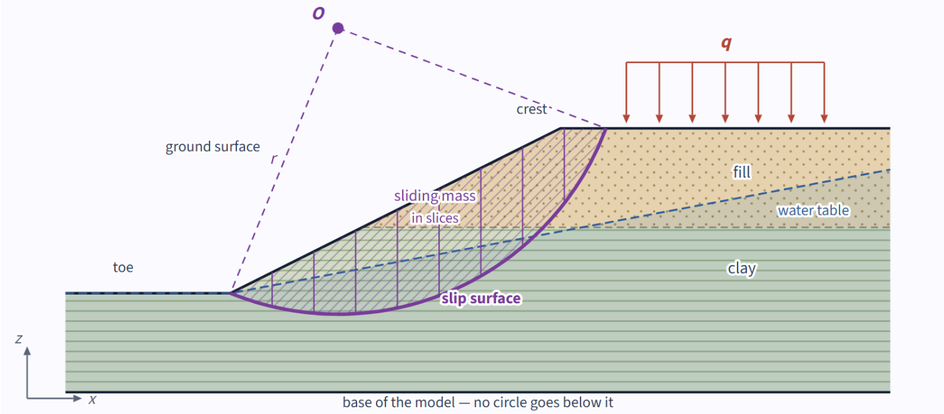
Geotechnical lecture presentation · Slope stability

Slope stability: limit equilibrium (slices)

Lecture: the limit equilibrium slice method; a detailed hand solution of one slip circle and one slice, the remaining slices and the topics with tables.

Slice method · Ordinary · Bishop · Spencer · Critical circle · Water · load · earthquake · crack · Reinforcement · Hoek–Brown



What will we learn in this lecture?

The idea of **limit equilibrium**: the mass above the slip surface is a rigid body in which the strength is mobilized in the same proportion; **FS = strength / required stress**

The division of the mass into **slices**: the weight W of each slice, the base inclination α, the base length L, the water force U

To solve **the three methods** (ordinary/Fellenius, simplified Bishop, Spencer) in detail on a single slip circle and for **a single slice**

How the same calculation is repeated for the **remaining 32 slices** and the tables of results; the search for the critical circle

Effects of water, load, earthquake, tension crack, reinforcement (nails, anchors, geogrid) and rock (Hoek–Brown)

LECTURE FLOW

First the **theory**, then a **hand solution** with a single circle and a single slice:

1 · **Theory**: FS, slice, three methods

2 · **Problem statement**: single circle, single slice

3 · **Solution**: slice 22 by hand; the rest by **table**

Model: a 2:1 slope 10 m high; fill on top (γ = 18, c′ = 5, φ′ = 30°), clay below (γ = 17.5, c′ = 20, φ′ = 18°); no water, load or earthquake. Forces are per meter of section (kN/m).

Drained, undrained, increasing with depth

THEORY

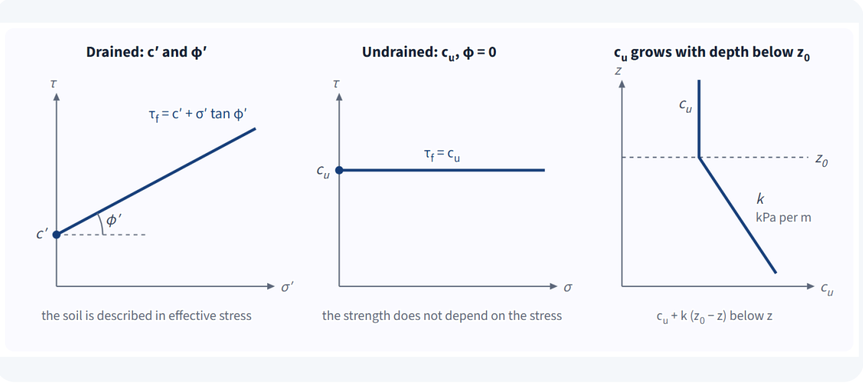
The shear strength of a **drained** soil is given by the Mohr–Coulomb envelope in terms of effective stress σ′ = σ − u and is defined by the parameters c′ and φ′.

In saturated clay loaded faster than drainage can occur, the strength is the **undrained strength** c_u, independent of normal stress, and φ = 0 is taken; in many natural clays c_u increases linearly with depth.

The strength of a slice is read at the level of the base midpoint; the specific criterion for rock is separate, and the choice of drained or undrained belongs to the engineer and constitutes the first assumption of the analysis.

$$\tau_f = c' + \sigma' \tan \varphi' \qquad \sigma' = \sigma - u \qquad \text{(drained)}$$

$$\tau_f = c_u \ (\varphi = 0) \qquad c_u(z) = c_u + k\,(z_0 - z) \ \ (z < z_0) \qquad \text{(undrained)}$$



Strength / required stress; moment about the circle center

THEORY

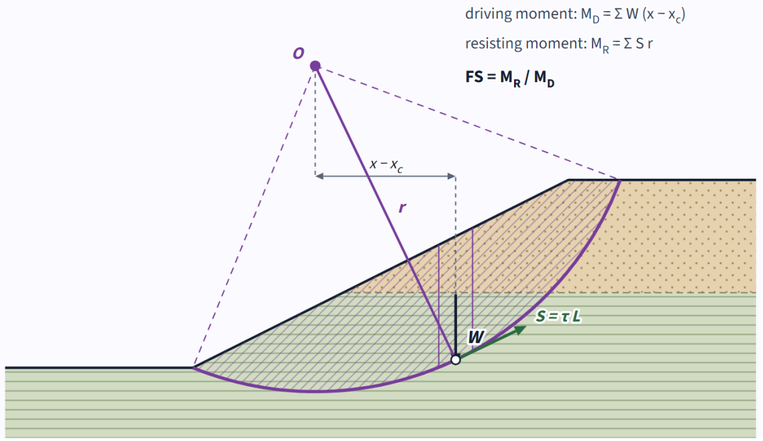
The factor of safety (FS) is the ratio of the shear strength available in the soil to the shear stress required for equilibrium, and it is assumed to take the same value at every point along the slip surface.

On a circular surface the governing equilibrium is the moment equilibrium about the centre O; dividing the moments by the radius, the ratio is written as FS = ΣR / ΣD, and both sums are in kN/m.

The three methods differ in how the effective normal force (N′) acting on each base is found: ordinary slices obtain it from weight alone, Bishop from vertical equilibrium, and Spencer from both force and moment equilibrium.

$$FS = \frac{\tau_f}{\tau} \qquad \tau = \frac{c' + \sigma' \tan \varphi'}{FS}$$

$$FS = \frac{M_R}{M_D} = \frac{\sum R}{\sum D} \qquad R = cL + N' \tan \varphi' \qquad D = \frac{M_D}{r}$$



Geometry, weight, base forces

THEORY

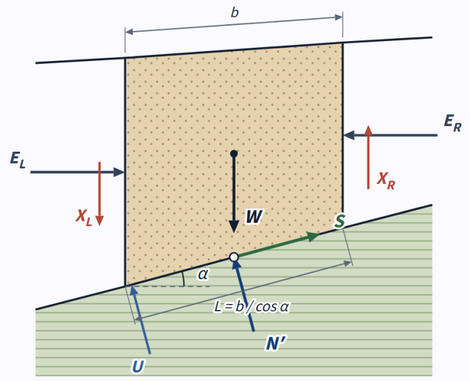
The mass between the slip surface and the ground surface is divided into slices by vertical lines according to the number of slices or the maximum slice width.

Every significant intersection point within the mass is a mandatory slice boundary; thus each base passes through a single soil and each slice carries a single type of load.

At the vertical through the middle of the slice, the base inclination α is taken positive on the crest side of the centre and negative on the toe side of the slope.

Effective normal force N′ = N – U; negative N′ indicates the base is in tension, and in the simple slice and Bishop methods N′ is kept at zero.

$$\sin\alpha = \frac{x_m - x_c}{r} \qquad L = \frac{b}{\cos\alpha} \qquad W = \sum_i b\, h_i\, \gamma_i \qquad N' = N - U\ ,\ U = uL$$



W	weight of the slice
N′	effective normal force
U = uL	pore force on the base
S	shear force (strength used)
E, X	forces between slices

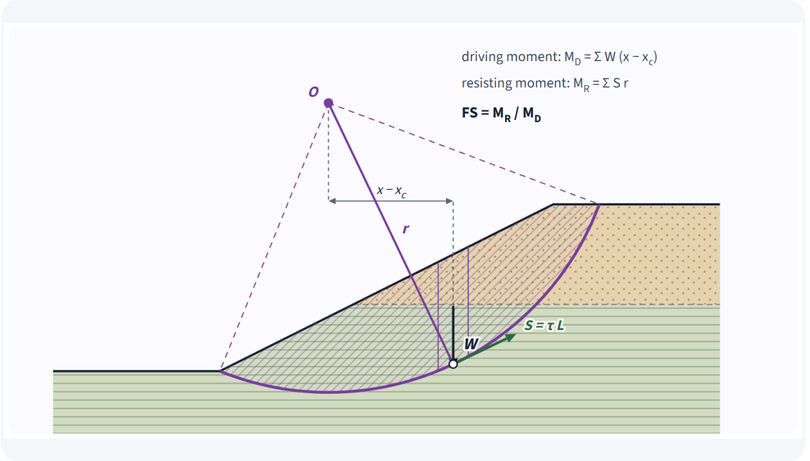
No interslice force

THEORY

In the **Fellenius** (Swedish) method the interslice forces are neglected entirely and each slice is in equilibrium only in the direction normal to its base.

This equilibrium gives the normal force from the weight and the loads; the factor of safety is obtained from the ratio of moments and, in homogeneous soil, reduces to FS = Σ [c b / cos α + W cos α tan φ] / Σ W sin α.

