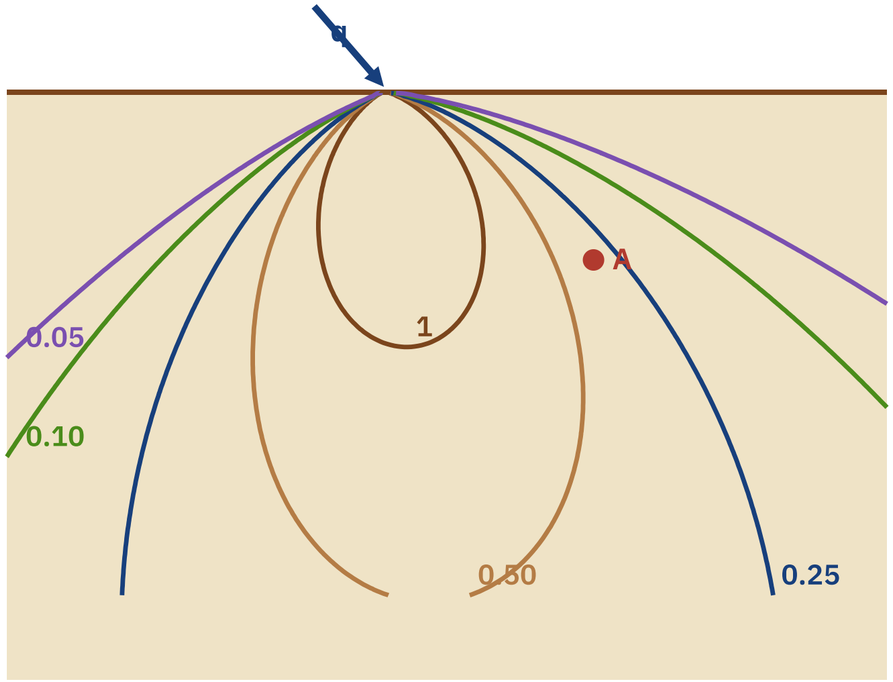
Geotechnical lecture presentation · Stress distribution in soil

Stress under a horizontal line load

Lecture: vertical stress in the soil caused by horizontal and inclined line loads; the introductory example solved end-to-end by hand with its components, and depth and lateral profiles presented in tables.

- Horizontal line load
- Components of an inclined load
- Sign convention
- Superposition
- Iso-stress contours



Introduction

What will we learn in this lecture?

The vertical stress due to a **horizontal** line load at the surface (wall base friction, braking force, arch thrust): $\Delta\sigma_z = (2q/\pi)\, x\, z^2/(x^2 + z^2)^2$

Sign convention: on the side the load pushes toward ($x > 0$) the stress increases, on the other side it decreases, and directly under the load it is zero; the maximum value occurs at $x/z = 1/\sqrt{3}$

Resolution of an inclined line load into its components: $q_V = q \cos\alpha$, $q_H = q \sin\alpha$; superposition of the two stresses

Effect of slope, depth and lateral position on stress; **superposition** for multiple loads

The module's opening example ($q = 10$ kN/m, $\alpha = 20°$) **end to end by hand**; the separate contribution of each component in tables

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example: $q = 10$ kN/m, $\alpha = 20°$ from the vertical; point A: $x = 5$ m, $z = 4$ m.

Horizontal and inclined line loads

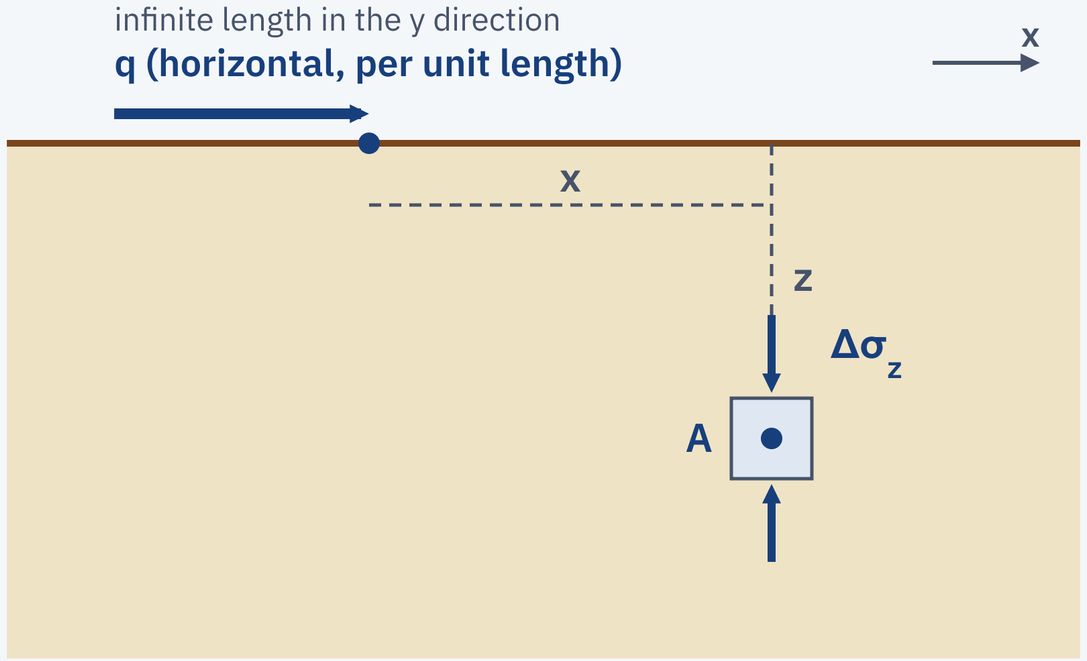
THEORY

A line load can also push along the surface instead of pressing down on it; friction at the base of a wall, railway braking force and arch thrust are examples of these.

