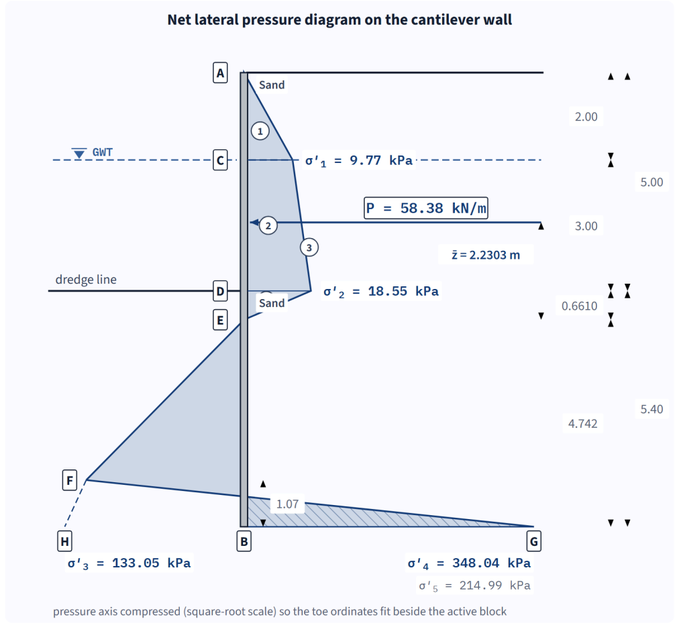
GeoTerva

Geotechnical lecture presentation · Sheet piles and braced cuts

Cantilever sheet pile wall in sand

Lecture: design of a cantilever sheet pile wall in sand; pressures above and below the excavation base, embedment depth, maximum moment, section check, earthquake and displacement; a worked example on each topic.

- Cantilever sheet pile
- Net pressure diagram
- Embedment depth · maximum moment
- EN 1993 · AISC 360
- Earthquake · displacement



Introduction

What will we learn in this lecture?

- **how a cantilever sheet pile rotates** and which pressures develop above, below and at the tip of the excavation base
- **Active pressure** above the excavation base (water, surcharge) and the **net pressure line** below it; zero point E
- **Embedment depth** from horizontal and moment equilibrium: a fourth-degree equation for L_4, $D_{theoretical}$
- Ways of adding a **margin of safety** to the design depth (factor f, K_p/FS_p, specified depth)
- The **maximum moment** (where shear is zero) and verification of the sheet pile with **EN 1993-1-1 / AISC 360-22**
- The effect of water, surcharge, sloping ground surface and **earthquake**; **displacement** by the modulus of subgrade reaction method

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example wall: L = 5 m of sand above the excavation base (γ = 15.9; γ_{sat} = 19.33 kN/m³; φ′ = 32°), the groundwater level is 2 m below the top and at the same level on both faces; the sheet pile is driven into the same sand. Forces are per metre of wall.

How does it work, how does it rotate?

THEORY

A sheet pile wall is a row of interlocking steel piles; it retains the soil above the excavation base (L) and stays standing by its **embedment depth D**. A **cantilever** wall has no other support: like a fence post, it is held only by the **passive resistance** of the soil into which it is driven; for this reason it is generally economical up to about 6 m from the excavation base.

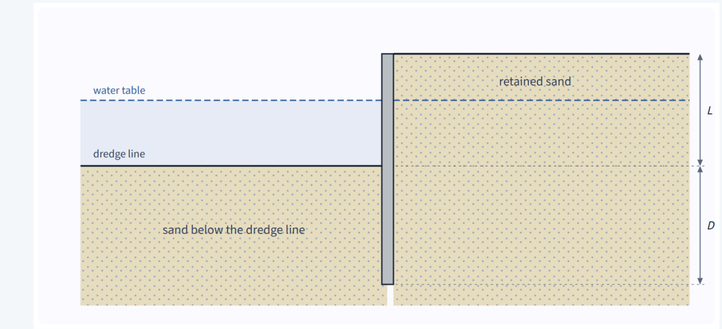
The wall rotates rigidly about point O (near the tip). **Zone A** (above the excavation base): only the retained soil, active pressure.

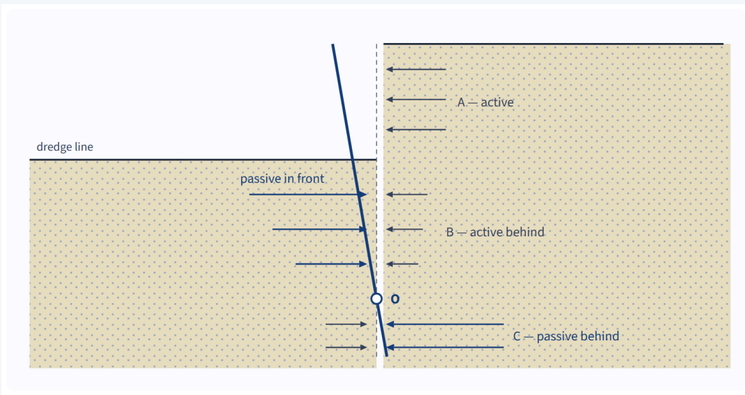
Zone B (excavation base–O): active behind, passive (pushed) in front.

Zone C (lower O): the sides swap roles; passive at the back, active at the front.

The actual distribution is curved; the source replaces it with **a linear approximation**: the net pressure below the excavation base is a single straight line passing through the zero point E; the end return is a block of height L_5.

Rankine pressures, **without wall friction**, are based on the effective vertical stress; if the water is at the same level on both faces, the hydrostatic pressures cancel each other out.





σ'_1 and σ'_2

THEORY

Above the excavation base there is only retained sand; Rankine active pressure = effective vertical stress × K_a. Water is at L_1 below the wall top, and the remaining $L_2 = L - L_1$ is below water ($\gamma' = \gamma_{sat} - \gamma_w$):

$$K_a = \tan^2\left(45^\circ - \frac{\varphi'}{2}\right) \qquad K_p = \tan^2\left(45^\circ + \frac{\varphi'}{2}\right)$$

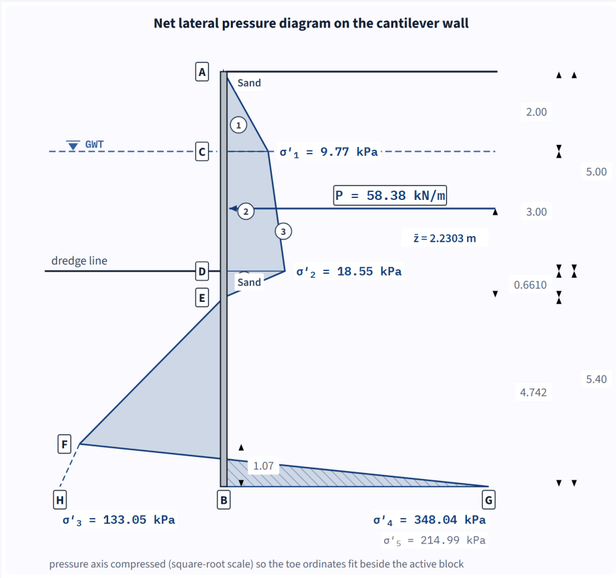
$$\sigma'_1 = \gamma L_1 K_a \qquad \sigma'_2 = (\gamma L_1 + \gamma' L_2)\, K_a$$

If the water is at the same level on both sides, the hydrostatic pressures **cancel**; only these effective pressures load the wall.

