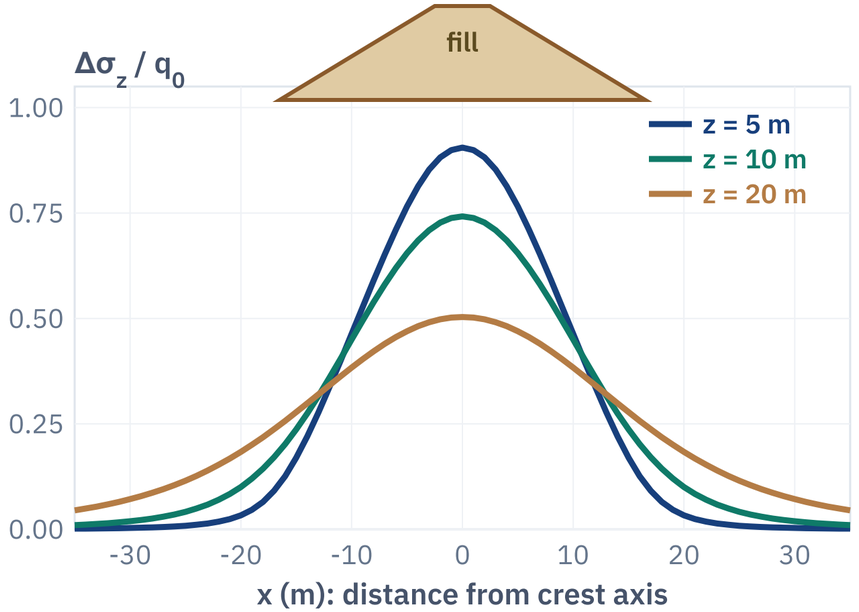
Geotechnical lecture presentation · Stresses in the soil mass

Stress beneath the embankment

Lecture: vertical stress beneath an embankment (trapezoidal load); the Osterberg solution, superposition of strips and hand-worked examples.

$q_0 = \gamma H$ | Osterberg I_2 | Three strips | Below and beside the slope | Multiple embankments



Introduction

What will we learn in this lecture?

The increase in vertical stress beneath a long **trapezoidal embankment** on the ground (highway or railway embankment, fill): load $q_0 = \gamma H$, the problem is two-dimensional

The Osterberg solution: for a half-fill, $\Delta\sigma_z = q_0\, I_2$; I_2 depends only on B_1/z and B_2/z

Under the crest, the **summation** of the two halves; beside the embankment, the **subtraction** of the small half from the large half

The **sum of three strips** giving the same result: the crest strip + two triangular strips; the module works this way

that the stresses of multiple fills are **superposed**

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example embankment: H = 7 m, γ = 17.5 kN/m³, crest 5 m, slopes 14 m. A_1 is beneath the centre of the crest, A_2 is beneath a point 5 m from the left toe; both at depth z = 5 m.

Trapezoidal embankment (fill)

