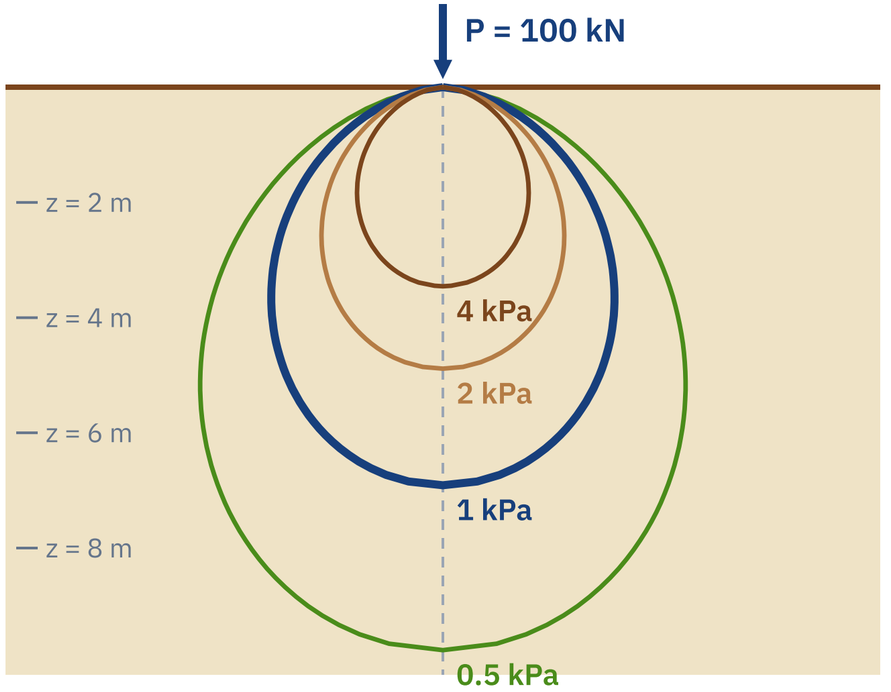
Geotechnical lecture presentation · Stress distribution in soil

Stress under a point load (Boussinesq)

Lecture: stresses produced in the soil by a vertical point load at the surface; the starting point is solved by hand end to end, depth and lateral profiles with tables.

Boussinesq solution | Influence factor I_1 | Horizontal stresses | Superposition | Pressure bulbs



Introduction

What will we learn in this lecture?

The three normal stresses at a point due to a **vertical point load** at the surface of a semi-infinite, homogeneous, isotropic and linear elastic medium: $\Delta\sigma_z$, $\Delta\sigma_x$, $\Delta\sigma_y$

That the vertical stress **does not depend on μ or the modulus of elasticity**; that horizontal stresses depend on μ

Influence factor I_1: $\Delta\sigma_z = (P/z^2)\, I_1$ and the dependence of I_1 on r/z alone

Superposition: adding the stresses of several individual loads; **isostress curves** (pressure bulbs) and $0.4775\ P/z^2$ beneath the load axis

The module's starting point **end to end by hand**; depth and lateral variation, horizontal stresses in tables

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example: P = 5 kN; point A: x = 3 m, y = 4 m, z = 6 m (r = 5 m); μ = 0.5.

Boussinesq semi-infinite medium

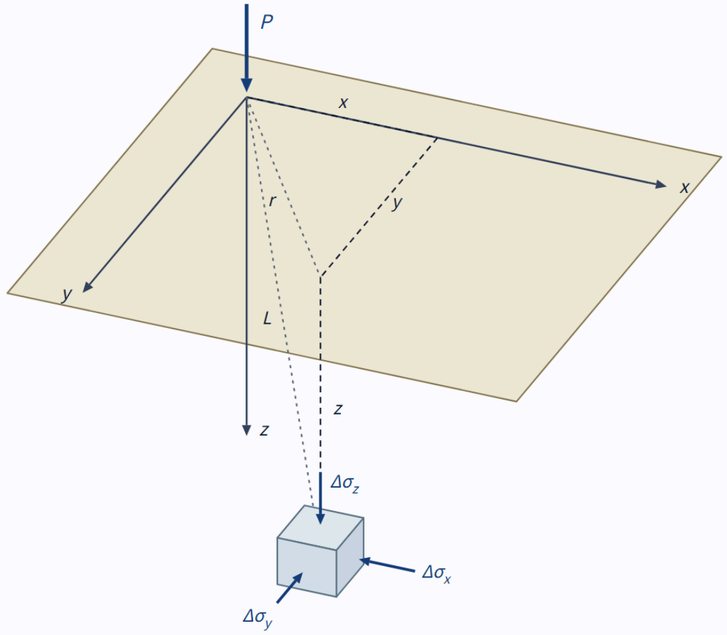
THEORY

Boussinesq (1883) solved the stresses produced by a point load acting on the surface of a semi-infinite medium in a homogeneous, elastic and isotropic medium.

According to the assumptions of the solution, the soil is a **semi-infinite medium**: it extends without limit laterally and downward beneath a plane surface.

The medium is homogeneous, isotropic and linearly elastic; it is defined by **Poisson's ratio** μ, and the elastic modulus does not appear in the stresses.

P is a vertical point load acting on the surface; the stresses found are the increases caused by the load, and the stresses from the soil's own weight are added to them.



Three normal stresses at a point

THEORY

With the coordinates in the figure (**r** = horizontal distance of the point from the load axis, **L** = distance from the load), the normal stresses at the point are:

$$\Delta\sigma_x = \frac{P}{2\pi}\left\{\frac{3x^2 z}{L^5} - (1-2\mu)\left[\frac{x^2-y^2}{L\,r^2(L+z)} + \frac{y^2 z}{L^3 r^2}\right]\right\}$$

$$\Delta\sigma_y = \frac{P}{2\pi}\left\{\frac{3y^2 z}{L^5} - (1-2\mu)\left[\frac{y^2-x^2}{L\,r^2(L+z)} + \frac{x^2 z}{L^3 r^2}\right]\right\}$$

$$\Delta\sigma_z = \frac{3P}{2\pi}\,\frac{z^3}{L^5} = \frac{3P}{2\pi}\,\frac{z^3}{(r^2+z^2)^{5/2}} \qquad r = \sqrt{x^2+y^2},\;\; L = \sqrt{r^2+z^2}$$

Effect of Poisson's ratio

THEORY

Horizontal stresses depend on Poisson's ratio μ; **vertical stress**, however, depends on neither μ nor the modulus of elasticity.

