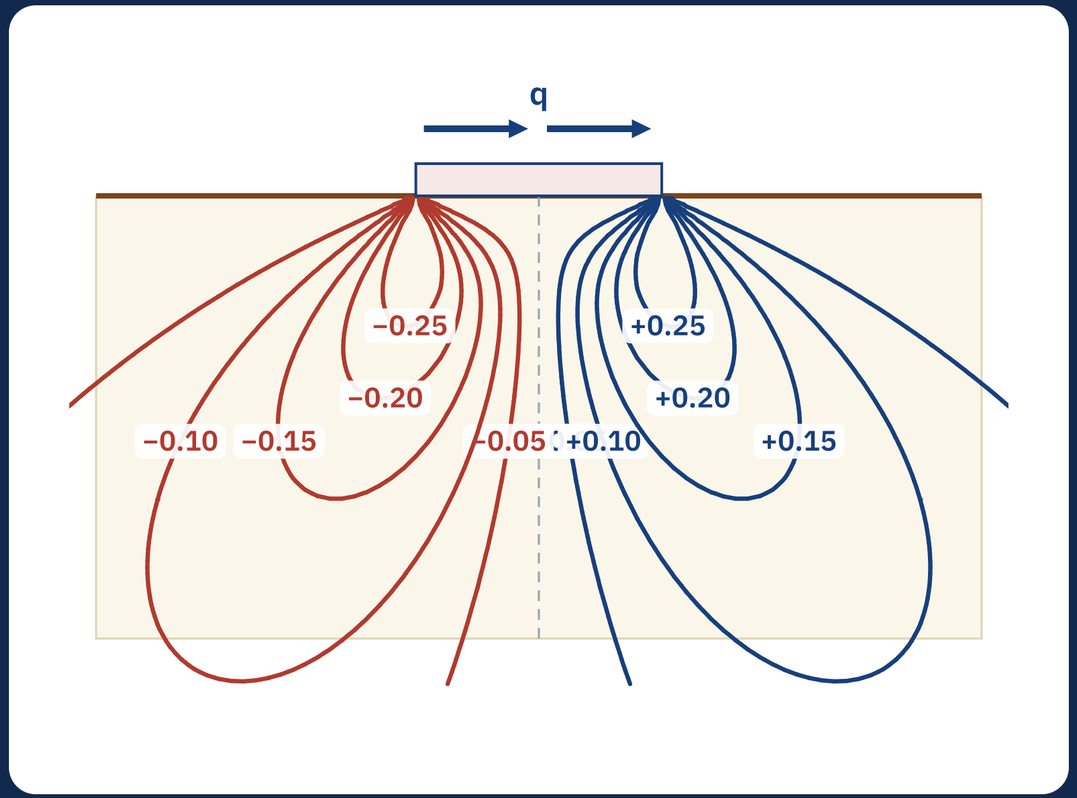
Stress under a horizontal strip load

Lecture: horizontal and inclined strip loads pushing along the surface; a point manually, inclination, depth and horizontal profiles.

- Integral of the horizontal line load
- Inclined load = vertical + horizontal
- Increase at the front, decrease at the back
- Superposition
- Can be solved by hand



What will we learn in this lecture?

The vertical stress increase beneath a **thrusting** (horizontal) and **inclined** strip load along the surface: the integral of the horizontal line load over the strip width

The equation of the horizontal strip and that the stress is **a function of x alone**: increase ahead of the load, decrease behind it, zero beneath the axis

Resolving the inclined load into components $\mathbf{q_v = q \cos \alpha}$ and $\mathbf{q_h = q \sin \alpha}$ and **superposing** the vertical and horizontal strip solutions

One point **by hand end to end** (A in front of and B behind the load); the effect of the inclination angle, depth and multiple strips with tables

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example: q = 150 kN/m², inclined at α = 30° from the vertical; B = 3 m; point A at x = +2.25 m, z = 3 m.

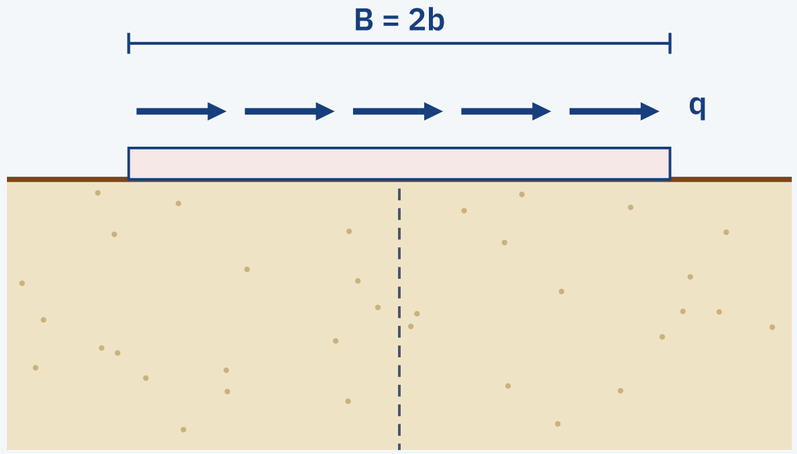
A strip load also pushes along the surface

THEORY

Friction beneath a sliding footing, the thrust at the base of a retaining wall, the braking force of vehicles on a road: a strip also **pushes** along the surface

The module gives the vertical stress increase beneath a **flexible strip** carrying a uniform horizontal pressure q and, by **superposition**, beneath an inclined strip; the strip has width B = 2b and is infinitely long (plane strain)