The module calculates the vertical stress increase under a horizontal line load and under an inclined line load by superposing the two components.

The flexible line load is infinitely long in the y direction, q is the load per unit length, and plane strain conditions apply in every cross-section.

The soil is a homogeneous, isotropic, linearly elastic half-space; $\Delta\sigma_z$ is only the stress increase produced by the load.



$\Delta\sigma_z$ of a horizontal line load

THEORY

For a horizontal line load, the elastic solution gives the vertical stress increase at point A as $\Delta\sigma_z = (2q/\pi) \times z^2/(x^2 + z^2)^2$.

The sign of the stress depends on the value of x: the vertical stress increases on the side toward which the load pushes (x > 0), decreases on the other side and is zero directly under the load (x = 0).

In dimensionless form this becomes $\Delta\sigma_z/(q/z) = (2/\pi)(x/z) / [(x/z)^2 + 1]^2$; a single curve is used for every q and every z.

$$\Delta\sigma_z = \frac{2q}{\pi} \frac{x\, z^2}{(x^2 + z^2)^2}$$

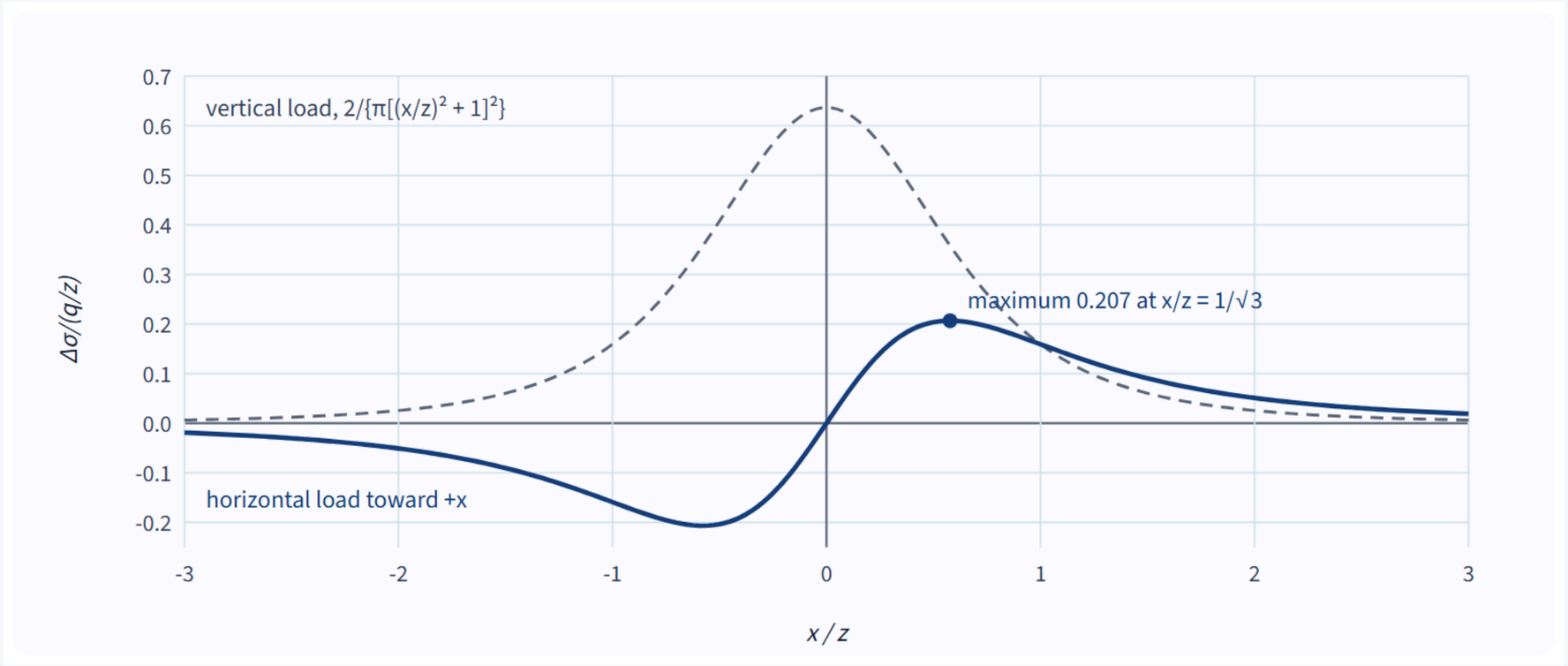
$$\frac{\Delta\sigma_z}{q/z} = \frac{2}{\pi} \frac{x/z}{[(x/z)^2 + 1]^2}$$

$\Delta\sigma_z/(q/z)$ VALUES (MODULE)

x/z	$\Delta\sigma_z/(q/z)$	x/z	$\Delta\sigma_z/(q/z)$
0.00	0.000	0.70	0.201
0.10	0.062	0.80	0.189
0.20	0.118	0.90	0.175
0.30	0.161	1.00	0.159
0.40	0.189	1.25	0.121
0.50	0.204	1.50	0.090
0.60	0.207	2.00	0.051

$\Delta\sigma_z/(q/z)$ versus x/z

THEORY



Δσz/(q/z) against x/z below a horizontal line load (full line) and, dashed, below a vertical one of the same intensity. The horizontal load acts toward +x.

