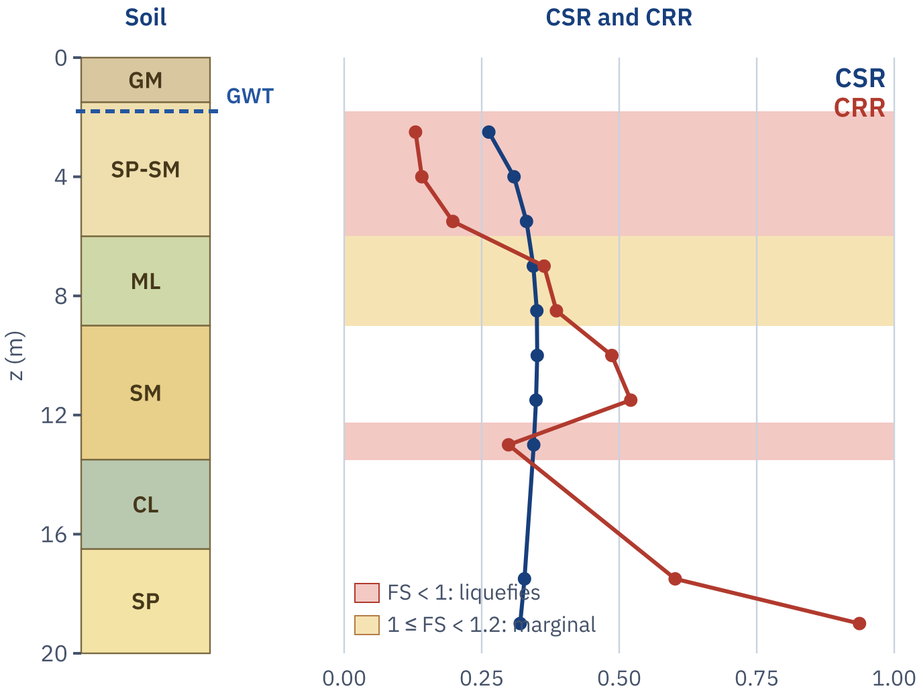
Geotechnical lecture presentation · Liquefaction

Liquefaction triggering with SPT

Lecture: liquefaction triggering from SPT (Idriss–Boulanger); one borehole and one test point are solved by hand end to end, the remaining points with tables.

Corrections · CSR · CRR · MSF · Kσ · FS and zones · Solvable by hand



Based on the GeoTerva Theory page · geoterva.com

What will we learn in this lecture?

- The **stress-based** evaluation of liquefaction: earthquake demand **CSR** is compared with soil resistance **CRR**; **FS = CRR / CSR**
- Corrections from the raw SPT blow count N_{30} to $\mathbf{(N_1)_{60cs}}$: energy, rod length, overburden and fines
- CSR (r_d, σ_v/σ'_v), the CRR curve, **magnitude scaling** MSF and the **overburden coefficient** K_σ
- Which tests are excluded from the calculation by **screening**; how the zones and the liquefiable thickness are found
- At a borehole, the **single test point end to end by hand**, and the others with the same steps in the table

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example: borehole BH-1 (GWT 1.80 m; 0–1.5 m fill, 1.5–6 m loose sand …); a_{max} = 0.35 g, M = 7.5, hammer energy ratio 60%, $FS_{required}$ = 1.20.

Which test is evaluated?