The soil is a **homogeneous, isotropic, linear elastic** half-space. $\Delta\sigma_z$ is only the increase in load and **can be negative** (a decrease)



Section: horizontal load pushes in the +x direction; the strip extends infinitely in the y direction

From horizontal line load to strip

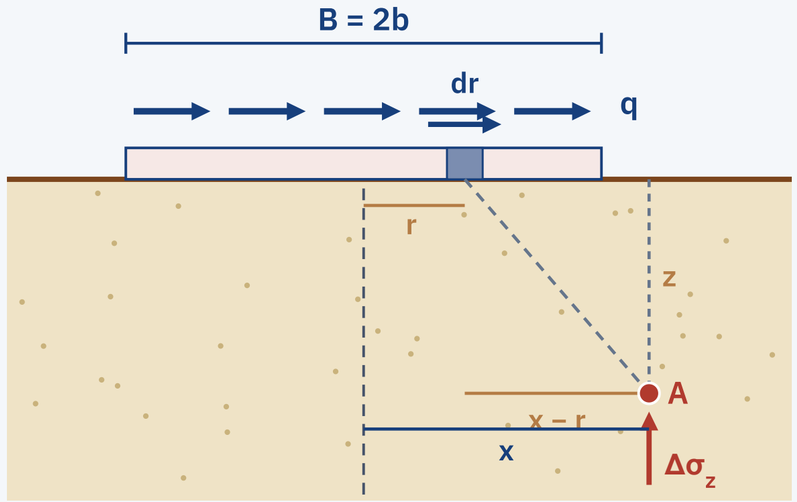
THEORY

As in the vertical strip, the elementary strip of width dr carries **q dr** per unit length and behaves like a **horizontal line load** acting in the +x direction

$$d\sigma_z = \frac{2\,(q\,dr)\,(x-r)\,z^2}{\pi\,[(x-r)^2+z^2]^2}$$

If u = x − r is taken, the antiderivative is −1/(u² + z²). Over the interval −b ≤ r ≤ b, the integral reduces to the difference of two line loads:

$$\Delta\sigma_z = \frac{q\,z^2}{\pi}\left[\frac{1}{(x-b)^2+z^2} - \frac{1}{(x+b)^2+z^2}\right]$$



$\Delta\sigma_z = 4bqxz^2 / \{\pi[(x^2 + z^2 - b^2)^2 + 4b^2z^2]\}$

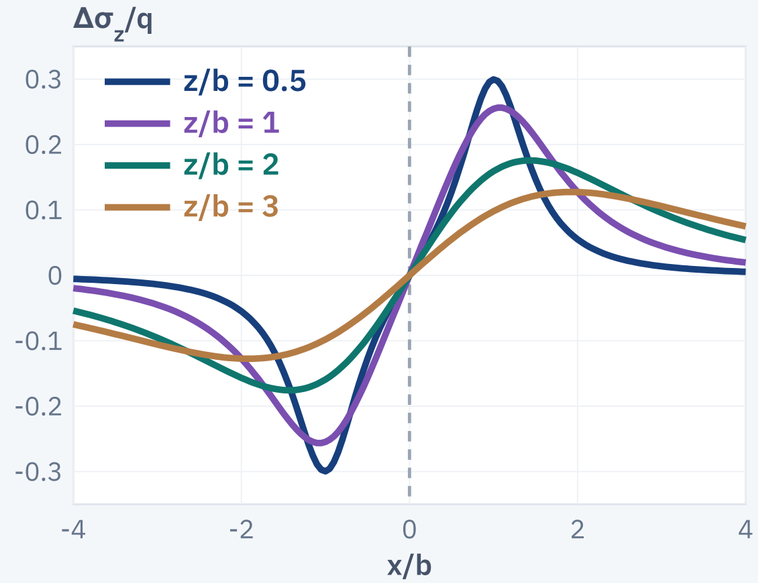
THEORY

The product of the two denominators becomes $(x^2 + z^2 - b^2)^2 + 4b^2z^2$; the **equation of the horizontal strip** is obtained (b = B/2):

$$\Delta\sigma_z = \frac{4\,b\,q\,x\,z^2}{\pi\,[(x^2+z^2-b^2)^2+4b^2z^2]} \qquad \frac{\Delta\sigma_z}{q} = \frac{4XZ^2}{\pi\,[(X^2+Z^2-1)^2+4Z^2]}$$

The stress is a **single function** of x: equal in magnitude and opposite in sign at points +x and –x. **In front of** the load the soil is pushed down, **behind** it it is lifted; it is zero at every depth beneath the strip axis

With X = x/b, Z = z/b, **a single family of curves** is valid for every q and B. Numerical verification (q = 100, B = 4, x = 1, z = 1): the integral of the horizontal line load along the strip gives **12.7324 kPa**, the equation **12.7324 kPa**