The method is simple, stable and on the safe side; its error increases with the depth of the circle and the pore water pressure, and for a deep circle it may give a result well below the other two methods.



$$N' = W\cos\alpha - uL \qquad R = cL + N'\tan\varphi' \qquad D = W\sin\alpha$$

$$FS = \frac{\sum R}{\sum D} = \frac{\sum \left[cL + (W\cos\alpha - uL)\tan\varphi'\right]}{\sum W\sin\alpha}$$

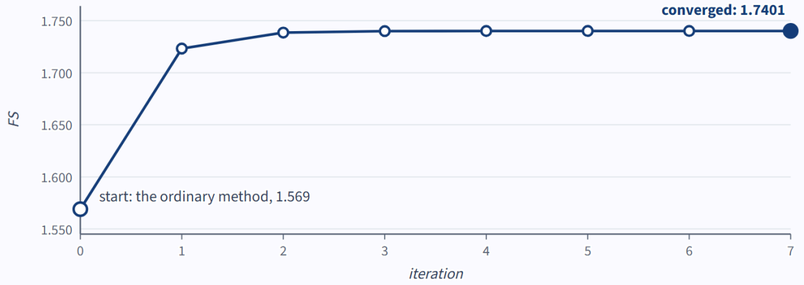
Vertical equilibrium of each slice; iterative solution

THEORY

In the simplified Bishop method the interslice shear forces are neglected, but the vertical equilibrium of each slice is satisfied and the shear force at the base is taken equal to the mobilized strength.

Since FS appears on both sides of the equation, the equation is solved by iteration starting from the value of the ordinary method of slices; a few steps suffice.

Negative **N′** is held at zero and $\mathbf{m_\alpha}$ is not allowed to fall below 0.2; this limitation is important in the toe region of deep circles.



$$N' = \frac{W - uL\cos\alpha - cL\sin\alpha/FS}{m_\alpha} \qquad m_\alpha = \cos\alpha + \frac{\sin\alpha\tan\varphi'}{FS}$$

$$FS = \frac{\sum [cL + N'\tan\varphi']}{\sum D}$$

Force and moment equilibrium; forces between parallel slices

THEORY

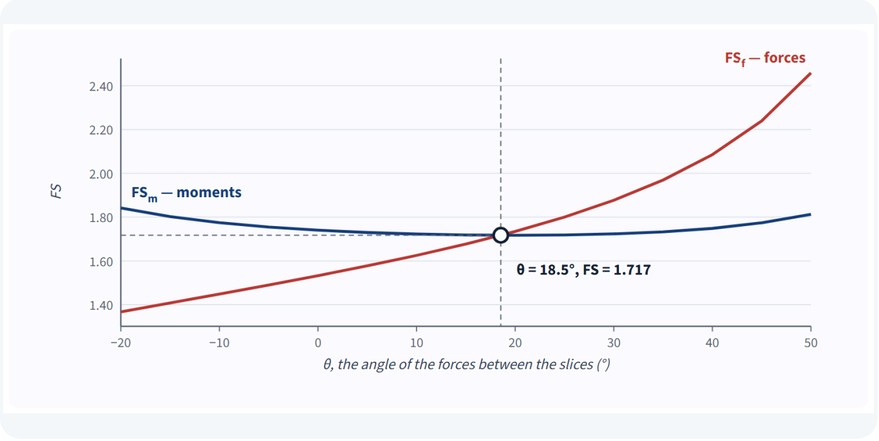
The **Spencer** method satisfies both force and moment equilibrium; the inter-slice forces are parallel to each other and the shear force is a constant fraction of the normal force.

Since the lateral faces at the two ends of the mass are free, ΣΔE = 0 gives the force equilibrium and a factor of safety FS_f is obtained for each angle θ.

Moments about the centre of the circle give a second value: FS_m = Σ[c L + N′ tan φ] / Σ D; $FS_f(\theta)$ and $FS_m(\theta)$ intersect at a single angle, and this constitutes the solution.

$$X = E \tan\theta \qquad \Delta E = \frac{cL + \tan\varphi' \left(W\cos\alpha - U\right) - FS\, W \sin\alpha}{FS(\cos\alpha + \tan\theta \sin\alpha) - \tan\varphi'(\tan\theta\cos\alpha - \sin\alpha)}$$

$$N' = W\cos\alpha - U + \Delta E\,(\tan\theta\cos\alpha - \sin\alpha) \qquad \sum \Delta E = 0 \;\Rightarrow\; FS_f(\theta)$$



What it provides, what it neglects

THEORY

Method	Interslice force	Force equilibrium	Moment equilibrium	Surface	FS (same circle)
Ordinary (Fellenius)	none	base normal only	✓	circle	**1.569**
Simple Bishop	normal only (no shear)	vertical per slice	✓	circle	**1.740**
Spencer	parallel: X = E tanθ	horizontal + vertical	✓	each surface	**1.717**

The ordinary method of slices is the default method that can be checked by hand calculation and is conservative; its result is the lowest of the three methods.

The Bishop method is the usual choice for circular surfaces; its result differs from that of the Spencer method on a circular surface by only a few percent.

The Spencer method satisfies the complete equilibrium and is used for non-circular surfaces; it is the most computationally demanding method, and for some circles no solution is found.

Problem statement

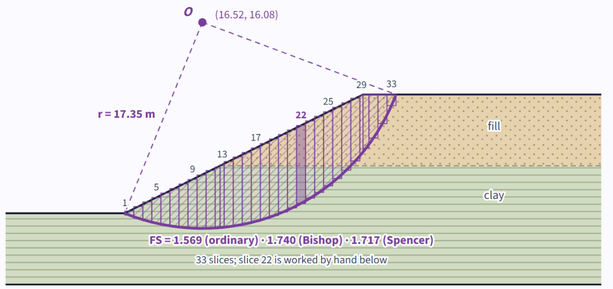
EXAMPLE

Given

Section	**ground surface** $(0;0)$–$(10;0)$–$(30;10)$–$(50;10)$ **m; base** $z=-6$ **m; above** $z=4$ **m fill, below clay**
Soils	**fill** $\gamma=18$ **kN/m³,** $c'=5$ **kPa,** $\varphi'=30°$**; clay** $\gamma=17.5$ **kN/m³,** $c'=20$ **kPa,** $\varphi'=18°$**; no water, load or earthquake**
Slip circle	**center** $x_c=16.52$ **m,** $z_c=16.08$ **m; radius** $r=17.35$ **m (critical circle, see what was found in the search)**

REQUIRED

The points $\mathbf{x_a}$ and $\mathbf{x_b}$ where the circle intersects the soil, its lowest point, and **slicing**.



Solution

SOLUTION

1 Intersection of the circle with the ground surface: $(x - x_c)^2 + (z - z_c)^2 = r^2$

$$x_a = x_c - \sqrt{r^2 - (z_a - z_c)^2} = 16.518 - \sqrt{301.08 - (0 - 16.080)^2} = \mathbf{10.00}\ \mathrm{m} \quad (z_a = 0)$$

$$x_b = x_c + \sqrt{r^2 - (z_b - z_c)^2} = 16.518 + \sqrt{301.08 - (10 - 16.080)^2} = \mathbf{32.77}\ \mathrm{m} \quad (z_b = 10)$$

2 Lowest point and slicing

$$z_{min} = z_c - r = 16.080 - 17.352 = -1.27\ \mathrm{m} \ \ (> -6\ \mathrm{m})$$

$$b = \frac{x_b - x_a}{30} = \frac{22.771}{30} = 0.759\ \mathrm{m} \ \Rightarrow\ 30\ \mathrm{slice}$$

The circle intersects the mass between x = 10.00 (foot) and 32.77 m (top); the edges at x = 10, 18 and 30 m split 3 more slices: **33 slices**.

Check the solution in GeoTerva ↗

Problem statement

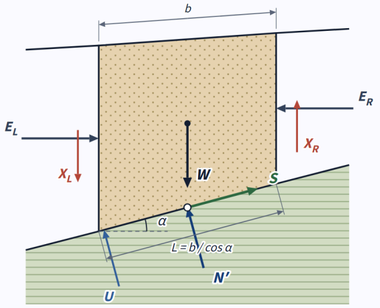
EXAMPLE

Given

Slice	**Slice 22 of the circle in Example 1: midpoint $x_m = 24.800$ m, width $b = 0.759$ m**
Soil	**fill $\gamma = 18$ kN/m³ ($z > 4$ m); clay $\gamma = 17.5$ kN/m³, $c' = 20$ kPa, $\varphi' = 18°$ (base in clay); $u = 0$**

REQUIRED

The **α**, **L**, **W** values of the slice; **N′**, **R** and **D** (ordinary method).



Solution (1/2)

SOLUTION

1 Base slope and length

$$\sin\alpha = \frac{x_m - x_c}{r} = \frac{24.800 - 16.518}{17.352} = 0.4773 \;\Rightarrow\; \alpha = \mathbf{28.51}^\circ,\ \cos\alpha = 0.8787$$

$$L = \frac{b}{\cos\alpha} = \frac{0.759}{0.8787} = \mathbf{0.864}\ \text{m}$$

2 Column height and weight (fill + clay)

$$z_{surface} = \frac{x_m - 10}{2} = 7.3998\ \text{m};\quad z_{base} = z_c - r\cos\alpha = 0.8327\ \text{m}$$

$$h_1 = 7.3998 - 4 = 3.3998\ \text{m (embankment (fill))};\quad h_2 = 4 - 0.8327 = 3.1673\ \text{m (clay)}$$

$$W = b\,(h_1\gamma_1 + h_2\gamma_2) = 0.7590(3.3998 \cdot 18 + 3.1673 \cdot 17.5) = \mathbf{88.52}\ \text{kN/m}$$

Slice 22 is the **heaviest slice** (W = 88.52 kN/m); its base is inclined at 28.5° and remains in the clay (c′ = 20 kPa, φ′ = 18°).

Solution (2/2)

SOLUTION

3 Normal force and resistance (u = 0)

$$N' = W\cos\alpha = 88.52 \times 0.8787 = \mathbf{77.79}\ \mathrm{kN/m}$$

$$R = cL + N'\tan\varphi' = 20(0.864) + 77.79(0.3249) = 17.27 + 25.27 = \mathbf{42.55}\ \mathrm{kN/m}$$

4 Driving effect

$$D = W\sin\alpha = 88.52 \times 0.4773 = \mathbf{42.25}\ \mathrm{kN/m}$$

Slice 22: **R = 42.55** kN/m, **D = 42.25** kN/m. The R and D values of each slice are found with **the same three steps**; the factor of safety is the ratio of their totals: **FS = ΣR/ΣD** (table on the following pages).