For μ = 0.5 (no volume change, undrained clay) the bracketed terms vanish and $\Delta\sigma_x = 3P\, x^2 z/(2\pi L^5)$, $\Delta\sigma_y = 3P\, y^2 z/(2\pi L^5)$.

On the load axis (r = 0) both horizontal stresses approach $-P(1-2\mu)/(4\pi z^2)$; this is a small reduction.

$$\mu = 0.5: \qquad \Delta\sigma_x = \frac{3P\,x^2 z}{2\pi L^5}, \qquad \Delta\sigma_y = \frac{3P\,y^2 z}{2\pi L^5}$$

$$r = 0: \qquad \Delta\sigma_x = \Delta\sigma_y = -\frac{P\,(1-2\mu)}{4\pi z^2}$$

$\Delta\sigma_z = (P/z^2)\, I_1$

THEORY

Since $L^2 = z^2[(r/z)^2 + 1]$, the equation can be written with a coefficient depending only on r/z.

The influence factor I_1 depends only on r/z; therefore a single table or curve is used for every P and every z.

$$L^2 = z^2 \left[(r/z)^2 + 1\right]$$

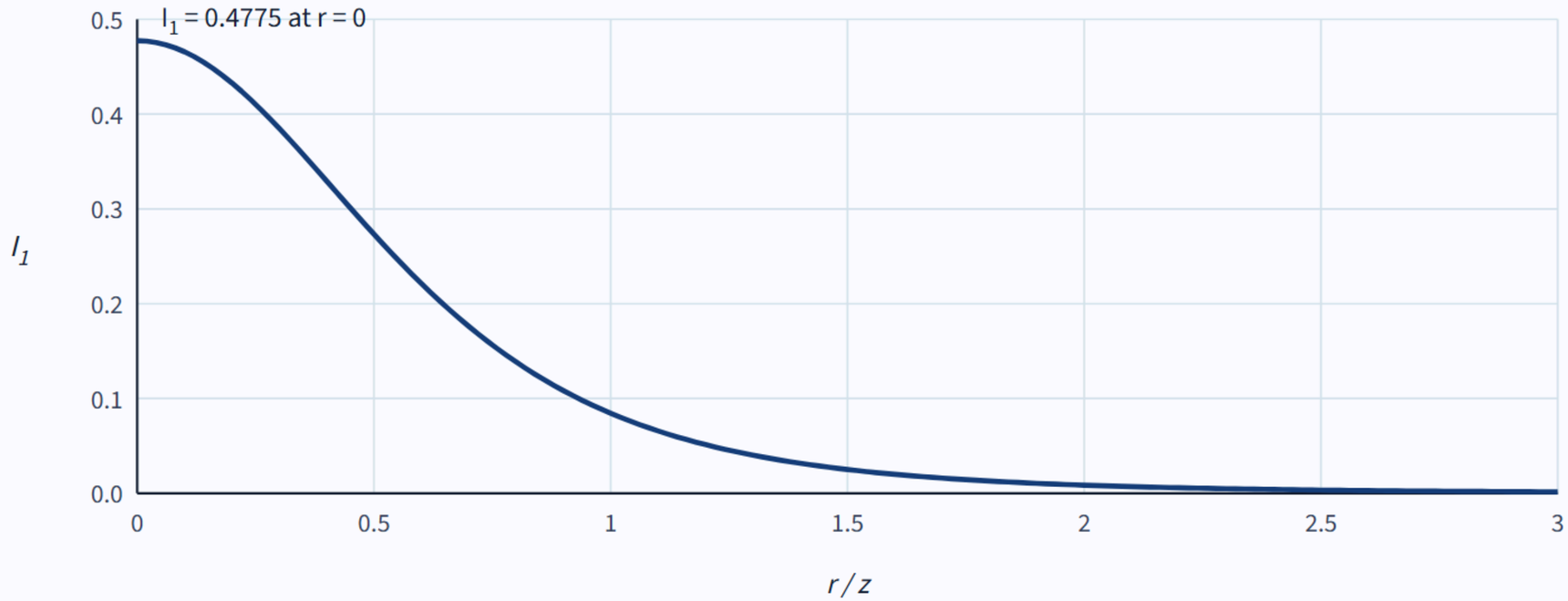
$$\Delta\sigma_z = \frac{P}{z^2} I_1 \qquad I_1 = \frac{3}{2\pi} \frac{1}{\left[(r/z)^2 + 1\right]^{5/2}}$$

I_1 VALUES (MODULE)

r/z	I_1	r/z	I_1
0.0	0.4775	1.0	0.0844
0.1	0.4657	1.2	0.0513
0.2	0.4329	1.4	0.0317
0.3	0.3849	1.6	0.0200
0.4	0.3295	1.8	0.0129
0.5	0.2733	2.0	0.0085
0.6	0.2214	2.5	0.0034
0.7	0.1762	3.0	0.0015
0.8	0.1386	4.0	0.0004
0.9	0.1083	5.0	0.0001

I_1 and r/z

THEORY



The influence factor I1 against r/z. It is largest on the load axis, 3/(2π) = 0.4775, and falls to 0.0844 at r/z = 1 and below 0.002 beyond r/z = 3.

