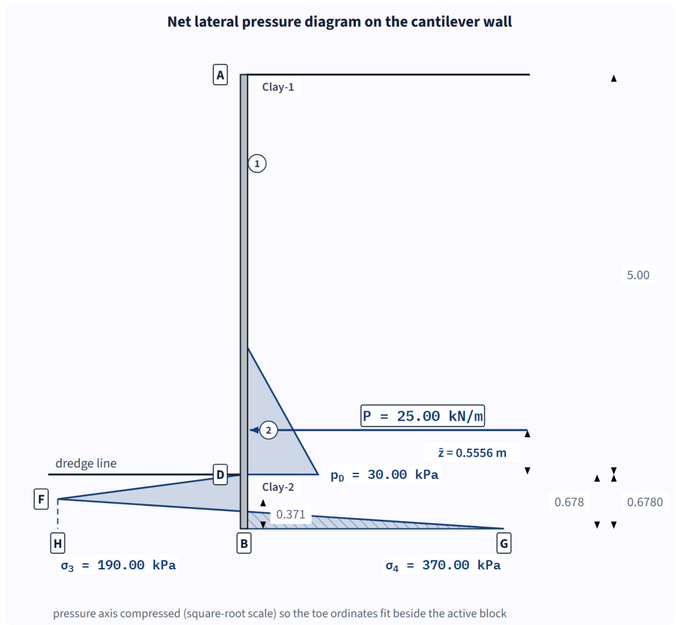
Geotechnical lecture presentation · Sheet piles and braced cuts

Cantilever sheet pile wall in clay

Lecture: design of a cantilever sheet pile wall in clay (undrained net pressure, embedment depth, moment, section, earthquake, base heave); a worked example for each topic.

- Cantilever sheet pile
- Undrained clay (φ = 0)
- Embedment depth · maximum moment
- Base heave
- Earthquake · displacement



Introduction

What will we learn in this lecture?

Why a cantilever sheet pile in clay is analysed by **undrained, total stress ($\varphi = 0$)** analysis; $K_a = K_p = 1$

The active pressure of the retained clay is $\sigma_v - 2c_u$, the **tension crack z_c** and the resultant P_1 above the excavation base

In clay, below the excavation base, a **constant net pressure $\sigma_6 = 4c_u - (\gamma L_1 + \gamma' L_2)$** and at the toe σ_7

Quadratic equation for D from horizontal and moment equilibrium; z' and M_{max}

Design depth (f, K_p/FS_p, specified depth); the effect of the **water-filled crack** and of a different water level

Sheet pile verification (EN 1993-1-1, AISC 360-22), **earthquake**, displacement and **base heave** check

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example wall: above the excavation base L = 5 m of clay (γ = 18 kN/m³, c_u = 30 kPa), below it stiffer clay (c_u = 70 kPa); no water, no surcharge. In the hand solution the tension crack is dry; in the module default it is water-filled. Forces are per metre of wall.

How does it work, how does it rotate?

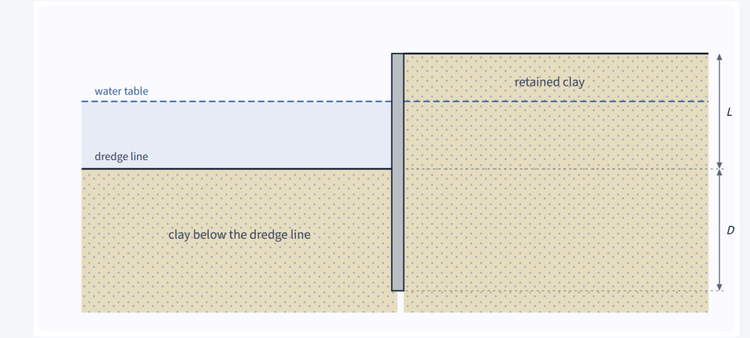
THEORY

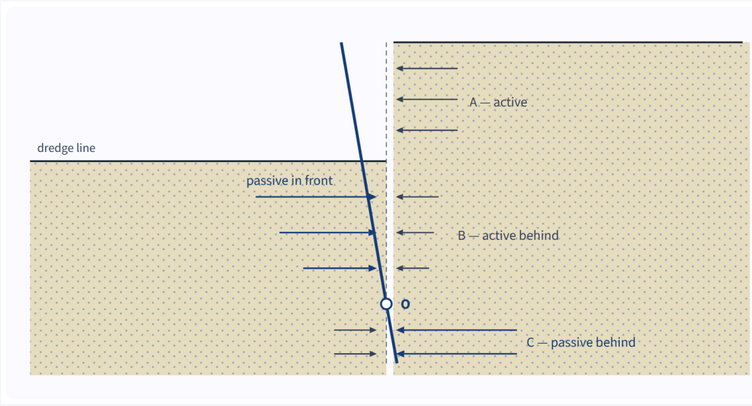
The sheet pile wall retains the soil above the excavation base (L) and stays standing by means of the **embedment depth D**. A **cantilever** wall has no other support; it is held only by the passive resistance of the soil into which it is driven, so it is economical up to about 6 m above the excavation base.

The wall rotates rigidly about point O (near the tip): **A** (above the excavation base) active; **B** (excavation base–O) active behind, passive in front; **C** (below O) passive behind, active in front.

The source replaces the actual (curved) distribution with **a linear idealization**: the net pressure below the excavation base and the rotational block at the tip; horizontal and moment equilibrium give the embedment depth.

In clay a rapidly loaded soil cannot drain: the analysis is performed **undrained, in total stress** ($\varphi = 0$, c_u); both Rankine coefficients equal 1. The long-term drained condition (c', φ') is not checked in this module.