Stress of the horizontal strip: the load pushes in the +x direction

Inclined load = vertical component + horizontal component

THEORY

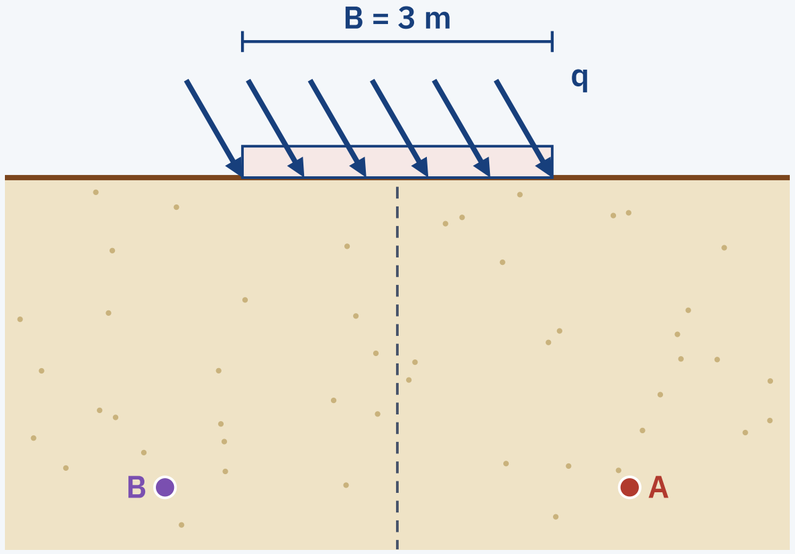
The inclined load is the pressure q acting at an angle **α** from the vertical (α is positive toward the +x direction; α = 90° is fully horizontal, α = 0 is a vertical strip)

$$q_v = q\cos\alpha \qquad q_h = q\sin\alpha$$

The vertical component gives the stress of the **vertical strip**, the horizontal component that of the **horizontal strip**; the two are added

$$\Delta\sigma_z = \Delta\sigma_{z(v)} + \Delta\sigma_{z(h)} \qquad \Delta\sigma_{z(v)} = q_v\, I_v \qquad \Delta\sigma_{z(h)} = q_h\, I_h$$

I_v is an even function of x and I_h is an odd function of x: at point A in front of the load the horizontal component is **added**, at point B behind it, it is **subtracted**



q = 150 kN/m², α = 30°, B = 3 m; A in front, B behind (x = ±2.25 m, z = 3 m)

Horizontal strip: $\Delta\sigma_z/q_h$ values

THEORY

z/b \ x/b	0.0	0.5	1.0	1.5	2.0	2.5	3.0	4.0
0.25	0.0000	0.0551	0.3134	0.0605	0.0165	0.0070	0.0037	0.0014
0.50	0.0000	0.1273	0.2996	0.1469	0.0551	0.0255	0.0138	0.0055
1.00	0.0000	0.1567	0.2546	0.2107	0.1273	0.0739	0.0449	0.0196
1.50	0.0000	0.1273	0.2037	0.2022	0.1567	0.1098	0.0753	0.0374
2.00	0.0000	0.0959	0.1592	0.1754	0.1567	0.1254	0.0955	0.0540
3.00	0.0000	0.0551	0.0979	0.1219	0.1273	0.1198	0.1058	0.0749
4.00	0.0000	0.0343	0.0637	0.0845	0.0959	0.0988	0.0955	0.0795

$\Delta\sigma_{z(h)} = q_h \cdot$ (table value) for $x > 0$; at points $x < 0$ the value is the same with the sign reversed. The $x = 0$ column is zero at every depth; the peak value decreases with depth and moves away from the strip axis.

Superposition

THEORY

Since the soil is **elastic**, the stress from each strip is found separately and summed. Each strip is defined by its own q, inclination α, width B and centre line; at most eight strips can be entered in the module.

The strips **cannot overlap** (they may meet at the edge). The stress for each strip is found with **its own x value**, i.e. the distance of point A from the centre line of that strip; the vertical and horizontal components can also be summed separately

$$\Delta\sigma_z = \sum_k \left(q_{v,k} I_{v,k} + q_{h,k} I_{h,k}\right) \qquad x_k = x_A - x_{c,k}$$

If the horizontal load of a strip is in the −x direction (α < 0), q_h is negative: the leading point is now the point on the **−x** side of the strip

Problem statement

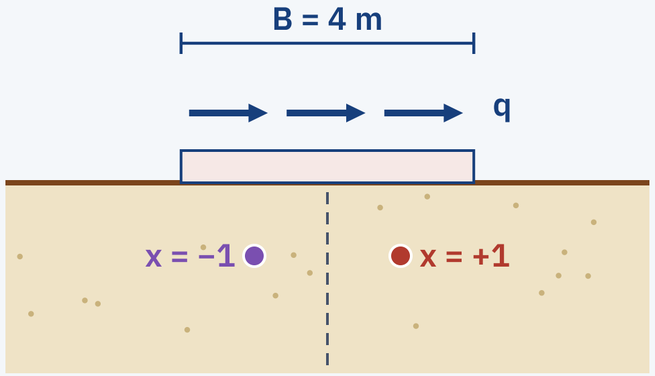
EXAMPLE

Given

Strip load	**Flexible, horizontal (in the $+x$ direction): $q = 100$ kN/m², $B = 4$ m ($b = B/2 = 2$ m); axis line $x = 0$**
Points	**at $z = 1$ m depth, $x = +1$ m (front of the load) and $x = -1$ m (behind)**

REQUIRED

$\Delta\sigma_z$ and **$\Delta\sigma_z/q$**: with the closed-form equation of the strip; verification with the **difference form**.