Uniform surcharge q (traffic, storage) adds q to the effective stress at every depth: the diagram starts with $\sigma'_0 = qK_a$, and every ordinate increases by qK_a: $\sigma'_2 = (q + \gamma L_1 + \gamma' L_2)K_a$.

The surcharge q below the excavation base remains in the vertical stress on the retained side: the net pressure starts from a larger σ'_2 and the passive pressure at the toe increases by qK_p; the excavation face carries no surcharge.

A limited load (strip, point) does not add a constant $K_a q$: the lateral pressure is taken from the semi-infinite medium and, for a rigid wall, is **doubled by the mirror method**; the module numerically integrates the net diagram.



Net pressure diagram of the worked example (as drawn by the module).

Problem statement

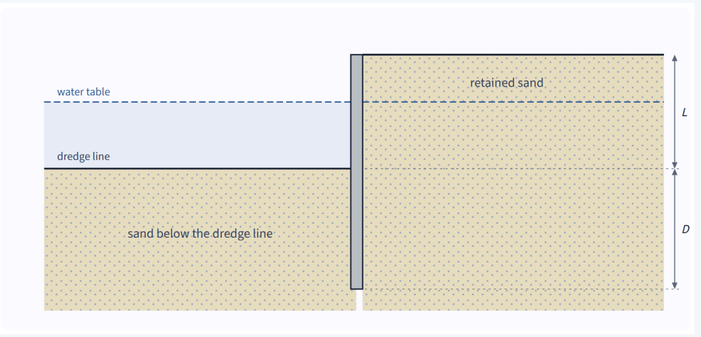
EXAMPLE

Given

Soil	**sand: $\gamma = 15.9$, $\gamma_{sat} = 19.33$ kN/m³, $\varphi' = 32^\circ$ (retained and embedded parts are the same sand)**
Wall	**$L = 5$ m; groundwater $L_1 = 2$ m below the top, at the same level on both sides; no surcharge**

REQUIRED

K_a, **K_p**, γ'; **σ'_1** and **σ'_2** (at the excavation base).



Solution

SOLUTION

1 Pressure coefficients and submerged unit weight

$$K_a = \tan^2(45^\circ - 16^\circ) = \mathbf{0.3073} \qquad K_p = \tan^2(45^\circ + 16^\circ) = \mathbf{3.2546}$$

$$\gamma' = \gamma_{sat} - \gamma_w = 19.33 - 9.81 = 9.52\ \mathrm{kN/m^3}$$

2 Active pressure at the water level and at the excavation base

$$\sigma'_1 = \gamma L_1 K_a = 15.9(2)(0.3073) = \mathbf{9.77}\ \mathrm{kPa}$$

$$\sigma'_2 = (\gamma L_1 + \gamma' L_2)\, K_a = (15.9(2) + 9.52(3))\,(0.3073)$$

$$\sigma'_v(L) = 60.36\ \mathrm{kPa} \ \Rightarrow\ \sigma'_2 = \mathbf{18.55}\ \mathrm{kPa}$$

The active pressure is **9.77 kPa** at the water level and **18.55 kPa** at the excavation base; passive coefficient K_p = 3.2546. Since the submerged unit weight (γ′ = 9.52 kN/m³) is used below water, the pressure increase slows down.

Check the solution in GeoTerva ↗

Zero point E, P and z̄, fourth-degree equation

THEORY

Below the excavation base there is soil on both sides; active pressure at the back, passive pressure at the front, and the passive grows faster: the net pressure starts at σ'_2, decreases linearly with depth and becomes zero at point E:

$$L_3 = \frac{\sigma'_2}{\gamma'\,(K_p - K_a)} \qquad \sigma'_5 = \sigma'_v(L)K_p + \gamma' L_3(K_p - K_a)$$

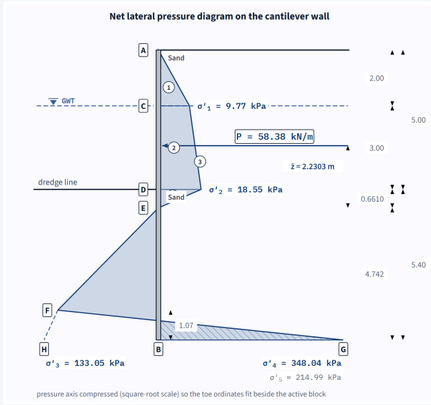
Above E the resultant of the diagram, **P** (area), acts at **z̄** above E. L_4 = the depth of the tip below E; from the conditions $\Sigma F_H = 0$ and $\Sigma M = 0$ about the tip:

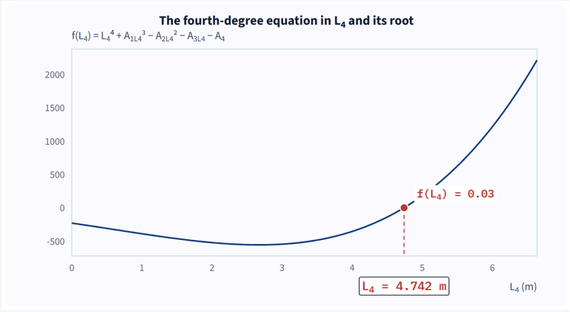
$$\sigma'_3 = L_4(K_p - K_a)\gamma' \quad \sigma'_4 = \sigma'_5 + \gamma' L_4(K_p - K_a) \quad L_5 = \frac{\sigma'_3 L_4 - 2P}{\sigma'_3 + \sigma'_4}$$

$$L_4^4 + A_1 L_4^3 - A_2 L_4^2 - A_3 L_4 - A_4 = 0$$

$$A_1 = \frac{\sigma'_5}{m} \quad A_2 = \frac{8P}{m} \quad A_3 = \frac{6P\,(2\bar{z}m + \sigma'_5)}{m^2} \quad A_4 = \frac{P\,(6\bar{z}\sigma'_5 + 4P)}{m^2} \quad (m = \gamma'(K_p - K_a))$$

The theoretical embedment depth **$D_{theoretical} = L_3 + L_4$**: the depth at which the passive resistance is fully mobilized and no margin remains.