Check the solution in GeoTerva ↗

Problem statement

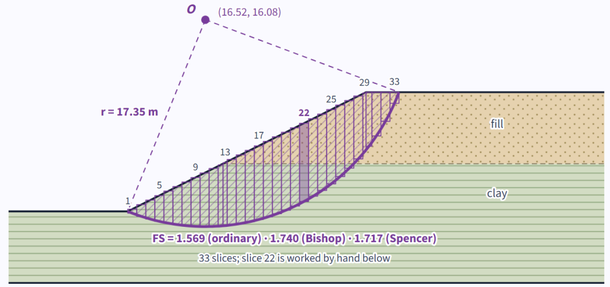
EXAMPLE

Given

Slices	**The circle in Example 1, 33 slices; each slice is solved with the three steps in Example 2**
Method	**Ordinary (Fellenius): no interslice forces**

REQUIRED

Table of all slices: **W, N′, R, D** and their totals; **FS = ΣR / ΣD**.



Solution (1/3)

SOLUTION

No	x_m (m)	b (m)	α (°)	L (m)	W (kN/m)	c′ (kPa)	φ′ (°)	N′ (kN/m)	R (kN/m)	D (kN/m)
1	10.00	0.001	−22.1	0.001	0.00	20	18	0.00	0.03	−0.00
2	10.38	0.758	−20.7	0.810	4.49	20	18	4.20	17.57	−1.59
3	11.14	0.759	−18.1	0.798	13.08	20	18	12.43	20.01	−4.06
4	11.90	0.759	−15.4	0.787	21.15	20	18	20.39	22.37	−5.63
5	12.66	0.759	−12.9	0.779	28.74	20	18	28.02	24.67	−6.40
6	13.41	0.759	−10.3	0.771	35.84	20	18	35.27	26.89	−6.41
7	14.17	0.759	−7.8	0.766	42.49	20	18	42.10	29.00	−5.74
8	14.93	0.759	−5.2	0.762	48.68	20	18	48.47	30.99	−4.45
9	15.69	0.759	−2.7	0.760	54.42	20	18	54.36	32.86	−2.59
10	16.45	0.759	−0.2	0.759	59.72	20	18	59.72	34.59	−0.23
11	17.21	0.759	2.3	0.760	64.58	20	18	64.53	36.16	2.57
12	17.79	0.411	4.2	0.412	36.85	20	18	36.75	20.19	2.71
13	18.17	0.348	5.5	0.349	32.15	20	18	32.01	17.39	3.07
14	18.73	0.759	7.3	0.765	73.11	20	18	72.51	38.87	9.31
15	19.49	0.759	9.9	0.770	76.77	20	18	75.64	39.98	13.13
16	20.25	0.759	12.4	0.777	79.97	20	18	78.11	40.92	17.18
17	21.00	0.759	15.0	0.786	82.70	20	18	79.89	41.67	21.38

Solution (2/3)

SOLUTION

No	x_m (m)	b (m)	α (°)	L (m)	W (kN/m)	c′ (kPa)	φ′ (°)	N′ (kN/m)	R (kN/m)	D (kN/m)
18	21.76	0.759	17.6	0.796	84.94	20	18	80.97	42.23	25.68
19	22.52	0.759	20.2	0.809	86.67	20	18	81.32	42.60	29.99
20	23.28	0.759	22.9	0.824	87.86	20	18	80.91	42.77	34.25
21	24.04	0.759	25.7	0.842	88.49	20	18	79.74	42.76	38.37
22	**24.80**	**0.759**	**28.5**	**0.864**	**88.52**	**20**	**18**	**77.79**	**42.55**	**42.25**
23	25.56	0.759	31.4	0.889	87.89	20	18	75.02	42.16	45.80
24	26.32	0.759	34.4	0.920	86.56	20	18	71.43	41.60	48.89
25	27.08	0.759	37.5	0.956	84.44	20	18	67.00	40.90	51.38
26	27.84	0.759	40.7	1.001	81.43	20	18	61.72	40.08	53.11
27	28.59	0.758	44.1	1.056	77.34	20	18	55.53	39.17	53.83
28	29.35	0.760	47.7	1.129	72.09	5	30	48.51	33.65	53.33
29	29.87	0.267	50.3	0.418	23.72	5	30	15.15	10.84	18.25
30	30.25	0.492	52.3	0.805	40.15	5	30	24.55	18.20	31.76
31	30.87	0.759	55.8	1.351	50.12	5	30	28.16	23.01	41.46
32	31.63	0.759	60.6	1.545	33.40	5	30	16.41	17.20	29.09
33	32.39	0.759	66.2	1.878	12.72	5	30	5.14	12.36	11.64
Σ					**1,841.11**			**1,613.77**	**1,006.25**	**641.34**

Solution (3/3)

SOLUTION

1 Sums and factor of safety

$$\sum R = 1{,}006.25\ \text{kN/m} \qquad \sum D = 641.34\ \text{kN/m} \qquad \sum W = 1{,}841.1\ \text{kN/m}$$

$$FS = \frac{\sum R}{\sum D} = \frac{1{,}006.25}{641.34} = \mathbf{1.569}$$

For this circle, the ordinary method gives **FS = 1.569**. In the slices on the toe side (α < 0) the base is inclined against sliding: D is negative and reduces ΣD. Slice 22 is row 22 of the table and agrees with the hand-calculated values.

ΣR/ΣD: ratio of **resisting** effects to driving effects; if FS > 1, sliding along this circle is not expected.

The ordinary method gives the lowest value; Bishop and Spencer are higher on the same circle (following examples).

Check the solution in GeoTerva ↗

Problem statement

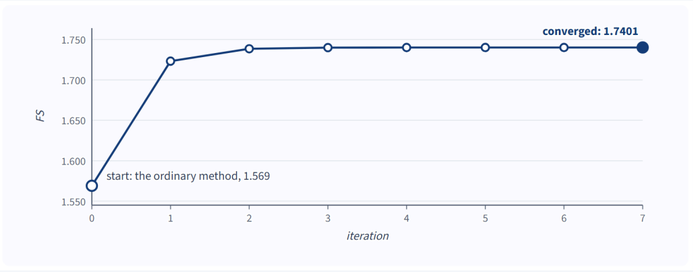
EXAMPLE

Given

Slice and circle	**The 22nd slice in Example 2 and the circle in Example 1; initial value $FS_0 = 1.569$ in Example 3**
Method	$N' = \dfrac{W - cL\sin\alpha/FS}{m_\alpha}$, $m_\alpha = \cos\alpha + \dfrac{\sin\alpha\tan\varphi'}{FS}$ **(u = 0)**

REQUIRED

For slice 22, **m_α**, **N′**, **R**; **iteration** and **FS** for the whole circle.



Solution (1/4)

SOLUTION

1 First iteration (FS_0 = value of the ordinary method = 1.5690)

$$m_\alpha = \cos\alpha + \frac{\sin\alpha\tan\varphi'}{FS_0} = 0.8787 + \frac{0.4773(0.3249)}{1.5690} = 0.9776$$

$$N' = \frac{W - cL\sin\alpha/FS_0}{m_\alpha} = \frac{88.52 - 20(0.864)(0.4773)/1.5690}{0.9776} = 85.17\ \text{kN/m}$$

$$R = 20(0.864) + 85.17(0.3249) = 44.95\ \text{kN/m}$$

Solution (2/4)

SOLUTION

2 With the converged value (FS = 1.7401)

$$m_\alpha = 0.8787 + \frac{0.4773(0.3249)}{1.7401} = \mathbf{0.9679}$$

$$N' = \frac{88.52 - 20(0.864)(0.4773)/1.7401}{0.9679} = \mathbf{86.56}\ \mathrm{kN/m}$$

$$R = cL + N' \tan\varphi' = 20(0.864) + 86.56(0.3249) = \mathbf{45.40}\ \mathrm{kN/m} \qquad D = 42.25\ \mathrm{kN/m}$$

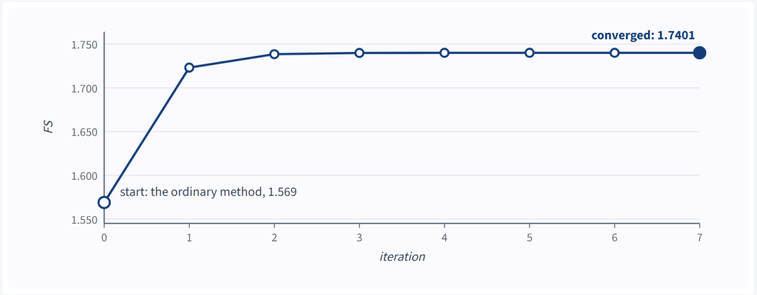
The normal force of slice 22 increases from 77.79 in the ordinary method to **86.56** kN/m (vertical equilibrium between slices); the resistance 42.55 → **45.40** kN/m. The same steps are repeated for 33 slices.

Solution (3/4)

SOLUTION

2 At each iteration ΣR is recomputed, $FS_{k+1} = \Sigma R_k / \Sigma D$

Iteration k	FS_k	Difference
0 (ordinary)	1.5690	
1	1.7232	0.1542
2	1.7385	0.0153
3	1.7399	0.0014
4	1.7401	0.0001
5	1.7401	0.0000
6	1.7401	0.0000
7	**1.7401**	0.0000



In 7 iterations **FS = 1.740**; ΣR = 1,115.97, ΣD = 641.34 kN/m. It is 10.9% higher than the ordinary method (1.569).