Solution (1/2)

SOLUTION

1 Quantities in the denominator (b = B/2 = 2 m)

$$x^2 + z^2 - b^2 = 1 + 1 - 4 = -2$$

$$(x^2 + z^2 - b^2)^2 + 4b^2z^2 = 4 + 4(4)(1) = \mathbf{20}$$

2 Influence value and stress

$$\frac{\Delta\sigma_z}{q} = \frac{4b\,x\,z^2}{\pi\,[\ldots]} = \frac{4(2)(\pm1)(1)^2}{\pi\,(20)} = \pm\frac{8}{\pi\,(20)} = \pm\mathbf{0.1273}$$

$$\Delta\sigma_z = \pm0.1273\,(100) = \pm\mathbf{12.73}\ \mathrm{kPa}$$

x = +1 m (in front of the load): $\Delta\sigma_z$ = **+12.73 kPa**, the soil is pushed downward. **x = −1 m** (behind it): **−12.73 kPa**, the stress decreases. Below the axis, $\Delta\sigma_z$ = 0 at every depth.

Solution (2/2)

SOLUTION

3 Difference form, x = +1 m

$$\Delta\sigma_z = \frac{q\,z^2}{\pi}\left[\frac{1}{(x-b)^2+z^2} - \frac{1}{(x+b)^2+z^2}\right] = \frac{100}{\pi}\left[\frac{1}{2} - \frac{1}{10}\right] = \frac{100}{\pi}(0.4) = \mathbf{12.73}\ \text{kPa}$$

4 Difference form, x = −1 m

$$\Delta\sigma_z = \frac{100}{\pi}\left[\frac{1}{(-1-2)^2+1} - \frac{1}{(-1+2)^2+1}\right] = \frac{100}{\pi}\left[\frac{1}{10} - \frac{1}{2}\right] = -\mathbf{12.73}\ \text{kPa}$$

Both forms give the **same value**: ±12.73 kPa. The module (α = 90°) gives the same value; the sign arises only from whether the point lies in front of or behind the load.

Check the solution in GeoTerva ↗

Problem statement

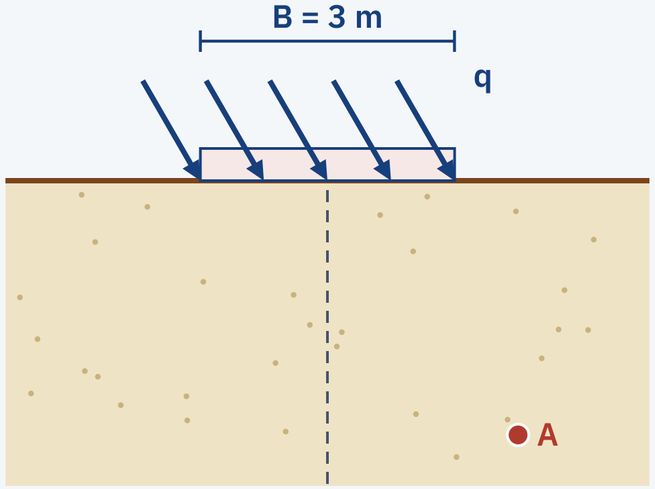
EXAMPLE

Given

Strip load	**Flexible, inclined at $\alpha = 30^\circ$ from the vertical (in the $+x$ direction): $q = 150$ kN/m², $B = 3$ m ($b = 1.5$ m); axis line $x = 0$**
Point A	**In front of the load: $x = +2.25$ m, $z = 3$ m ($x/b = 1.5$; $z/b = 2$)**

REQUIRED

$\Delta\sigma_{z(v)}$, $\Delta\sigma_{z(h)}$ and total **$\Delta\sigma_z$**: by resolving the load into its components.



Solution (1/3)

SOLUTION

1 Components of the load and dimensionless distances

$$q_v = 150\cos 30^\circ = \mathbf{129.90}\ \text{kPa} \qquad q_h = 150\sin 30^\circ = \mathbf{75.00}\ \text{kPa}$$

$$\frac{x}{b} = \frac{2.25}{1.5} = 1.5 \qquad \frac{z}{b} = \frac{3}{1.5} = 2$$

2 Vertical component: arctangents and algebraic term (x – b = 0.75 m; x + b = 3.75 m)

$$A_1 = \tan^{-1}\frac{3}{0.75} = 1.3258 \qquad A_2 = \tan^{-1}\frac{3}{3.75} = 0.6747$$

$$T = \frac{3(3)\,(5.0625 - 9 - 2.25)}{(5.0625 + 9 - 2.25)^2 + 9(9)} = \frac{-55.6875}{220.535} = -0.2525$$

Point A lies **outside** the strip (x = 2.25 m > b = 1.5 m): the two arctangents are taken directly, no π correction is needed.

Solution (2/3)

SOLUTION

3 Stress due to the vertical component

$$I_v = \frac{1}{\pi}\left[A_1 - A_2 - T\right] = \frac{1}{\pi}\left[1.3258 - 0.6747 - (-0.2525)\right] = \mathbf{0.2876}$$

$$\Delta\sigma_{z(v)} = 0.2876\,(129.90) = \mathbf{37.36}\ \text{kPa}$$

4 Stress from the horizontal component (b = 1.5 m)

$$x^2 + z^2 - b^2 = 5.0625 + 9 - 2.25 = 11.8125 \qquad 4b\,x\,z^2 = 4(1.5)(2.25)(9) = 121.5$$

$$I_h = \frac{121.5}{\pi\left[(11.8125)^2 + 4(2.25)(9)\right]} = \frac{121.5}{\pi\,(220.535)} = \mathbf{0.1754}$$

$$\Delta\sigma_{z(h)} = 0.1754\,(75.00) = \mathbf{13.15}\ \text{kPa}$$

Since the load is inclined at 30°, the **vertical component of 37.36 kPa** and the **horizontal component of 13.15 kPa** produce stress; the effect of the horizontal component at A is positive (in front).