Problem statement

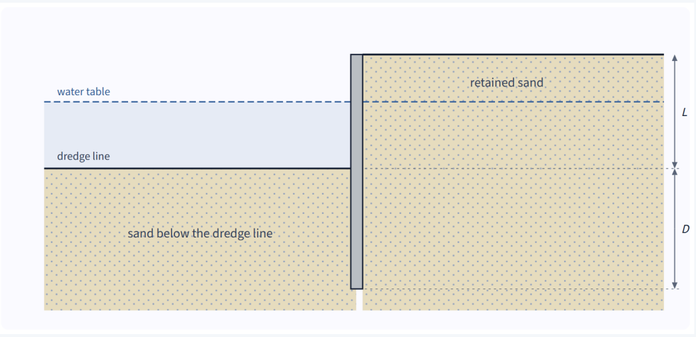
EXAMPLE

Given

Given	**The wall in Example 1:** $\sigma'_1 = 9.77$, $\sigma'_2 = 18.55$ **kPa;** $K_a = 0.3073$, $K_p = 3.2546$, $\gamma' = 9.52$ **kN/m³**

REQUIRED

The zero point of the net pressure is **L_3**; the resultant of the diagram above E is **P** and its arm is **z̄**.



Solution

SOLUTION

1 Zero point

$$L_3 = \frac{\sigma'_2}{\gamma'(K_p - K_a)} = \frac{18.55}{9.52(3.2546 - 0.3073)} = \frac{18.55}{28.059} = \mathbf{0.6610}\ \text{m}$$

2 Parts of the active diagram (from top to E): P = Σ area, z̄ = Σ(area × arm)/P

Piece	Figure	Area (kN/m)	Arm to E (m)	Area × arm
1	triangle: 0–2 m	9.77	4.328	42.28
2	rectangular: 2–5 m	29.31	2.161	63.34
3	triangle: 2–5 m	13.16	1.661	21.86
4	triangle: 5 m–E	6.13	0.441	2.70
Total		**58.38**		**130.19**

P = 58.38 kN/m; z̄ = 130.19 / 58.38 = **2.2303 m** (above E). The net pressure becomes zero 0.661 m below the excavation base.

Check the solution in GeoTerva ↗

Problem statement

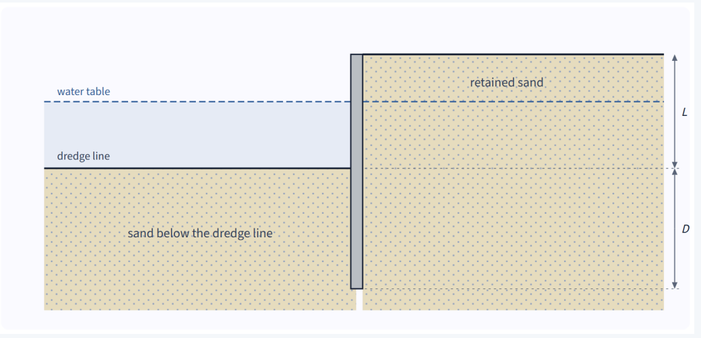
EXAMPLE

Given

Given	**Example 2:** $L_3 = 0.661$ **m,** $P = 58.38$ **kN/m,** $\bar{z} = 2.2303$ **m;** $\sigma'_v(L) = 60.36$ **kPa**
Coefficients	$K_a = 0.3073$**,** $K_p = 3.2546$**,** $m = \gamma'(K_p - K_a) = 28.06$ **kN/m³**

REQUIRED

σ′5, A1…A4; root **L4**; **Dtheoretical**; σ′3, σ′4, **L5**.



Solution (1/2)

SOLUTION

1 Ordinate at the tip σ′$_5$ (passive of the retained side + net correct)

$$\sigma'_5 = \sigma'_v(L)K_p + \gamma' L_3(K_p - K_a) = 60.36(3.2546) + 28.059(0.6610) = \mathbf{214.99}\ \text{kPa}$$

2 Coefficients of the equation

$$A_1 = \sigma'_5/m = 214.99/28.059 = 7.6623 \qquad A_2 = 8P/m = 8(58.38)/28.059 = 16.6439$$

$$A_3 = 6P\left(2\bar{z}m + \sigma'_5\right)/m^2 = 151.328 \qquad A_4 = P\left(6\bar{z}\sigma'_5 + 4P\right)/m^2 = 230.633$$

3 Positive root (equation solved numerically; satisfies f = 0)

$$L_4^4 + 7.662L_4^3 - 16.644L_4^2 - 151.33L_4 - 230.63 = 0$$

$$L_4 = \mathbf{4.742}\ \text{m} \qquad D_{theoretical} = L_3 + L_4 = 0.6610 + 4.742 = \mathbf{5.40}\ \text{m}$$

Theoretical embedment depth **D = 5.40 m** (from the excavation base); L_4 = 4.742 m. At this depth the passive resistance is fully mobilized and there is **no margin at all**.

Solution (2/2)

SOLUTION

4 End pressure block

$$\sigma'_3 = L_4\, m = 4.742(28.059) = \mathbf{133.05}\ \text{kPa}$$

$$\sigma'_4 = \sigma'_5 + L_4 m = 214.99 + 133.05 = \mathbf{348.04}\ \text{kPa}$$

$$L_5 = \frac{\sigma'_3 L_4 - 2P}{\sigma'_3 + \sigma'_4} = \frac{133.05(4.742) - 2(58.38)}{133.05 + 348.04} = \mathbf{1.0686}\ \text{m}$$

Diagram complete: the net line through E, the block turning from σ′$_3$ = 133.0 kPa to σ′$_4$ = 348.0 kPa at **L_5 = 1.069 m** above the tip. Check: the left-hand side of the fourth-degree equation is only 0.03 at the root (zero relative to the scale).

Check the solution in GeoTerva ↗

Ways to add a margin of safety

THEORY

There is no allowance at the theoretical depth. The source proposes two approaches (figure at side):

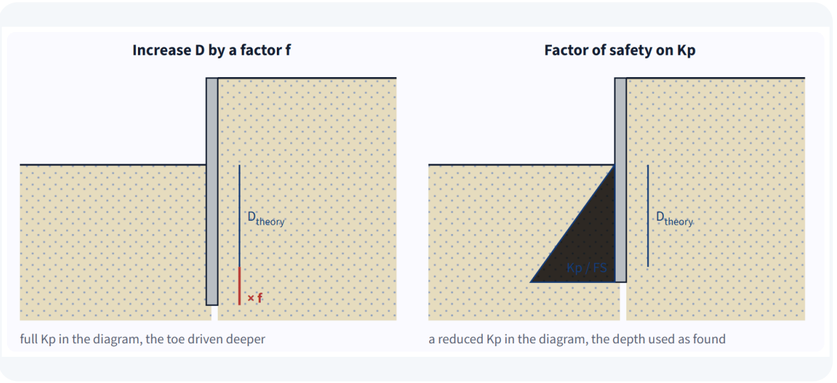
Increasing the depth: the diagram is set up with full K_p and the depth is increased by 20–30%: $D_{design} = f \cdot D_{theoretical}$, f = 1.2–1.3. The pressures and moment are those of the theoretical wall.

