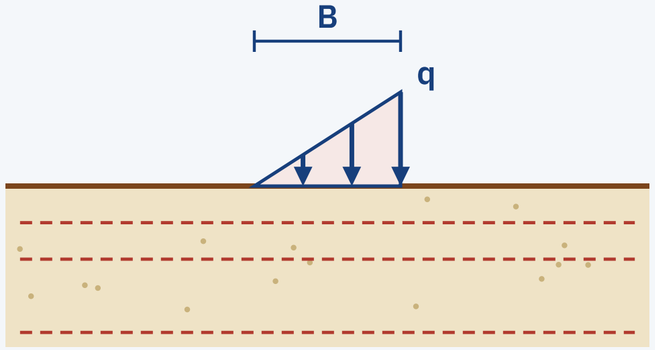
EXAMPLE

Given

Strip load	$q = 100$ **kN/m²,** $B = 2$ **m:** $2x/B = x$ **(m),** $2z/B = z$ **(m)**
Lines	**horizontal lines at depth** $z = 0.5,\ 1,\ 2,\ 4$ **m;** $x = -2$ **...** 4 **m**
Hand calculation	**at** $z = 2$ **m,** $x = 2B/3 = 1.333$ **m (centroid of the load)**

REQUIRED

At every depth, the variation of $\Delta\sigma_z$ along x, **the location and value of the peak**; how the peak shifts with depth.



Solution (1/3)

SOLUTION

z (m)	x = -2 m	x = -1 m	x = 0 m	x = 0.5 m	x = 1 m	x = 2 m	x = 2 m	x = 3 m	x = 4 m
0.5	0.08	0.41	7.49	25.74	47.97	64.48	42.20	1.52	0.19
1	0.53	2.17	12.73	26.20	40.92	47.26	35.24	6.22	1.19
2	2.49	6.43	15.92	22.35	27.49	28.70	25.00	12.06	4.57
4	5.67	8.94	12.73	14.31	15.29	15.45	14.76	11.54	7.75

Near the surface (z = 0.5 m) the stress follows the load and is larger near the loaded edge (about 65 kPa); with increasing depth the curve **spreads** and the peak **decreases** (15.5 kPa at z = 4 m). To the left of the strip (x < 0) the values are small but not zero.

Solution (2/3)

SOLUTION

1 x = 2B/3 = 1,333 m, z = 2 m

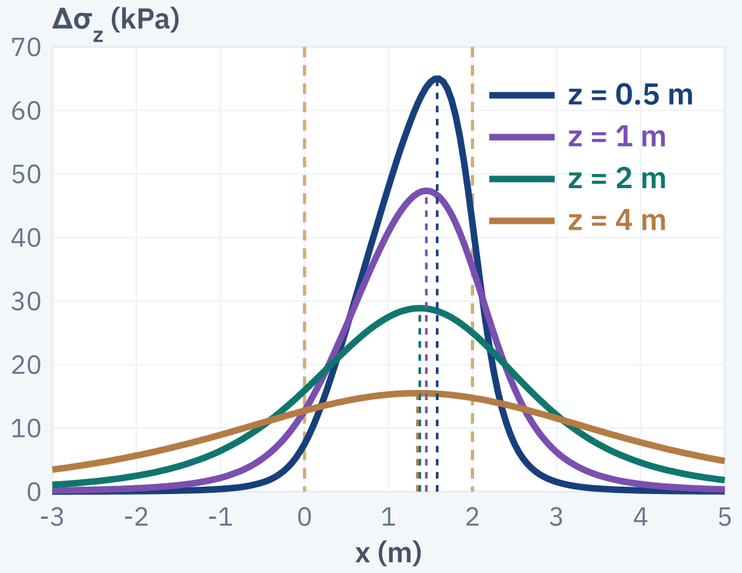
$$\alpha = \tan^{-1}\frac{1.333}{2} - \tan^{-1}\frac{-0.667}{2} = 0.9098$$

$$\delta = -0.3218 \;\Rightarrow\; \sin 2\delta = -0.6000$$

$$\frac{\Delta\sigma_z}{q} = \frac{1}{2\pi}\left[(1.333)(0.9098) - (-0.6000)\right] = 0.2885$$

$$\Delta\sigma_z = \mathbf{28.85}\ \text{kPa}$$

For z = 2 m the peak of the curve is 28.87 kPa; the value below the centroid of the load (28.85 kPa) is very close to it.