Solution (3/3)

SOLUTION

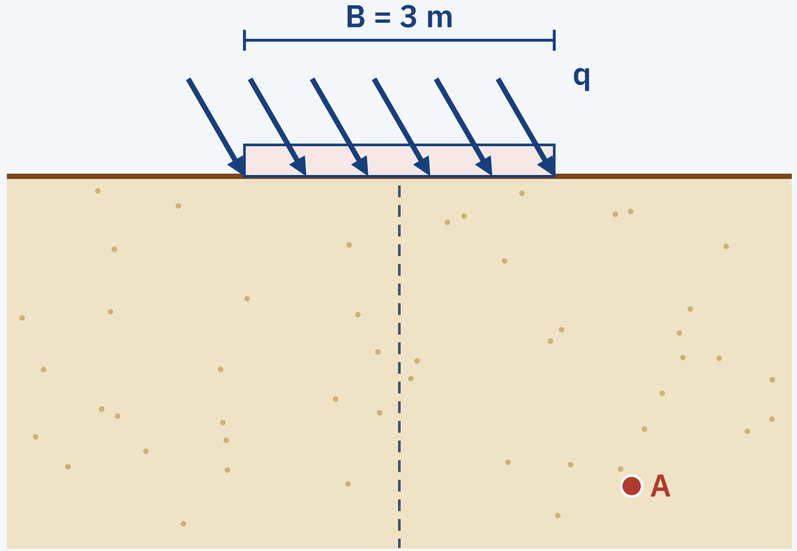
5 Superposition

$$\Delta\sigma_z = \Delta\sigma_{z(v)} + \Delta\sigma_{z(h)} = 37.36 + 13.15 = \mathbf{50.52}\ \text{kPa}$$

6 Verification for the horizontal component using the difference form

$$I_h = \frac{z^2}{\pi}\left[\frac{1}{(x-b)^2+z^2} - \frac{1}{(x+b)^2+z^2}\right]$$

$$= \frac{9}{\pi}\left[\frac{1}{9.5625} - \frac{1}{23.0625}\right] = \mathbf{0.1754}$$



Δσ$_z$ = 50.52 kPa; the module gives the same value (vertical 37.36, horizontal 13.15). If the vertical load were 150 kPa (α = 0), it would be only 43.14 kPa at A.

Check the solution in GeoTerva ↗

Problem statement

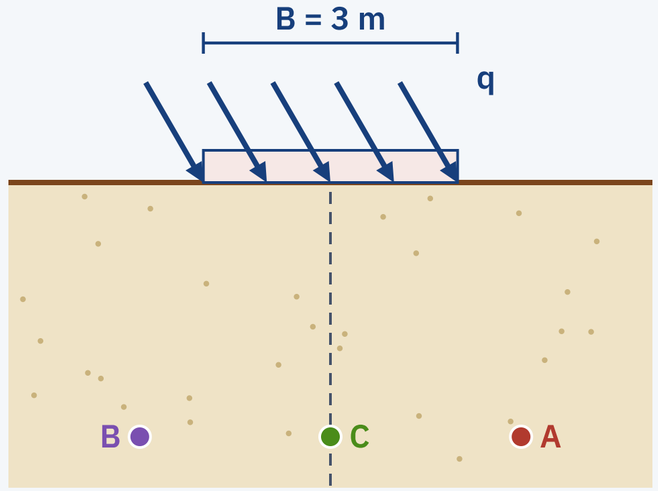
EXAMPLE

Given

Strip load	**The strip load in Example 2: $q = 150$ kN/m², $B = 3$ m, $\alpha = 30^\circ$**
Points	**B: $x = -2.25$ m (behind the load); C: $x = 0$ (axis); $z = 3$ m**
Inclination	$\alpha = 0,\ 15^\circ,\ 30^\circ,\ 45^\circ,\ 60^\circ,\ 75^\circ,\ 90^\circ$ **(at A and B)**

REQUIRED

$\Delta\sigma_z$ at points **B** and **C** by symmetry; variation of the stress at A and B as α changes.



Solution (1/2)

SOLUTION

1 Point B (x = −2.25 m): vertical component doubled, horizontal component single

$$\Delta\sigma_{z(v)}(B) = \Delta\sigma_{z(v)}(A) = 37.36\ \text{kPa} \qquad \Delta\sigma_{z(h)}(B) = -\Delta\sigma_{z(h)}(A) = -13.15\ \text{kPa}$$

$$\Delta\sigma_z(B) = 37.36 + (-13.15) = \mathbf{24.21}\ \text{kPa}$$

2 Point C (axis, x = 0): horizontal component zero; vertical component by the under-axis formula

$$I_v = \frac{2}{\pi}\left[\tan^{-1}\frac{b}{z} + \frac{bz}{b^2+z^2}\right] = \frac{2}{\pi}\left[0.4636 + 0.4000\right] = 0.5498$$

$$\Delta\sigma_z(C) = 0.5498\,(129.90) = \mathbf{71.42}\ \text{kPa}$$

In front of the load (A) 50.52, **behind** it (B) 24.21, **below the axis** (C) 71.42 kPa: at the same depth, the horizontal component adds 13.15 kPa at A and reduces it by the same amount at B. The module gives the same values for A and B.