$\sigma_v - 2c_u$, tension crack z_c, P_1

THEORY

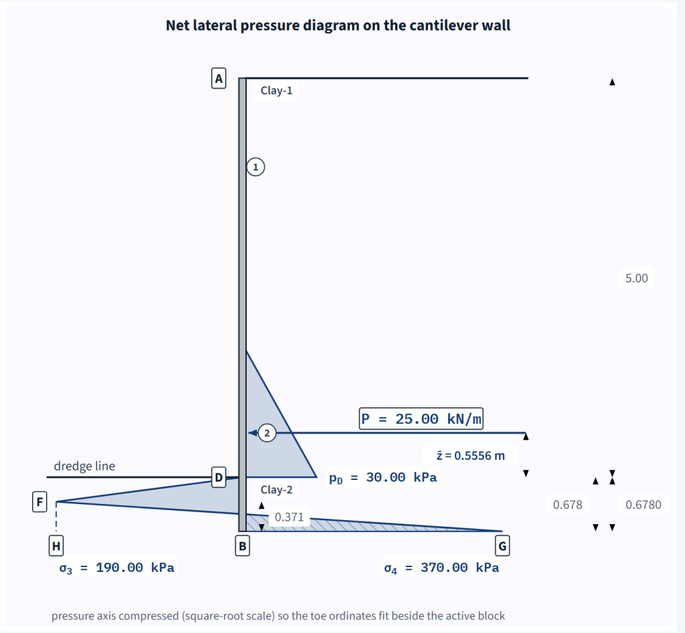
The clay retained above the excavation base pushes with its undrained active pressure: total vertical stress minus $2c_u$ ($K_a = 1$ at $\varphi = 0$):

$$\sigma_a(z) = q + \gamma z - 2c_u \qquad z_c = \frac{2c_u - q}{\gamma}$$

$$P_1 = \tfrac{1}{2}\,\sigma_a(L)\,(L - z_c) \qquad \bar{z}_1 = \frac{L - z_c}{3}$$

Near the surface σ_a is negative: the clay would pull on the wall. Such **suction is not counted**; the pressure is zero down to the depth z_c (tension crack) and increases linearly below it. In a single clay the diagram is a triangle from z_c to the excavation base; P_1 acts at $\bar{z}_1$ above the excavation base.

The crack may be filled with water (the module default): the hydrostatic water pressure along the crack, $\tfrac{1}{2}\gamma_w z_c^2$, is added to P_1; if the crack is taken as **dry** (the source equations), it is not added. Below the water table, water fills the crack to a constant level.



Net pressure diagram (opening wall with water-filled crack).

Problem statement

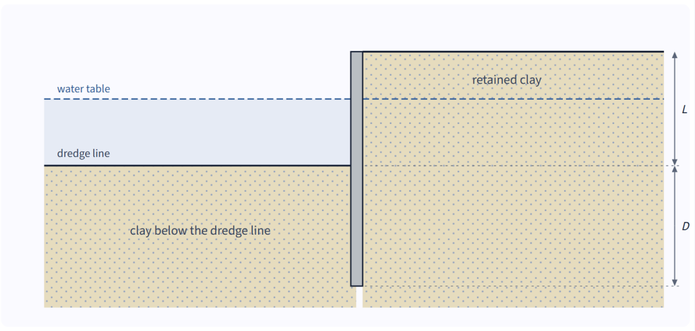
EXAMPLE

Given

Soil	**retained clay: $\gamma = 18$ kN/m³, $c_u = 30$ kPa; below it stiff clay $c_u = 70$ kPa**
Wall	**$L = 5$ m; no water, no surcharge; tension crack dry (equations of the source)**

REQUIRED

z_c, $\sigma_a(L)$, P_1 and its lever arm **$\bar{z}_1$** (from the excavation base).



Solution

SOLUTION

1 Tension crack depth and active pressure at the excavation base

$$z_c = \frac{2c_u}{\gamma} = \frac{2(30)}{18} = \mathbf{3.333}\ \text{m}$$

$$\sigma_a(L) = \gamma L - 2c_u = 18(5) - 60 = \mathbf{30}\ \text{kPa}$$

2 Resultant of the triangular diagram and its arm

$$P_1 = \tfrac{1}{2}\,\sigma_a(L)(L - z_c) = \tfrac{1}{2}(30)(1.667) = \mathbf{25.00}\ \text{kN/m}$$

$$\bar{z}_1 = \frac{L - z_c}{3} = \frac{1.667}{3} = \mathbf{0.5556}\ \text{m}\ \ (\text{above the excavation base})$$

In the first **3.33 m** of depth the clay gives no active pressure (crack); at the excavation base σ_a reaches 30 kPa. **P_1 = 25.0 kN/m**, 0.556 m above the excavation base.

Check the solution in GeoTerva ↗

Constant net pressure σ_6 and σ_7 at the tip

THEORY

Clay loaded rapidly, as by a wall, cannot drain: φ = 0, coefficients equal 1, and the lateral pressures at the surface are the vertical total stress ∓ 2c_u:

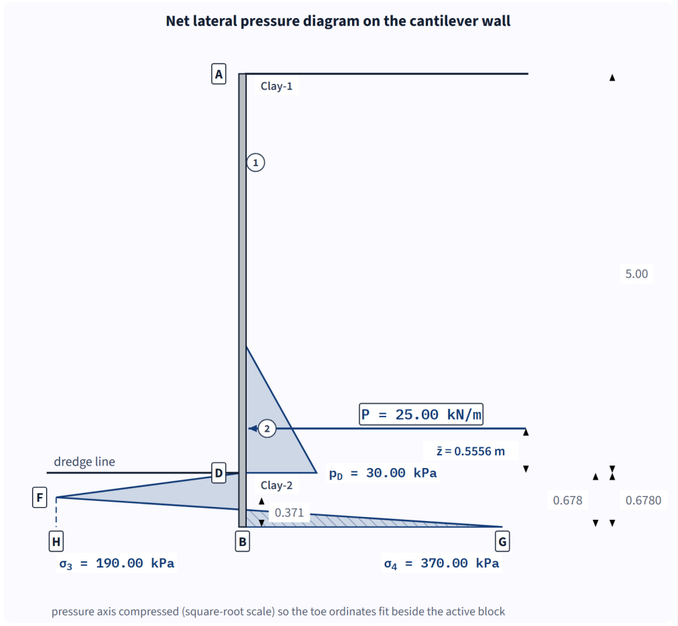
$$\sigma_a = \sigma_v - 2c_u \qquad \sigma_p = \sigma_v + 2c_u$$

Just below the excavation base the retained side pushes with the load it carries ($\gamma L_1 + \gamma' L_2$) − 2c_u; the excavation face, which as yet carries no load, resists with 2c_u. Below, both faces gain the same load and **the difference does not change**: the net pressure is constant and directed toward the retained soil. In the turning zone the sides swap:

$$\sigma_6 = 4c_u - (\gamma L_1 + \gamma' L_2) \qquad \sigma_7 = 4c_u + (\gamma L_1 + \gamma' L_2)$$

> σ_6 must be positive: a clay wall that does not exceed the load at the excavation base of 4c_u cannot be retained, and the module calculation stops.