Solution (3/3)

SOLUTION

z (m)	2z/B	Peak: x (m)	Peak: 2x/B	Peak: $\Delta\sigma_z$ (kPa)	Crest: $\Delta\sigma_z/q$
0.5	0.5	1.579	1.579	**65.04**	0.6504
1	1	1.450	1.450	**47.36**	0.4736
2	2	1.372	1.372	**28.87**	0.2887
4	4	1.344	1.344	**15.49**	0.1549
8	8	1.336	1.336	**7.90**	0.0790

The peak starts slightly inside the higher edge near the surface ($2x/B \approx 1.58$) and approaches the **centroid of the load** ($2x/B = 1.333$) as depth increases: 1.336 at z = 8 m. The peak value **decreases** with depth: 65.0 kPa → 7.9 kPa.

Check the solution in GeoTerva ↗

Problem statement

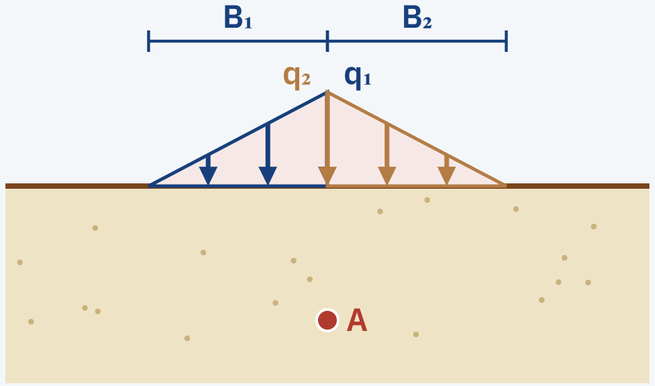
EXAMPLE

Given

1st strip	$q_1 = 100$ **kN/m², $B_1 = 2$ m, left edge $x_{L1} = 0$, zero at the left (0 … 2 m)**
2nd strip	$q_2 = 100$ **kN/m², $B_2 = 2$ m, left edge $x_{L2} = 2$ m, zero on the right (2 … 4 m); the two strips merge at $x = 2$**
Points	$z = 1.5$ **m at $x = 2$ m (below the crest) and $x = 0$ (left zero edge); $x = 0$ … 5 m**

REQUIRED

The sum of $\Delta\sigma_z$ by hand at two points; to show that the stress from a symmetric load is **symmetric**.



Solution (1/2)

SOLUTION

1 Below the crest (x = 2 m): both strips are at their own high edge

$$x_1 = 2 - 0 = 2\ \text{m} \qquad x_2 = x_{L2} + B - x_A = 2 + 2 - 2 = 2\ \text{m}$$

$$\Delta\sigma_{z(1)} = \Delta\sigma_{z(2)} = 29.52\ \text{kPa}\ \ (\text{Example 2}) \ \Rightarrow\ \Delta\sigma_z = \mathbf{59.03}\ \text{kPa}$$

2 Below the left zero edge (x = 0)

$$x_1 = 0 - 0 = 0 \ \Rightarrow\ \Delta\sigma_{z(1)} = 15.28\ \text{kPa}\ \ (\text{Example 2})$$

$$x_2 = 2 + 2 - 0 = 4\ \text{m} \ \Rightarrow\ \Delta\sigma_{z(2)} = 2.85\ \text{kPa}; \qquad \Delta\sigma_z = 15.28 + 2.85 = \mathbf{18.13}\ \text{kPa}$$

Under the peak the stress is **59.03 kPa**, under the left zero edge **18.13 kPa**. The module gives the same values for both strips.