Superposition

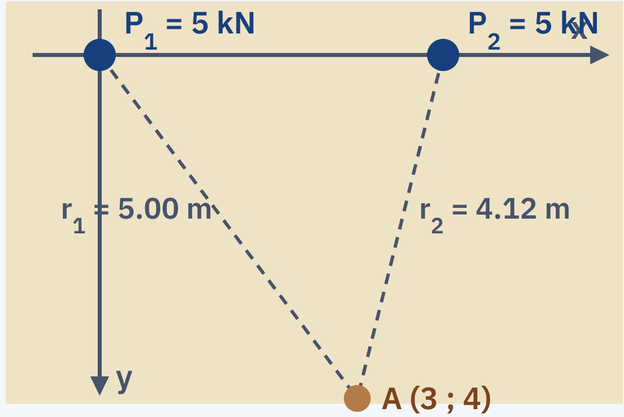
THEORY

For multiple point loads on a linear elastic soil, stresses are summed by superposition: $\Delta\sigma_z = \Sigma\Delta\sigma_{z,k}$, $\Delta\sigma_x = \Sigma\Delta\sigma_{x,k}$, $\Delta\sigma_y = \Sigma\Delta\sigma_{y,k}$.

Each load is considered separately and the point is taken at a distance ($x - x_{L,k}$, $y - y_{L,k}$) from it; thus r_k and L_k are the load's own values.

$$\Delta\sigma_z = \sum_k \frac{3P_k}{2\pi}\,\frac{z^3}{L_k^5}, \qquad L_k = \sqrt{(x - x_{L,k})^2 + (y - y_{L,k})^2 + z^2}$$

The module accepts at most eight loads and no two may be at the same location; the equal-stress contours in the section view are drawn in the vertical plane $y = y_A$.



Stress bulbs of a point load

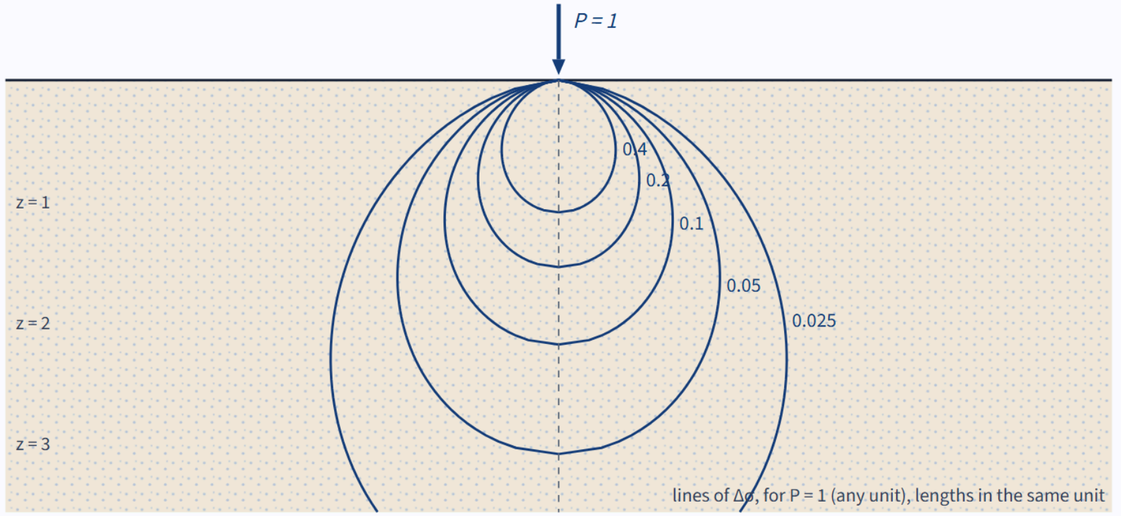
THEORY

By joining the points with equal $\Delta\sigma_z$ values, onion-shaped isostress contours are obtained.

Along the load axis the stress decreases with the square of the depth: $\Delta\sigma_z = 0.4775\ P/z^2$.

In the region very close to the load the solution gives very large stresses: a real load is never a point.

$$r = \sqrt{\left(\frac{3P\,z^3}{2\pi\,\Delta\sigma_z}\right)^{2/5} - z^2}$$



Problem statement

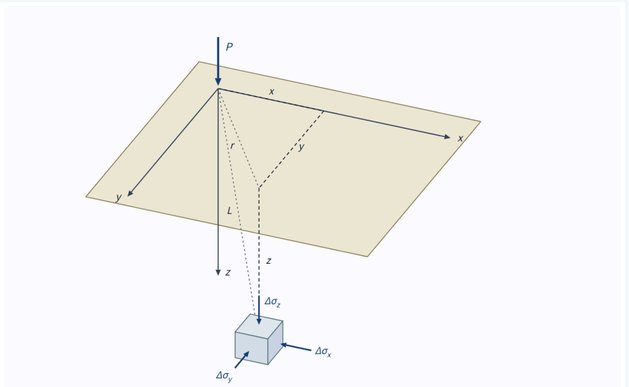
EXAMPLE

Given

Load	**vertical point load $P = 5$ kN, at the surface, at the coordinate origin**
Point A	$x = 3$ m, $y = 4$ m, $z = 6$ m
Soil	**homogeneous, elastic semi-infinite medium; $\mu = 0.5$**

REQUIRED

r, **L**, **r/z**, influence factor $\mathbf{I_1}$ and vertical stress increase $\mathbf{\Delta\sigma_z}$; direct check of the result with the equation.



Solution (1/2)

SOLUTION

1 Horizontal distance r, distance to load L and r/z

$$r = \sqrt{3^2 + 4^2} = \mathbf{5\ m} \qquad L = \sqrt{r^2 + z^2} = \sqrt{5^2 + 6^2} = \mathbf{7.810\ m}$$

$$r/z = 5/6 = \mathbf{0.8333}$$

2 Influence factor I_1

$$I_1 = \frac{3}{2\pi} \cdot \frac{1}{\left[(r/z)^2 + 1\right]^{5/2}} = \frac{0.47746}{\left[0.8333^2 + 1\right]^{2.5}}$$

$$I_1 = \frac{0.47746}{1.6944^{2.5}} = \frac{0.47746}{3.7374} = \mathbf{0.12775}$$

The point is **5 m** from the load axis and **6 m** deep; since r/z = 0.8333, I_1 is 0.27 times the value under the load axis (0.4775).