THEORY

A test is evaluated if it is **below the groundwater table (GWT)**, not deeper than $\mathbf{z_{max}}$ (default 20 m) and in soil that **behaves like sand**

Soil with plasticity index PI ≥ PI_{lim} (default 7) is **clay-like**; if PI is not given, the symbol CL, CH, OL, OH or PT indicates this

Clay-like soil may **soften** under cyclic loading but does not liquefy in the sense used here: it is left to the cyclic softening check

Tests that are not evaluated remain in all tables **with their reasons**. The consequences of liquefaction (settlement, lateral spreading, loss of bearing capacity) are outside the scope of this version

Total, pore water and effective vertical stress

THEORY

γ above the GWT; **γ_{sat}** below it; summed layer by layer. Pore water pressure is **hydrostatic** from the GWT downward (z_W is the GWT depth of the borehole)

The effective stress σ'_V is used both in CSR and in the overburden corrections

$$\sigma_v = \sum \gamma_i \, h_i \qquad u = \gamma_w \, (z - z_w) \qquad \sigma'_v = \sigma_v - u$$

We compute the effective stress only for tests below the water table: above the GWT u = 0 and the test is screened out.

From N_{30} to N_{60}

THEORY

The raw blow count N_{30} (last 30 cm) is brought to the standard rod with **60% energy**: C_E = ER/60 (hammer), C_B (borehole diameter), C_S (sample tube)

C_R depends on rod length: rod length = z + length above the surface (here 1.5 m)

$$N_{60} = N_{30} \cdot C_E \cdot C_B \cdot C_S \cdot C_R$$

Rod length (m)	< 3	3 – 4	4 – 6	6 – 10	≥ 10
C_R	0.75	0.80	0.85	0.95	1.00

$N_{60} \rightarrow (N_1)_{60} \rightarrow (N_1)_{60cs}$

THEORY

C_N brings the measurement to an effective stress of 1 atm (P_a = 101.3 kPa); since the exponent m depends on the result itself, it is found **by iteration**; $C_N \leq 1.7$

Fines increase the sand-like resistance: using FC (percent, the test's own value if measured, otherwise the layer value), the clean-sand equivalent is obtained

$$C_N = \left(\frac{P_a}{\sigma'_v}\right)^m \leq 1.7 \qquad m = 0.784 - 0.0768\sqrt{(N_1)_{60cs}} \qquad (N_1)_{60} = C_N N_{60}$$

$$\Delta(N_1)_{60} = \exp\left[1.63 + \frac{9.7}{FC + 0.01} - \left(\frac{15.7}{FC + 0.01}\right)^2\right] \qquad (N_1)_{60cs} = (N_1)_{60} + \Delta(N_1)_{60}$$

FC (%)	0	5	10	15	20	25	30	35	50
$\Delta(N_1)_{60}$	0.00	0.00	1.15	3.26	4.48	5.07	5.36	5.51	5.61

Earthquake demand: CSR and r_d

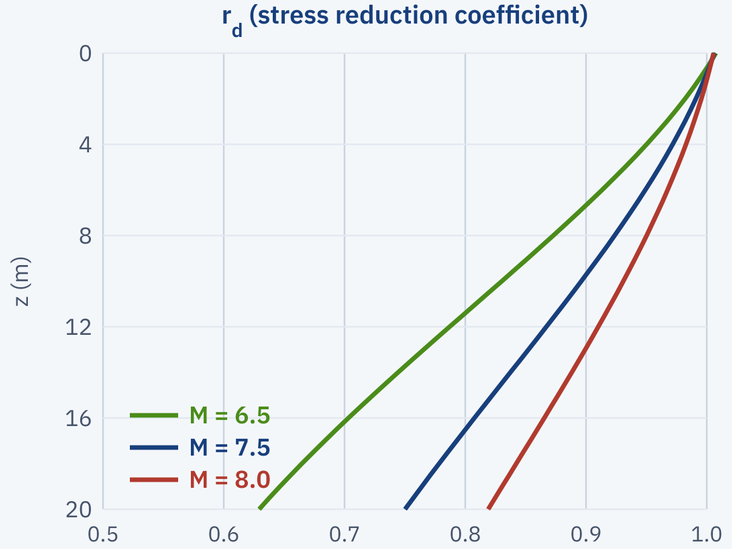
THEORY

CSR is the ratio of the cyclic shear stress applied by the earthquake to the soil to the effective stress; 0.65 carries the average stress, σ_V/σ'_V carries the effect of depth

r_d (stress reduction coefficient) decreases with depth and magnitude; it is 1 at the surface

$$CSR = 0.65\,\frac{\sigma_v}{\sigma'_v}\,a_{max}\,r_d \qquad r_d = \exp[\alpha(z) + \beta(z)M]$$

$$\alpha = -1.012 - 1.126\sin\left(\frac{z}{11.73} + 5.133\right) \qquad \beta = 0.106 + 0.118\sin\left(\frac{z}{11.28} + 5.142\right)$$