> The zero point is E, the excavation base itself; below the excavation base the net pressure is constant at σ_6 and at the tip returns to σ_7 with a block of height L_4.



Problem statement

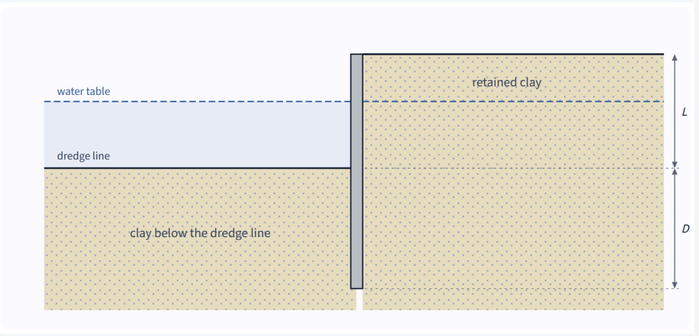
EXAMPLE

Given

Given	**Example 1:** $\gamma = 18$ **kN/m³,** $L = 5$ **m (no water):** $\gamma L = 90$ **kPa; in the embedded clay** $c_u = 70$ **kPa**

REQUIRED

The constant net pressure $\boldsymbol{\sigma_6}$ below the excavation base and $\boldsymbol{\sigma_7}$ at the tip; is σ_6 positive?



Solution

SOLUTION

1 Net pressure below the excavation base (toward the retained side)

$$\sigma_6 = 4c_u - (\gamma L_1 + \gamma' L_2) = 4(70) - 90 = \mathbf{190}\ \mathrm{kPa}\ > 0\ \checkmark$$

2 End rotation ordinate

$$\sigma_7 = 4c_u + (\gamma L_1 + \gamma' L_2) = 4(70) + 90 = \mathbf{370}\ \mathrm{kPa}$$

Net pressure σ_6 = **190 kPa**, positive: the stiff clay can retain the wall. At the tip the pressure reverses to σ_7 = 370 kPa. Below the excavation base the same net pressure of **190 kPa** acts at every depth (constant diagram).

Check the solution in GeoTerva ↗

L_4 from horizontal equilibrium, quadratic equation for D

THEORY

Above the excavation base the resultant is P_1 (arm $\bar{z}_1$); below it, σ_6 acts over the entire embedment (D) and the tip block (height L_4) turns the pressure back to σ_7. Horizontal equilibrium gives L_4:

$$P_1 - \sigma_6 D + \tfrac{1}{2} L_4(\sigma_6 + \sigma_7) = 0 \;\Rightarrow\; L_4 = \frac{\sigma_6 D - P_1}{4c_u}$$

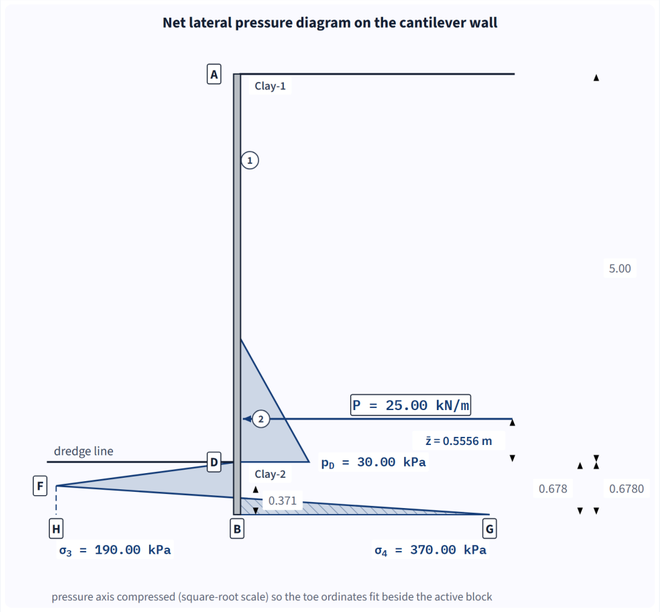
Placing L_4 from the moment equilibrium about the tip leaves a **quadratic** equation for D:

$$P_1(D + \bar{z}_1) - \frac{\sigma_6 D^2}{2} + \frac{4c_u}{3} L_4^2 = 0$$

$$\sigma_6(\sigma_6 - 6c_u)D^2 + P_1(12c_u - 2\sigma_6)D + P_1(P_1 + 12c_u\bar{z}_1) = 0$$

$$z' = \frac{P_1}{\sigma_6} \qquad M_{max} = P_1(z' + \bar{z}_1) - \frac{\sigma_6 z'^2}{2}$$

Shear is zero at the depth z′ (below the excavation base) where the constant net pressure removes P_1; the moment is largest here. $D_{theoretical}$ is the positive root of this equation; the passive resistance has fully mobilized, with no reserve.



Problem statement

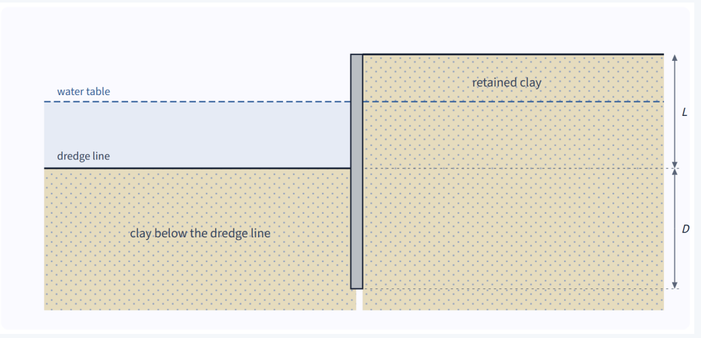
EXAMPLE

Given

Given	**Example 1–2:** $P_1 = 25.00$ **kN/m,** $\bar{z}_1 = 0.5556$ **m;** $\sigma_6 = 190$ **kPa,** $\sigma_7 = 370$ **kPa;** $c_u = 70$ **kPa**

REQUIRED

Coefficients of the equation; $D_{theoretical}$; height of the tip block L_4.