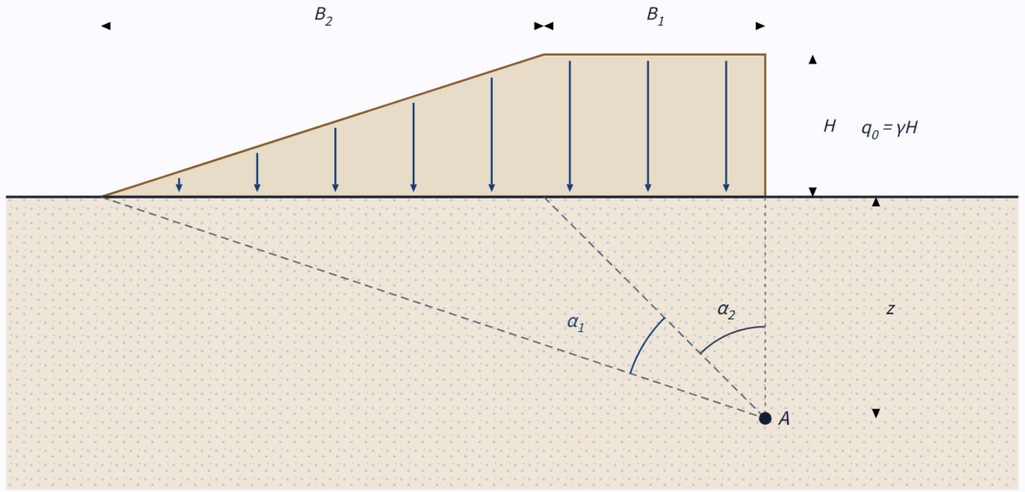
THEORY

A highway or railway embankment, a fill, a long heap: the load of a **trapezoidal-section** fill on the ground

The weight per unit area is $\mathbf{q_0 = \gamma H}$ under the crest and decreases linearly to **zero** at the toe

The embankment is long: the problem is **two-dimensional** (plane strain), as with strip loads

The embankment acts as a flexible load on the ground surface, its own stiffness being neglected; the soil is a homogeneous, isotropic, linearly elastic half-space; $\Delta\sigma_z$ is only the increment from the embankment



$\Delta\sigma_z = q_0\, I_2$

THEORY

For the **half embankment** where point A lies below the vertical passing through the inner edge of width B_1 ($q_0 = \gamma H$; angles in radians):

$$\Delta\sigma_z = \frac{q_0}{\pi}\left[\frac{B_1 + B_2}{B_2}\,(\alpha_1 + \alpha_2) - \frac{B_1}{B_2}\,\alpha_2\right]$$

$$\alpha_1 = \tan^{-1}\frac{B_1 + B_2}{z} - \tan^{-1}\frac{B_1}{z} \qquad \alpha_2 = \tan^{-1}\frac{B_1}{z}$$

In brief, **$\Delta\sigma_z = q_0\, I_2$**: the influence value I_2 depends only on **B_1/z** and **B_2/z** (Osterberg 1957)

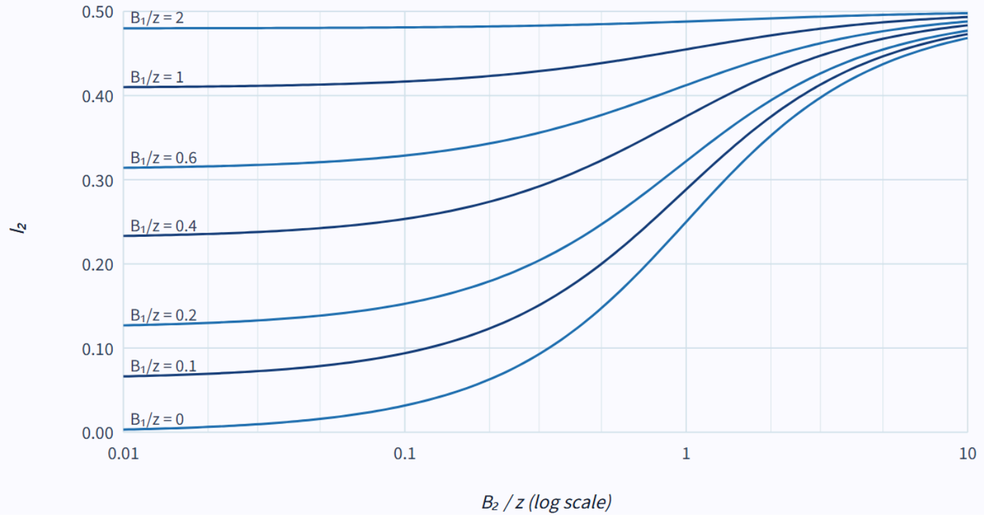
I_2 and the two halves

THEORY

A point below the crest divides the embankment into **two halves**, left and right; the I_2 values are added: $\Delta\sigma_z = q_0(I_{2,left} + I_{2,right})$

For a point **beside** the fill, a small half is **subtracted** from a large half