Components and superposition

THEORY

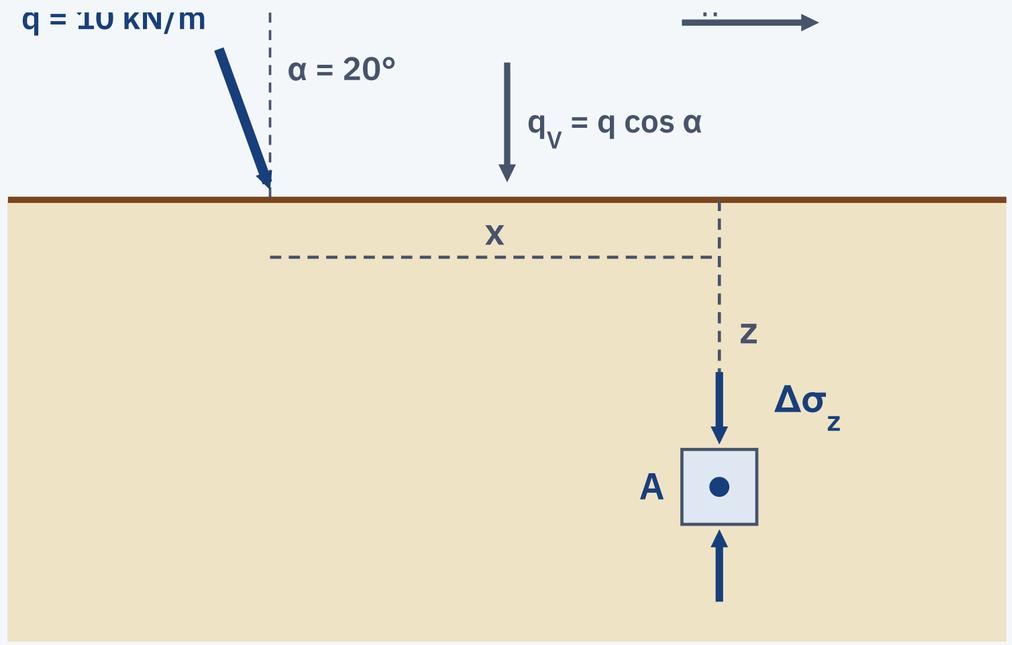
An inclined line load is the sum of the components of an inclined line load q at an angle α from the vertical, and since the soil is elastic, the stresses of the components are added.

α = 90° gives a fully horizontal load, α = 0 gives a vertical load.

Additional line loads, each defined by its own magnitude, inclination and position, are entered one by one via Add line load (at most eight; two cannot be at the same position).

$$q_V = q\cos\alpha, \qquad q_H = q\sin\alpha$$

$$\Delta\sigma_z = \frac{2q_V\, z^3}{\pi(x^2+z^2)^2} + \frac{2q_H}{\pi}\,\frac{x\, z^2}{(x^2+z^2)^2}$$



Problem statement

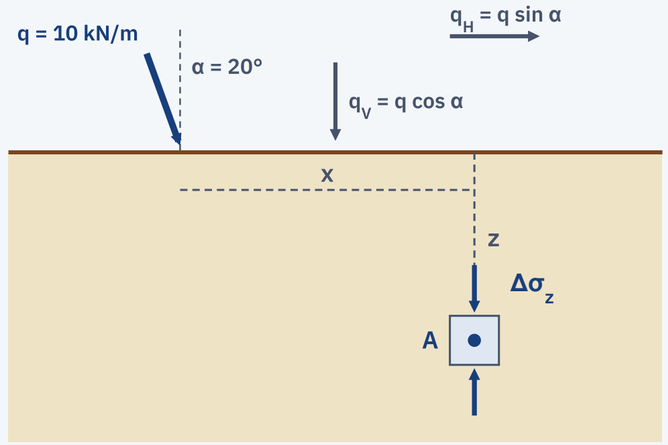
EXAMPLE

Given

Load	**line load $q = 10$ kN/m, inclined $\alpha = 20^\circ$ from the vertical (horizontal component in the $+x$ direction), $x_L = 0$**
Point A	$x = 5$ m, $z = 4$ m
Soil	**homogeneous, elastic half-space; plane strain**

REQUIRED

q_V, **q_H**; the **$\Delta\sigma_z$** value of each component and the total **$\Delta\sigma_z$**.