Solution (2/2)

SOLUTION

3 Vertical stress increase

$$\Delta\sigma_z = \frac{P}{z^2} I_1 = \frac{5}{6^2}\,(0.12775) = 0.13889\,(0.12775) = \mathbf{0.01774}\ \text{kPa}$$

4 Check: direct Boussinesq equation

$$\Delta\sigma_z = \frac{3P}{2\pi}\frac{z^3}{L^5} = \frac{3(5)(6^3)}{2\pi\,(7.810)^5}$$

$$= \frac{3{,}240}{2\pi\,(29{,}062)} = \mathbf{0.01774}\ \text{kPa}$$

$\Delta\sigma_z$ = 0.01774 kPa (I_1 = 0.12775). Since the load is small, the stress is also small; the module gives the same value at this point.

Check the solution in GeoTerva ↗

Problem statement

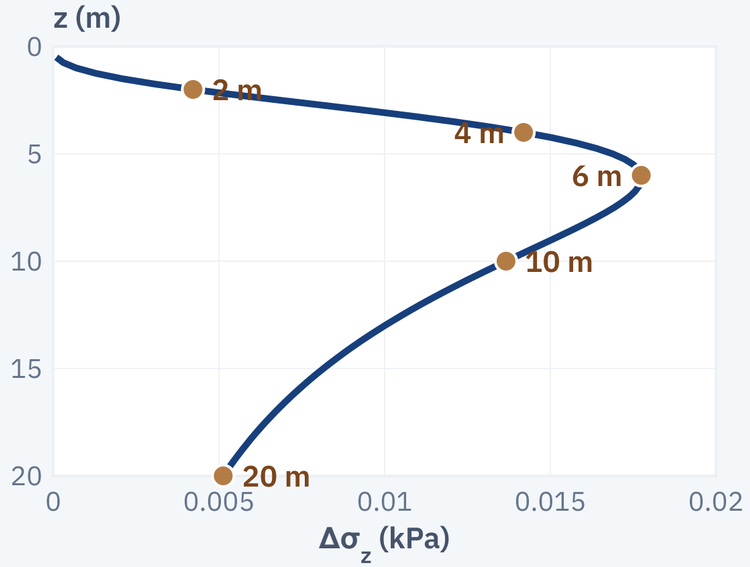
EXAMPLE

Given

Load and point	$P = 5$ **kN;** $x = 3$ **m,** $y = 4$ **m, hence** $r = 5$ **m (the same at every depth)**
Depths	$z = 2; 4; 6; 10; 20$ **m**

REQUIRED

For each depth, **r/z**, $\mathbf{I_1}$ and $\mathbf{\Delta\sigma_z}$; comparison with $0.4775\ P/z^2$ under the load axis; the depth at which the stress is largest.



Solution (1/2)

SOLUTION

1 z = 2 m (r = 5 m): r/z = 2.5

$$I_1 = \frac{0.47746}{\left[2.50^2 + 1\right]^{2.5}} = \frac{0.47746}{141.529} = 0.00337$$

$$\Delta\sigma_z = \frac{5}{2^2}\,(0.00337) = \mathbf{0.00422}\ \mathrm{kPa}$$

2 z = 4 m (r = 5 m): r/z = 1.25

$$I_1 = \frac{0.47746}{\left[1.25^2 + 1\right]^{2.5}} = \frac{0.47746}{10.511} = 0.04542$$

$$\Delta\sigma_z = \frac{5}{4^2}\,(0.04542) = \mathbf{0.01419}\ \mathrm{kPa}$$

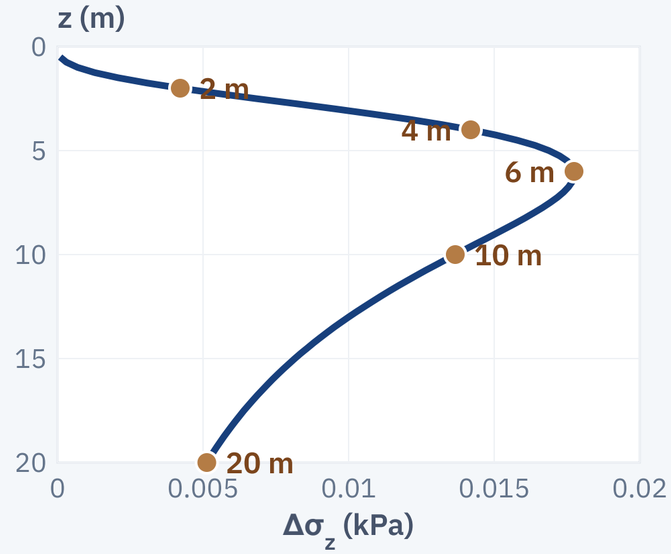
For the same r = 5 m, as z increases, r/z decreases and I_1 increases; but P/z² decreases. The product first increases, then decreases: 0.00422 at z = 2 m and 0.01419 kPa at z = 4 m.

Solution (2/2)

SOLUTION

z (m)	r/z	I_1	$\Delta\sigma_z$ (kPa)	0.4775 P/z² (kPa)
2	2.500	0.0034	0.00422	0.59683
4	1.250	0.0454	0.01419	0.14921
6	0.833	0.1278	**0.01774**	0.06631
10	0.500	0.2733	0.01367	0.02387
20	0.250	0.4103	0.00513	0.00597

The maximum stress occurs at depth z = r√(3/2) = **6.12 m** (0.01775 kPa); z = 6 m is very close to this depth (the module's opening point). At 5 m from the axis, the stress at shallow depths is much smaller than that beneath the load axis and approaches it as depth increases (z = 20 m: 0.00513 and 0.00597 kPa).