Factor of safety on K_p: the diagram is constructed from the outset with K_p/FS_p (FS_p = 1.5–2), and the resulting depth is used as is. Every ordinate below the excavation base changes, and **the moment also increases**.

Given depth: if the depth comes from the engineer rather than from the analysis, it is verified against the depth required by the chosen method ($D_{given} \geq D_{design,required}$) and the mobilized factor of safety $FS_{p,mob}$ is found: dividing K_p by this value makes the diagram exactly in equilibrium at the given depth.

$$D_{given} \geq D_{design,req} \qquad D(K_p/FS_{p,mob}) = D_{given}$$

At the given depth the **internal forces** are studied for two conditions: the required-depth condition and the depth equilibrium given by $K_p/FS_{p,mob}$. The moment for the given condition may exceed the other (a long wall deflects more with a partially mobilized passive); the sheet pile is selected according to **the larger of the two**.



Problem statement

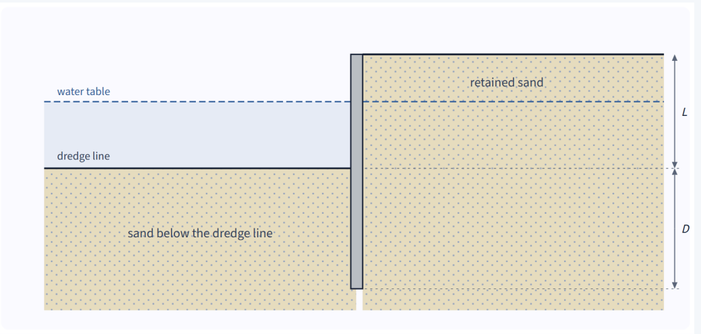
EXAMPLE

Given

Wall	**The wall in Examples 1–3 ($D_{theoretical} = 5.40$ m, $M_{max} = 209.58$ kN·m/m)**
Roads	**$f = 1.3$; $FS_p = 1.5$ and 2.0; given depths 6.0 and 8.0 m; sheet pile: $W_{el} = 1\,400$ cm³/m, S275 (EN 1993-1-1, $\gamma_F = 1.35$)**

REQUIRED

For each route, **D_{design}**, total length, **M_{max}** and sheet-pile utilization ratio **η**.



Solution

SOLUTION

1 Path f: $D_{design} = 1.3 \times 5.403 = 7.023$ m (moment unchanged)

Road	D_{design} (m)	L + D (m)	M_{max} (kN·m/m)	η (EN 1993)	Note
f = 1.3 ($D = f \cdot D_{theoretical}$)	7.02	12.02	209.58	0.735	theoretical moment of the wall
$FS_p = 1.5$ ($K_p/1.5$)	7.10	12.10	263.53	0.924	$K_p = 2.170$
$FS_p = 2.0$ ($K_p/2$)	8.84	13.84	325.91	1.143	**η > 1**
Given D = 6.0 m	6.00	11.00	227.74	0.799	D < 7.02: insufficient; $FS_{p,mob} = 1.18$
Given D = 8.0 m	8.00	13.00	294.92	1.034	D ≥ 7.02; $FS_{p,mob} = 1.76$; **η > 1**

On the K_p/FS_p path, because the diagram is weakened from the outset, the depth (7.10 m) and the moment (263.5 kN·m/m) increase together; with $FS_p = 2$ the moment is 325.9 kN·m/m and the sheet pile is inadequate. Although the **given D = 8 m** is longer than the design depth, the moment (294.9) **increases** due to the partially mobilized passive pressure: increasing the depth does not always reduce the moment.

Check the solution in GeoTerva ↗

Where the cut is zero

THEORY

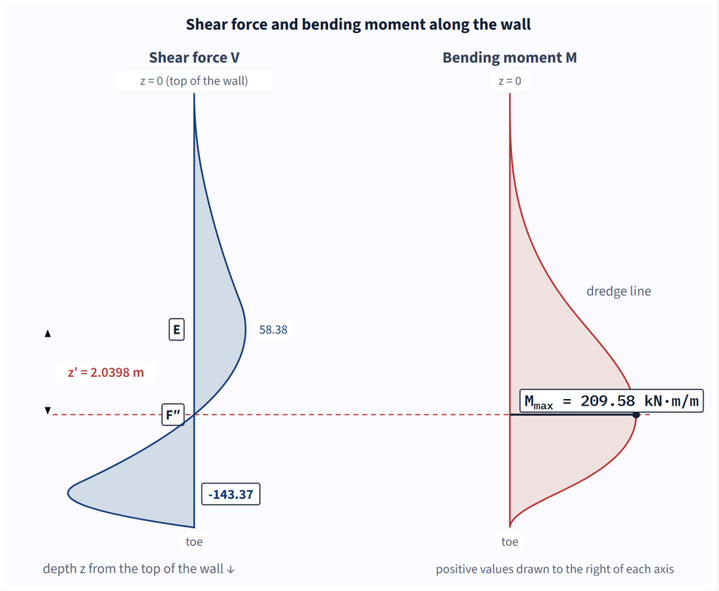
The bending moment is greatest where the shear force is zero. Below E the net passive triangle ½γ′(K$_p$ − K$_a$)z′² grows; P is balanced against E at **z′** below E:

$$z' = \sqrt{\frac{2P}{\gamma'(K_p - K_a)}} \qquad M_{max} = P\,(\bar{z} + z') - \tfrac{1}{2}\gamma' z'^2\,(K_p - K_a)\,\frac{z'}{3}$$

The shear reaches its maximum at E (net pressure zero) and is zero at depth z′; the moment is greatest here and returns to zero at the tip.

M$_{max}$ is the moment of the theoretical wall (f path); in the K$_p$/FS$_p$ path the diagram is built with the reduced K$_p$, so M$_{max}$ is larger.

Sheet pile selection is made with **S = M$_{max}$/σ$_{allow}$** (classical) or by limit-state verification (EN 1993-1-1, AISC 360-22 below).