Solution

SOLUTION

1 Coefficients

$$a = \sigma_6(\sigma_6 - 6c_u) = 190(190 - 420) = -43{,}700$$

$$b = P_1(12c_u - 2\sigma_6) = 25(840 - 380) = 11{,}500$$

$$c = P_1(P_1 + 12c_u\bar{z}_1) = 25(25 + 840(0.5556)) = 12{,}291.7$$

2 Positive root and end block

$$-43{,}700D^2 + 11{,}500D + 12{,}291.7 = 0 \;\Rightarrow\; D_{theoretical} = \mathbf{0.678}\ \mathrm{m}$$

$$L_4 = \frac{\sigma_6 D - P_1}{4c_u} = \frac{190(0.678) - 25}{280} = \mathbf{0.3708}\ \mathrm{m}$$

The theoretical embedment depth is only **D = 0.678 m**: since the embedded clay is stiff (c_u = 70 kPa) and the crack is dry, very little embedment below the excavation base is sufficient. The end block is 0.371 m.

Check the solution in GeoTerva ↗

Problem statement

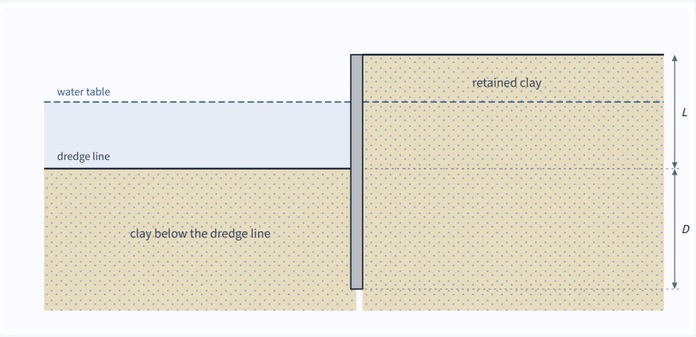
EXAMPLE

Given

Given	**Example 1–3:** $P_1 = 25.00$ **kN/m,** $\bar{z}_1 = 0.5556$ **m,** $\sigma_6 = 190$ **kPa**

REQUIRED

The **z′** and $\mathbf{M_{max}}$ at which the shear is zero.



Solution

SOLUTION

1 Depth at which shear is zero (from the excavation base)

$$z' = \frac{P_1}{\sigma_6} = \frac{25}{190} = \mathbf{0.1316}\ \mathrm{m}$$

2 Maximum moment

$$M_{max} = P_1(z' + \bar{z}_1) - \frac{\sigma_6 z'^2}{2} = 25(0.1316 + 0.5556) - 95(0.1316)^2 = \mathbf{15.53}\ \mathrm{kN{\cdot}m/m}$$

M_{max} = 15.53 kN·m/m, only 0.13 m below the excavation base. Since the net pressure is constant, the shear force decreases linearly along z′, and the moment reaches its maximum value very close to the excavation base.

Check the solution in GeoTerva ↗

Ways to add a margin of safety

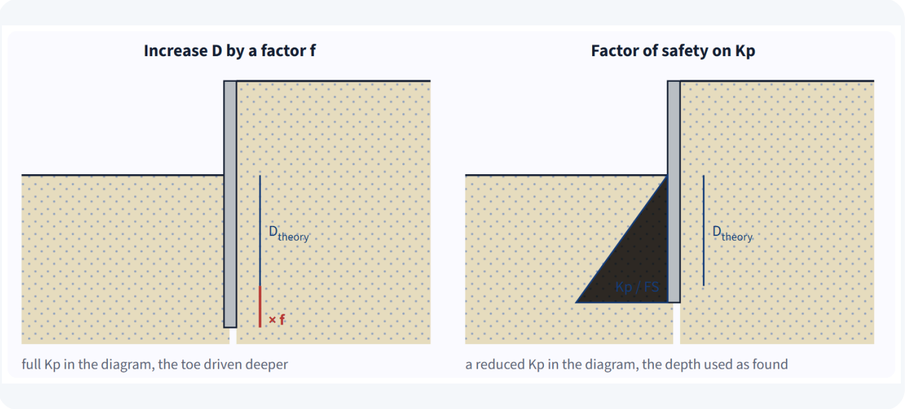
THEORY

There is no allowance at the theoretical depth. Two approaches (figure):

Increasing the depth: $D_{design} = f \cdot D_{theoretical}$; for clay f = 1.4–1.6 is taken (the module default is 1.5). The pressures and moment are those of the theoretical wall.

Factor of safety on passive pressure: the diagram is set up from the start with σ_p/FS_p ($\sigma_p = \sigma_v + 2c_u$); the resulting depth is used as is, and **the moment increases**.

Given depth: checked against the required depth ($D_{given} \geq D_{design,required}$); with $\sigma_p/FS_{p,mob}$ the diagram is fully balanced at the given depth. Internal forces are studied for both cases, the sheet pile is selected for the **larger moment**; the moment of the given depth may exceed that of the required depth.



Problem statement

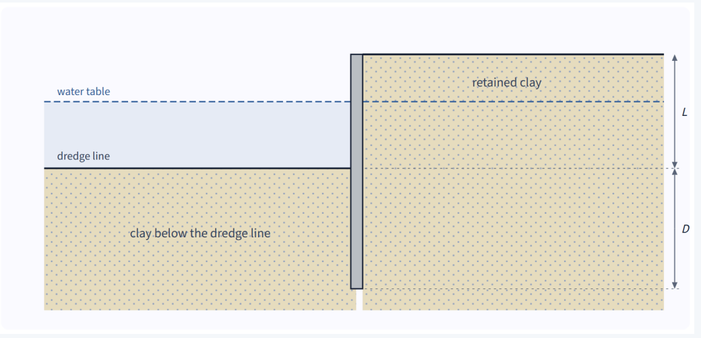
EXAMPLE

Given

Wall	**wall opening with a water-filled tension crack (module default): $L = 5$ m, $c_u = 30/70$ kPa; if the crack is dry, D = 0.678 m (Example 3)**
Roads	**$f = 1.5$; $FS_p = 1.5$; given $D = 5.0$ m; sheet pile S275, $W_{el} = 1\,400$ cm³/m ($\gamma_F = 1.35$)**

REQUIRED

D_{design} of each approach, total length, **M_{max}** and sheet-pile utilization ratio **η**.