Check the solution in GeoTerva ↗

Problem statement

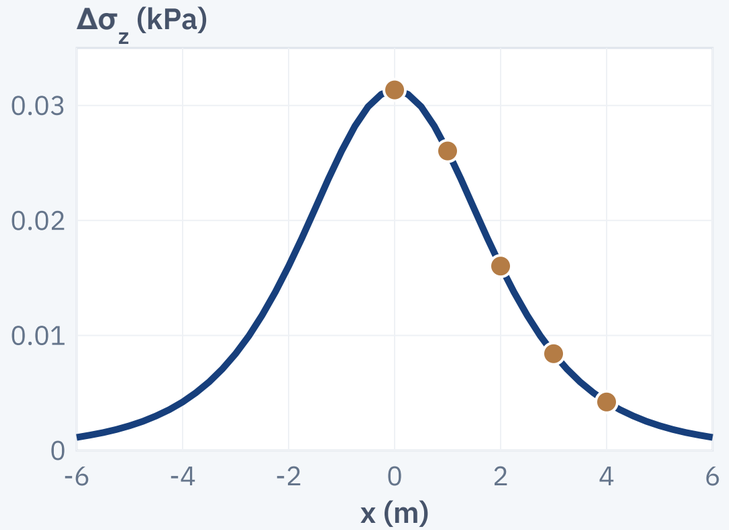
EXAMPLE

Given

Load	$P = 5$ **kN, at the coordinate origin**
Points	$z = 2$ **m,** $y = 3$ **m;** $x = 0, 1, 2, 3, 4$ **m**
Soil	$\mu = 0.5$ **(vertical stress independent of** μ**)**

REQUIRED

For each point, **r**, **r/z**, $\mathbf{I_1}$ and $\mathbf{\Delta\sigma_z}$; the ratio of the value under the load (x = 0) to the farthest value (x = 4 m).



Solution (1/2)

SOLUTION

1 Horizontal distance and r/z (x = 2 m, y = 3 m)

$$r = \sqrt{x^2 + y^2} = \sqrt{2^2 + 3^2} = \sqrt{13} = \mathbf{3.606}\ \text{m}$$

$$\frac{r}{z} = \frac{3.606}{2} = \mathbf{1.8028} \qquad \left(\frac{r}{z}\right)^2 + 1 = 4.250$$

2 Influence factor and stress

$$I_1 = \frac{0.47746}{4.250^{2.5}} = \frac{0.47746}{37.237} = 0.01282$$

$$\Delta\sigma_z = \frac{5}{2^2}\,(0.01282) = \mathbf{0.01603}\ \text{kPa}$$

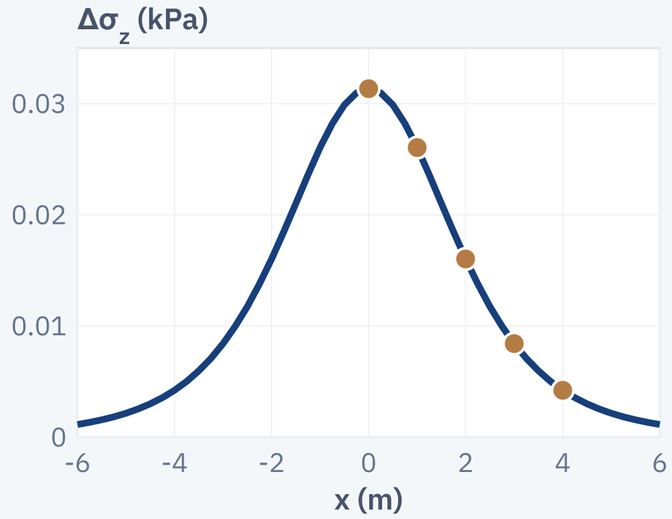
At x = 2 m, r = 3.606 m, r/z = 1.803, **$\Delta\sigma_z$ = 0.01603 kPa**. The other x values are found with the same three steps (table on the next page).

Solution (2/2)

SOLUTION

x (m)	r (m)	r/z	I_1	$\Delta\sigma_z$ (kPa)
0	3.000	1.500	0.0251	0.03134
1	3.162	1.581	0.0208	0.02604
2	3.606	1.803	0.0128	**0.01603**
3	4.243	2.121	0.0067	0.00841
4	5.000	2.500	0.0034	0.00422

At the point under the load (x = 0, r = 3 m) **0.03134 kPa**; at x = 4 m (r = 5 m) **0.00422 kPa**: ratio 7.4. At the same depth the stress decreases rapidly as the horizontal distance increases; the curve is highest around x = 0 (since y = 3 m, the load is still 3 m away).



Check the solution in GeoTerva ↗

Problem statement

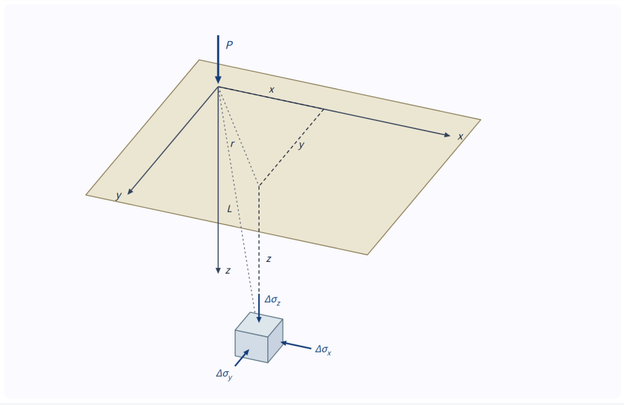
EXAMPLE

Given

Load and point	$P = 5$ **kN; A:** $x = 3$ **m,** $y = 4$ **m,** $z = 6$ **m (** $r = 5$ **m,** $L = 7.810$ **m)**
Poisson's ratio	$\mu = 0.5; 0.3; 0.2; 0$

REQUIRED

Δσ$_x$ and **Δσ$_y$** (by hand with the short equation for μ = 0.5, with the full equation for μ = 0.3); the effect of μ on vertical stress.