Solution (1/2)

SOLUTION

1 Components of the inclined load (α = 20°)

$$q_V = q\cos\alpha = 10\cos 20^\circ = \mathbf{9.397}\ \mathrm{kN/m}$$

$$q_H = q\sin\alpha = 10\sin 20^\circ = \mathbf{3.420}\ \mathrm{kN/m}$$

2 Stress from the vertical component (x = 5, z = 4)

$$\Delta\sigma_{z(V)} = \frac{2q_V z^3}{\pi(x^2+z^2)^2} = \frac{2(9.397)(4^3)}{\pi(5^2+4^2)^2} = \frac{1{,}202.8}{\pi\,(1{,}681)} = \mathbf{0.22776}\ \mathrm{kPa}$$

The vertical component alone gives **0.22776 kPa**; the contribution of the horizontal component is on the next page.

Solution (2/2)

SOLUTION

3 Stress of the horizontal component (x = 5 m: the side pushed by the load, sign positive)

$$\Delta\sigma_{z(H)} = \frac{2q_H}{\pi}\,\frac{xz^2}{(x^2+z^2)^2} = \frac{2(3.420)(5)(4^2)}{\pi(5^2+4^2)^2} = \frac{547.23}{\pi\,(1{,}681)} = \mathbf{0.10362}\ \text{kPa}$$

4 Superposition

$$\Delta\sigma_z = \Delta\sigma_{z(V)} + \Delta\sigma_{z(H)} = 0.22776 + 0.10362 = \mathbf{0.33138}\ \text{kPa}$$

$\Delta\sigma_z$ = 0.33138 kPa (vertical 0.22776 + horizontal 0.10362). Check: for x/z = 1.25, $\Delta\sigma_{z(V)}/(q_V/z)$ = 0.0970, $\Delta\sigma_{z(H)}/(q_H/z)$ = 0.1212; the products are 0.22776 and 0.10362 kPa. The module gives the same value.

Check the solution in GeoTerva ↗

Problem statement

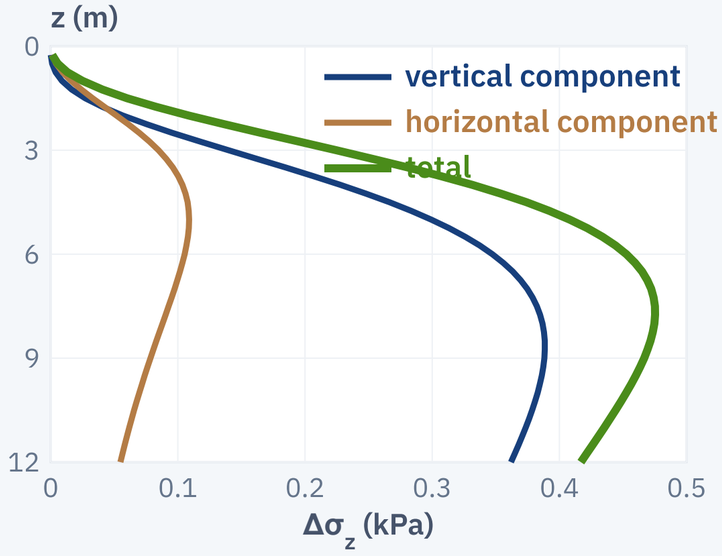
EXAMPLE

Given

Load	**Load in Example 1:** $q = 10$ **kN/m,** $\alpha = 20^\circ$**,** $x_L = 0$
Points	$x_A = 5$ **m;** $z = 1.2; 2.4;$ **...;** 12 **m (profile rows of the module)**

REQUIRED

For each depth $\Delta\sigma_{z(V)}$, $\Delta\sigma_{z(H)}$ and the total; detailed solution for z = 5 m; the depths at which the components are largest.



Solution (1/2)

SOLUTION

1 Vertical component (x = 5, z = 5: $x^2 + z^2 = 50$)

$$\Delta\sigma_{z(V)} = \frac{2(9.397)(5^3)}{\pi(50)^2} = \frac{2{,}349.2}{7{,}854.0} = \mathbf{0.29911}\ \text{kPa}$$

2 Horizontal component

$$\Delta\sigma_{z(H)} = \frac{2(3.420)(5)(5^2)}{\pi(50)^2} = \frac{855.05}{7{,}854.0} = \mathbf{0.10887}\ \text{kPa}$$

3 Total

$$\Delta\sigma_z = 0.29911 + 0.10887 = \mathbf{0.40798}\ \text{kPa}$$

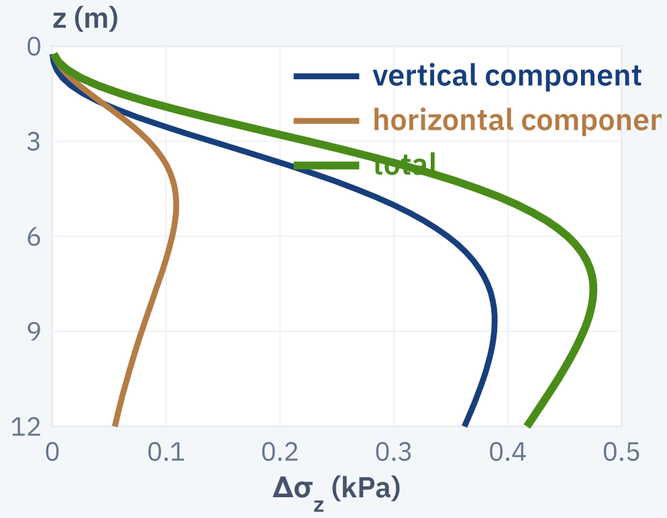
At z = 5 m, $\Delta\sigma_z$ = 0.40798 kPa; it is greater than the value at z = 4 m (0.33138 kPa). The table is on the next page.