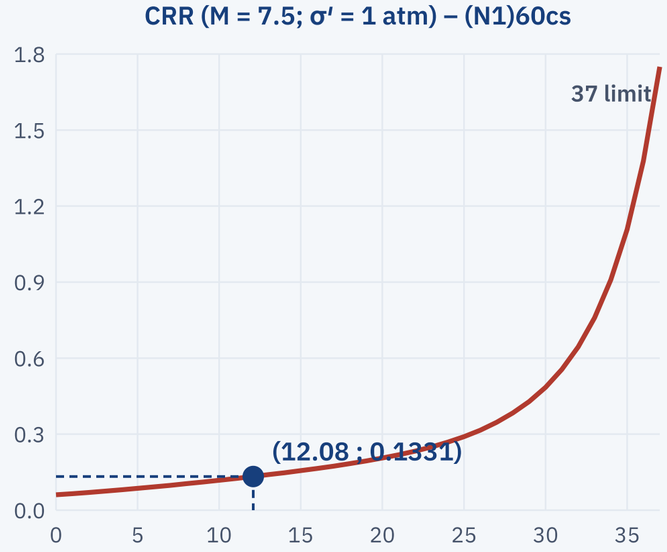
Soil resistance: CRR curve

THEORY

$CRR_{M=7.5;\,\sigma'=1}$ depends only on $\mathbf{(N_1)_{60cs}}$ (Idriss–Boulanger curve); it is valid for M = 7.5 and σ'_V = 1 atm

The curve is used up to $\mathbf{(N_1)_{60cs} = 37}$; a denser test is evaluated at this limit and a warning is given

$$CRR_{7.5,1} = \exp\left[\frac{n}{14.1} + \left(\frac{n}{126}\right)^2 - \left(\frac{n}{23.6}\right)^3 + \left(\frac{n}{25.4}\right)^4 - 2.8\right] \qquad n = (N_1)_{60cs}$$



MSF and K_σ

THEORY

MSF magnitude scaling: CRR is for M = 7.5; at other magnitudes it carries the difference in the number of cycles. For M < 7.5, MSF > 1

K_σ overburden coefficient: CRR is for 1 atm; as σ'_v increases, the resistance ratio decreases. $K_\sigma \leq 1.1$

$$MSF = 1 + (MSF_{max} - 1)\left(8.64\, e^{-M/4} - 1.325\right) \qquad MSF_{max} = 1.09 + \left(\frac{(N_1)_{60cs}}{31.5}\right)^2 \leq 2.2$$

$$K_\sigma = 1 - C_\sigma \ln \frac{\sigma'_v}{P_a} \leq 1.1 \qquad C_\sigma = \frac{1}{18.9 - 2.55\sqrt{(N_1)_{60cs}}} \leq 0.3$$

FS = CRR / CSR; zone and thickness

THEORY

CRR = $CRR_{7.5;1}$ · MSF · K_σ; **FS = CRR / CSR**. FS < 1: **liquefaction is triggered**; 1 ≤ FS < $FS_{required}$ (1.20): **marginal**; the required value is an input taken from the project specification

Each evaluated test represents its own layer: down to the **midpoint** with the test above and the test below; the interval stops at the layer boundary and at the GWT