The contribution of each half is at most **0.5**



Any point, any inclinations

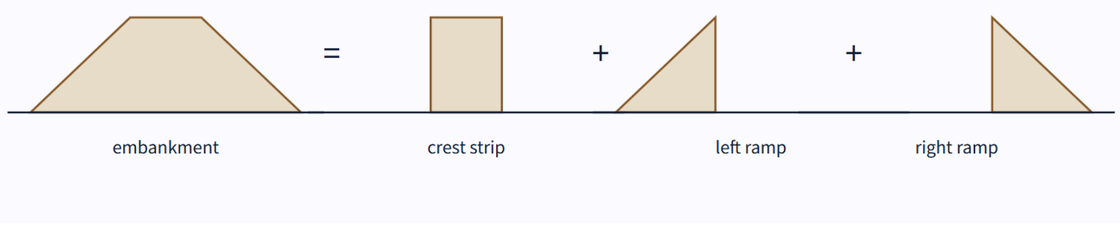
THEORY

Trapezoid = a **uniform strip** of width B_c and intensity q_0 + **two triangular strips** rising from zero at the heel to q_0 at the crest edges

The module works this way: there is no case distinction and no chart; $q_0(I_{2,left} + I_{2,right})$ under the crest is obtained exactly, and this check is also shown in the calculation steps

The slope widths may differ; $\mathbf{B_c = 0}$ gives a triangular-section embankment

$$\Delta\sigma_z = \Delta\sigma_{z(\text{crest})} + \Delta\sigma_{z(\text{left slope})} + \Delta\sigma_{z(\text{right slope})}$$



Superposition

THEORY

Each fill is defined by its own **γ, H, crest, slopes** and the position of the crest axis x_c (at most eight in the module)

The ground covered tip to tip by two fills **cannot be shared**; the two tips may touch each other

Since the soil is elastic, the stress of each fill is calculated **with respect to the crest axis** $(x - x_c)$ and the stresses are summed

$$\Delta\sigma_z = \sum_k \Delta\sigma_{z(k)} \qquad \Delta\sigma_{z(k)} = q_{0,k}\,[\,\text{crest} + \text{left slope} + \text{right slope}\,]$$

Problem statement

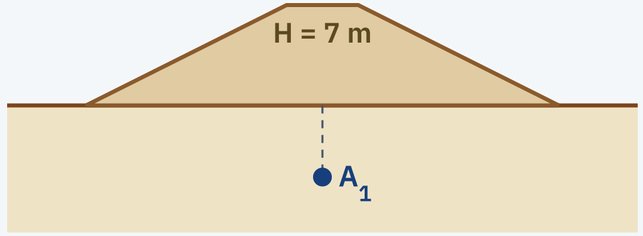
EXAMPLE

Given

Fill	$H = 7$ m, $\gamma = 17.5$ kN/m³; crest $B_c = 5$ m, slopes $L_1 = L_2 = 14$ m
Point A_1	below the crest axis, $z = 5$ m

REQUIRED

$\Delta\sigma_z$: by the Osterberg solution (two halves) and by the sum of the three strips.



Solution (1/2)

SOLUTION

1 Load, ratios and angles (each half: B_1 = 2.5 m; B_2 = 14 m; z = 5 m)

$$q_0 = \gamma H = 17.5\,(7) = \mathbf{122.5}\ \text{kPa} \qquad B_1/z = 0.5 \qquad B_2/z = 2.8$$

$$\alpha_2 = \tan^{-1}(2.5/5) = 0.4636 \qquad \alpha_1 = \tan^{-1}(16.5/5) - \alpha_2 = 1.2766 - 0.4636 = 0.8129$$

2 Influence value and stress

$$I_2 = \frac{1}{\pi}\left[\frac{16.5}{14}(1.2766) - \frac{2.5}{14}(0.4636)\right] = \frac{1.4217}{\pi} = \mathbf{0.4525}$$

$$\Delta\sigma_z = q_0\,(I_{2,\text{left}} + I_{2,\text{right}}) = 122.5\,(0.4525 + 0.4525) = \mathbf{110.87}\ \text{kPa}$$

$\Delta\sigma_z$ = 110.87 kPa at 5 m below the centre of the crest: 90.5% of the embankment load (q_0 = 122.5 kPa).