Solution (2/2)

SOLUTION

x (m)	1st strip (kPa)	2nd strip (kPa)	Total (kPa)
0	15.28	2.85	**18.13**
1	33.41	10.10	**43.50**
2	29.52	29.52	**59.03**
3	10.10	33.41	**43.50**
4	2.85	15.28	**18.13**
5	0.97	4.47	**5.44**

The total is **symmetric** about the axis of the embankment (x = 2 m): 18.13 kPa at x = 0 and x = 4 m, 43.50 kPa at x = 1 and x = 3 m. The values of the two strips mirror each other: the value of strip 1 at x equals that of strip 2 at 4 − x.

Check the solution in GeoTerva ↗

Problem statement

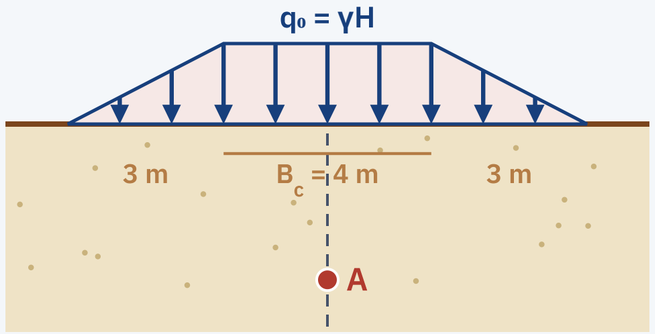
EXAMPLE

Given

Fill	**Crest width $B_c = 4$ m ($x = -2 \ldots 2$), two slopes at 3 m + 3 m horizontal width; $q_0 = \gamma H = 100$ kN/m²**
Weathering	**uniform strip $B = 4$ m (vertical strip solution) + two triangular strips $B = 3$ m (zero at the outer edge)**
Point A	**below the axis $x = 0$, $z = 3$ m**

REQUIRED

The share of the three parts and the **total $\Delta\sigma_z$**; the depth profile and a comparison with the calculation of the same load as a single wide uniform strip.



Solution (1/3)

SOLUTION

1 Uniform (crest) section: x = 0, b = 2 m, z = 3 m

$$\frac{\Delta\sigma_z}{q} = \frac{2}{\pi}\left[\tan^{-1}\frac{b}{z} + \frac{bz}{b^2+z^2}\right] = \frac{2}{\pi}\left[0.5880 + 0.4615\right] = 0.6682 \;\Rightarrow\; \Delta\sigma_{z,c} = \mathbf{66.82}\ \text{kPa}$$

2 Left slope: zero edge x = −5 m, B = 3 m; for A, x = 0 − (−5) = 5 m

$$\alpha = \tan^{-1}\frac{5}{3} - \tan^{-1}\frac{5-3}{3} = 0.4424 \qquad \delta = 0.5880 \;\Rightarrow\; \sin 2\delta = 0.9231$$

$$\frac{\Delta\sigma_z}{q} = \frac{1}{2\pi}\left[(3.333)(0.4424) - (0.9231)\right] = 0.0878 \;\Rightarrow\; \Delta\sigma_{z,s} = \mathbf{8.78}\ \text{kPa}$$

Since the right slope is a mirror, it gives the same value for A: x = x_L + B − x_A = 2 + 3 − 0 = 5 m. Total: 66.82 + 2(8.78) = **84.37 kPa**; the module's uniform strip and triangular solutions give the same value.

Solution (2/3)

SOLUTION

x (m)	Peak (kPa)	Left slope	Right slope	Total
0	66.82	8.78	8.78	**84.37**
1	60.71	4.57	16.15	**81.43**
2	44.80	2.44	25.00	**72.24**
5	7.72	0.51	15.92	**24.15**
8	1.51	0.16	2.49	**4.16**