Solution

SOLUTION

1 $D_{design} = f \cdot D_{theoretical}$ (moment unchanged); the diagram is reconstructed along the K_p path

Road	D_{design} (m)	L + D (m)	M_{max} (kN·m/m)	η (EN 1993)	Note
Crack dry, f = 1.5 (Example 3)	1.017	6.02	15.53	0.054	$D_{theoretical}$ = 0.678 m
Water-filled crack, f = 1.5	3.433	8.43	181.91	0.638	$D_{theoretical}$ = 2.288 m
Water-filled, FS_p = 1.5	2.775	7.77	187.51	0.657	depth and moment increase
Water-filled, given D = 5.0 m	5.000	10.00	209.71	0.735	$FS_{p,mob}$ = 5.19

A water-filled crack introduces a much larger load: P_1 25.0 → 79.5 kN/m, $D_{theoretical}$ 0.68 → 2.29 m, M_{max} 15.5 → 181.9 kN·m/m (Example 6). The given D = 5.0 m is longer than the design depth (3.43 m) and mobilizes only 1/5.2 of the passive resistance; however, the required-depth condition yields a moment (209.7 kN·m/m) **larger** than that moment (181.9): the sheet pile is selected according to the larger one.

Check the solution in GeoTerva ↗

Water in the tension crack and different water levels

THEORY

Water-filled crack (the module default): the hydrostatic pressure along the crack $P_w = \frac{1}{2}\gamma_w z_c^2$ is added to the active pressure (its moment arm at the lower third of the crack); total $P_1 = P_w + \frac{1}{2}\sigma_a(L)(L - z_c)$. If the crack is set to **dry**, the equations of the source.

Same water level on both sides: hydrostatic pressures above the excavation base cancel; below the water table the submerged unit weight γ′ of the clay is used, and without a water table the natural γ applies everywhere.

Different levels (beyond the source): net water pressure above the excavation base is added to the active pressure (no seepage). Since the clay is analyzed in terms of total stress ($\gamma L_1 + \gamma' L_2$), $\Delta q = q_r - q_e$ is used instead, the difference between the total vertical stresses on the two sides; water levels within the clay do not enter the undrained analysis.

$$u_{net}(z) = \gamma_w(z - D_w) - \gamma_w(z - D_{w,exc}) \quad (\text{each term} \geq 0)$$

$$\sigma_6 = 4c_u - \Delta q \qquad \Delta q = q_r - q_e$$

$$P_1 = \tfrac{1}{2}\gamma_w z_c^2 + \tfrac{1}{2}\sigma_a(L)(L - z_c)$$

Problem statement

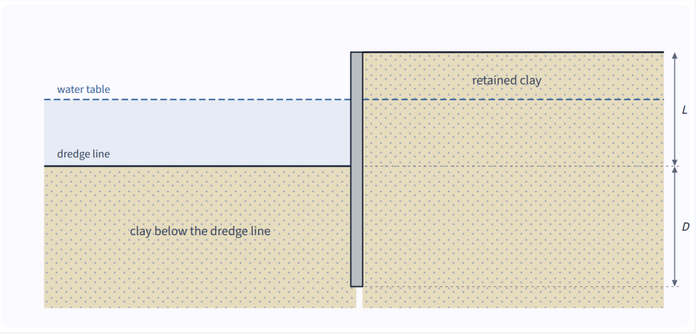
EXAMPLE

Given

Wall	**Wall from Examples 1–5 ($L = 5$ m, $\gamma = 18$, $c_u = 30/70$ kPa)**
Cases	**(a) crack dry; (b) crack filled with water (default); (c) water at 2 m behind, at 6 m on the excavation side (net water pressure)**

REQUIRED

For (b), $\mathbf{P_1}$ and $\mathbf{\bar{z}_1}$ (by hand); the $\mathbf{D_{theoretical}}$ and $\mathbf{M_{max}}$ values for the three cases.



Solution

SOLUTION

1 (b) Water pressure in the crack is added (crack depth z_c = 3.333 m)

$$P_w = \tfrac{1}{2}\gamma_w z_c^2 = \tfrac{1}{2}(9.81)(3.333)^2 = 54.50\ \text{kN/m} \quad (\text{arm } 5 - \tfrac{2}{3}z_c = 2.778\ \text{m})$$

$$P_1 = 54.50 + 25.00 = \mathbf{79.50}\ \text{kN/m} \qquad \bar{z}_1 = \frac{54.50(2.778) + 25.00(0.5556)}{79.50} = \mathbf{2.0790}\ \text{m}$$

2 Summary of the three cases (same equations; for D and M the quadratic equation is solved)

Condition	P_1 (kN/m)	$\bar{z}_1$ (m)	σ_6 (kPa)	$D_{theoretical}$ (m)	M_{max} (kN·m/m)
(a) Crack dry	25.00	0.556	190	0.678	15.53
(b) Crack filled with water	79.50	2.079	190	2.288	181.91
(c) Different water level	115.70	1.754	187	2.765	238.79

A water-filled crack raises P_1 from 25.0 to **79.5 kN/m**: the embedment goes from 0.68 to **2.29 m**, the moment from 15.5 to **181.9 kN·m/m**. A different water level (net water pressure) gives P_1 = 115.7 kN/m, D = 2.77 m, M = 238.8 kN·m/m. Ignoring the water in the tension crack falls on the unsafe side.

Check the solution in GeoTerva ↗

EN 1993-1-1 and AISC 360-22

THEORY

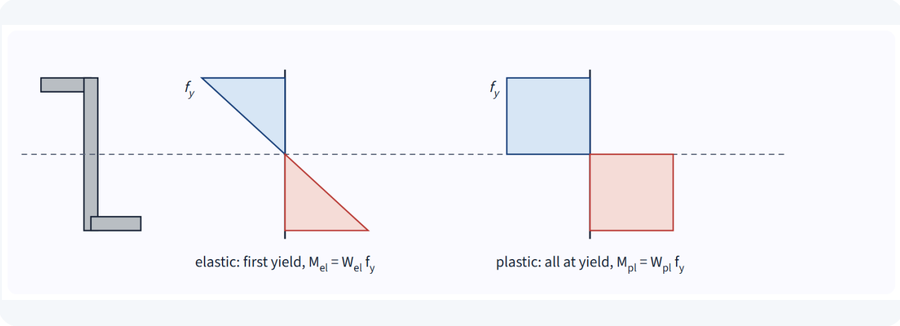
Classical route: S = M_{max}/σ_{allow}. The module performs a limit-state verification: the moment of the net pressure diagram is a characteristic action; the partial factor raises it to the design value:

$$M_{Ed} = \gamma_F M_{max} \quad W_{req} = \frac{M_{Ed}\gamma_{M0}}{f_y} \quad M_{c,Rd} = \frac{W f_y}{\gamma_{M0}} \quad \eta = \frac{M_{Ed}}{M_{c,Rd}} \le 1$$

The table gives **W_{el}** (first yield, up to class 3) and **W_{pl}** (fully plastic, class 1–2). The embedment depth contribution is not multiplied separately by the steel code.

$$\text{LRFD: } M_u = \gamma_H M_{max} \le \phi_b M_n,\ \phi_b = 0.90 \qquad \text{ASD: } M_a = M_{max} \le \frac{M_n}{\Omega_b},\ \Omega_b = 1.67$$

$M_n = F_y Z$ (compact) or $F_y S$ at first yield; the soil pressure is taken as the lateral earth pressure H: γ_H = 1.6 in LRFD, 1.0 in ASD.