Solution (2/2)

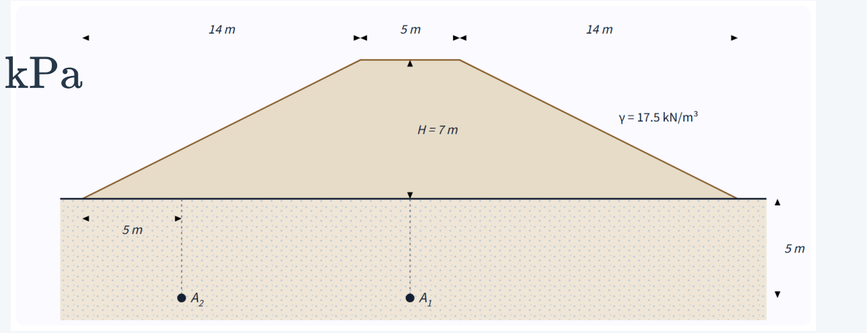
SOLUTION

1 Crest strip (q_0, B_c = 5 m): $F(u) = \tan^{-1}(u/z) + zu/(u^2 + z^2)$

$$\Delta\sigma_z = \frac{q_0}{\pi}\left[F(2.5) - F(-2.5)\right] = \frac{122.5}{\pi}\left(2 \times 0.8636\right) = \mathbf{67.35}\ \text{kPa}$$

2 Left slope (L_1 = 14 m; x' = 16.5 m from the left toe): $G(u) = (x'/2)[zu/(u^2 + z^2) + \tan^{-1}(u/z)] + z^3/[2(u^2 + z^2)]$

$$\Delta\sigma_z = \frac{2q_0}{\pi L_1}\left[G(16.5) - G(2.5)\right] = \frac{245}{14\pi}\left(13.0316 - 9.1251\right) = \mathbf{21.76}\ \text{kPa}$$



3 Right slope (symmetric) and total

$$\Delta\sigma_z = 67.35 + 21.76 + 21.76 = \mathbf{110.87}\ \text{kPa}$$

The sum of the three strips is the same as the Osterberg solution: **110.87 kPa**. The module works this way and also shows the Osterberg value under the crest as a check.

Check the solution in GeoTerva ↗

Problem statement

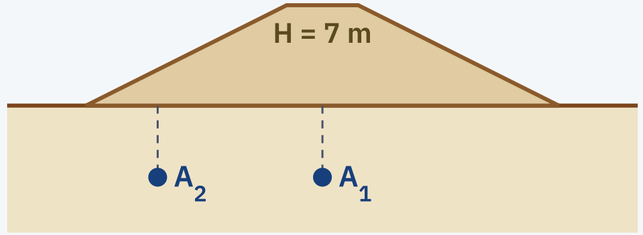
EXAMPLE

REQUIRED

$\Delta\sigma_z$: by adding and subtracting the I_2 values (three parts) and with three strips.

Given

Fill	**Embankment in Example 1: $q_0 = 122.5$ kPa**
Point A_2	**5 m inward from the left toe ($x = -11.5$ m), $z = 5$ m**



Solution (1/2)

SOLUTION

1 Partition on the vertical through A_2: x = −11.5 m; height 5 m × 7/14 = 2.5 m → q = 2.5 × 17.5 = 43.75 kPa

Piece	q (kPa)	B_1 (m)	B_2 (m)	B_1/z	B_2/z	I_2	$\Delta\sigma_z$ (kPa)
(+) small ramp on the left	43.75	0	5	0	1.0	0.25000	**10.94**
(+) full height on the right: rectangle + ramp	122.5	14	14	2.8	2.8	0.49669	**60.84**
(−) triangle added below the slope	78.75	0	9	0	1.8	0.33859	**26.66**

2 I_2 values (z = 5 m)

$$I_2(0;\,5) = \frac{1}{\pi}\,\frac{5}{5}\,\tan^{-1}1 = 0.25000 \qquad I_2(14;\,14) = \frac{1}{\pi}\left[2\,(1.3941) - 1.2278\right] = 0.49669$$