Solution (2/2)

SOLUTION

α (°)	q_v	q_h	A: $\Delta\sigma_{z(v)}$	A: $\Delta\sigma_{z(h)}$	A: $\Delta\sigma_z$	B: $\Delta\sigma_z$
0	150.00	0.00	43.14	0.00	**43.14**	**43.14**
15	144.89	38.82	41.67	6.81	**48.48**	**34.86**
30	129.90	75.00	37.36	13.15	**50.52**	**24.21**
45	106.07	106.07	30.51	18.60	**49.11**	**11.91**
60	75.00	129.90	21.57	22.78	**44.35**	**−1.21**
75	38.82	144.89	11.17	25.41	**36.58**	**−14.24**
90	0.00	150.00	0.00	26.31	**26.31**	**−26.31**

As the inclination increases, the vertical component decreases and the horizontal component increases. At **A**, the total is greatest around α ≈ 30° (50.5 kPa); at α = 90° only the horizontal component remains (26.3 kPa). At **B**, the stress **switches to decrease** at α = 60° with -1.21 kPa and becomes −26.3 kPa at α = 90°.

Check the solution in GeoTerva ↗

Problem statement

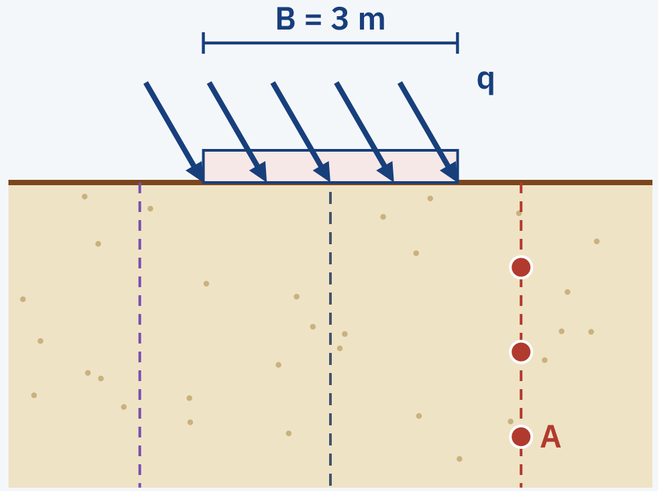
EXAMPLE

Given

Strip load	**The inclined strip in Example 2: $q = 150$ kN/m², $\alpha = 30^\circ$, $B = 3$ m**
Lines	**The ordinate of A is $x = +2.25$ m and the ordinate of B is $x = -2.25$ m**
Depth	$z = 0.5$ **...** 12 **m**

REQUIRED

Variation of $\Delta\sigma_{z(v)}$, $\Delta\sigma_{z(h)}$ and the total with depth; the **depth at which the horizontal component is greatest**.

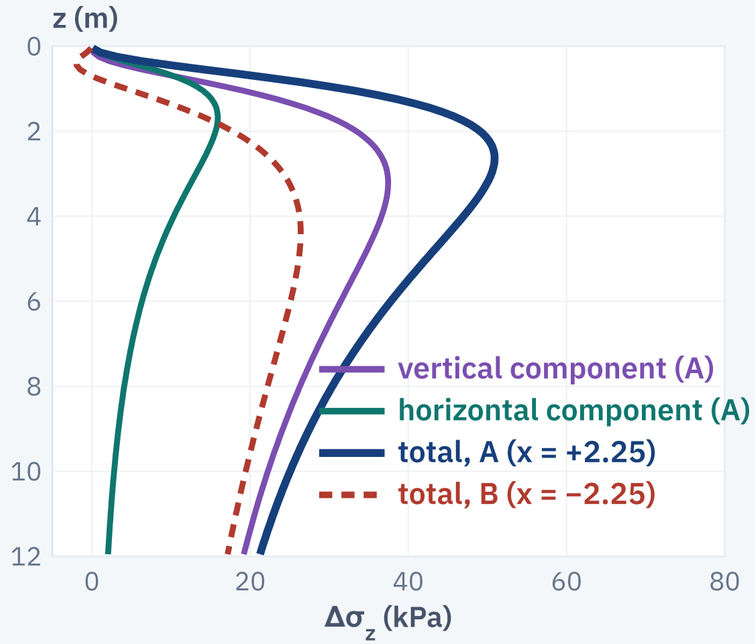


Solution

SOLUTION

z (m)	z/b	A: $\Delta\sigma_{z(v)}$	A: $\Delta\sigma_{z(h)}$	A: $\Delta\sigma_z$	B: $\Delta\sigma_z$
0.5	0.33	5.17	6.93	**12.09**	**−1.76**
1.0	0.67	18.01	13.69	**31.71**	**4.32**
1.5	1.00	27.77	15.81	**43.57**	**11.96**
2.0	1.33	33.43	15.64	**49.08**	**17.79**
3.0	**2.00**	**37.36**	**13.15**	**50.52**	**24.21**
4.0	2.67	36.62	10.36	**46.98**	**26.26**
5.0	3.33	34.23	8.07	**42.30**	**26.16**
6.0	4.00	31.45	6.34	**37.79**	**25.11**
8.0	5.33	26.31	4.09	**30.40**	**22.22**
10.0	6.67	22.25	2.81	**25.06**	**19.44**
12.0	8.00	19.14	2.03	**21.17**	**17.11**

Below A the **vertical component** peaks at z ≈ 3.25 m (37.5 kPa); the **horizontal component** is largest at a shallower depth, z ≈ 1.65 m (15.9 kPa), and then decays as 1/z². Below B the stress near the surface is negative (-1.76 kPa at z = 0.5 m).