Solution (4/4)

SOLUTION

No	α (°)	W	m_α	N′	R
1	−22.1	0.00	0.8566	0.01	0.03
2	−20.7	4.49	0.8693	8.95	19.11
3	−18.1	13.08	0.8928	17.83	21.76
4	−15.4	21.15	0.9141	25.78	24.12
5	−12.9	28.74	0.9333	32.92	26.27
6	−10.3	35.84	0.9505	39.38	28.22
7	−7.8	42.49	0.9656	45.23	30.02
8	−5.2	48.68	0.9788	50.55	31.67
9	−2.7	54.42	0.9900	55.39	33.20
10	−0.2	59.72	0.9993	59.80	34.61
11	2.3	64.58	1.0066	63.81	35.93
12	4.2	36.85	1.0110	36.10	19.98
13	5.5	32.15	1.0133	31.35	17.18
14	7.3	73.11	1.0156	70.88	38.34
15	9.9	76.77	1.0172	73.99	39.45
16	12.4	79.97	1.0168	76.77	40.49
17	15.0	82.70	1.0143	79.24	41.46
18	17.6	84.94	1.0097	81.39	42.37
19	20.2	86.67	1.0028	83.22	43.22
20	22.9	87.86	0.9937	84.71	44.01
21	25.7	88.49	0.9821	85.83	44.74
22	**28.5**	**88.52**	**0.9679**	**86.56**	**45.40**
23	31.4	87.89	0.9508	86.84	46.00
24	34.4	86.56	0.9307	86.59	46.53
25	37.5	84.44	0.9072	85.70	46.98
26	40.7	81.43	0.8798	84.02	47.33
27	44.1	77.34	0.8480	81.24	47.52
28	47.7	72.09	0.9183	75.89	49.46
29	50.3	23.72	0.8941	25.50	16.81
30	52.3	40.15	0.8741	43.83	29.33
31	55.8	50.12	0.8363	56.09	39.14
32	60.6	33.40	0.7803	37.85	29.58
33	66.2	12.72	0.7076	11.00	15.74
Σ		**1,841.11**			**1,115.97**

Check the solution in GeoTerva ↗

Problem statement

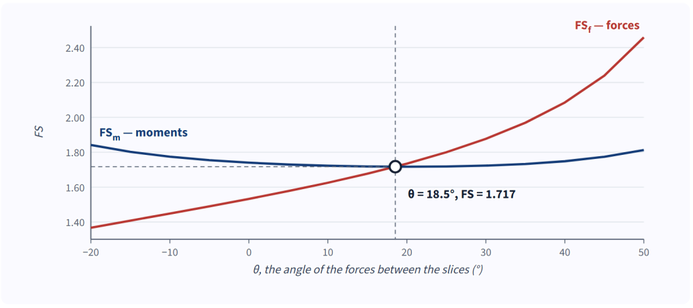
EXAMPLE

Given

Slice and circle	**Slice 22 in Example 2, the circle in Example 1; the interslice forces are parallel, $X = E \tan\theta$**
Solution angle	**$\theta = 18.54°$ (the angle where the two curves below intersect), $FS = 1.717$**

REQUIRED

For slice 22: **ΔE**, **N′**, **R**, **E** and **X**; θ–FS curves and the entire circle.



Solution (1/4)

SOLUTION

1 Normal force difference of the slice ΔE (θ = 18.54°, tan θ = 0.3354, FS = 1.717)

$$\text{allowance} = cL + \tan\varphi' \, W\cos\alpha - FS\, W\sin\alpha = 17.275 + 25.274 - 72.542 = -29.99$$

$$\text{denominator}_1 = FS(\cos\alpha + \tan\theta\sin\alpha) = 1.7837$$

$$\text{denominator}_2 = -\tan\varphi'(\tan\theta\cos\alpha - \sin\alpha) = 0.0593$$

$$\Delta E = \frac{\text{allowance}}{\text{denominator}_1 + \text{denominator}_2} = \frac{-29.99}{1.8430} = \mathbf{-16.27}\ \text{kN/m}$$

Solution (2/4)

SOLUTION

2 Normal force, resistance and interslice forces

$$N' = W\cos\alpha + \Delta E(\tan\theta\cos\alpha - \sin\alpha) = 77.786 + (-16.27)(-0.1826) = \mathbf{80.76}\ \mathrm{kN/m}$$

$$R = cL + N'\tan\varphi' = \mathbf{43.51}\ \mathrm{kN/m}$$

$$E_{foot} = 254.69\ \mathrm{kN/m} \quad \Rightarrow \quad E_{top} = E_{foot} + \Delta E = 238.41\ \mathrm{kN/m}$$

$$X_{foot} = E\tan\theta = \mathbf{85.42}$$

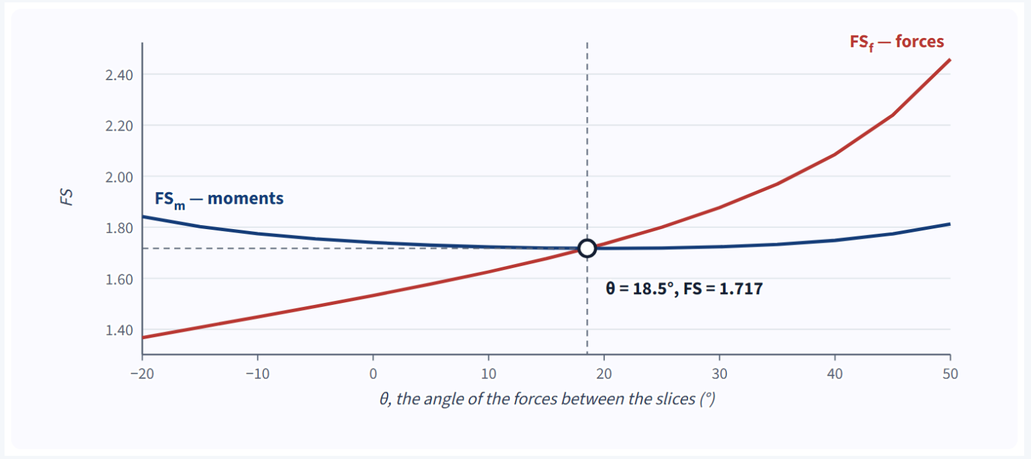
In Spencer, the normal force on slice 22 is **80.76** kN/m (ordinary 77.79, Bishop 86.56); the interslice force acting toward the toe is E = 254.69 kN/m.

Solution (3/4)

SOLUTION

3 FS_f ($\Sigma\Delta E = 0$) and FS_m (moment) for each θ; the intersection is the solution

θ (°)	FS_m (moment)	FS_f (force)
-20	1.842	1.367
-15	1.802	1.408
-10	1.774	1.449
-5	1.755	1.490
0	1.740	1.533
5	1.730	1.577
10	1.723	1.625
15	1.718	1.677
20	1.717	1.735
25	1.718	1.800
30	1.723	1.877
35	1.733	1.969
40	1.748	2.085
45	1.774	2.239
50	1.812	2.458
18.54	**1.717**	**1.717**



Solution (4/4)

SOLUTION

No	ΔE	E (foot)	N′	R
1	0.02	−0.00	0.02	0.03
2	17.12	0.02	15.62	21.28
3	21.59	17.14	26.01	24.42
4	24.46	38.73	34.81	27.06
5	26.08	63.19	42.35	29.33
6	26.67	89.27	48.84	31.30
7	26.41	115.94	54.44	33.01
8	25.43	142.35	59.29	34.51
9	23.87	167.78	63.49	35.83
10	21.80	191.65	67.12	36.99
11	19.32	213.45	70.24	38.01
12	9.30	232.78	39.17	20.97
13	7.19	242.08	33.72	17.95
14	13.38	249.26	75.26	39.76
15	10.03	262.64	77.24	40.50
16	6.48	272.67	78.84	41.16
17	2.77	279.15	80.07	41.73
18	−1.03	281.92	80.95	42.23
19	−4.89	280.89	81.47	42.65
20	−8.75	276.01	81.62	43.01
21	−12.57	267.25	81.39	43.29
22	**−16.27**	**254.69**	**80.76**	**43.51**
23	−19.80	238.41	79.67	43.67
24	−23.06	218.61	78.07	43.76
25	−25.94	195.55	75.88	43.79
26	−28.29	169.61	72.98	43.74
27	−29.89	141.32	69.14	43.59
28	−30.84	111.42	64.36	42.80
29	−11.02	80.59	21.27	14.37
30	−19.70	69.57	36.10	24.87
31	−26.62	49.87	45.17	32.83
32	−18.69	23.25	29.61	24.82
33	−4.56	4.56	8.70	14.41
Σ	0.00			**1,101.18**

θ = 18.54°; ΣR = 1,101.18, ΣD = 641.34 → **FS = 1.717**. E_{max} = 281.9 kN/m.

Check the solution in GeoTerva ↗

Weight, pore water pressure, water in the slope

THEORY

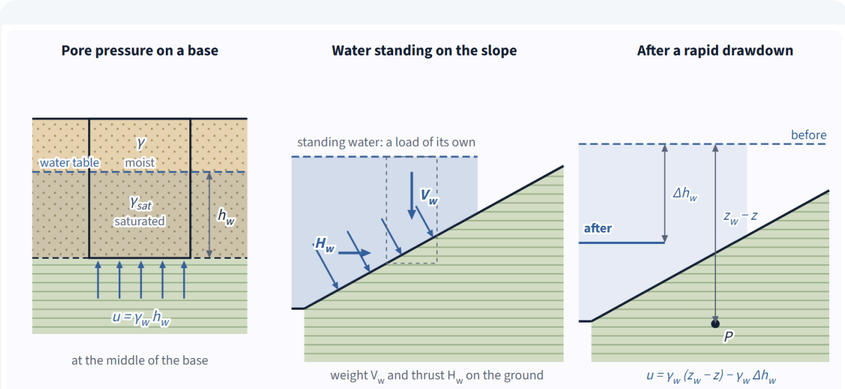
Water enters the analysis in three ways: the weight of the saturated soil, the pore water pressure that reduces the strength at the base of each slice, and the water standing on the slope as a load in its own right where it is located.

Below the groundwater line the pore water pressure is hydrostatic; since seepage is not analyzed, a sloping groundwater level is an approximation to a flow net, and the submerged unit weight is never used.

When the pore water pressure is known as a proportion of the overburden, a ratio $\mathbf{r_u}$ is assigned to the soil; this applies to an embankment during the construction stage or to an embankment over soft clay.