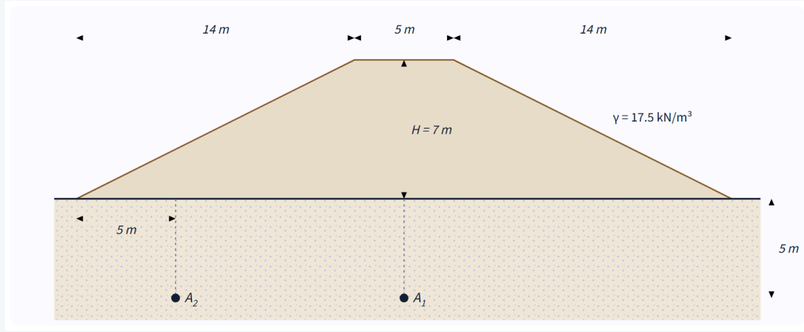
$$I_2(0;\,9) = \frac{1}{\pi}\,\tan^{-1}1.8 = 0.33859 \qquad \Delta\sigma_z = 10.94 + 60.84 - 26.66 = \mathbf{45.12}\ \text{kPa}$$

At point A_2, **$\Delta\sigma_z$ = 45.12 kPa** ($\Delta\sigma_z/q_0$ = 0.3683): about 41% of that below the crest centre (110.87 kPa).

Solution (2/2)

SOLUTION

Strip	Load	Width (m)	Position relative to the point	$\Delta\sigma_z$ (kPa)
Crest strip	q_0 = 122.5	5	x = −11.5: u = x ± 2.5 = −9 and −14	2.195
Left slope (triangular)	q_0 = 122.5	14	x′ = −11.5 + 16.5 = 5 (from the left toe)	42.304
Right slope (triangular)	q_0 = 122.5	14	x′ = 16.5 + 11.5 = 28	0.620
Total				**45.119**



The sum of the three strips is **45.12 kPa**; it is the same as the 45.12 kPa found via the component I_2 method. Most of the stress beneath the point is carried by the **left slope** (42.30 kPa); the crest strip contributes only 2.19 kPa.

Check the solution in GeoTerva ↗

Problem statement

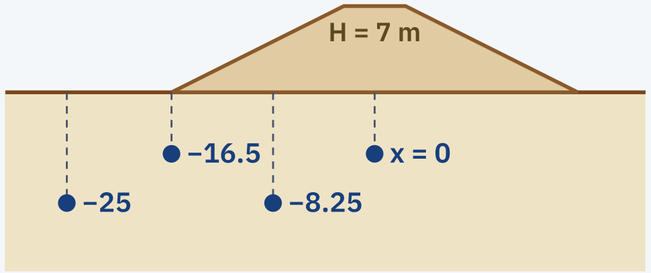
EXAMPLE

REQUIRED

At every location **$\Delta\sigma_z$** and $\Delta\sigma_z/q_0$; interpretation of the variation with depth.

Given

Fill	**Embankment in Example 1 ($q_0 = 122.5$ kPa)**
Positions	**$x = 0$ (crest axis); -8.25 (mid-slope); -16.5 (left toe); -25 m (8.5 m outside the toe)**
Depths	$z = 2; 5; 10; 15; 20; 30$ **m**



Solution (1/2)

SOLUTION

z (m)	x = 0 m	x = -8.25 m	x = -16.50 m	x = -25 m	x = 0: $\Delta\sigma_z/q_0$
2	120.81	72.03	5.52	0.08	0.986
5	110.87	70.42	13.17	1.03	0.905
10	90.88	64.71	22.78	5.08	0.742
15	74.26	57.80	28.12	10.02	0.606
20	61.67	51.17	30.28	14.12	0.503
30	45.06	40.38	29.66	18.60	0.368

Below the crest axis, the stress starts close to q_0 and decreases with depth (120.8 → 45.1 kPa). **At the slope midpoint** 72.0 → 40.4 kPa; **at the left toe** it is small near the surface (5.5 kPa) and increases at depth (29.7 kPa); 8.5 m outside the toe it rises from 0.08 kPa to 18.6 kPa.