Solution (1/3)

SOLUTION

1 L^5 (L = 7,810 m)

$$L^5 = 61^{2.5} = 29{,}062\ \mathrm{m}^5 \qquad 2\pi L^5 = 182{,}602$$

2 μ = 0.5: the terms in the square brackets vanish

$$\Delta\sigma_x = \frac{3P\,x^2 z}{2\pi L^5} = \frac{3(5)(3^2)(6)}{182{,}602} = \frac{810}{182{,}602} = \mathbf{0.00444}\ \mathrm{kPa}$$

$$\Delta\sigma_y = \frac{3P\,y^2 z}{2\pi L^5} = \frac{3(5)(4^2)(6)}{182{,}602} = \frac{1{,}440}{182{,}602} = \mathbf{0.00789}\ \mathrm{kPa}$$

Check: for μ = 0.5, $\Delta\sigma_x + \Delta\sigma_y = 3P\ r^2 z/(2\pi L^5) = 0.00444 + 0.00789 =$ **0.01232 kPa**; both are the values given by the module.

Solution (2/3)

SOLUTION

3 Terms of the full equation (x = 3, y = 4, z = 6, r² = 25, L = 7.810)

$$\frac{3x^2 z}{L^5} = \frac{162}{29{,}062} = 0.005574 \qquad \frac{3y^2 z}{L^5} = \frac{288}{29{,}062} = 0.009910$$

$$\frac{x^2 - y^2}{L\,r^2(L+z)} = \frac{-7}{2{,}696.5} = -0.002596 \qquad \frac{y^2 z}{L^3 r^2} = \frac{96}{11{,}910.6} = 0.008060$$

$$\frac{x^2 z}{L^3 r^2} = \frac{54}{11{,}910.6} = 0.004534$$

4 μ = 0.3: 1 − 2μ = 0.4

$$\Delta\sigma_x = \frac{5}{2\pi}\left[0.005574 - 0.4\,(-0.002596 + 0.008060)\right] = \mathbf{0.00270}\ \text{kPa}$$

$$\Delta\sigma_y = \frac{5}{2\pi}\left[0.009910 - 0.4\,(0.002596 + 0.004534)\right] = \mathbf{0.00562}\ \text{kPa}$$

Solution (3/3)

SOLUTION

μ	$\Delta\sigma_x$ (10^{-3} kPa)	$\Delta\sigma_y$ (10^{-3} kPa)	$\Delta\sigma_z$ (10^{-3} kPa)
0.5	4.436	7.886	17.744
0.3	2.697	5.617	17.744
0.2	1.827	4.482	17.744
0.0	0.088	2.212	17.744

$\Delta\sigma_z$ does not depend on μ ($17.744 \cdot 10^{-3}$ kPa is the same in every row); the horizontal stresses decrease as μ decreases: $\Delta\sigma_x = 4.436$ at $\mu = 0.5$, $2.697 \cdot 10^{-3}$ kPa at $\mu = 0.3$. At this point all three stresses are compressive.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Load	$P = 100$ **kN, vertical point load**
Points	**on the load axis,** $z = 1; 2; 3; 5; 10$ **m**
Soil	$\mu = 0.3$

REQUIRED

For each depth, $\Delta\sigma_z$ ($I_1 = 3/(2\pi)$ for r = 0) and the horizontal stresses $\Delta\sigma_x = \Delta\sigma_y$.

Solution (1/2)

SOLUTION

1 Vertical stress (r = 0 ⇒ r/z = 0 ⇒ I_1 = 3/(2π) = 0.4775)

$$\Delta\sigma_z = \frac{P}{z^2} I_1 = \frac{100}{2^2} (0.4775) = \mathbf{11.937}\ \text{kPa}$$

2 Horizontal stresses (limit value for r = 0, μ = 0.3)

$$\Delta\sigma_x = \Delta\sigma_y = -\frac{P(1-2\mu)}{4\pi z^2} = -\frac{100\,(0.4)}{4\pi\,(2^2)} = \mathbf{-0.796}\ \text{kPa}$$

at z = 2 m on the axis **$\Delta\sigma_z$ = 11.937 kPa**; horizontal stresses are **negative** (–0.796 kPa): a small decrease. It is about 15 times smaller than the vertical stress.

Solution (2/2)

SOLUTION

z (m)	$\Delta\sigma_z = 0.4775\ P/z^2$ (kPa)	$\Delta\sigma_x = \Delta\sigma_y$ (kPa)	$\Delta\sigma_z$ ratio (z = 1 m = 1)
1	47.746	−3.183	1.0000
2	11.937	−0.796	0.2500
3	5.305	−0.354	0.1111
5	1.910	−0.127	0.0400
10	0.477	−0.032	0.0100

When the depth **doubles, the stress falls to one quarter** (z = 1 → 2 m: 47.75 → 11.94 kPa). Since the stresses are increments, they are added to the stress from the soil's own weight: for example, if γ = 18 kN/m³, then at z = 2 m σ_{v0} = 36 kPa and the vertical stress after loading becomes 47.94 kPa.

Check the solution in GeoTerva ↗

Problem statement

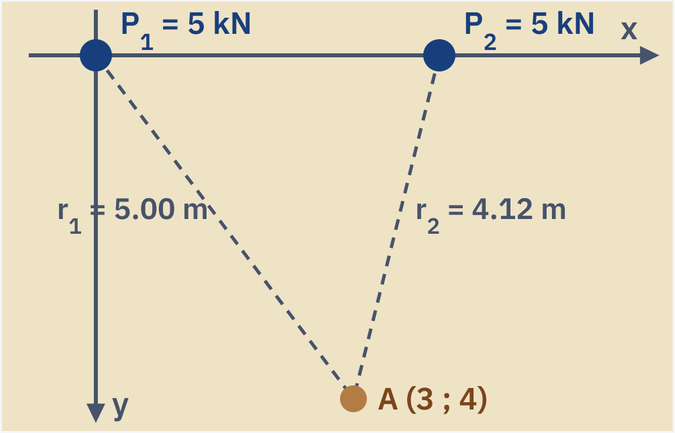
EXAMPLE

Given

Loads	$P_1 = 5$ **kN at point** $(0; 0)$**;** $P_2 = 5$ **kN at point** $(4; 0)$
Point A	$x = 3$ **m,** $y = 4$ **m; depth** $z = 6$ **m (then** $2; 4; 10; 20$ **m)**
Soil	$\mu = 0.5$

REQUIRED

The **$\Delta\sigma_z$** value of each load and their sum; the total **$\Delta\sigma_x$**, **$\Delta\sigma_y$**; the depth profile.