Solution (2/2)

SOLUTION

z (m)	$\Delta\sigma_{z(V)}$ (kPa)	$\Delta\sigma_{z(H)}$ (kPa)	Total $\Delta\sigma_z$ (kPa)
1.2	0.01479	0.02243	**0.03721**
2.4	0.08740	0.06628	**0.15368**
3.6	0.19370	0.09792	**0.29161**
4.8	0.28667	0.10869	**0.39536**
6.0	0.34726	0.10533	**0.45259**
7.2	0.37817	0.09559	**0.47376**
8.4	0.38829	0.08412	**0.47241**
9.6	0.38559	0.07309	**0.45868**
10.8	0.37563	0.06330	**0.43893**
12.0	0.36194	0.05489	**0.41683**

The stress from the horizontal component is greatest at depth z = x = 5 m ($q_H/(2\pi x)$ = 0.10887 kPa), then decreases; the vertical component is greatest at z = $\sqrt{3}\cdot$x = 8.66 m (0.3886 kPa). The total is at most 0.4753 kPa at z ≈ 7.7 m.



Check the solution in GeoTerva ↗

Problem statement

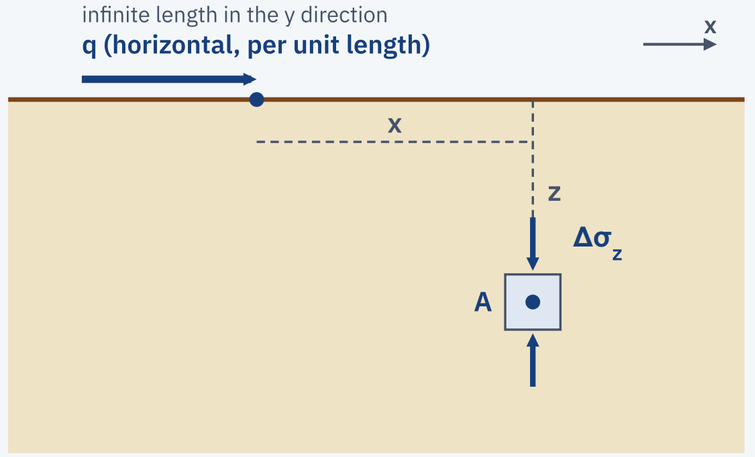
EXAMPLE

Given

Load	**horizontal line load $q = 20$ kN/m ($\alpha = 90°$), in the $+x$ direction, $x_L = 0$**
Points	$z = 3$ m; $x = -6; -3; -1.73; 0; 1.73; 3; 6$ m ($x/z = -2 \ldots 2$)

REQUIRED

For each point, **x/z**, **$\Delta\sigma_z/(q/z)$** and **$\Delta\sigma_z$**; comparison with a vertical load of the same intensity; the maximum value.



Solution (1/2)

SOLUTION

1 x = 3 m (x/z = 1): side pushed by the load

$$\Delta\sigma_z/(q/z) = \frac{2}{\pi}\cdot\frac{1}{(1^2+1)^2} = \frac{1}{2\pi} = 0.15915$$

$$\Delta\sigma_z = 0.15915\,(20/3) = \mathbf{1.0610}\ \mathrm{kPa} \qquad (x = -3\ \mathrm{m}:\ -\mathbf{1.0610}\ \mathrm{kPa})$$

2 Maximum value: x/z = 1/√3 = 0.5774 (x = 1.732 m)

$$\Delta\sigma_z/(q/z) = \frac{2}{\pi}\cdot\frac{0.5774}{(1/3+1)^2} = 0.20675$$

$$\Delta\sigma_z = 0.20675\,(20/3) = \mathbf{1.3783}\ \mathrm{kPa}$$

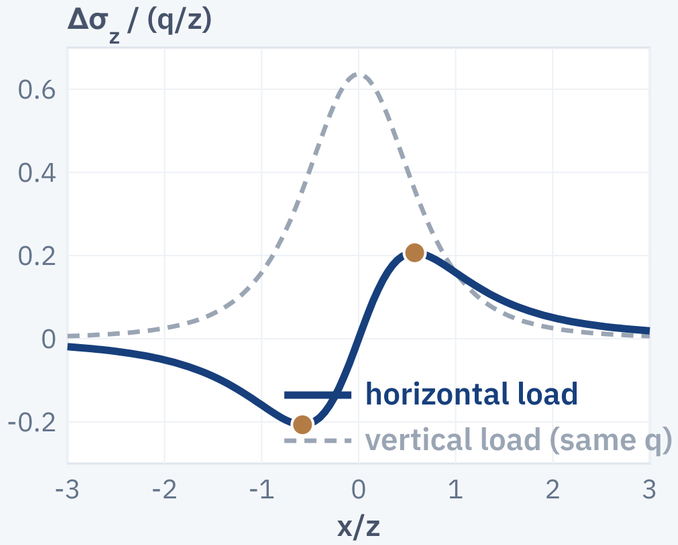
Under a horizontal load (x = 0) the stress is **zero**; it increases on the pushed side and decreases by the same amount on the opposite side. The maximum value is 0.2067·(q/z) = 1.378 kPa at x/z = 0.5774.