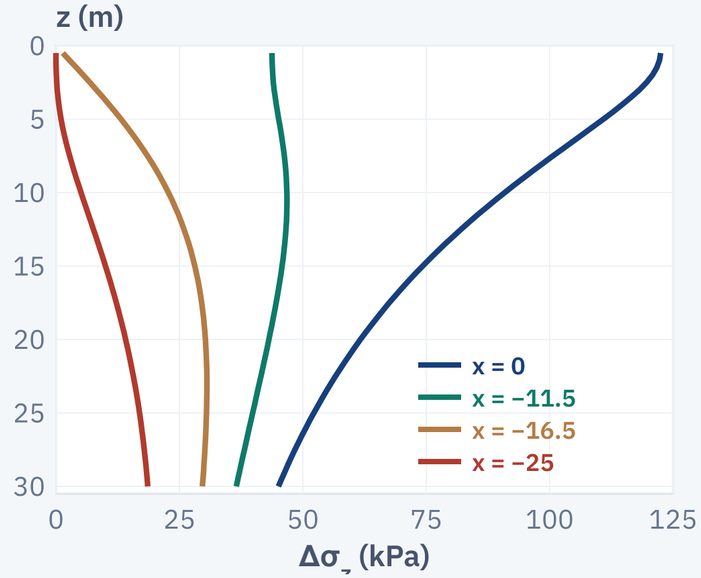
Solution (2/2)

SOLUTION

1 Maximum stress at and outside the heel (depth scanned up to 120 m)

Location	maximum $\Delta\sigma_z$ (kPa)	depth z (m)
x = −16.5 (toe)	30.57	23.2
x = −25 (outside)	19.62	39.9

Below the crest axis, the stress decreases with depth in one direction; at the **toe** and **outside** it first increases, then decreases: the embankment load spreads laterally with depth. At the toe the maximum value is 30.6 kPa at z ≈ 23.2 m; 8.5 m outside it is deeper (z ≈ 39.9 m) and smaller (19.6 kPa).



Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Table	$B_1/z = 0$ **...** 2 **and** $B_2/z = 0.1$ **...** 10 **for** I_2
Manual	$B_1 = B_2 = z$**:** $I_2(1;\ 1)$

REQUIRED

I_2 table; limit value; detailed calculation for $B_1 = B_2 = z$.

Solution (1/2)

SOLUTION

$B_2/z \setminus B_1/z$	0.0	0.1	0.2	0.4	0.6	1.0	2.0
0.1	0.0317	0.0939	0.1527	0.2534	0.3287	0.4166	0.4809
0.5	0.1476	0.2001	0.2470	0.3230	0.3769	0.4385	0.4847
1.0	0.2500	0.2885	0.3221	0.3751	0.4123	0.4548	0.4879
2.0	0.3524	0.3749	0.3943	0.4250	0.4465	0.4714	0.4916
5.0	0.4372	0.4465	0.4546	0.4674	0.4764	0.4869	0.4958
10.0	0.4683	0.4730	0.4770	0.4834	0.4879	0.4933	0.4978

Each value is less than **0.5**; as B_1/z increases and B_2/z increases, I_2 approaches 0.5 (for $B_1 \to \infty$, $I_2 = 0.5$). Since the two halves add together beneath the crest, $\Delta\sigma_z$ is at most q_0.