If neighbouring intervals give the same result, they merge into one **zone**; the **liquefiable thickness** of a borehole is the sum of the zones with FS < 1

$$CRR = CRR_{7.5,\,1} \cdot MSF \cdot K_\sigma \qquad FS = \frac{CRR}{CSR}$$

Problem statement

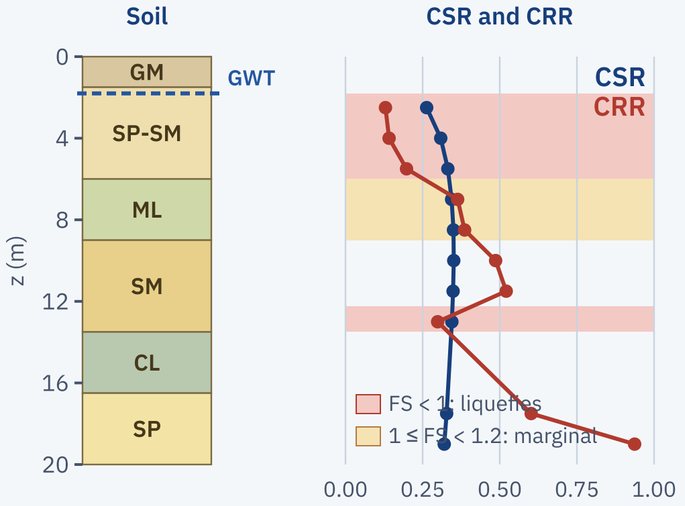
EXAMPLE

Given

Borehole BH-1	**GWT** $z_w = 1.80$ **m; 0–1.5 m fill (GM):** $\gamma = 18.5$**; 1.5–6 m loose sand (SP-SM):** $\gamma = 18.0$**,** $\gamma_{sat} = 19.5$ **kN/m³, FC = 8 %**
Test	$z = 4.0$ **at** $N_{30} = 10$ **m (below the GWT)**
Earthquake and equipment	$a_{max} = 0.35g$**,** $M = 7.5$**; ER = 60%** ($C_E = C_B = C_S = 1$)**, rod length =** $z + 1.5$ **m;** $FS_{req} = 1.20$

REQUIRED

Stresses, **N_{60}**, **$(N_1)_{60cs}$** (by iteration), **CSR**, **CRR** and **FS**.



Solution (1/5)

SOLUTION

1 Total vertical stress (layer by layer; GWT 1.80 m)

$$\sigma_v = 18.5(1.5) + 18.0(1.80 - 1.5) + 19.5(4.0 - 1.80) = 27.75 + 5.40 + 42.90 = \mathbf{76.05}\ \mathrm{kPa}$$

2 Pore water pressure and effective stress

$$u = \gamma_w(z - z_w) = 9.81(4.0 - 1.80) = \mathbf{21.58}\ \mathrm{kPa}$$

$$\sigma'_v = \sigma_v - u = 76.05 - 21.58 = \mathbf{54.47}\ \mathrm{kPa}$$

The test is **2.20 m** below the GWT: σ_V = 76.05, u = 21.58, **σ'_V = 54.47 kPa**. This point is where the borehole's smallest FS occurs (figure on the problem statement page).

Solution (2/5)

SOLUTION

3 Rod length correction and N_{60}

$$L = z + 1.5 = 5.5\ \text{m} \Rightarrow C_R = 0.85 \qquad N_{60} = 10(1)(1)(1)(0.85) = \mathbf{8.50}$$

4 Increase in fines

$$\Delta(N_1)_{60} = \exp[1.63 + 9.7/8.01 - (15.7/8.01)^2] = \exp[-1.0008] = \mathbf{0.3676}$$

5 Overburden correction: since C_N depends on $(N_1)_{60cs}$, iteration is required (σ'_v = 54.47 kPa)

Iteration	n (input)	m	C_N	$(N_1)_{60} = C_N N_{60}$	$(N_1)_{60cs} = + \Delta$
1	8.5000	0.5601	1.4158	12.0339	12.4015
2	12.4015	0.5135	1.3754	11.6912	12.0587
3	12.0587	0.5173	1.3786	11.7185	12.0861
…	…	…	…	…	…
8	12.0840	0.5170	1.3784	11.7165	12.0840

It converges in 8 iterations: **C_N = 1.3784**, $(N_1)_{60}$ = 11.716, **$(N_1)_{60cs}$ = 12.08**.