Check the solution in GeoTerva ↗

Problem statement

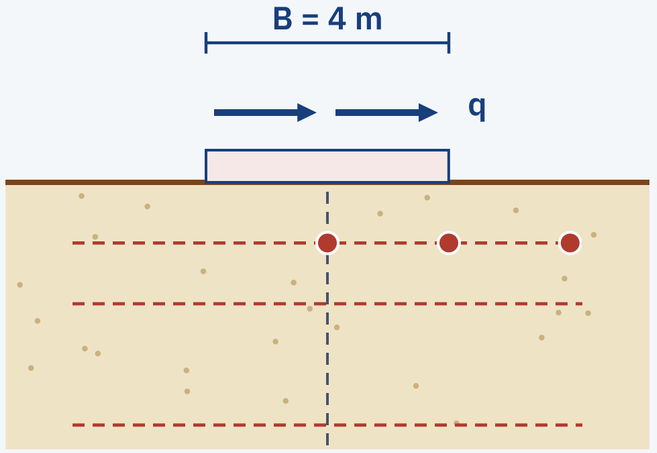
EXAMPLE

Given

Strip load	**Horizontal: $q = 100$ kN/m², $B = 4$ m ($b = 2$ m), in the $+x$ direction**
Lines	**Horizontal lines at depth $z = 1,\ 2,\ 4,\ 6$ m; $x = 0, 1, 2, 3, 4, 6, 8$ m**
Hand calculation	**$z = 1$ m, $x = 2$ m (edge) point**

REQUIRED

The variation of **$\Delta\sigma_z$** along x at each depth; the **location of the peak value** and how its magnitude varies with depth.



Solution (1/2)

SOLUTION

1 Quantities in the denominator (b = 2 m)

$$x^2 + z^2 - b^2 = 4 + 1 - 4 = 1 \qquad (x^2 + z^2 - b^2)^2 + 4b^2z^2 = 1 + 16 = 17$$

$$4b\,x\,z^2 = 4(2)(2)(1) = 16$$

2 Influence value and stress

$$\frac{\Delta\sigma_z}{q} = \frac{16}{\pi\,(17)} = \mathbf{0.2996}$$

$$\Delta\sigma_z = 0.2996\,(100) = \mathbf{29.96}\ \mathrm{kPa}$$

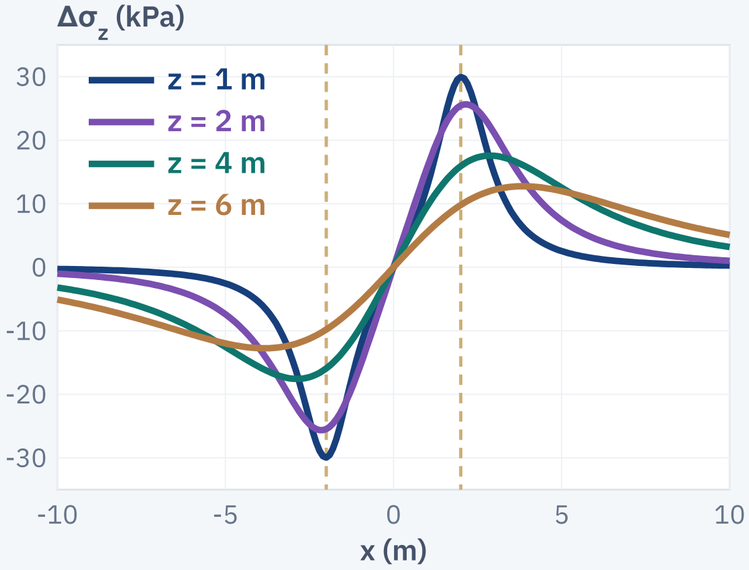
At the front edge of the strip (x = b), at depth z = 1 m, $\Delta\sigma_z$ = **29.96 kPa**: this is very close to the **peak value** on the z = 1 m line (x = 2.01 m, 29.96 kPa).

Solution (2/2)

SOLUTION

z (m)	x = 1 m	x = 2 m	x = 3 m	x = 4 m	x = 6 m	x = 8 m	Peak: x (m)	Peak: $\Delta\sigma_z$
1	12.73	29.96	14.69	5.51	1.38	0.55	**2.01**	**29.96**
2	15.67	25.46	21.07	12.73	4.49	1.96	**2.15**	**25.65**
4	9.59	15.92	17.54	15.67	9.55	5.40	**2.87**	**17.57**
6	5.51	9.79	12.19	12.73	10.58	7.49	**3.84**	**12.75**

The peak value, near the surface (z = 1 m), is **30.0 kPa** just below the strip edge; with increasing depth it **decreases** (12.7 kPa at z = 6 m) and **moves away from the strip** (x = 2.0 m → 3.8 m). The x = 0 column is always zero; at points x < 0 the values are the same, with the sign reversed (single symmetry).