Problem statement

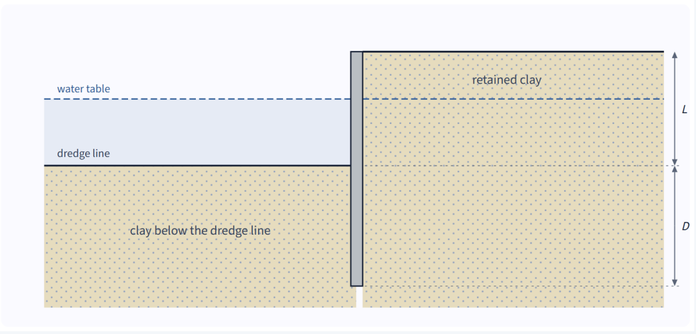
EXAMPLE

Given

Section	**S275 ($f_y = 275$ MPa); $W_{el} = 1\,400$ cm³/m**
Moment	**Wall with water-filled crack: $M_{max} = 181.91$ kN·m/m; EN: $\gamma_F = 1.35$, $\gamma_{M0} = 1.00$; AISC: $\gamma_H = 1.6$, $\Omega_b = 1.67$ ($M_n = F_y S$)**

REQUIRED

For EN 1993-1-1: **M_{Ed}**, **W_{req}**, $M_{c,Rd}$, **η**; for AISC LRFD and ASD: η.



Solution (1/2)

SOLUTION

1 EN 1993-1-1

$$M_{Ed} = 1.35(181.91) = \mathbf{245.58}\ \mathrm{kN\cdot m/m}$$

$$W_{req} = \frac{M_{Ed}\gamma_{M0}}{f_y} = \frac{245.58(1.00)}{275\,000} = \mathbf{893.0}\ \mathrm{cm^3/m}$$

$$M_{c,Rd} = 1\,400 \cdot 10^{-6}(275\,000) = 385.00\ \mathrm{kN\cdot m/m} \qquad \eta = \frac{245.58}{385.00} = \mathbf{0.638}\ \leq 1\ \checkmark$$

The selected sheet pile is adequate: η = **0.638**; required W = 893 cm³/m < 1 400 cm³/m.

Solution (2/2)

SOLUTION

1 AISC 360-22: $M_n = F_y S = 275\,000\ (1\,400 \cdot 10^{-6}) = 385.0$ kN·m/m

$$\text{LRFD: } M_u = 1.6(181.91) = 291.06\ \text{kN·m/m} \leq \phi_b M_n = 0.90(385.0) = 346.5 \quad \eta = \mathbf{0.840}$$

$$\text{ASD: } M_a = 181.91\ \text{kN·m/m} \leq \frac{M_n}{\Omega_b} = \frac{385.0}{1.67} = 230.5 \quad \eta = \mathbf{0.789}$$

Adequate also with AISC: LRFD η = **0.840**, ASD η = **0.789**; in EN 1993-1-1, 0.638. The ratios differ because the safety philosophies of the codes differ.

Check the solution in GeoTerva ↗

Quasi-static undrained wedge

THEORY

The earthquake preserves the static limit-equilibrium framework by adding constant inertial forces ($k_h W$ horizontal, $k_v W$ vertical) to the soil wedges. Mononobe–Okabe cannot be applied in clay (φ = 0); the module takes an **undrained wedge with a 45° critical plane** (GeoTerva assumption):

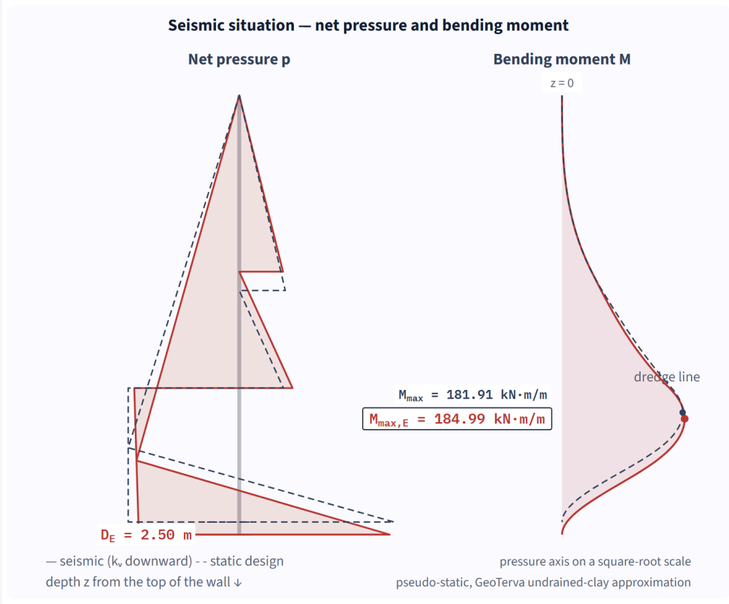
$$\tan\theta = \frac{k_h}{1 \mp k_v} \qquad K_{AE}^{c} = 1 + \tan\theta \qquad K_{PE}^{c} = 1 - \tan\theta$$

$$\text{pressure: } (1 \mp k_v)\,(1 \pm \tan\theta)\,\sigma_v \;\mp\; 2c_u = [(1 \mp k_v) \pm k_h]\,\sigma_v \;\mp\; 2c_u$$

The pore water in clay always moves together with the soil (restricted); free water occurs only in highly permeable soil. **Both directions of k_v** are checked.