Problem statement

EXAMPLE

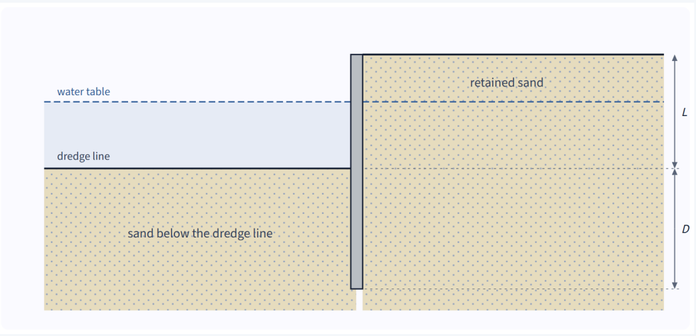
Given

Given

Example 2–3: $P = 58.38$ **kN/m,** $\bar{z} = 2.2303$ **m;** $m = \gamma'(K_p - K_a) = 28.06$ **kN/m³**

REQUIRED

The depth **z′** at which shear is zero (below E) and $\mathbf{M_{max}}$; its depth from the top of the wall.



Solution

SOLUTION

1 Where shear is zero (below E)

$$z' = \sqrt{\frac{2P}{m}} = \sqrt{\frac{2(58.38)}{28.059}} = \mathbf{2.0398}\ \mathrm{m}$$

2 Maximum moment

$$M_{max} = P(\bar{z} + z') - \tfrac{1}{6}\, m\, z'^3$$

$$= 58.38(2.2303 + 2.0398) - \tfrac{1}{6}(28.059)(2.0398)^3 = \mathbf{209.58}\ \mathrm{kN{\cdot}m/m}$$

M_{max} = 209.58 kN·m/m, at a depth L + L_3 + z′ = 5 + 0.661 + 2.040 = **7.70 m** below the wall top. The sheet pile is selected for this moment.

Check the solution in GeoTerva ↗

Same setup, different inputs

THEORY

No water: the same setup with natural unit weight; $\sigma'_2 = \gamma L K_a$, $L_3 = \sigma'_2/(\gamma(K_p - K_a))$; γ replaces every γ′ in the equations, and the embedment equation keeps the same form.

Uniform surcharge q: every ordinate increases by qK_a; the net line starts from the larger σ'_2, and the passive qK_p at the end increases accordingly (previous section).

Different water levels on the two sides (beyond the source): hydrostatic pressures are not subtracted; in the no-seepage case the net water pressure is added. The net is piecewise linear, and the E and equilibrium equations are solved numerically on the diagram.

Inclined retained surface (β, infinite): the horizontal component of the Rankine active coefficient for an inclined fill is used on the retained side; it remains on the excavation side and on the flat passive surface at the end (conservative side). For β ≥ φ′ there is no solution.

$$u_{net}(z) = \gamma_w(z - D_w) - \gamma_w(z - D_{w,exc}) \quad (\text{each term} \geq 0)$$

$$p(z) = K_a\sigma'_{v,r}(z) - K_p\sigma'_{v,e}(z) + u_{net}(z)$$

$$K_{a,\beta} = \cos\beta\,\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi'}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi'}} \qquad K_{a,h} = K_{a,\beta}\cos\beta$$

Problem statement

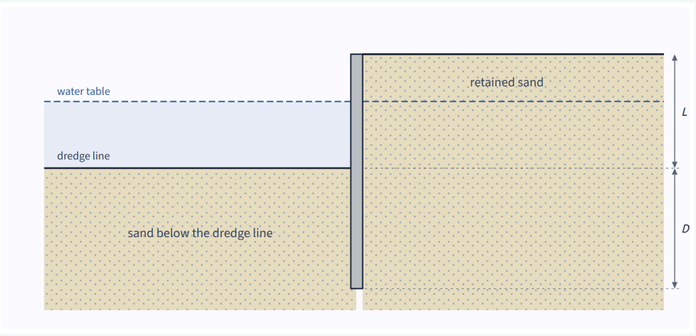
EXAMPLE

Given

Wall	**The wall in Examples 1–5 (f = 1.3; S275, $W_{el} = 1\,400$ cm³/m)**
Changes	**(a) no water; (b) uniform surcharge $q = 10$ kPa; (c) water on the front face at $z = 6$ m (excavation dry, at the back 2 m); (d) retained surface inclined at $\beta = 15°$**

REQUIRED

For each case **L_3**, P, **$D_{theoretical}$**, D_{design}, **M_{max}**, η; hand verification when there is no water.



Solution

SOLUTION

1 No water (manual): $\sigma'_2 = \gamma L K_a$, $L_3 = \sigma'_2/(\gamma(K_p - K_a))$

$$\sigma'_2 = 15.9(5)(0.3073) = 24.43\ \text{kPa} \qquad L_3 = \frac{24.43}{15.9(3.2546 - 0.3073)} = \mathbf{0.521}\ \text{m}$$

2 Summary of the five cases (the others are solved by the module with the same method)

Condition	L_3 (m)	P (kN/m)	$D_{theoretical}$ (m)	D_{design} (m)	M_{max} (kN·m/m)	η (EN 1993)
Excavation (water 2 m, same on both sides)	0.661	58.38	5.40	7.02	209.6	0.735
(a) No water	0.521	67.43	4.53	5.89	212.1	0.744
(b) Surcharge q = 10 kPa	0.770	75.94	6.15	7.99	305.5	**1.071**
(c) Water on the front face z = 6 m	1.319	126.29	8.00	10.39	543.8	**1.907**
(d) Inclined surface β = 15°	0.713	63.00	5.64	7.33	232.1	0.814

If there is no water, γ replaces γ′: the zero point moves closer and the embedment depth **decreases** to 4.53 m. **Surcharge** increases D to 6.15 m and the moment to 306 kN·m/m (η > 1); a **different water level** is the worst case: M = 544 kN·m/m (P = 126 kN/m). Slope β = 15°: $K_{a,h}$ = 0.3289 (level: 0.3073).