Check the solution in GeoTerva ↗

Problem statement

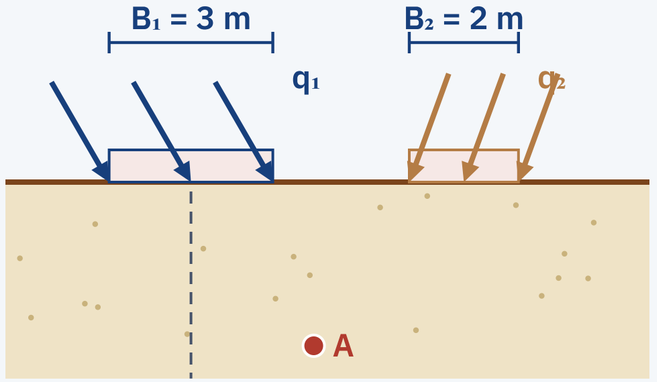
EXAMPLE

Given

1st strip	$q_1 = 150$ **kN/m²,** $\alpha_1 = 30^\circ$ **(+**x**),** $B_1 = 3$ **m,** $x_{c1} = 0$ **(–1.5 … 1.5 m) — the strip in Example 2**
2nd strip	$q_2 = 100$ **kN/m²,** $\alpha_2 = -20^\circ$ **(−**x **direction),** $B_2 = 2$ **m,** $x_{c2} = 5$ **m (4 … 6 m)**
Point A	$x = 2.25$ **m,** $z = 3$ **m (between the two strips)**

REQUIRED

2. the strip's share (vertical, horizontal) and the **total $\Delta\sigma_z$**; depth profile.



Solution (1/3)

SOLUTION

1 2nd strip: $x_2 = 2.25 - 5 = -2.75$ m ($|x_2| = 2.75$ m), b = 1 m; $\alpha_2 = -20°$

$$q_v = 100\cos 20^\circ = \mathbf{93.97}\ \text{kPa} \qquad q_h = 100\sin(-20^\circ) = -\mathbf{34.20}\ \text{kPa}$$

2 Vertical component (symmetry: for $|x_2| = 2.75$ m)

$$A_1 = \tan^{-1}\frac{3}{1.75} = 1.0427 \quad A_2 = \tan^{-1}\frac{3}{3.75} = 0.6747 \quad T = \frac{2(3)(-2.4375)}{(15.5625)^2 + 36} = -0.0526$$

$$\Delta\sigma_{z(v)} = \frac{93.97}{\pi}[A_1 - A_2 - T] = 0.1339\,(93.97) = \mathbf{12.58}\ \text{kPa}$$

Solution (2/3)

SOLUTION

3 Horizontal component: q_h negative, x_2 negative → $\Delta\sigma_{z(h)}$ positive

$$I_h(|x|) = \frac{4b\,x\,z^2}{\pi\left[(x^2+z^2-b^2)^2+4b^2z^2\right]} = \frac{4(1)(2.75)(9)}{\pi\left[(15.5625)^2+36\right]} = \mathbf{0.1133}$$

$$\Delta\sigma_{z(h)} = (-34.20)\,(-0.1133) = \mathbf{3.87}\ \text{kPa}$$

4 Superposition (1st strip: Example 2)

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} = (37.36 + 13.15) + (12.58 + 3.87)$$
$$= 50.52 + 16.45 = \mathbf{66.97}\ \text{kPa}$$

$\Delta\sigma_z$ = 66.97 kPa (vertical components 49.94, horizontal components 17.03 kPa). The module gives the same values for both strips.

Solution (3/3)

SOLUTION

z (m)	1st strip (kPa)	2nd strip (kPa)	Total (kPa)	share of strip 2
1	31.71	4.25	**35.96**	%12
2	49.08	12.18	**61.26**	%20
3	50.52	16.45	**66.97**	%25
4	46.98	17.48	**64.45**	%27
6	37.79	15.83	**53.62**	%30
8	30.40	13.38	**43.79**	%31
12	21.17	9.71	**30.89**	%31

In the near-surface layer, strip 1 dominates; at **z = 12 m** the share of strip 2 rises to 31% of the total. The horizontal components of the two strips add in the same direction at A (both toward A): the loads push toward each other.

Check the solution in GeoTerva ↗

Summary

Key points

Horizontal strip: $\Delta\sigma_z = 4bqxz^2 / \{\pi[(x^2 + z^2 - b^2)^2 + 4b^2z^2]\}$, $b = B/2$; equivalent form $(qz^2/\pi)[1/((x - b)^2 + z^2) - 1/((x + b)^2 + z^2)]$. An **odd function** of x: + in front, – behind, 0 under the axis

Inclined strip: $q_v = q \cos\alpha$, $q_h = q \sin\alpha$; $\Delta\sigma_z = \Delta\sigma_{z(v)} + \Delta\sigma_{z(h)}$. The vertical component is doubled at x, the horizontal component is single

Example ($q = 150$ kPa, $\alpha = 30°$, $B = 3$ m; $z = 3$ m): A ($x = +2.25$): 37.36 + 13.15 = **50.52 kPa**; B ($x = -2.25$): 37.36 – 13.15 = **24.21 kPa**; horizontal strip ($q = 100$, $B = 4$; $x = \pm1$, $z = 1$): **±12.73 kPa**

Effects: as α increases, the total at A is greatest at about $\alpha \approx 30°$; at B the stress decreases for $\alpha \geq 60°$; for a strip load it decreases as the peak deepens and moves away from the strip

Superposition: each strip with its own x and α values; in the two inclined strips example 50.52 + 16.45 = **66.97 kPa**

References

Das, B.M. (2018). Principles of Geotechnical Engineering, 9th ed. Cengage Learning.

NOTE

All numerical examples have been generated with the module's own calculation engine; the module yields the same values. The module opens with q = 150 kN/m², α = 30°, B = 3 m and A (x = +2.25 m, z = 3 m); entering x = −2.25 m gives point B, and entering α = 90° gives a fully horizontal strip.

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own stripes (q, inclination, B, centreline) and point A in GeoTerva; the module returns the stress, the iso-stress contours, the depth profile, the step-by-step solution and the report together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com