Solution (2/2)

SOLUTION

1 Angles ($z = 1$; $B_1 = B_2 = 1$)

$$\alpha_2 = \tan^{-1} 1 = 0.7854$$

$$\alpha_1 = \tan^{-1} 2 - \tan^{-1} 1 = 1.1071 - 0.7854 = 0.3218$$

2 Influence value

$$I_2 = \frac{1}{\pi}\left[\frac{2}{1}(1.1071) - \frac{1}{1}(0.7854)\right] = \frac{1.4289}{\pi} = \mathbf{0.45483}$$

$I_2(1; 1) = 0.45483$: it is the same as the value $B_1/z = 1$, $B_2/z = 1$ in the table. For the point under this half, $\Delta\sigma_z = 0.455\ q_0$.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Common	$\gamma = 17.5$ **kN/m³,** $H = 7$ **m (**$q_0 = 122.5$ **kPa); crest below the axis** $z = 2; 5; 10; 20$ **m**
Shapes	**(1)** $B_c = 5$, $L = 14$**; (2)** $B_c = 0$, $L = 16.5$ **(same 33 m base); (3)** $B_c = 5$, $L = 7$**; (4)** $B_c = 20$, $L = 14$

REQUIRED

Δσ$_z$ below the crest axis; effect of shape; hand check of the triangular fill.

Solution (1/2)

SOLUTION

Format	Base (m)	z = 2 m	z = 5 m	z = 10 m	z = 20 m	z = 5: $\Delta\sigma_z/q_0$
Trapezoidal (opening)	33	120.81	110.87	90.88	61.67	0.905
Triangle (same base 33 m)	33	113.09	99.55	80.01	53.79	0.813
Vertical slope (L = 7 m)	19	119.33	102.06	73.59	43.47	0.833
Wide crest (20 m)	48	122.38	120.87	113.47	91.29	0.987

At the same height, a **wide crest** carries the load deeper (z = 20 m: 91.3 kPa), while **triangular** and **steep-sided** embankments carry less (53.8 and 43.5 kPa). At z = 5 m the order is: triangular 99.6 < steep-sided 102.1 < trapezoidal 110.9 < wide crest 120.9 kPa.

Solution (2/2)

SOLUTION

1 Each half: B_1 = 0; B_2 = 16.5 m (z = 5 m)

$$\alpha_2 = 0 \qquad \alpha_1 = \tan^{-1}\frac{16.5}{5} = 1.2766$$

$$I_2 = \frac{1}{\pi}\,\frac{16.5}{16.5}\,\alpha_1 = 0.40634$$

2 Stress

$$\Delta\sigma_z = q_0\,(2\,I_2) = 122.5\,(2 \times 0.40634) = \mathbf{99.55}\ \mathrm{kPa}$$

99.55 kPa at 5 m below the crest of the triangular fill: 11.32 kPa less than the trapezoidal fill with the same base (110.87 kPa); the triangular section carries less fill volume than the trapezoid on the same base.

Check the solution in GeoTerva ↗

Problem statement

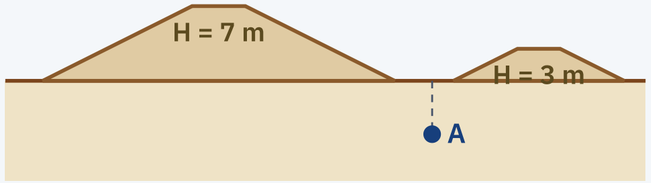
EXAMPLE

Given

Fill 1	$\gamma = 17.5$, $H = 7$ m, $B_c = 5$, $L_1 = L_2 = 14$ m; crest axis $x = 0$
Fill 2	$\gamma = 17.5$, $H = 3$ m, $B_c = 4$, $L_1 = L_2 = 6$ m; crest axis $x = 30$ m
Point A	$x = 20$ m (between the two embankments), $z = 5$ m; also $2 - 30$ m

REQUIRED

Three strips of each embankment, $\Delta\sigma_z$ for each embankment and for the total; depth profile.