Solution (1/3)

SOLUTION

1 Load 1 (at the start): identical to Example 1

$$r_1 = 5\ \text{m}, \quad \frac{r_1}{z} = 0.8333, \quad I_{1,1} = 0.12775 \ \Rightarrow\ \Delta\sigma_{z,1} = \frac{5}{36}(0.12775) = \mathbf{0.01774}\ \text{kPa}$$

2 Load 2 (at point (4 ; 0)): distance of A from it is (–1 ; 4) m

$$r_2 = \sqrt{(3-4)^2 + 4^2} = \sqrt{17} = 4.123\ \text{m}, \qquad \frac{r_2}{z} = 0.6872$$

$$I_{1,2} = \frac{0.47746}{\left[0.6872^2 + 1\right]^{2.5}} = \frac{0.47746}{2.6299} = 0.18155$$

$$\Delta\sigma_{z,2} = \frac{5}{6^2}\,(0.18155) = \mathbf{0.02522}\ \text{kPa}$$

Since load 2 is closer to A (4.12 m) than load 1 (5 m), it produces a larger stress.

Solution (2/3)

SOLUTION

3 Vertical stress: superposition

$$\Delta\sigma_z = \Delta\sigma_{z,1} + \Delta\sigma_{z,2} = 0.01774 + 0.02522 = \mathbf{0.04296}\ \text{kPa}$$

4 Horizontal stresses (μ = 0.5): for load 2, x = −1, y = 4 (from its own distance)

$$\Delta\sigma_{x,2} = \frac{3(5)(1)(6)}{2\pi\,(53)^{2.5}} = 0.000700 \qquad \Delta\sigma_{y,2} = \frac{3(5)(16)(6)}{2\pi\,(53)^{2.5}} = 0.011207$$

$$\Delta\sigma_x = 0.004436 + 0.000700 = \mathbf{0.00514} \qquad \Delta\sigma_y = 0.007886 + 0.011207 = \mathbf{0.01909}\ \text{kPa}$$

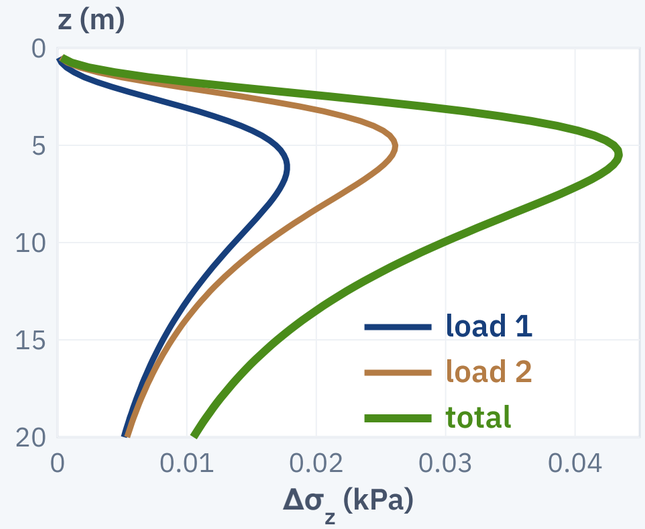
$\Delta\sigma_z$ = 0.04296 kPa, $\Delta\sigma_x$ = 0.00514, $\Delta\sigma_y$ = 0.01909 kPa; in a linear elastic soil the stresses from the loads are summed.

Solution (3/3)

SOLUTION

z (m)	r_1/z	$\Delta\sigma_{z,1}$	r_2/z	$\Delta\sigma_{z,2}$	Total $\Delta\sigma_z$ (kPa)
2	2.500	0.00422	2.062	0.00945	0.01367
4	1.250	0.01419	1.031	0.02442	0.03862
6	0.833	0.01774	0.687	0.02522	**0.04296**
10	0.500	0.01367	0.412	0.01612	0.02979
20	0.250	0.00513	0.206	0.00538	0.01051

At every depth the total is the sum of the two loads; the maximum total is around z ≈ 5 m (z = 4 m: 0.03862, z = 6 m: 0.04296 kPa). Load 2 contributes more than load 1 at every depth.



Check the solution in GeoTerva ↗

Problem statement

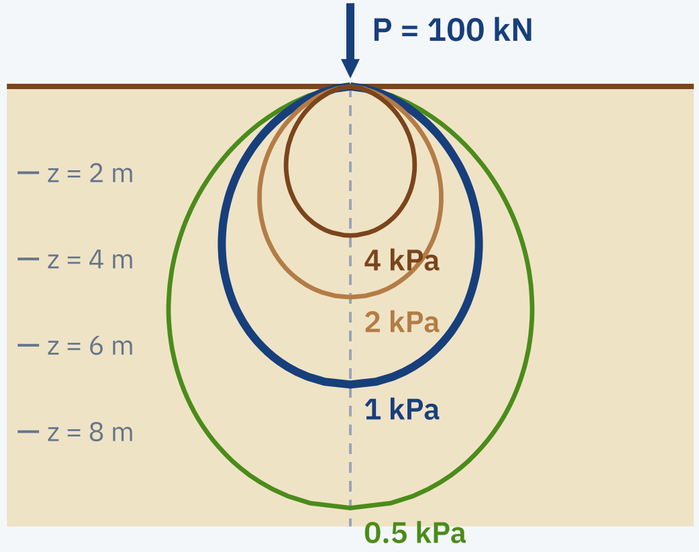
EXAMPLE

Given

Load	$P = 100$ **kN, vertical point load**
Line	**Points where** $\Delta\sigma_z = 1$ **equals kPa;** $z = 1 - 6$ **m**
Additional lines	0.5**;** 2**;** 4 **kPa (figure)**

REQUIRED

At each depth, the distance of the contour from the load axis is **r**; the depth at which the contour closes on the axis; the check that the value of r actually gives $\Delta\sigma_z$ = 1 kPa.