Check the solution in GeoTerva ↗

EN 1993-1-1 and AISC 360-22

THEORY

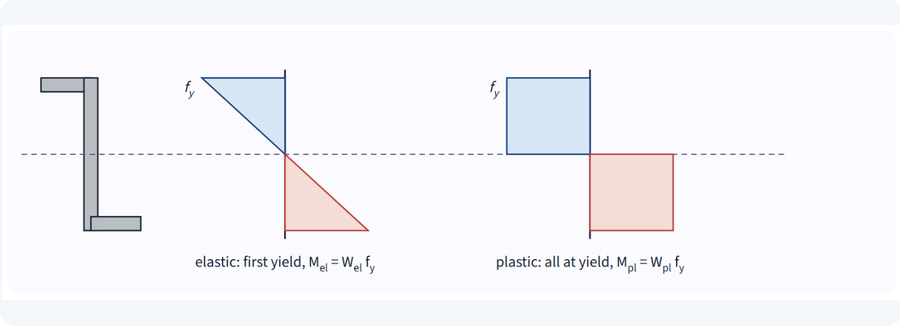
Classical route: S = M_{max}/σ_{allow}. The module replaces this with limit-state verification; the moment of the net pressure diagram is a characteristic action, and the partial factor raises it to the design value:

$$M_{Ed} = \gamma_F M_{max} \qquad W_{req} = \frac{M_{Ed}\gamma_{M0}}{f_y} \qquad M_{c,Rd} = \frac{W f_y}{\gamma_{M0}} \qquad \eta = \frac{M_{Ed}}{M_{c,Rd}} \le 1$$

The table has two section moduli: **W_{el}** up to first yield (valid up to class 3), **W_{pl}** up to the fully plastic moment (class 1–2). The embedment depth allowance is not multiplied separately by the steel code.

$$\text{LRFD: } M_u = \gamma_H M_{max} \le \phi_b M_n,\ \phi_b = 0.90 \qquad \text{ASD: } M_a = M_{max} \le \frac{M_n}{\Omega_b},\ \Omega_b = 1.67$$

$M_n = F_yZ$ (compact section) or F_yS at first yield. Earth pressure is taken as lateral earth pressure H: in the LRFD combination γ_H = 1.6, in ASD 1.0.



Problem statement

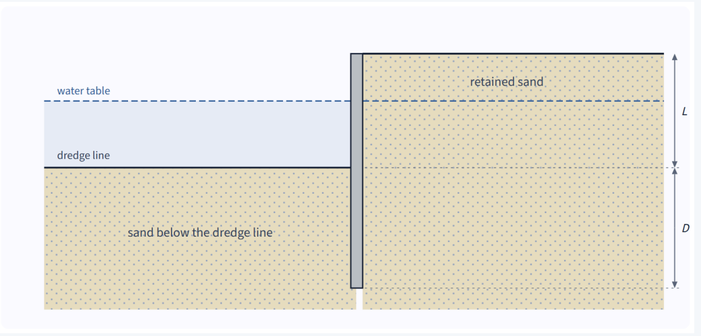
EXAMPLE

Given

Section	**S275 ($f_y = 275$ MPa); elastic section modulus $W_{el} = 1\,400$ cm³/m**
Moment	**Example 5: $M_{max} = 209.58$ kN·m/m; EN 1993-1-1: $\gamma_F = 1.35$, $\gamma_{M0} = 1.00$; AISC: $\gamma_H = 1.6$, $\Omega_b = 1.67$ ($M_n = F_y S$)**

REQUIRED

For EN 1993-1-1: **M_{Ed}**, **W_{req}**, $M_{c,Rd}$, **η**; for AISC LRFD and ASD: η.



Solution (1/2)

SOLUTION

1 EN 1993-1-1

$$M_{Ed} = \gamma_F M_{max} = 1.35(209.58) = \mathbf{282.93}\ \mathrm{kN\cdot m/m}$$

$$W_{req} = \frac{M_{Ed}\gamma_{M0}}{f_y} = \frac{282.93(1.00)}{275\,000} = \mathbf{1{,}028.8}\ \mathrm{cm^3/m}$$

$$M_{c,Rd} = \frac{W_{el} f_y}{\gamma_{M0}} = 1\,400 \cdot 10^{-6}(275\,000) = 385.00\ \mathrm{kN\cdot m/m} \qquad \eta = \frac{282.93}{385.00} = \mathbf{0.735} \le 1\ \checkmark$$

The selected sheet pile is adequate according to EN 1993-1-1: η = **0.735**; required W = 1,029 cm³/m < 1 400 cm³/m. If W_{pl} is given in the table and the section class is 1–2, the plastic moment may be used (assuming W_{pl} = 1 650 cm³/m, η = 0.624).

Solution (2/2)

SOLUTION

1 AISC 360-22: nominal flexural strength $M_n = F_y S = 275\,000\ (1\,400 \cdot 10^{-6}) = 385.0$ kN·m/m

$$\text{LRFD: } M_u = 1.6(209.58) = 335.32\ \text{kN·m/m} \ \leq\ \phi_b M_n = 0.90(385.0) = 346.5 \quad \eta = \mathbf{0.968}$$

$$\text{ASD: } M_a = 209.58\ \text{kN·m/m} \ \leq\ \frac{M_n}{\Omega_b} = \frac{385.0}{1.67} = 230.5 \quad \eta = \mathbf{0.909}$$

Also adequate with AISC: LRFD η = **0.968** (first yield), ASD η = **0.909**. The same moment gives a utilization ratio of 0.735 in EN 1993-1-1 and 0.968 in LRFD: the ratios differ because the codes use different safety philosophies.

Check the solution in GeoTerva ↗

Pseudo-static Mononobe–Okabe framework

THEORY

The earthquake preserves the static limit-equilibrium framework by adding constant inertial forces ($k_h W$ horizontal, $k_v W$ vertical) to the soil wedges. Vertical wall, horizontal surface, no wall friction (the same idealization as the static Rankine design):

$$\tan\theta = \frac{k_h}{1 \mp k_v} \qquad K_{AE} = \frac{\cos^2(\varphi' - \theta)}{\cos^2\theta \left[1 + \sqrt{\frac{\sin\varphi' \sin(\varphi' - \theta)}{\cos\theta}}\right]^2} \qquad K_{PE} = \ldots \left[1 - \sqrt{\ldots}\right]^2$$

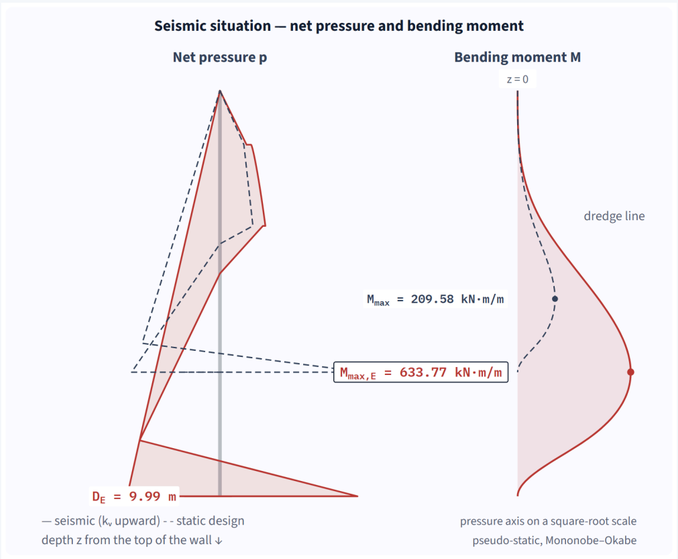
$$\text{under water:} \quad \tan\theta_w = \frac{\gamma_{sat}}{\gamma_{sat} - \gamma_w} \cdot \frac{k_h}{1 \mp k_v}$$

Pressures $(1 \mp k_v)K_{AE}\sigma'_v$ and $(1 \mp k_v)K_{PE}\sigma'_v$; **both directions of k_v** are checked. At $\theta = 0$, K_a and K_p; if $\theta > \varphi'$, there is no equilibrium wedge.