Solution (3/5)

SOLUTION

6 Stress reduction coefficient r_d (z = 4.0 m, M = 7.5)

$$\alpha = -1.012 - 1.126 \sin\left(\frac{4}{11.73} + 5.133\right) = -0.1971$$

$$\beta = 0.106 + 0.118 \sin\left(\frac{4}{11.28} + 5.142\right) = 0.0225$$

$$r_d = \exp[\alpha + \beta M] = \exp[-0.1971 + 0.0225(7.5)] = \mathbf{0.9718}$$

7 Cyclic stress ratio

$$CSR = 0.65 \frac{\sigma_v}{\sigma'_v} a_{max}\, r_d = 0.65 \left(\frac{76.05}{54.47}\right)(0.35)(0.9718) = \mathbf{0.3087}$$

The earthquake imposes a cyclic shear stress at this depth equal to **30.9** percent of the effective stress (CSR = 0.3087).

Solution (4/5)

SOLUTION

8 Base resistance for M = 7.5 and σ′ = 1 atm ($(N_1)_{60cs}$ = 12.08)

$$CRR_{7.5,1} = \exp\left[\frac{12.08}{14.1} + \left(\frac{12.08}{126}\right)^2 - \left(\frac{12.08}{23.6}\right)^3 + \left(\frac{12.08}{25.4}\right)^4 - 2.8\right]$$

$$CRR_{7.5,1} = \exp[-2.0168] = \mathbf{0.1331}$$

9 Magnitude and overburden coefficients

$$MSF_{max} = 1.09 + (12.08/31.5)^2 = 1.2372$$

$$MSF = 1 + (1.2372 - 1)(8.64\,e^{-7.5/4} - 1.325) = \mathbf{1.0000}$$

$$C_\sigma = 1/(18.9 - 2.55\sqrt{12.08}) = 0.0996 \qquad K_\sigma = 1 - 0.0996\ln(54.47/101.325) = \mathbf{1.0619}$$

$CRR_{7.5;1}$ = **0.1331**; MSF = **1.0000**; K_σ = **1.0619**: the product of these three numbers gives CRR (next slide).

Solution (5/5)

SOLUTION

10 Resistance and factor of safety

$$CRR = CRR_{7.5,1} \cdot MSF \cdot K_\sigma = 0.1331(1.0000)(1.0619) = \mathbf{0.1413}$$

$$FS = \frac{CRR}{CSR} = \frac{0.1413}{0.3087} = \mathbf{0.458}$$

FS = 0.458 < 1: in this test, liquefaction is **triggered**. The test represents the interval 3.25 – 4.75 m (midpoints of the test above at 2.5 m and the one below at 5.5 m). The same ten steps are repeated for each test.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Borehole BH-1	**13 SPT:** $z = 1.0$ **–** 19.0 **m; GWT** 1.80 **m;** $z_{max} = 20$ **m;** $PI_{lim} = 7$
Layers	**GM (0–1.5), SP-SM (1.5–6), ML** $PI = 5$ **(6–9), SM (9–13.5), CL** $PI = 18$ **(13.5–16.5), SP (16.5–20)**

REQUIRED

For each test, the **evaluated / not evaluated** decision and its justification.