When the water in front of the slope draws down rapidly because it can drain, the support disappears, the pore water pressure is preserved, and the factor of safety corresponds to the lowest value immediately after the drawdown.

$$u = \gamma_w h_w \qquad u = r_u \frac{W}{b} \qquad H_w = \frac{\gamma_w}{2}\left[(z_w - z_L)^2 - (z_w - z_R)^2\right]$$



Problem statement

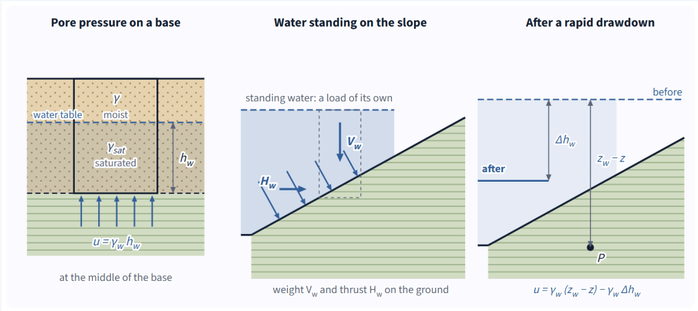
EXAMPLE

Given

Water line	$(0;0)$–$(20;2.5)$–$(50;7)$ **m, hydrostatic; $\gamma_w = 9.81$ kN/m³; saturated unit weights fill 20, clay 19.5 kN/m³**
Slice	**same circle; the slice at $x_m = 24.800$ m (with water it becomes the 24th slice, not the 33rd)**

REQUIRED

In the slice **z_w, h_w, u, U**; the new **W, N′, R, D**; over the whole circle **FS**.



Solution (1/2)

SOLUTION

1 Water height and pore water pressure

$$z_w = 2.5 + \frac{24.800 - 20}{30}(4.5) = 3.2199 \text{ m}$$

$$h_w = z_w - z_{base} = 3.2199 - 0.8327 = \mathbf{2.3872} \text{ m}$$

$$u = \gamma_w h_w = 9.81(2.3872) = \mathbf{23.42} \text{ kPa} \qquad U = uL = 23.42(0.864) = \mathbf{20.23} \text{ kN/m}$$

2 Weight: moist above water, saturated below

$$W = b\left[h_1\gamma_1 + h_{dry}\gamma_{moist} + h_w\gamma_{sat}\right]$$

$$W = 0.7590[3.3998(18) + 0.7801(17.5) + 2.3872(19.5)] = \mathbf{92.14} \text{ kN/m}$$

Water both raises the weight from 88.52 kN/m dry to **92.14** kN/m and produces a water force **U = 20.23** kN/m at the base.

Solution (2/2)

SOLUTION

3 Effective normal force, resistance, driving effect (slice 22)

$$N' = W\cos\alpha - U = 92.14(0.8787) - 20.23 = \mathbf{60.74}\ \mathrm{kN/m}$$

$$R = 20(0.864) + 60.74(0.3249) = \mathbf{37.01}\ \mathrm{kN/m} \qquad D = W\sin\alpha = \mathbf{43.98}\ \mathrm{kN/m}$$

4 Full circle (36 slices, same circle) and comparison with the dry case

Method	ΣU	ΣR	ΣD	FS (water)	FS (dry)
Ordinary	476.94	879.96	655.41	**1.343**	1.569
Bishop	476.94	981.07	655.41	**1.497**	1.740
Spencer	476.94	970.22	655.41	**1.480**	1.717

For the same circle, water reduces FS from 1.569 to **1.343** (ordinary); if the critical circle is searched again with the water line, it becomes shallower and equals **1.255 / 1.568 / 1.564** (ordinary / Bishop / Spencer).

Check the solution in GeoTerva ↗

Strip and point load; vertical, perpendicular, inclined

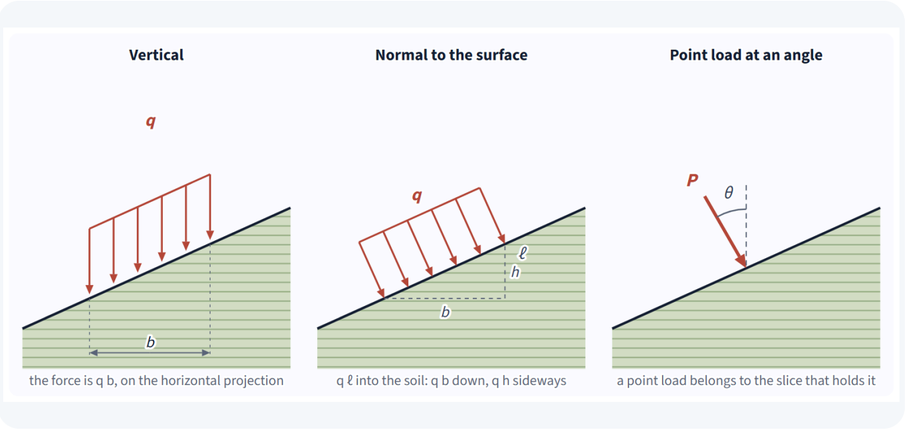
THEORY

A vertical or inclined strip load is taken over its horizontal projection, and the force on the part of horizontal width b is q b.

A load acting perpendicular to the surface is taken over the surface length; the force on a segment of length ℓ is q ℓ, with vertical component q b and horizontal component q h.

Each slice carries the portion of a load falling between its edges, distributed according to horizontal width, and the point load is assigned to the slice containing its point of application.

$$F = q\,b \qquad F_n = q\,\ell \;\Rightarrow\; F_z = q\,b,\; F_x = q\,h \qquad P_x = P\sin\theta,\; P_z = P\cos\theta$$



Inertial forces at the centroid of the slice

THEORY

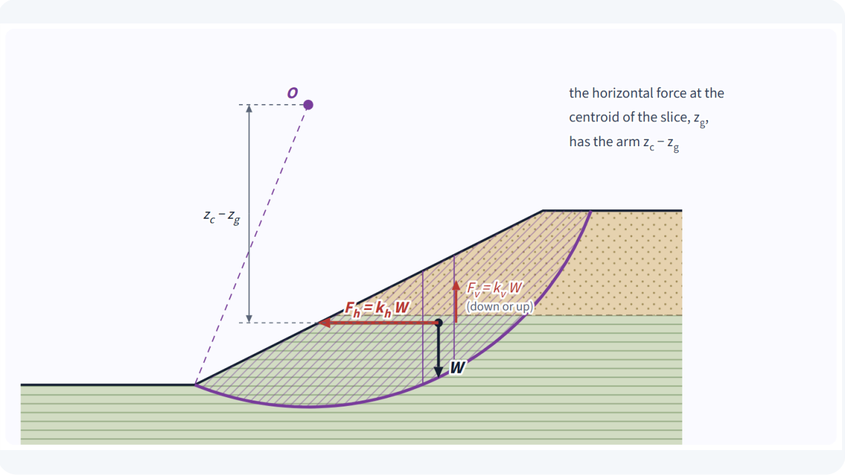
In the **pseudo-static** approach, earthquake shaking is represented by a constant acceleration applied to the sliding mass and expressed as a fraction of g.

In each slice, the inertial forces of the soil mass ($\mathbf{W_{soil}}$) are applied at the centre of gravity ($\mathbf{z_g}$) found from the moist and saturated portions of the soil within the slice, since the water accumulated on the slice does not carry this force.

In the critical-case option, both horizontal and both vertical directions are tested and the lowest factor of safety is adopted; the coefficients are provided by the user, and the static and **pseudo-static** FS are reported together.

$$F_h = k_h W_{soil} \qquad F_v = k_v W_{soil}$$

$$N = (W + F_v)\cos\alpha \mp F_h \sin\alpha \qquad D = (W + F_v)\sin\alpha + F_h \frac{z_c - z_g}{r}$$



Problem statement

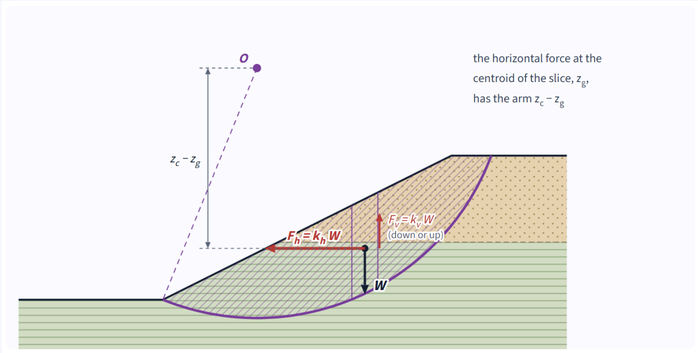
EXAMPLE

Given

Earthquake	$k_h = 0.15$, $k_v = 0$; **horizontal force in the direction of the toe (critical direction)**
Slice and circle	**Slice 22 and the circle in Example 2; the centroid is needed for the moment arm z_g**

REQUIRED

In slice 22: **F_h, N′, R, D**; for the entire circle, **$FS_{earthquake}$** by three methods.



Solution (1/2)

SOLUTION

1 Inertial force and moment arm

$$F_h = k_h W = 0.15(88.52) = \mathbf{13.28}\ \mathrm{kN/m}$$

$$z_g = \sum W_i z_{g,i}/W = (46.45(5.700) + 42.07(2.416))/88.52 = 4.139\ \mathrm{m}$$

$$\text{moment arm} = (z_c - z_g)/r = (16.080 - 4.139)/17.352 = 0.6882$$

2 Normal force, resistance, driving effect

$$N' = W\cos\alpha - F_h \sin\alpha = 77.79 - 6.34 = \mathbf{71.45}\ \mathrm{kN/m}$$

$$R = 20(0.864) + 71.45(0.3249) = \mathbf{40.49}\ \mathrm{kN/m}$$

$$D = W\sin\alpha + F_h\,\frac{z_c - z_g}{r} = 42.25 + 13.28(0.6882) = \mathbf{51.39}\ \mathrm{kN/m}$$

The driving effect goes 42.25 → **51.39** kN/m (+22%), the resistance 42.55 → **40.49** kN/m: the horizontal force reduces the normal force and adds a sliding moment.