In sand of medium permeability, the pore water moves together with the soil (**constrained**): the inertia is in the saturated mass and the angle increases. If there is free water in the excavation, the Westergaard hydrodynamic pressure (in the opposite direction) is added.

Check: the net pressure diagram is set up as in statics; the two equilibrium equations give $\mathbf{D_E}$. The earthquake case is a situation tied to the excavation; no coefficient is placed on K_{PE}: the wall withstands it if $\mathbf{D_E \leq D_{design}}$. In a cantilever, seismic pressures are distributed like the static ones, proportional to effective stress.

In the seismic combination, effects are taken with 1.0: $M_{Ed,E} = M_{max,E}$.



Problem statement

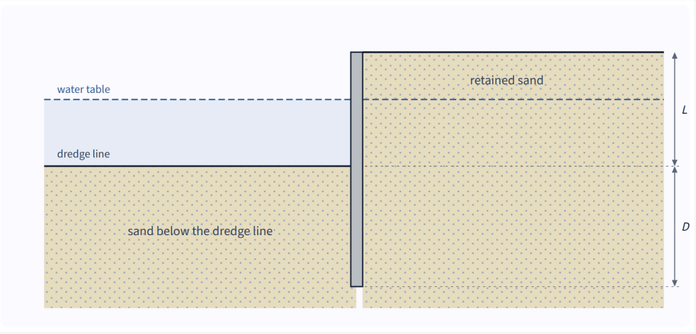
EXAMPLE

Given

Wall	**The wall in Example 1–5: $D_{design} = 7.02$ m, $M_{max} = 209.58$ kN·m/m; S275, $W_{el} = 1\,400$ cm³/m**
Earthquake	**$k_h = 0.15$, $k_v = \pm 0.075$ (two directions); pore water restricted; 3 m free water in the excavation**

REQUIRED

Seismic angles and coefficients; $\mathbf{D_E}$ (two k_V directions), $\mathbf{M_{max,E}}$; can the static wall withstand this earthquake?



Solution (1/2)

SOLUTION

1 Seismic angles and coefficients ($\varphi' = 32°$; static $K_a = 0.3073$, $K_p = 3.2546$)

$$\tan\theta = 0.15/(1-0.075) = 0.1622 \;\Rightarrow\; \theta = 9.21^\circ$$

$$\tan\theta_w = \frac{19.33}{19.33-9.81}(0.1622) = 0.3293 \;\Rightarrow\; \theta_w = 18.22^\circ$$

k_v	$f_v = 1 - k_v$	θ (at top)	θ_w (in water)	K_{AE} / K_{PE} (above)	K_{AE} / K_{PE} (in water)
+0.075	0.925	9.21°	18.22°	0.411 / 2.948	0.562 / 2.589
−0.075	1.075	7.94°	15.82°	0.395 / 2.992	0.514 / 2.693

Under water the angle is larger (inertia in the saturated mass): K_{AE} rises from 0.411 to 0.562 and K_{PE} falls from 2.948 to 2.589; active pressure increases and passive resistance decreases.

Solution (2/2)

SOLUTION

2 Seismic net pressure diagram and embedment depth D_E

Condition	L_3 (m)	D_E / $D_{theoretical}$ (m)	M_{max} (kN·m/m)	D ≤ 7.02?
Static (f = 1.3)	0.661	— / 5.40	209.58	D_{design} = 7.02
Earthquake, k_v = +0.075	1.756	9.987	**633.77**	**No**
Earthquake, k_v = −0.075	1.497	8.988	603.87	**No**

3 Section and result

$$W_{req,E} = \frac{M_{Ed,E}}{f_y} = \frac{633.77}{275\,000} = 2{,}305\ \text{cm}^3/\text{m} > 1\,400 \qquad \eta_E = \mathbf{1.646} > 1 \ \text{✗}$$

The static wall **does not withstand** this earthquake: the seismic diagram requires an embedment **2.96 m** deeper (D_E = 9.99 > 7.02 m) and **3.0 times the static moment** (633.8 kN·m/m); the sheet pile is also insufficient (η_E = 1.65). For the earthquake combination, the embedment length and the section must be increased together.

Check the solution in GeoTerva ↗

Modulus of subgrade reaction method: elastic beam on spring foundation

THEORY

Limit equilibrium gives the embedment depth and the moment, but not **how much the wall displaces** (it describes the failure state at which the passive resistance is fully mobilized). For the displacement in the working state, the wall is taken as an elastic beam (EI) and the soil on each face as independent springs (Winkler):

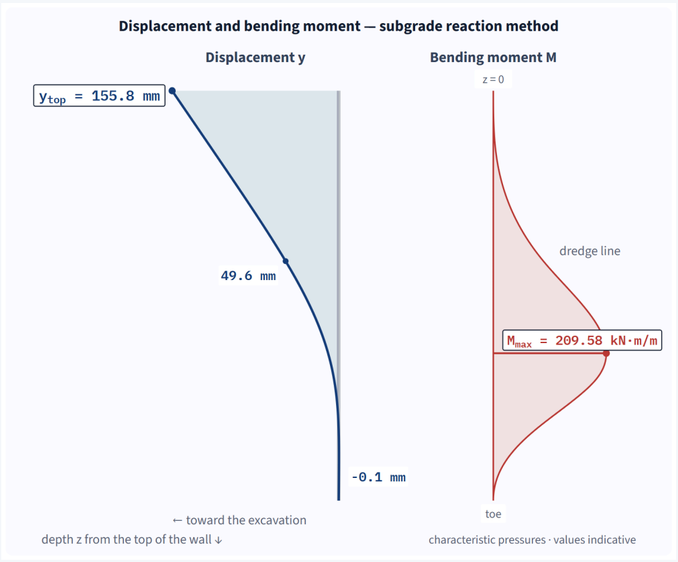
$$EI\frac{d^4y}{dz^4} = p_R(z) - p_E(z) + u_{net}(z)$$

$$p_R = K_0\sigma'_{v,r} - k_h y \;\; (K_a\sigma'_{v,r} \le p_R \le K_p\sigma'_{v,r}) \qquad p_E = K_0\sigma'_{v,e} + k_h y \;\; (\dots) \qquad K_0 = 1 - \sin\varphi'$$

The springs start from the at-rest pressure, vary in proportion to displacement, and do not exceed the active and passive limits. k_h is a **model parameter** rather than a soil property; indicative values for sand: loose 4.8–16, medium dense 9.6–80, dense 64–128 MN/m³.