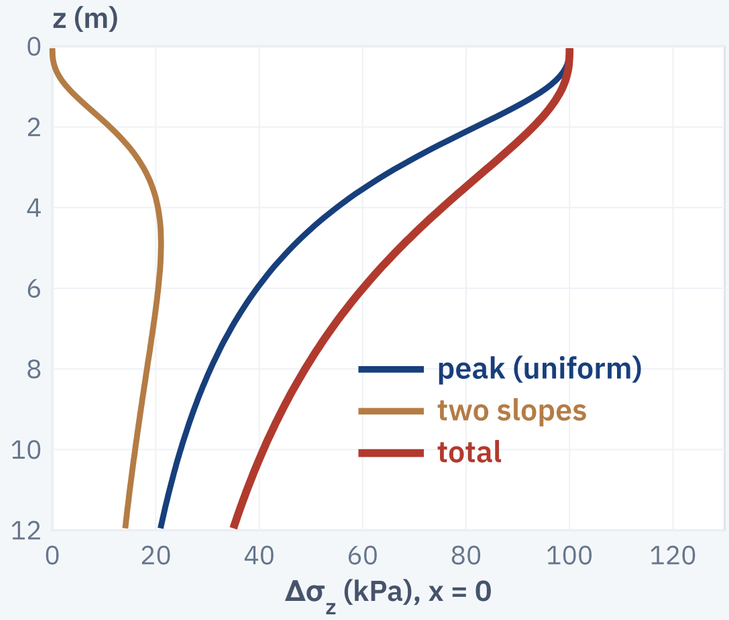
At depth z = 3 m, the stress beneath the embankment is **84.37 kPa** on the axis, **24.15 kPa** at the toe of the slope (x = 5 m), and **4.16 kPa** at 3 m beyond the toe (x = 8 m). At points on the right (x > 0), the right slope dominates; at those on the left, the left slope dominates; the axis of symmetry (x = 0) is taken as the **axis of symmetry**.

Solution (3/3)

SOLUTION

z (m)	Peak (uniform)	Two slopes	Embankment summation	Single strip (B = 10 m)
1	95.95	2.79	**98.73**	99.68
2	81.83	11.13	**92.96**	97.73
3	66.82	17.55	**84.37**	93.68
4	54.98	20.42	**75.40**	88.10
6	39.58	20.48	**60.06**	75.54
8	30.58	18.30	**48.87**	64.17
12	20.84	14.04	**34.88**	47.74

At the surface the stress q_0 is close to 100 kPa; with increasing depth the share of the crest decreases while the **share of the slopes** increases (largest at z = 4 – 6 m, 20.5 kPa). If the same load were a single wide uniform strip (B = 10 m), 75.5 kPa would be obtained at z = 6 m: the three triangular slopes carry half of the load.



Check the solution in GeoTerva ↗

Summary

Key points

Angle form: $\Delta\sigma_z = (q/2\pi)[(2x/B)\,\alpha - \sin 2\delta]$; $\alpha = \tan^{-1}(x/z) - \tan^{-1}[(x - B)/z]$, $\delta = \tan^{-1}[(x - B)/z]$; x is measured from the edge at zero, δ is negative if the higher edge is on the +x side; the result is valid for every x

Example (q = 100 kPa, B = 2 m; A: x = −1, z = 1.5 m): α = 29.74° (0.5191 rad), δ = -63.43° → $\Delta\sigma_z/q$ = **0.0447**; **$\Delta\sigma_z$ = 4.47 kPa**; the same value in the mirrored strip

Edges (z = 1.5 m): 29.52 kPa beneath the high edge, 15.28 kPa beneath the zero edge. The **peak** decreases with depth and shifts toward the centroid of the load (x = 2B/3)

Superposition: two mirrored strip loads symmetric about the axis give a triangle (59.03 kPa below the crest); the uniform strip plus two triangular fills form the embankment (84.37 kPa at z = 3 m below the axis)

References

Das, B.M. (2018). Principles of Geotechnical Engineering, 9th ed. Cengage Learning.

NOTE

All numerical examples were generated with the module's own calculation engine; the module yields the same values. The module opens with q = 100 kN/m², B = 2 m and A (x = –1 m, z = 1.5 m); the strip with its zero edge on the right is the mirror image of this strip.

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own triangular strips (q, B, left edge, direction) and point A in GeoTerva; the module provides the stress, isostress contours, depth profile, step-by-step solution and report together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com