Solution (1/2)

SOLUTION

1 Overlap check (toes: embankment 1 → 16.5 m; embankment 2 → 22 m) and three strips of each embankment

Fill	q_0 (kPa)	dx = x − x_c	Crest strip	Left slope	Right slope	$\Delta\sigma_z$ (kPa)
1 (H = 7 m)	122.5	20	0.282	0.133	3.654	**4.069**
2 (H = 3 m)	52.5	−10	1.160	3.992	0.276	**5.427**

2 Superposition

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} = 4.069 + 5.427 = \mathbf{9.497}\ \mathrm{kPa}$$

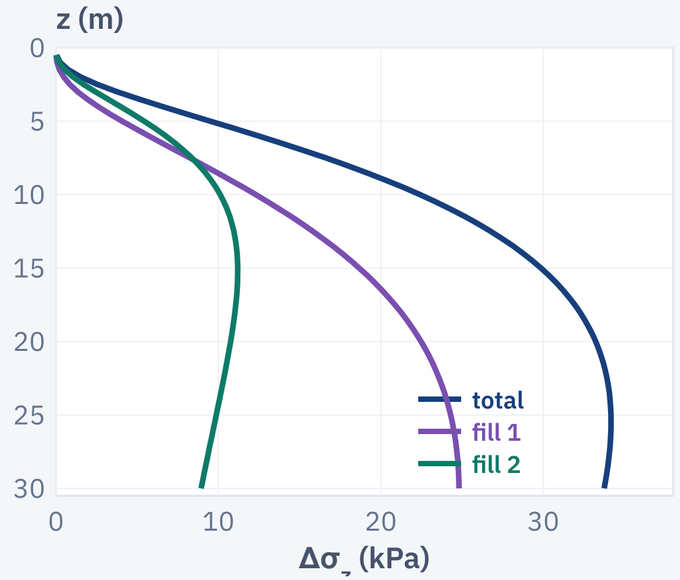
Point A is 3.5 m from the right toe of embankment 1 and 2 m from the left toe of embankment 2. The low but nearby **embankment 2** contributes most of the total (5.43 kPa); the total is **$\Delta\sigma_z$ = 9.50 kPa**.

Solution (2/2)

SOLUTION

z (m)	Fill 1	Fill 2	Total
2	0.48	1.06	**1.53**
5	4.07	5.43	**9.50**
10	12.30	10.11	**22.41**
15	18.67	11.19	**29.86**
20	22.48	10.75	**33.23**
30	24.82	8.94	**33.76**

The total stress reaches its maximum value (34.18 kPa) at z ≈ 25.5 m; with depth the share of high fill 1 increases (z = 30 m: 24.82 and 8.94 kPa).



Check the solution in GeoTerva ↗

Summary

Key points

Load: $q_0 = \gamma H = 122.5$ kPa; **Osterberg:** $\Delta\sigma_z = q_0\, I_2(B_1/z, B_2/z)$, each half at most 0.5; below the crest the two halves are summed

A_1 (below crest, z = 5 m): $I_2 = 0.4525$; $\Delta\sigma_z = 122.5\ (2 \times 0.4525) =$ **110.87 kPa**; three strips: 67.35 + 21.76 + 21.76

A_2 (below slope): 10.94 + 60.84 − 26.66 = **45.12 kPa**; beside the embankment the smaller half is subtracted from the larger half; the three strips give the same result

Location: at the toe and outside, the stress first increases then decreases; beneath the crest it decreases in one direction. **Shape:** a wide crest carries more load to depth; a triangular fill gives 99.6 kPa at the same base (trapezoidal 110.9)

Two fills: stresses are computed about the crest axes and summed: 4.07 + 5.43 = 9.50 kPa

References

Das, B.M. (2018). Principles of Geotechnical Engineering, 9th ed. Cengage Learning.

Osterberg, J.O. (1957). Influence values for vertical stresses in a semi-infinite mass due to an embankment loading. Proc. 4th Int. Conf. on Soil Mechanics and Foundation Engineering, London, Vol. 1, 393–394.

NOTE

All numerical examples were generated with the module's own calculation engine; the module gives the same values. The module opens with an H = 7 m, γ = 17.5 kN/m³ embankment and point A_1 (x = 0; z = 5 m).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your embankments (γ, H, crest, slopes, position), point A and the depth in GeoTerva; the module provides the three strips, the Osterberg check, the $\Delta\sigma_z$ value, the equal-stress contours, the depth profile, the step-by-step solution and the report together.

Open the module → **Theory page →**

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com