Solution

SOLUTION

z (m)	N_{30}	Soil	PI	GWT?	Clay-like?	Decision
1.0	9	GM	–	**above**	–	**not evaluated**: above the GWT
2.5	7	SP-SM	–	below	no	is evaluated
4.0	10	SP-SM	–	below	no	is evaluated
5.5	16	SP-SM	–	below	no	is evaluated
7.0	21	ML	5	below	no	is evaluated
8.5	22	ML	5	below	no	is evaluated
10.0	27	SM	–	below	no	is evaluated
11.5	29	SM	–	below	no	is evaluated
13.0	25	SM	–	below	no	is evaluated
14.5	13	CL	18	below	yes	**not evaluated**: clay-like (PI ≥ 7)
16.0	15	CL	18	below	yes	**not evaluated**: clay-like (PI ≥ 7)
17.5	40	SP	–	below	no	is evaluated
19.0	44	SP	–	below	no	is evaluated

3 of the 13 tests are screened out: 1.0 m (above the GWT) and 14.5 and 16.0 m (CL, PI = 18 ≥ 7). The ML layer (PI = 5 < 7) is not considered clay-like; the 7.0 and 8.5 m tests **are evaluated** (FC = 62 %). The screened-out tests still appear in the tables.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Borehole	**BH-1 and the parameters from Example 1; ten steps identical for every test**
Zone rule	**each test extends in its own layer to the midpoint of the neighbouring tests; it stops at the layer boundary and at the GWT**

REQUIRED

Table of all tests; **liquefied zones**, **liquefied thickness** and the smallest FS.

Solution (1/2)

SOLUTION

z (m)	N_{30}	Soil	FC %	σ_v	σ'_v	N_{60}	C_N	$(N_1)_{60cs}$	CSR	CRR	FS	Condition
1.0	9	GM	20	–	–	6.75	1.700	15.95	–	–	–	*Above GWT*
2.5	7	SP-SM	8	46.8	39.9	5.95	1.652	10.19	0.263	0.130	0.493	Liquefies
4.0	**10**	**SP-SM**	**8**	**76.1**	**54.5**	**8.50**	**1.378**	**12.08**	**0.309**	**0.141**	**0.458**	**Liquefies**
5.5	16	SP-SM	8	105.3	69.0	15.20	1.191	18.46	0.332	0.198	0.596	Liquefies
7.0	21	ML	62	134.1	83.0	19.95	1.079	27.13	0.344	0.363	1.056	At the limit
8.5	22	ML	62	162.6	96.8	22.00	1.017	27.98	0.350	0.386	1.102	At the limit
10.0	27	SM	18	191.9	111.4	27.00	0.966	30.18	0.351	0.487	1.386	Safe
11.5	29	SM	18	221.6	126.4	29.00	0.924	30.89	0.349	0.521	1.494	Safe
13.0	25	SM	18	251.2	141.4	25.00	0.878	26.03	0.345	0.299	0.867	Liquefies
14.5	13	CL	88	–	–	13.00	0.816	16.12	–	–	–	*clay-like (PI ≥ 7)*
16.0	15	CL	88	–	–	15.00	0.788	17.34	–	–	–	*clay-like (PI ≥ 7)*
17.5	40	SP	4	338.7	184.6	40.00	0.812	32.50	0.328	0.602	1.835	Safe
19.0	44	SP	4	369.4	200.7	44.00	0.799	35.15	0.320	0.937	2.928	Safe

Smallest FS = 0.458 (z = 4.0 m). Liquefaction zones: **1.80 – 6.00 m** and **12.25 – 13.50 m**; at the boundary: 6.00 – 9.00 m. Liquefied thickness = 5.45 m. The 4.0 m row is Example 1 solved by hand.