Solution (2/2)

SOLUTION

3 Three methods on the same circle (k_h = 0.15, k_v = 0)

Method	ΣR	ΣD	FS (static)	FS (earthquake)
Ordinary	967.97	840.48	1.569	**1.152**
Bishop	1,083.01	840.48	1.740	**1.289**
Spencer	1,069.09	840.48	1.717	**1.272**

k_h = 0.15 reduces FS by about 27% (ordinary 1.569 → **1.152**). Under earthquake action the critical circle also changes: in the model on the page (k_h = 0.15, k_v = 0.05, critical directions) the search result becomes 1.138 / 1.265 / 1.250.

Check the solution in GeoTerva ↗

Water in the crack and at depth

THEORY

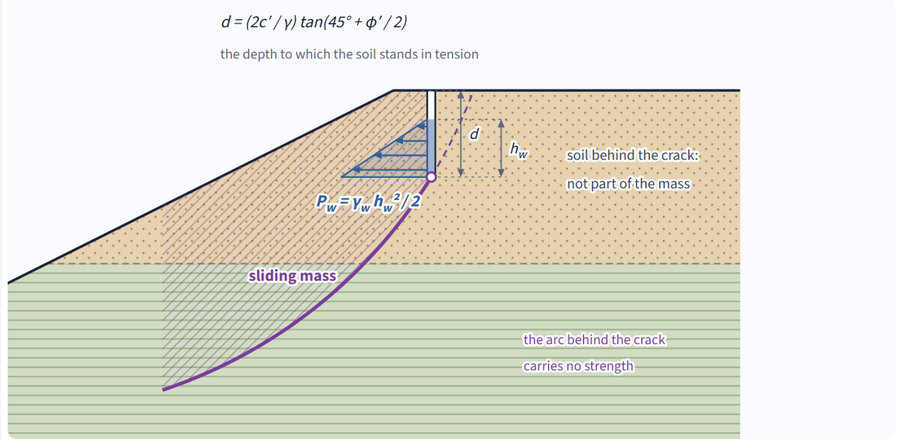
The tension crack depth is found as d = 2c′/γ·tan(45° + φ′/2); in the undrained case this depth becomes $2c_u/\gamma$.

When the crack is active, the slip surface terminates at the point where the depth of the sliding mass equals the crack depth; the soil behind the crack is not part of the sliding mass, and the portion of the arc within the crack carries no resistance.

Water standing in the crack up to height h_W exerts pressure with hydrostatic force; a water-filled crack lowers the factor of safety, and this condition is more often overlooked.

$$d = \frac{2c'}{\gamma}\tan\left(45^\circ + \frac{\varphi'}{2}\right) \qquad d = \frac{2c_u}{\gamma}$$

$$P_w = \frac{\gamma_w h_w^2}{2} \qquad z_f = z_{base} + \frac{h_w}{3} \qquad D_{crack} = \frac{P_w\,(z_c - z_f)}{r}$$



Problem statement

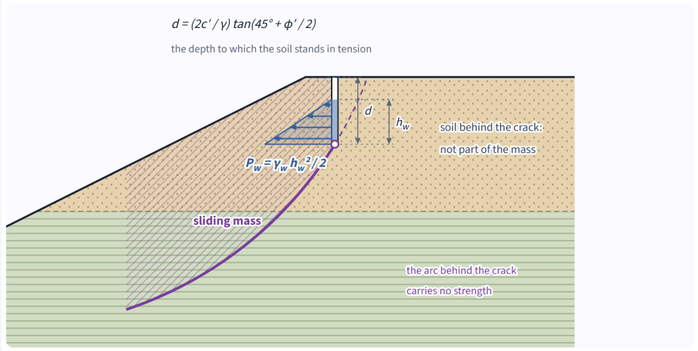
EXAMPLE

Given

Crack	**at the top, vertical crack at $d = 2$ m depth (the model's initial value); two conditions, dry and water-filled ($h_w = 2$ m)**
Circle	**The circle in Example 1; ends at the base of the crack**

REQUIRED

The **x** location of the crack; **P_w**, **z_f**, **D_{crack}**; **FS** by three methods.



Solution (1/2)

SOLUTION

1 Location of the crack: it ends where the circle reaches elevation z = 10 − 2 = 8 m

$$x = x_c + \sqrt{r^2 - (8 - z_c)^2} = 16.518 + \sqrt{301.08 - (8 - 16.080)^2} = \mathbf{31.87}\ \text{m}$$

2 Water in the crack (h_W = 2 m): force and moment

$$P_w = \frac{\gamma_w h_w^2}{2} = \frac{9.81(2)^2}{2} = \mathbf{19.62}\ \text{kN/m} \qquad z_f = 8 + \frac{2}{3} = 8.667\ \text{m}$$

$$D_{crack} = \frac{P_w(z_c - z_f)}{r} = \frac{19.62(16.080 - 8.667)}{17.352} = \mathbf{8.38}\ \text{kN/m}$$

The Rankine depth is 0.96 m in the fill and 3.15 m in the clay; the module takes the crack depth as the value entered by the engineer (here 2 m). A water-filled crack adds **8.38** kN/m to ΣD.

Solution (2/2)

SOLUTION

3 Uncracked, dry crack and water-filled crack (same circle; 34 slices in the crack)

Method	FS without crack	ΣR	ΣD (dry)	FS dry crack	ΣD (water)	FS filled crack
Ordinary	1.569	991.18	625.52	**1.585**	633.91	**1.564**
Bishop	1.740	1,096.39	625.52	**1.753**	633.91	**1.727**
Spencer	1.717	1,083.00	625.52	**1.731**	633.91	**1.702**

A dry crack changes FS very little (the driving mass above and the resistance in the arc go together: 1.569 → 1.585); a **water-filled crack** lowers it (1.564). Water in the crack is the most commonly forgotten case.

Check the solution in GeoTerva ↗

Center grid, radii, refinement

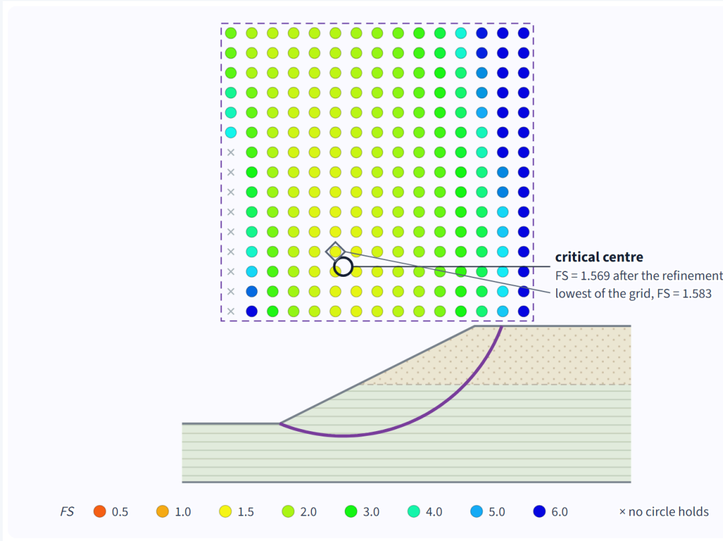
THEORY

The critical circle search is carried out over an $n_x \times n_z$ grid of centers placed over the slope, and at each center a set of equally spaced radii is tried.

Each circle is divided into slices and solved, and the lowest factor of safety found for each center is stored together with the corresponding radius; in the worked model, 4650 circles were tested with a 15 × 15 grid of centers and 12 radii.

$$FS(x_c, z_c) = \min_j FS(x_c, z_c, r_j) \qquad FS_{crit} = \min_{x_c, z_c} FS(x_c, z_c)$$

Worked model	Value
Grid ($n_x \times n_z \times$ radius)	15 × 15 × 12
Trial / accepted circle	4,650 / 3,487
The lowest of the grid	FS = 1.583
After refinement	**FS = 1.569**
Critical circle	x_c = 16.52, z_c = 16.08, r = 17.35 m



Broken line, Spencer only

THEORY

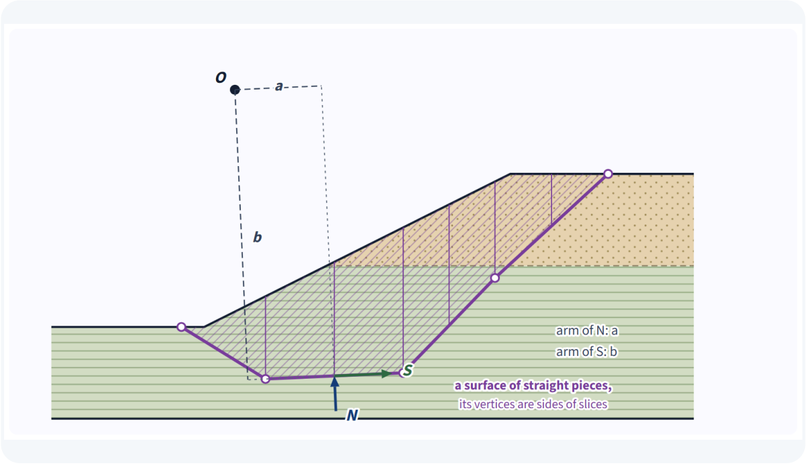
A non-circular surface may follow a weak layer with wedges descending from the slope crest and rising toward the toe, and is a polygonal surface with both ends at the ground surface.

The moment equation on a multiple linear slip surface is written about a point O chosen above the toe of the mass; the moment arm of N is a and that of S is b.