Solution (2/2)

SOLUTION

x (m)	x/z	$\Delta\sigma_z/(q/z)$	Horizontal load $\Delta\sigma_z$ (kPa)	Vertical load, same q (kPa)
−6.00	−2.000	−0.0509	**−0.3395**	0.1698
−3.00	−1.000	−0.1592	**−1.0610**	1.0610
−1.73	−0.577	−0.2067	**−1.3783**	2.3873
0.00	0.000	0.0000	**0.0000**	4.2441
1.73	0.577	0.2067	**1.3783**	2.3873
3.00	1.000	0.1592	**1.0610**	1.0610
6.00	2.000	0.0509	**0.3395**	0.1698

For horizontal load the curve is **antisymmetric** about x = 0 (odd function); the vertical load curve is even and is largest under the load (4.244 kPa). At x/z = 1 the two loads give the same magnitude (1.061 kPa).



Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Load	$q = 10$ **kN/m, inclined at** α **from vertical;** $x_L = 0$
Point A	$x = 5$ **m,** $z = 4$ **m**
Slopes	$\alpha = 0; 10; 20; 30; 45; 60; 90^\circ$

REQUIRED

For each slope angle, q_V, q_H, $\Delta\sigma_{z(V)}$, $\Delta\sigma_{z(H)}$ and the total; details for α = 45°; the slope angle at which the total is greatest.

Solution (1/2)

SOLUTION

1 Coefficients for unit load (x = 5, z = 4): each component is multiplied by its own coefficient

$$a = \frac{2z^3}{\pi(x^2+z^2)^2} = \frac{128}{5{,}281.0} = 0.024238$$

$$b = \frac{2xz^2}{\pi(x^2+z^2)^2} = \frac{160}{5{,}281.0} = 0.030297 \qquad \Delta\sigma_z = q\,(a\cos\alpha + b\sin\alpha)$$

2 α = 45° ($q_V = q_H$ = 7,071 kN/m)

$$\Delta\sigma_z = 10\,(0.024238)(0.7071) + 10\,(0.030297)(0.7071)$$
$$= 0.17139 + 0.21423 = \mathbf{0.38562}\ \text{kPa}$$

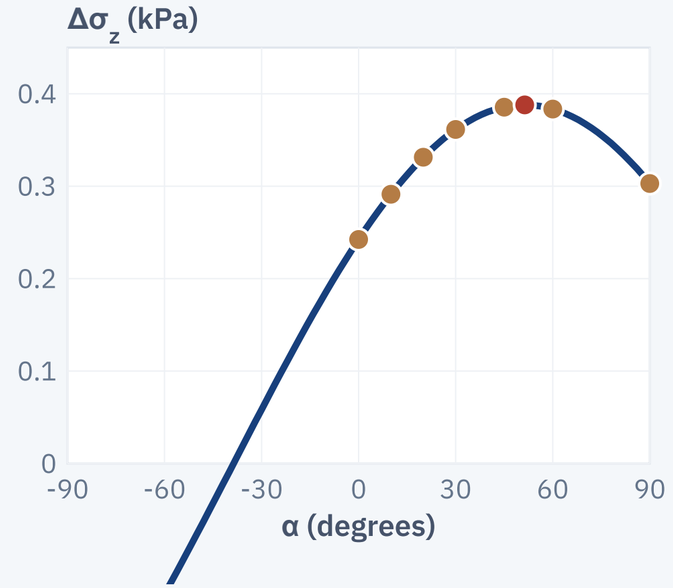
Check: x/z = 1.25, hence b/a = 1.25; the total is greatest under the condition tan α = b/a = x/z: for α = 51.34° $\Delta\sigma_z = q\sqrt{(a^2 + b^2)}$ = **0.38799 kPa**.

Solution (2/2)

SOLUTION

α (°)	q_V (kN/m)	q_H (kN/m)	$\Delta\sigma_{z(V)}$ (kPa)	$\Delta\sigma_{z(H)}$ (kPa)	Total $\Delta\sigma_z$ (kPa)
0	10.000	0.000	0.24238	0.00000	**0.24238**
10	9.848	1.736	0.23870	0.05261	**0.29131**
20	9.397	3.420	0.22776	0.10362	**0.33138**
30	8.660	5.000	0.20991	0.15149	**0.36139**
45	7.071	7.071	0.17139	0.21423	**0.38562**
60	5.000	8.660	0.12119	0.26238	**0.38357**
90	0.000	10.000	0.00000	0.30297	**0.30297**

A vertical load (α = 0) gives 0.24238 kPa; as the load inclines toward point A, the horizontal component comes into play and the total increases: the largest value is 0.3880 kPa at α ≈ 51.3°. At a fully horizontal load (α = 90°) it drops to 0.3030 kPa. A negative α turns the horizontal component away from A and reduces the total (figure).



Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Load	**The load in Example 1:** $q = 10$ **kN/m,** $\alpha = 20^\circ$ **(pushing in the** $+x$ **direction),** $x_L = 0$
Points	$z = 4$ **m;** $x = -10; -5; 0; 5; 10$ **m**

REQUIRED

For each point, $\Delta\sigma_{z(V)}$, $\Delta\sigma_{z(H)}$ and the total; details for x = −5 m and x = 0; the asymmetry of the curve.

Solution (1/2)

SOLUTION

1 x = −5 m (the side not pushed by the load): vertical component the same, horizontal component negative

$$\Delta\sigma_{z(V)} = \frac{2(9.397)(4^3)}{\pi((-5)^2+4^2)^2} = 0.22776\ \text{kPa}$$

$$\Delta\sigma_{z(H)} = \frac{2(3.420)(-5)(4^2)}{\pi(41)^2} = \mathbf{-0.10362}\ \text{kPa}$$

$$\Delta\sigma_z = 0.22776 + (-0.10362) = \mathbf{0.12414}\ \text{kPa}$$

2 x = 0 (directly beneath the load): horizontal component is zero

$$\Delta\sigma_{z(V)} = \frac{2q_V}{\pi z} = \frac{2(9.397)}{\pi(4)} = \mathbf{1.49557}\ \text{kPa}, \qquad \Delta\sigma_{z(H)} = 0$$

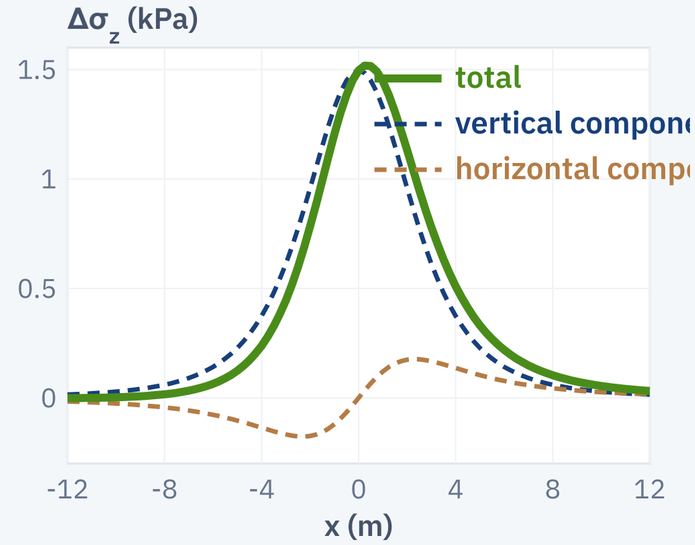
At x = +5 m, 0.3314 kPa; at x = −5 m, 0.1241 kPa: at the same distance, the stress is **2.67 times** larger on the side the load pushes toward. The vertical component is an even function, the horizontal component an odd function.

Solution (2/2)

SOLUTION

x (m)	$\Delta\sigma_{z(V)}$ (kPa)	$\Delta\sigma_{z(H)}$ (kPa)	Total $\Delta\sigma_z$ (kPa)
−10	0.02845	−0.02589	**0.00256**
−5	0.22776	−0.10362	**0.12414**
0	1.49557	0.00000	**1.49557**
5	0.22776	0.10362	**0.33138**
10	0.02845	0.02589	**0.05434**

The total curve is highest under the load (x = 0) (1.4956 kPa) and decays more slowly on the side toward which the load pushes: at x = 10 m 0.05434, at x = −10 m only 0.00256 kPa.



Check the solution in GeoTerva ↗

Problem statement

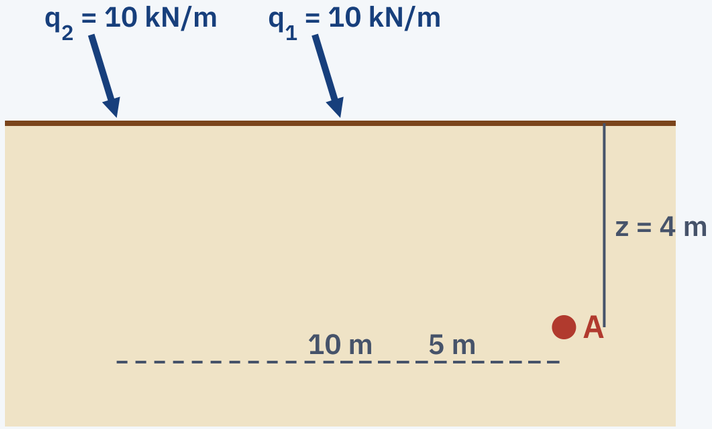
EXAMPLE

Given

Loads	**load 1:** $q_1 = 10$ **kN/m,** $\alpha_1 = 20^\circ$**,** $x_{L1} = 0$**; load 2:** $q_2 = 10$ **kN/m,** $\alpha_2 = 20^\circ$**,** $x_{L2} = -5$ **m**
Point A	$x_A = 5$ **m;** $z = 4$ **m (then** 2**;** 6**;** 8**;** 12 **m)**

REQUIRED

Contribution of load 2 (vertical and horizontal), sum of the two loads; depth profile.