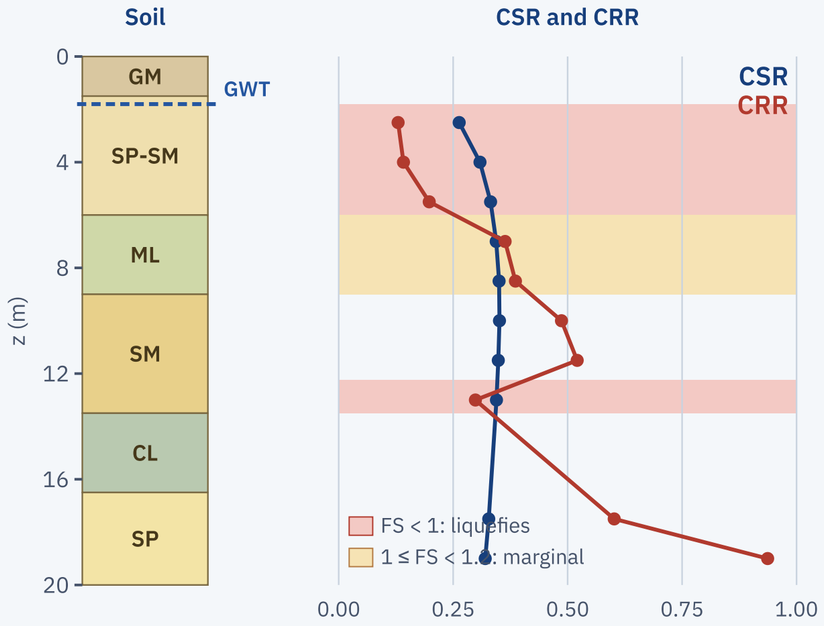
Solution (2/2)

SOLUTION

1 Each test represents its own interval; adjacent intervals with the same result are merged

Zone	Range (m)	Thickness (m)	Tests	FS range
Liquefiable 1	1.80 – 6.00	4.20	2.5; 4.0; 5.5 m	0.458 – 0.596
At the limit	6.00 – 9.00	3.00	7.0; 8.5 m	1,056 – 1,102
Liquefiable 2	12.25 – 13.50	1.25	13.0 m	0.867
Total liquefied		**5.45**		

Example: the 4.0 m test represents 3.25 – 4.75 m; the 2.5 m test covers 1.80 – 3.25 m (stops at the GWT), the 5.5 m test covers 4.75 – 6.00 m (stops at the layer boundary): total **4.20 m**. The liquefiable thickness of the borehole is 5.45 m.



Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Base case	**BH-1:** $a_{max} = 0.35g$, $M = 7.5$, **GWT** 1.80 **m, ER = 60%**
Variables	**only one at a time:** $a_{max} = 0.25g$ **/** $0.45g$**;** $M = 6.5$ **/** 8.0**; GWT** 3.0 **m /** 1.0 **m; ER 45% / 80%**

REQUIRED

The **smallest FS** and the **liquefied thickness** of BH-1; manual check of MSF in the 4.0 m test.

Example 4 · Sensitivity table

Solution (1/2)

SOLUTION

Condition	Change	Minimum FS	z (m)	Liquefied thickness (m)	At the limit (m)
Base	a_{max} 0.35g; M 7.5; GWT 1.8; ER 60	**0.458**	4.0	5.45	3.00
a_{max} decreases	0.25g	**0.641**	4.0	4.20	0.00
a_{max} increases	0.45g	**0.356**	4.0	8.45	3.25
Magnitude decreases	M 6.5	**0.510**	4.0	4.20	0.00
Magnitude increases	M 8.0	**0.436**	4.0	8.45	3.25
GWT deepens	3.0 m	**0.517**	4.0	4.00	3.00
GWT rises	1.0 m	**0.412**	2.5	5.75	3.00
Energy decreases	ER %45 (C_E 0.75)	**0.388**	4.0	15.20	0.00
Energy increases	ER %80 (C_E 1.33)	**0.560**	4.0	4.20	0.00

a_{max} is the most influential input: at 0.25g the liquefiable thickness is 4.20 m, at 0.45g it is 8.45 m. The **hammer energy** effect is unexpectedly large: with ER 45 %, N_{60} decreases and the liquefiable thickness rises to 15.20 m. **Knowing** the measured energy ratio is therefore important.