The surface with the lowest factor of safety is sought as a piecewise linear surface with n segments; movement is maintained only when the surface remains admissible, and the convexity constraint prevents drifting toward zigzag surfaces.

$$a = (X - X_O)\cos\alpha - (z - z_O)\sin\alpha \qquad b = (X - X_O)\sin\alpha + (z - z_O)\cos\alpha$$

$$\sum [Na + Sb] - \sum V(X - X_O) - \sum A(z_A - z_O) + M_{reinforcement} + M_{crack} = 0$$



Contribution of the passive and active elements to FS

THEORY

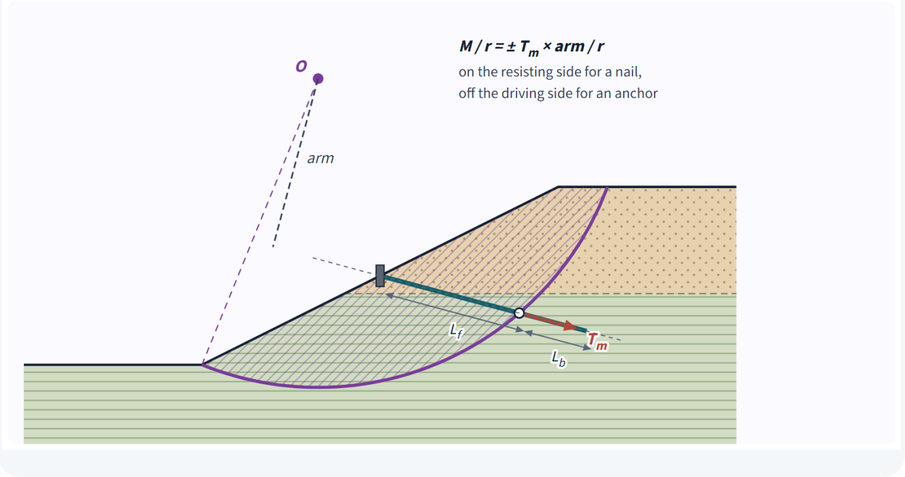
The reinforcement element is a line intersecting the slip surface, and the length of the slip surface in front of the point where it intersects the arc is denoted L_f, while the bonded length behind this point is denoted L_b.

The moment of a passive element (nail, geogrid) is added to the resisting side; since an active element (anchor) carries its load before movement, its moment is subtracted from the driving side.

For the same force, an active element increases the factor of safety more than a passive element; the force is taken the same in the static and pseudo-static cases.

$$\frac{M}{r} = \pm \frac{(x_P - x_c)F_z - (z_P - z_c)F_x}{r}$$

$$FS = \frac{\sum R + \sum M_{passive}/r}{\sum D - \sum M_{active}/r}$$



Smallest of the three values

THEORY

The tensile force of the soil nail is $T = \min[T_t, q_u \pi D_h L_b, q_u \pi D_h L_f + T_h]$, the smallest of the three values.

A nail is a passive element: it is mobilized as the soil mass moves, and the nail force divided by the spacing S_h is expressed as $T_m = T/S_h$.

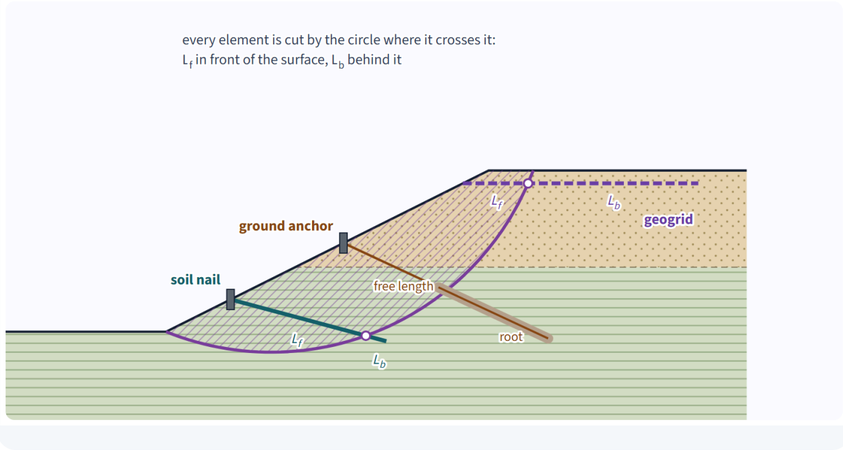
A nail whose head lies outside the circle or whose end lies within the mass carries no force for that circle.

No bond forms in the free length; the anchor is held by the root, and the force the anchor transmits by intersecting a slip circle is the smallest of three values: **$T = \min[P, T_t, q_u \pi D_h L_b]$**.

The anchor is an active element: the load is in the tendon before the mass moves, and in the Spencer equations the anchor is a given force; there it is not divided by **FS**.

$$\text{nail: } T = \min\left[T_t,\ q_u \pi D_h L_b,\ q_u \pi D_h L_f + T_h\right] \qquad T_m = \frac{T}{S_h}$$

$$\text{anchor: } T = \min\left[P,\ T_t,\ q_u \pi D_h L_b\right] \qquad T_m = \frac{T}{S_h}$$



Design strength, pullout, connection

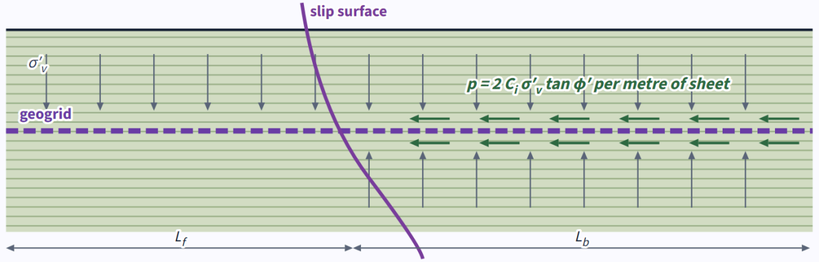
THEORY

The geogrid is defined by its length L, its long-term design strength per metre of section T_d, the interaction coefficient between soil and geogrid C_i, and the connection strength T_c when it is connected to the facing.

The pullout resistance per metre of length arises from friction on both faces under the effective overburden stress σ'_v; the geogrid is a passive element and acts in the plane of the sheet.

σ'_v is obtained from the weight of the soil above the plate — with saturated unit weight below the groundwater level — minus the pore water pressure; surface loads are not included, staying on the safe side.

$$p = 2\,C_i\,\sigma'_v\,\tan\varphi' \qquad T_m = \min\left[T_d,\ \int_{L_b} p\,dl,\ \int_{L_f} p\,dl + T_c\right]$$



Problem statement

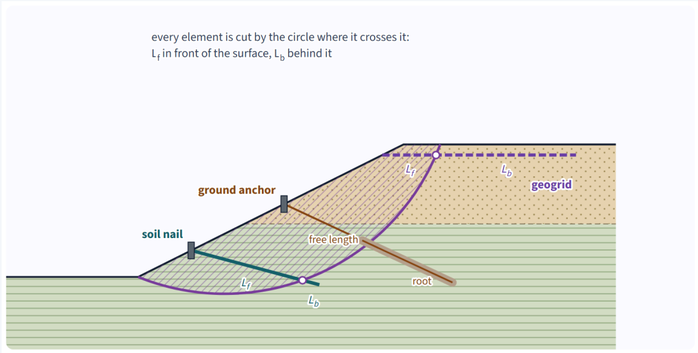
EXAMPLE

Given

Nails	**four nails, heads at $x = 14, 18, 22, 26$ m (at the surface), 15° inclined, $L = 14$ m; $T_t = 250$ kN, $q_u = 120$ kPa, $D_h = 120$ mm, $S_h = 1.5$ m**
Circle	**circle found by the search in the nailed model: center $(18.29;\ 13.90)$, $r = 18.18$ m**

REQUIRED

For each nail **L_f, L_b, T** and **T_m**; the moment contribution; **FS**.



Solution (1/2)

SOLUTION

1 Bond strength (unit length) and nail N3

$$q_u \pi D_h = 120\pi(0.120) = \mathbf{45.239}\ \mathrm{kN/m}$$

$$L_f = 11.282\ \mathrm{m},\ L_b = 2.718\ \mathrm{m} \qquad P(32.90;\ 3.08)$$

$$T = \min[250,\ 45.239(2.718) = 122.94,\ 45.239(11.282) = 510.4] = \mathbf{122.94}\ \mathrm{kN}$$

$$T_m = \frac{T}{S_h} = \frac{122.94}{1.5} = \mathbf{81.96}\ \mathrm{kN/m}$$

2 Moment of the force about the circle center

$$F_x = T_m \cos 15^\circ = 79.17, \quad F_z = -T_m \sin 15^\circ = -21.21\ \mathrm{kN/m}$$

$$\frac{M}{r} = \frac{(x_P - x_c)F_z - (z_P - z_c)F_x}{r} = \frac{(14.606)(-21.21) - (-10.822)(79.17)}{18.178} = \mathbf{30.09}\ \mathrm{kN/m}$$

Nail N3 carries a load of **81.96** kN/m holding the mass; its moment adds 30.09 kN/m to ΣR (passive element).

Solution (2/2)

SOLUTION

3 Forces of four nails (same method)

Nail	L_f (m)	L_b (m)	T (kN)	Governing	T_m (kN/m)	M/r (kN/m)
N1	–	–	0	tip in the mass: carries no load	0	0
N2	13.13	0.87	39.32	pullout at the rear	26.21	13.90
N3	11.28	2.72	122.94	pullout at the rear	81.96	30.09
N4	8.82	5.18	234.18	pullout at the rear	156.12	31.82

4 Nailed model: FS on the circle found by the search

Method	Without nail (excavation)	Nailed
Ordinary	1.569	**1.753**
Bishop	1.740	**2.018**
Spencer	1.717	**2.031**

The nails increase FS by about 12% (ordinary 1.569 → 1.753; Bishop 1.740 → 2.018).

Check the solution in GeoTerva ↗

Curvilinear strength envelope and the tangent at each base

THEORY

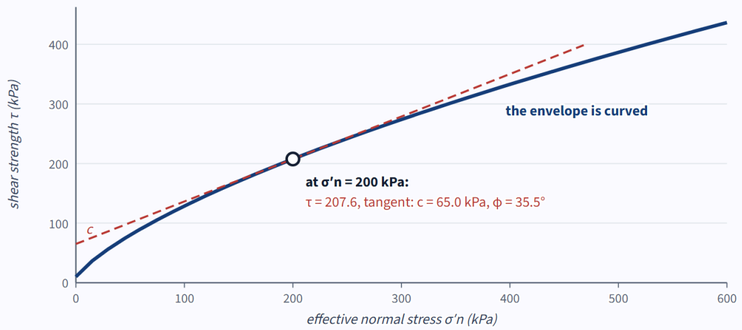
In the generalized Hoek–Brown criterion the rock mass is defined by σ_{ci}, GSI, m_i and D; D = 0 is taken for undisturbed mass, D = 1 for heavy blasting or loosening.