Check: the net pressure diagram is constructed as in statics; the two equilibrium equations give D_E. An earthquake is a condition attached to failure, so no factor is applied to the passive side: the wall holds if $D_E \le D_{design}$. In the cantilever, seismic pressures are distributed in proportion to effective stress, as in statics.

In the seismic combination, effects are taken with 1.0: $M_{Ed,E} = M_{max,E}$.



Problem statement

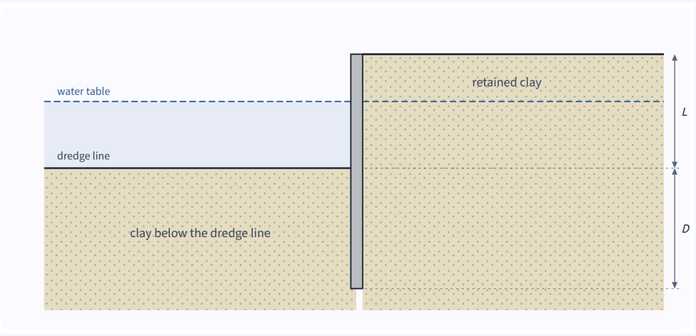
EXAMPLE

Given

Wall	**Example 6(b): wall with water-filled crack,** $D_{design} = 3.43$ **m,** $M_{max} = 181.91$ **kN·m/m;** $W_{el} = 1\,400$ **cm³/m**
Earthquake	$k_h = 0.15$, $k_v = \pm 0.075$ **(two directions)**

REQUIRED

Seismic coefficients; $\mathbf{D_E}$ (two k_V directions), $\mathbf{M_{max,E}}$; can the static wall withstand this earthquake?



Solution

SOLUTION

1 Seismic coefficients (static: active = passive = 1) and results for both directions

k_v	f_v	tan θ	active: $f_v + k_h$	passive: $f_v - k_h$	D_E (m)	$M_{max,E}$ (kN·m/m)
+0.075	0.925	0.1622	1.075	0.775	2.391	181.39
−0.075	1.075	0.1395	1.225	0.925	**2.503**	184.99

2 Check and section

$$D_E = 2.50\ \text{m} \ \leq\ D_{design} = 3.43\ \text{m}\ \checkmark$$

$$M_{Ed,E} = M_{max,E} = 184.99\ \text{kN·m/m} \qquad \eta_E = \frac{184.99}{385.00} = \mathbf{0.481}\ \leq 1\ \checkmark$$

The static design **resists** this earthquake: the seismic diagram requires only **2.50 m** of embedment (design 3.43 m); the moment is 185.0 kN·m/m (static 181.9), nearly the same, and the sheet pile is adequate. Since the net pressure in clay is constant, the earthquake changes the diagram very little.

Check the solution in GeoTerva ↗

Modulus of subgrade reaction method

THEORY

Limit equilibrium gives the embedment depth and the moment but not **how much the wall displaces**. For displacement in the working condition, the wall is an elastic beam (EI) and the soil on each face consists of independent springs (Winkler):

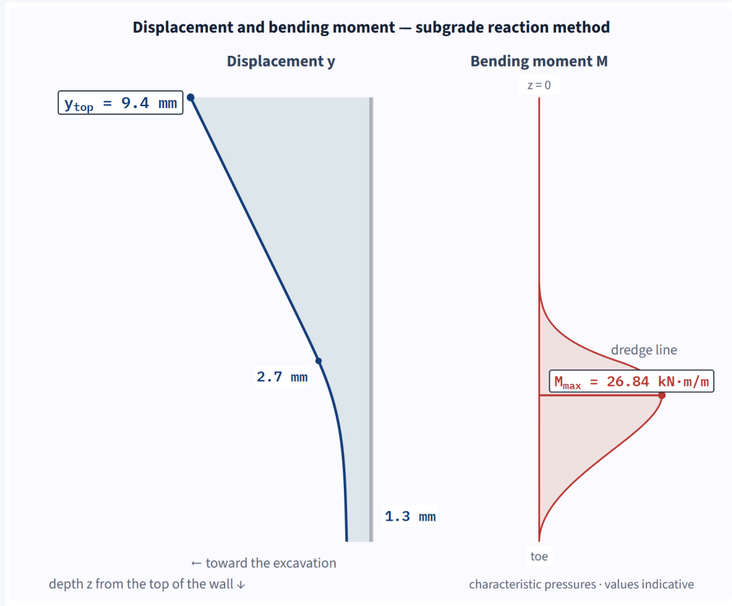
$$EI\frac{d^4y}{dz^4} = p_R(z) - p_E(z) + u_{net}(z)$$

$$p_R = K_0\sigma_{v,r} - k_h y \;\; (\sigma_a \le p_R \le \sigma_p) \qquad p_E = K_0\sigma_{v,e} + k_h y \;\; (\ldots) \qquad K_0 = 1 - \sin\varphi'$$

Springs start from the at-rest pressure, vary in proportion to displacement, and do not exceed the active and passive limits. k_h is a **model parameter** rather than a soil property. Since φ = 0 in clay, K_0 = 1.

The wall is at design length (L + D_{design}), free at both ends; the pressures are characteristic, with no factor of safety. The excavation is in a single stage, the wall is in place beforehand.

The solution is performed with finite elements, iterating until the springs reach the elastic/active/passive state; the result is given as an **unbounded estimate**.



Problem statement

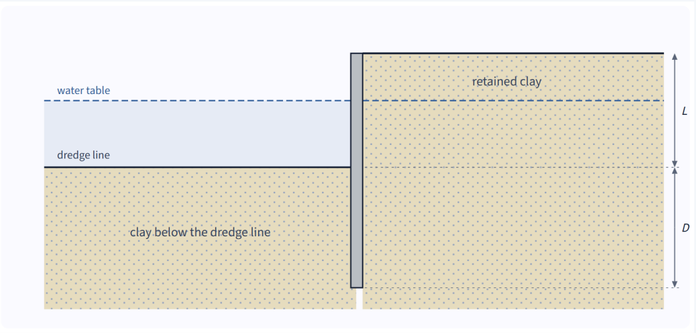
EXAMPLE

Given

Wall	**Example 6(b): design length $L + D = 8.43$ m**
Sheet pile	$I = 21\,000$ **cm⁴/m,** $E = 210\,000$ **MPa (**$EI = 44\,100$ **kN·m²/m)**
Soil	$k_h = 15$ **MN/m³ (upper clay),** 30 **MN/m³ (lower clay)**

REQUIRED

Displacement at the wall top, at the excavation base and at the tip; does the model's maximum moment equal the limit-equilibrium moment?