Solution (2/2)

SOLUTION

1 Magnitude scaling factor for M = 6.5 and M = 8.0 ($(N_1)_{60cs}$ = 12.08)

$$MSF_{6.5} = 1 + (1.2372 - 1)(8.64\, e^{-6.5/4} - 1.325) = \mathbf{1.0892}$$

$$MSF_{8.0} = 1 + (1.2372 - 1)(8.64\, e^{-8.0/4} - 1.325) = \mathbf{0.9631}$$

2 At the same test (4.0 m) FS

M	r_d	CSR	MSF	CRR	FS
6.5	0.9502	0.3018	1.0892	0.1539	**0.510**
7.5	0.9718	0.3087	1.0000	0.1413	**0.458**
8.0	0.9828	0.3122	0.9631	0.1361	**0.436**

A smaller magnitude means fewer cycles: MSF increases and FS becomes 0.458 → **0.510**; for M = 8.0, FS is 0.436. Both r_d and MSF depend on magnitude; the two act in the same direction.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Boreholes	**BH-1 (GWT 1.80 m), BH-2 (2.40 m), BH-3 (3.10 m); same parameters**
Section	**BH-1 $x = 0$, BH-2 $x = 32$, BH-3 $x = 70$ m**

REQUIRED

The **smallest FS** of each borehole, the **liquefied** and **marginal** thicknesses; project summary.

Solution

SOLUTION

Borehole	GWT (m)	Test	Screened	Minimum FS	z (m)	Liquefied zones (m)	Thickness (m)	At the limit (m)
BH-1	1.80	13	3	**0.458**	4.0	1.80–6.00; 12.25–13.50	5.45	3.00
BH-2	2.40	13	3	**0.697**	3.0	2.40–5.25	2.85	3.50
BH-3	3.10	13	6	**0.841**	6.0	5.00–8.25	3.25	1.75

Liquefaction is triggered **in all three of the three boreholes**. Smallest FS = 0.458 (BH-1, 4.0 m); the thickest liquefied soil is in BH-1 (5.45 m). Of 39 tests, 27 were evaluated and 12 were screened out (above the GWT or clay-like).

If a section line is present, the colour band is interpolated linearly between boreholes; the grey area is ground that is not evaluated.

Check the solution in GeoTerva ↗

Summary

Key points

FS = CRR / CSR; CSR = 0.65 (σ_v/σ'_v) a_{max} r_d; CRR = $CRR_{7.5;1}$ · MSF · K_σ

N_{60} = N_{30} C_E C_B C_S C_R; **$(N_1)_{60cs}$** = C_N N_{60} + Δ(FC), where C_N is found by iteration

Example (BH-1, 4.0 m): σ'_v 54.47 kPa; N_{60} 8.50; C_N 1.378; $(N_1)_{60cs}$ 12.08; CSR 0.3087; CRR 0.1413 (MSF 1.000, K_σ 1.062) → **FS 0.458**

Screening: tests above the GWT, below z_{max} and clay-like (PI ≥ 7) are not included in the calculation; **zone**: tests extend to their neighbouring midpoints

Sensitivity: a_{max} and the measured hammer energy are the inputs that change the result most; the consequences of liquefaction are a separate calculation

References

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Youd, T.L. et al. (2001). Liquefaction resistance of soils: summary report from the 1996 NCEER and 1998 NCEER/NSF workshops. J. Geotech. Geoenviron. Eng., 127(10).

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NOTE

All numerical examples have been produced with the module's own calculation engine; the module gives the same values. The module's opening project is these three boreholes (a_{max} = 0.35g, M = 7.5).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own boreholes, SPT results, GWT and earthquake parameters in GeoTerva; the module provides the corrections for each test, CSR, CRR, FS, the zones, the liquefiable thickness and the colour-banded section, together with the step-by-step solution and the report.

Open the module → **Theory page →**

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com