Solution (1/2)

SOLUTION

1 Solving the equation for r: $\Delta\sigma_z = 3P\, z^3/[2\pi(r^2 + z^2)^{5/2}] = 1$

$$r = \sqrt{\left(\frac{3(100)(4^3)}{2\pi\,(1)}\right)^{0.4} - 4^2} = \sqrt{(3{,}055.8)^{0.4} - 16}$$

$$r = \sqrt{24.777 - 16} = \mathbf{2.963}\ \mathrm{m}$$

2 Check: Boussinesq equation with the found r

$$\Delta\sigma_z = \frac{3(100)}{2\pi}\,\frac{4^3}{(2.963^2 + 4^2)^{5/2}} = \mathbf{1.000}\ \mathrm{kPa}$$

At z = 4 m the 1 kPa contour is **2.963 m** from the load axis; substituting the r value found into the Boussinesq equation gives exactly $\Delta\sigma_z$ = 1 kPa (other depths on the next page).

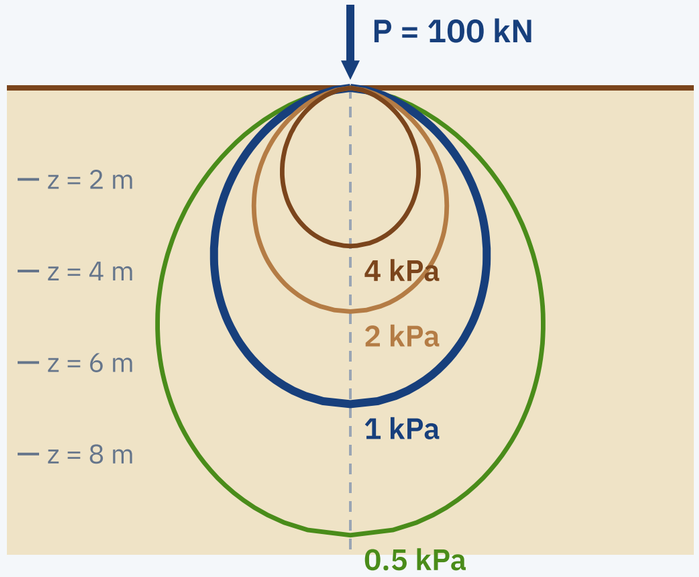
Solution (2/2)

SOLUTION

3 Depth at which the line closes on the axis (r = 0)

$$z_{axis} = \sqrt{\frac{3P}{2\pi\,\Delta\sigma_z}} = \sqrt{\frac{3(100)}{2\pi(1)}} = \sqrt{47.75} = \mathbf{6.91}\ \text{m}$$

z (m)	r (m)	Width 2r (m)	$\Delta\sigma_z$ from Boussinesq (kPa)
1	1.922	3.84	1.000
2	2.605	5.21	1.000
3	2.923	5.85	1.000
4	2.963	5.93	1.000
5	2.717	5.43	1.000
6	2.075	4.15	1.000



The 1 kPa contour starts at the load point on the surface, reaches its widest extent (r ≈ 2.98 m) at a depth of z ≈ 3.65 m, and closes on the axis at a depth of 6.91 m; the contours of smaller stresses extend deeper and wider.

Check the solution in GeoTerva ↗

Summary

Key points

Boussinesq: $\Delta\sigma_z = 3P\,z^3/(2\pi L^5) = (P/z^2)\,I_1$, $I_1 = (3/2\pi) / [(r/z)^2 + 1]^{5/2}$; $\Delta\sigma_z$ depends on neither μ nor E; $\Delta\sigma_x$, $\Delta\sigma_y$ depend on μ

Example (P = 5 kN, A (3 ; 4 ; 6)): r = 5 m; L = 7.810 m; r/z = 0.8333; $I_1 = 0.12775$; **$\Delta\sigma_z = 0.01774$ kPa**; for μ = 0.5, $\Delta\sigma_x = 0.00444$, $\Delta\sigma_y = 0.00789$ kPa

Depth: for the same r = 5 m, the largest stress occurs at $z = r\sqrt{(3/2)} = 6.12$ m; **below the load axis** $\Delta\sigma_z = 0.4775\,P/z^2$ (when z doubles, it becomes one quarter)

Superposition: each load is taken separately with its own r_k and L_k values, and the stresses are summed (in the example 0.01774 + 0.02522 = 0.04296 kPa)

Pressure bulb: contours of equal $\Delta\sigma_z$, $r = \sqrt{[(3Pz^3/(2\pi\Delta\sigma_z))^{2/5} - z^2]}$; for P = 100 kN the 1 kPa contour closes on the axis at a depth of 6.91 m

References

Boussinesq, J. (1883). Application des Potentiels à l’Étude de l’Équilibre et du Mouvement des Solides Élastiques. Gauthier-Villars, Paris.

Das, B.M. (2018). Principles of Geotechnical Engineering, 9th ed. Cengage Learning.

NOTE

All numerical examples have been produced with the module's own calculation engine; the module gives the same values. The module opens with a load P = 5 kN and point A (3 ; 4 ; 6).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own individual loads, the point of interest and Poisson's ratio in GeoTerva; the module provides $\Delta\sigma_z$, $\Delta\sigma_x$ and $\Delta\sigma_y$ values, the depth profile, isostress contours, the step-by-step solution and the report together.

Open the module → | **Theory page →**

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com