The wall is at design length (L + D_{design}), free at both ends; pressures are characteristic: full Rankine K_p, no factor of safety; K_0 Jaky (1944). The wall is in place before excavation; the excavation is a single stage.

The equation is solved with beam finite elements, iterating until the springs reach the elastic/active/passive state; the result is given as an **unbounded estimate**.



Problem statement

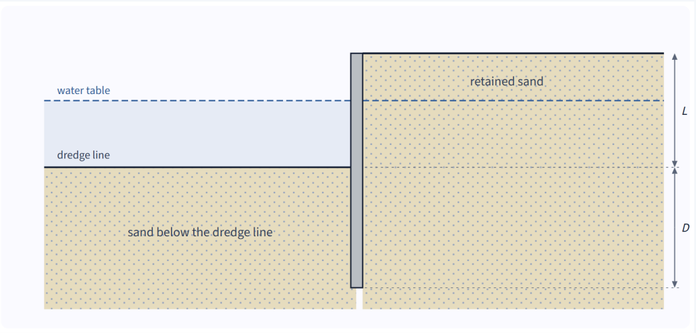
EXAMPLE

Given

Wall	**The wall of Examples 1–5, design length $L + D = 12.02$ m**
Sheet pile	$I = 21\,000$ **cm⁴/m,** $E = 210\,000$ **MPa (**$EI = 44\,100$ **kN·m²/m)**
Soil	$k_h = 20$ **MN/m³ (on both faces);** $K_0 = 1 - \sin 32^\circ$**; excavation in a single stage**

REQUIRED

Displacement at the wall top, at the excavation base and at the toe; does the maximum moment of the model agree with the limit-equilibrium moment?



Solution

SOLUTION

1 At-rest coefficient and model

$$K_0 = 1 - \sin 32^\circ = 0.4701 \qquad EI = 210\,000\,000(21\,000 \cdot 10^{-8}) = 44{,}100\ \mathrm{kN\cdot m^2/m}$$

$$EI\, y^{IV} = p_R - p_E \ \ (\text{finite element, springs elastic/active/passive})$$

2 Result (toward the excavation, mm)

$$y_{üst} = \mathbf{155.8}\ \mathrm{mm} \qquad y_{excavation\ base} = \mathbf{49.6}\ \mathrm{mm} \qquad y_{tip} = -0.1\ \mathrm{mm}$$

$$M_{max}^{model} = 209.58\ \mathrm{kN\cdot m/m}\ \ (z = 7.70\ \mathrm{m}) \ \approx\ M_{max}^{limit\ equilibrium} = 209.58$$

The wall top moves **155.8 mm** toward the excavation and 49.6 mm at the excavation base; the tip remains almost in place (-0.1 mm) — the wall rotates about a point near the tip. The model moment (209.58) is the same as the limit-equilibrium moment (209.58): active pressure behind, passive pressure in front, at their limits over the entire height where the moment develops. About 40% of the passive resistance is mobilized in the springs.

Check the solution in GeoTerva ↗

Scope of the solution

THEORY

Rankine pressures, without wall friction, based on effective vertical stress; the profile is sand throughout (a layer at every point); embedded clay is a separate module. For the embedded part, the source's closed-form solution for a single sand is used, while for multiple sands the same diagram is solved numerically.

Simplified net pressure diagram of the source: a single line through E and the end block; the pivot point O is not searched more precisely. Water is at the same level on both faces or at different levels (no seepage); seepage and its effect on passive resistance are not modeled.

Embedment depth carries the margin of its own path (f or FS_p) and is not multiplied again by the steel code. The sheet pile is checked in bending with the given section modulus; shear, local buckling and corrosion loss are not checked.

Displacement is given by the subgrade modulus method, with characteristic pressures, without limit; settlement behind the wall is not calculated.

A cantilever wall is rarely economical beyond D ≈ 1.5–2 L; if the theoretical embedment exceeds 3L the module warns. The anchored wall is covered in a different module (separate presentation).

Summary

Key points

A **cantilever sheet pile** stands on the passive resistance below the excavation base; active above the excavation base and net pressure below it: the line starting from σ'_2 and vanishing at E ($L_3 = \sigma'_2/(\gamma'(K_p - K_a))$) and the L_5 block at the tip

Equilibrium: P, $\bar{z}$ and $\Sigma F_H = 0$, $\Sigma M = 0 \rightarrow$ a fourth-degree equation for L_4; $\mathbf{D_{theoretical} = L_3 + L_4}$ (example: 5.40 m)

Numerator: D with f = 1.2–1.3 (example: 7.02 m, moment unchanged) or K_p/FS_p (depth and moment increase); at the given depth $FS_{p,mob}$ and **the larger of the two force cases**

$\mathbf{M_{max}}$ where the shear is zero: $z' = \sqrt{(2P/m)}$, $M_{max} = P(\bar{z} + z') - m\, z'^3/6$ (example: 209.58 kN·m/m)

Water, surcharge, slope: without water $\gamma' \rightarrow \gamma$; q increases every ordinate by qK_a; a different water level and surcharge greatly increase D and M

Section: EN 1993-1-1 $M_{Ed} = \gamma_F M_{max} \leq W f_y/\gamma_{M0}$ ($\eta = 0.735$); AISC LRFD/ASD (0.968 / 0.909)

Earthquake: K_{AE}, K_{PE}, θ_w; $D_E \leq D_{design}$ is sought: the example wall does not resist (D_E = 9.99 m, M = 634 kN·m/m)

Displacement: elastic beam on spring foundation; in the example the head ≈ 156 mm, the tip ≈ 0

References

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Mononobe, N. and Matsuo, H. (1929). On the determination of earth pressure during earthquakes. World Engineering Congress, Tokyo. Okabe, S. (1926). General theory of earth pressure. Journal of the Japan Society of Civil Engineering, 12(1).

Westergaard, H.M. (1933). Water pressures on dams during earthquakes. Transactions of the American Society of Civil Engineers, 98, 418–433.

NOTE

All numerical examples were produced with the module's own calculation engine; the module gives the same values. The step-by-step example on the module page is also this wall (L = 5 m, γ = 15.9; γ_{sat} = 19.33 kN/m³, φ′ = 32°, water at 2 m).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter the height of your own wall, the layered sand profile, the water, the surcharge, the sheet-pile section and the earthquake coefficients in GeoTerva; the module gives the pressure diagram, the embedment depth, the moment, the section check and the displacement together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com