Solution (1/2)

SOLUTION

1 Load 2 (x = 5 – (–5) = 10 m): the same components $q_V = 9.397$, $q_H = 3.420$ kN/m

$$\Delta\sigma_{z(V)} = \frac{2(9.397)(4^3)}{\pi(10^2+4^2)^2} = \frac{1{,}202.8}{42{,}273.3} = 0.02845$$

$$\Delta\sigma_{z(H)} = \frac{2(3.420)(10)(4^2)}{\pi(10^2+4^2)^2} = \frac{1{,}094.5}{42{,}273.3} = 0.02589 \qquad \Delta\sigma_{z(2)} = \mathbf{0.05434}\ \text{kPa}$$

2 Total: the contribution of load 1 is the value in Example 1

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} = 0.33138 + 0.05434 = \mathbf{0.38573}\ \text{kPa}$$

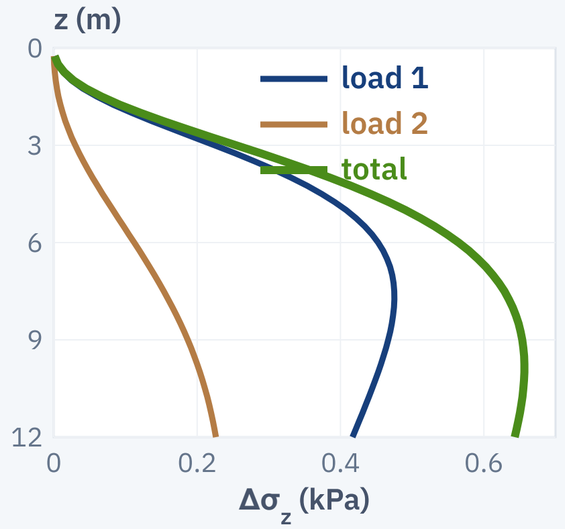
$\Delta\sigma_z$ = 0.38573 kPa (vertical components 0.25621, horizontal components 0.12951). Load 2 adds only 0.05434 kPa: it is 10 m away.

Solution (2/2)

SOLUTION

z (m)	Load 1: V + H	Load 2: V + H	Total $\Delta\sigma_z$ (kPa)
2	0.0569 + 0.0518 = 0.1087	0.0044 + 0.0081 = 0.0125	**0.1212**
4	0.2278 + 0.1036 = 0.3314	0.0285 + 0.0259 = 0.0543	**0.3857**
6	0.3473 + 0.1053 = 0.4526	0.0699 + 0.0424 = 0.1122	**0.5648**
8	0.3867 + 0.0880 = 0.4746	0.1139 + 0.0518 = 0.1657	**0.6403**
12	0.3619 + 0.0549 = 0.4168	0.1736 + 0.0527 = 0.2263	**0.6431**

At each depth the vertical and horizontal components of the loads are summed separately, and the result is the sum of the two loads. The total is 0.6403 kPa at z = 8 m and 0.6431 kPa at z = 12 m; the share of load 2 increases at depth.



Check the solution in GeoTerva ↗

Summary

Key points

Horizontal load: $\Delta\sigma_z = (2q/\pi)\, x\, z^2/(x^2 + z^2)^2$; it increases on the side it pushes ($x > 0$), decreases on the opposite side, and is zero beneath it; the maximum value is $0.2067 \cdot (q/z)$ for $x/z = 1/\sqrt{3}$

Inclined load: $q_V = q \cos\alpha$ and $q_H = q \sin\alpha$; $\Delta\sigma_z = \Delta\sigma_{z(V)} + \Delta\sigma_{z(H)}$ (in an elastic soil the stresses of the components are superposed)

Example ($q = 10$ kN/m, $\alpha = 20°$; A: $x = 5$, $z = 4$ m): $q_V = 9.397$, $q_H = 3.420$ kN/m; $\Delta\sigma_{z(V)} = 0.22776$ + $\Delta\sigma_{z(H)} = 0.10362$ = **0.33138 kPa**

Slope and depth: the total is greatest when $\tan\alpha = x/z$ ($\alpha = 51.3°$); the horizontal component peaks at $z = x$, the vertical component peaks at $z = \sqrt{3} \cdot x$

Two sides: 0.3314 at $x = +5$ m and 0.1241 kPa at $x = -5$ m; for **two loads** the components are summed separately (0.3857 kPa in the example)

References

Das, B.M. (2018). Principles of Geotechnical Engineering, 9th ed. Cengage Learning.

NOTE

All numerical examples were generated with the module's own calculation engine; the module gives the same values. The module opens with q = 10 kN/m, a load with α = 20° and point A (x = 5 m, z = 4 m).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own horizontal or inclined line loads, their slopes, positions and your point in GeoTerva; the module provides the contribution of the vertical and horizontal components, the total $\Delta\sigma_z$ value, the depth profile, the contour lines, the step-by-step solution and the report together.

Open the module → | **Theory page →**

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com