Since the shear strength at the slice base depends on the normal stress, the curvilinear envelope is transformed into the τ–σ'_n plane using the Balmer relations; the tangent yields the values of c_i and φ_i at the base.

$$\sigma'_1 = \sigma'_3 + \sigma_{ci}\left(m_b \frac{\sigma'_3}{\sigma_{ci}} + s\right)^a$$

$$m_b = m_i\, e^{\frac{GSI-100}{28-14D}} \qquad s = e^{\frac{GSI-100}{9-3D}} \qquad a = \frac{1}{2} + \frac{e^{-GSI/15} - e^{-20/3}}{6}$$

$$d = \frac{d\sigma'_1}{d\sigma'_3} \qquad \sin\varphi_i = \frac{d-1}{d+1} \qquad c_i = \tau - \sigma'_n \tan\varphi_i$$



Problem statement

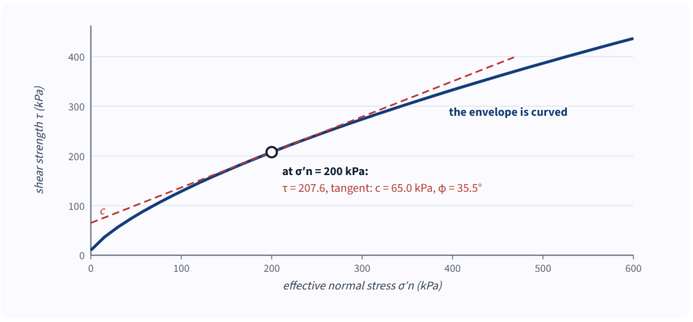
EXAMPLE

Given

Rock mass	$\sigma_{ci} = 5$ **MPa,** $GSI = 30$**,** $m_i = 8$**,** $D = 0$
Base	**effective normal stress** $\sigma'_n = 200$ **kPa**

REQUIRED

m_b, s, a; at this stress, **τ**, **φ_i** and **c_i**.



Solution (1/2)

SOLUTION

1 Rock mass constants

$$m_b = 8\,e^{(30-100)/28} = 0.6567 \qquad s = e^{(30-100)/9} = 0.000419$$

$$a = 0.5 + \frac{e^{-2} - e^{-20/3}}{6} = 0.5223$$

2 For σ'_n = 200 kPa, σ'_3 = 93.05 kPa (found iteratively)

$$d = 1 + a\,m_b\left(\frac{m_b\sigma'_3}{\sigma_{ci}} + s\right)^{a-1} = 3.767$$

$$\sigma'_1 = \sigma'_3 + \sigma_{ci}\left(\frac{m_b\sigma'_3}{\sigma_{ci}} + s\right)^{a} = 602.9\ \text{kPa}$$

$$\sigma'_n = \frac{\sigma'_1 + \sigma'_3}{2} - \frac{\sigma'_1 - \sigma'_3}{2}\,\frac{d-1}{d+1} = 200.0\ \text{kPa}\ \checkmark$$

Solution (2/2)

SOLUTION

3 Shear strength and equivalent Mohr–Coulomb parameters

$$\tau = (\sigma_1' - \sigma_3')\frac{\sqrt{d}}{d+1} = \mathbf{207.6}\ \mathrm{kPa}$$

$$\varphi_i = \arcsin\frac{d-1}{d+1} = \mathbf{35.48}^\circ \qquad c_i = \tau - \sigma_n' \tan\varphi_i = 207.6 - 200(0.7128) = \mathbf{65.0}\ \mathrm{kPa}$$

At this base the rock behaves as in Mohr–Coulomb with **c_i = 65.0 kPa, φ_i = 35.5°** (τ = 207.6 kPa); when the stress changes these values change too: each slice uses its own c and φ.

Check the solution in GeoTerva ↗

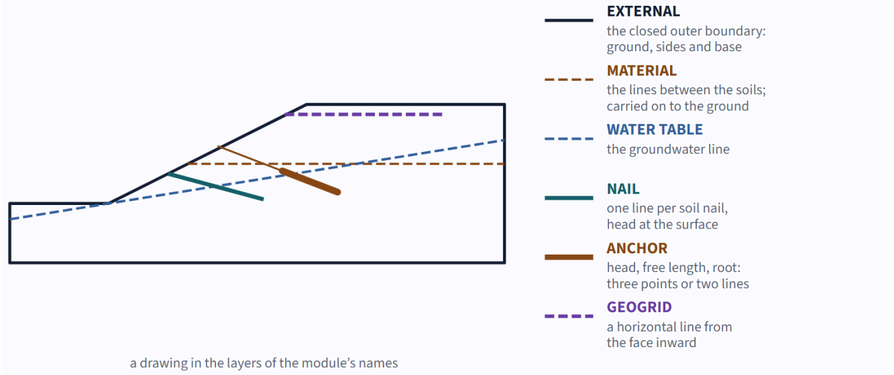
Reading by layer names

THEORY

DXF section layer names are read without regard to upper/lower case, spaces, hyphens and underscores; **EXTERNAL** gives the ground surface, the two sides and the base as a single closed polyline.

MATERIAL lines are the lines between soils; a line ending within the section is extended along the ground surface and a separate soil is assigned to each zone between two boundaries.

WATER TABLE is the groundwater line; **NAIL**, **ANCHOR** and **GEOGRID** are reinforcements, and the unit weight, strength, capacity, load and earthquake not shown in the drawing are entered in the Model tab.

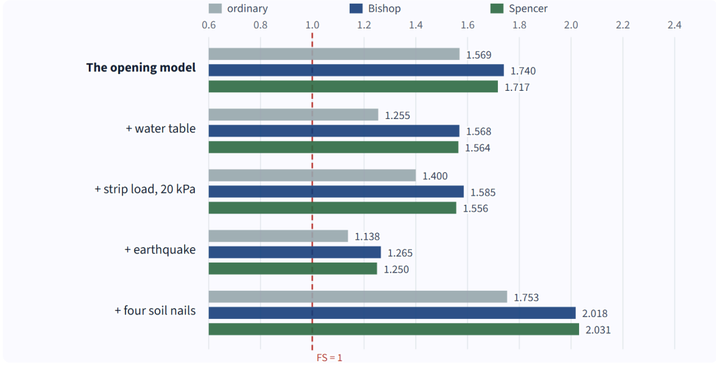


a drawing in the layers of the module's names

Results table

Same section, five models, three methods

Model (something is added, the circle is searched again)	Center (x_c; z_c)	r (m)	Slice	Ordinary	Bishop	Spencer
Model at opening (static)	16.52; 16.08	17.35	33	**1.569**	**1.740**	**1.717**
+ groundwater level rising toward the interior of the slope	17.12; 11.45	16.85	36	**1.255**	**1.568**	**1.564**
+ 20 kPa strip load, x = 25 – 32 m	16.25; 15.44	16.66	35	**1.400**	**1.585**	**1.556**
+ earthquake, kh 0.15, kv 0.05 (critical)	16.72; 17.45	18.70	34	**1.138**	**1.265**	**1.250**
+ four soil nails, 14 m long	18.29; 13.90	18.18	34	**1.753**	**2.018**	**2.031**



The ordinary method is always the lowest; Bishop and Spencer are close to each other (a few percent difference for a circle).

Water and earthquake lower FS the most (1.255 and 1.138, ordinary); four nails raise FS to 1.753. With nails, Bishop and Spencer exceed 2.0.

Assumptions

What the model assumes and what it does not

The **model** assumes plane strain: one metre of an infinitely long slope, the same strength in every direction (except for the isotropic rock criterion) and a rigid soil mass until movement begins.

For **water**, a hydrostatic line below a given line or the overburden's r_u ratio is used; no seepage analysis is performed, and in rapid drawdown $\bar{B} = 1$ is adopted as an instantaneous drawdown.

For **slices**, all quantities are taken at the vertical midline of the slice, negative effective normal force is kept at zero; earthquakes are handled with pseudo-static coefficients and the inertia of water accumulating at the surface is neglected.

The **Spencer** method assumes the inter-slice forces to be parallel to each other at a single angle for the whole mass; moment equilibrium is satisfied in all three methods, force equilibrium only in Spencer.

The **search** finds the surface with the lowest factor of safety near the starting surfaces; it is not proven that this is the lowest among all surfaces.

WHAT IT DOES NOT DO

- Three-dimensional effects (end of the slip, width of the slope)
- Deformation, progressive failure, strain softening
- Seepage and consolidation: pore water pressure is given, not calculated
- Dynamic analysis (Newmark): the pseudo-static calculation is a check
- Provision: FS is specified; the engineer and the specification determine the adequate value

Summary

Key points

FS = ΣR/ΣD; R = cL + N′ tan φ′, D = W sin α (moment about the circle centre). The three methods differ only in how they find **N′**

Slice 22 (example): α = 28.51°, L = 0.864, W = 88.52 → ordinary N′ = 77.79, R = 42.55, D = 42.25; Bishop N′ = 86.56, R = 45.40; Spencer N′ = 80.76, R = 43.51 kN/m

Same circle: **1.569 (ordinary) · 1.740 (Bishop) · 1.717 (Spencer)**; the critical circle was found by testing 4650 circles on a 15 × 15 × 12 grid

Water: u = $\gamma_w h_w$ reduces N′; **earthquake**: F_h reduces the normal and adds a moment; **water in the crack** adds a moment P_w to ΣD′

Reinforcement: passive (nails, geogrid) is taken on the resistance side, active (anchor) on the sliding side; **Hoek–Brown**: tangent c_i, φ_i at each base

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NOTE

All numerical examples have been produced with the module's own computation libraries; the module gives the same values. The worked model on the module page is also this section (2:1 slope 10 m high, clay over fill).

GeoTerva

Online geotechnical analysis

From theory to practice

Draw your own section or import it from DXF; enter the soils, water, loads, earthquake and reinforcement. The module searches for the critical circle and provides the factors of safety of the three methods, the slice tables, the step-by-step solution and the report.

Open the module → | **Theory page →**

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com