Solution

SOLUTION

1 Model

$$EI = 210\,000\,000(21\,000 \cdot 10^{-8}) = 44{,}100\ \mathrm{kN{\cdot}m^2/m} \qquad K_0 = 1 - \sin 0^\circ = 1$$

$$EI\,y^{IV} = p_R - p_E \ \ (\text{finite element, springs elastic/active/passive})$$

2 Result (toward the excavation)

$$y_{üst} = \mathbf{9.4}\ \mathrm{mm} \qquad y_{excavation\ base} = \mathbf{2.7}\ \mathrm{mm} \qquad y_{tip} = 1.3\ \mathrm{mm}$$

$$M_{max}^{model} = 26.84\ \mathrm{kN{\cdot}m/m}\ \ (z = 5.65\ \mathrm{m}) \qquad M_{max}^{limit\ equilibrium} = 181.91$$

The top of the wall moves toward the excavation by **9.4 mm**, the excavation base by 2.7 mm and the tip by 1.3 mm. The moment of the working-state model (26.8 kN·m/m) is **much smaller** than the limit-equilibrium moment (181.9): the springs use only 48% of the passive limit; limit equilibrium describes the moment of wall failure, and that is the **design moment**.

Check the solution in GeoTerva ↗

Heave of the excavation base

THEORY

However little embedment the lateral equilibrium provides, it says nothing about the excavation base. In clay the base **may heave**: the clay below the excavation is squeezed inward by the weight of the adjacent soil (reverse bearing capacity failure). The usual check for excavations compares the shear resistance on the failure surface with this weight:

$$FS = \frac{5.14\, c_u \left(1 + 0.2 \dfrac{B''}{L_{exc}}\right) + \dfrac{c_{side} H}{B'}}{\sigma_v(H) + q} \qquad B' = \min\left(T, \frac{B}{\sqrt{2}}\right),\ B'' = \sqrt{2}\, B'$$

B is the excavation width, L_{exc} the plan length (the shape term drops out for a long excavation), T the depth of the stiff layer below the base (if not entered, the surface is limited by the width alone); c_u is the clay below the base, c_{side} the average cohesion over the retained height, $\sigma_v(H)$ the total lateral stress at base level.

The embedment of the wall below its base is not counted (conservative). Surcharge and strip/point loads are added to q as equivalent uniform loads within B′. For multiple layers the formula is used with GeoTerva's layered extension.

$$FS_{req} = 1.5$$

The required factor of safety is an input; the check applies only to cohesive excavations.

Problem statement

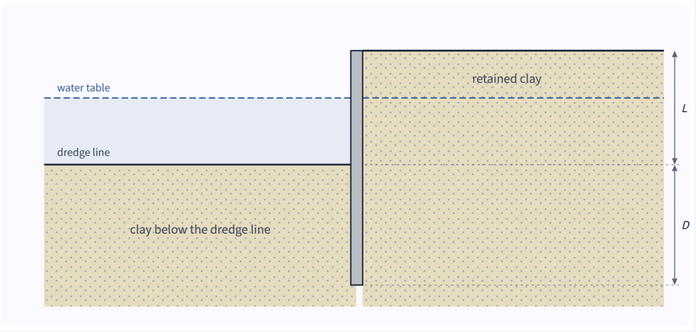
EXAMPLE

Given

Excavation	**width $B = 10$ m (long excavation, stiff layer not entered); wall as in Example 6 ($H = 5$ m)**
Soil	**clay below the base $c_u = 70$ kPa; average over the retained height $c_{side} = 30$ kPa; $\sigma_v(H) = 90$ kPa; no surcharge**

REQUIRED

B′, **FS** (≥ 1.5?); what if the excavation width were B = 5 m and 20 m?



Solution

SOLUTION

1 Effective width (L_{exc} not entered: no shape term)

$$B' = \frac{B}{\sqrt{2}} = \frac{10}{\sqrt{2}} = \mathbf{7.071}\ \mathrm{m}$$

2 Factor of safety

$$FS = \frac{5.14(70) + 30(5)/7.071}{90} = \frac{359.8 + 21.2}{90} = \mathbf{4.23}\ \geq 1.5\ \checkmark$$

$$B = 5\ \mathrm{m}:\ FS = 4.47 \qquad B = 20\ \mathrm{m}:\ FS = 4.12$$

The factor of safety against base heave is **4.23** (required 1.5): the base is safe. As the width increases the margin against lateral sliding decreases (4.12 for B = 20 m); the c_u value of the underlying clay is decisive.

Check the solution in GeoTerva ↗

Scope of the solution

THEORY

Each layer is analyzed in clay under **undrained total stress** (φ = 0): **short-term condition**. The long-term drained condition (c′, φ′) is not checked; it may be governing in permanent walls. A wall with sand fill is designed with the sand module.

Rankine pressures without wall adhesion: $K_a = K_p = 1$; active $\sigma_v - 2c_u$, passive $\sigma_v + 2c_u$; no suction above the excavation base (tension crack, by default water-filled or dry; below the water table the crack retains groundwater).

The source's simplified net pressure diagram: at each depth the pressures of the two faces, the end block from horizontal equilibrium, the depth from moments. Water the same or different on the two faces (no seepage); seepage is not modelled.

In an earthquake, clay is treated as a semi-static undrained wedge with a 45° critical plane (GeoTerva assumption). The sheet pile is verified in bending; shear, local buckling and corrosion are not checked. Displacement is given limitlessly by the modulus of subgrade reaction method.

Summary

Key points

Clay undrained ($\varphi = 0$, c_u): active $\sigma_v - 2c_u$, passive $\sigma_v + 2c_u$; suction near the surface is not counted: **tension crack $z_c = (2c_u - q)/\gamma$**; $P_1 = \frac{1}{2}\sigma_a(L)(L - z_c)$, $\bar{z}_1 = (L - z_c)/3$

Below the excavation base the net pressure is constant: $\sigma_6 = 4c_u - (\gamma L_1 + \gamma' L_2)$, at the toe $\sigma_7 = 4c_u + (\ldots)$; if $\sigma_6 > 0$ does not hold, the clay cannot support the wall

Equilibrium: $L_4 = (\sigma_6 D - P_1)/(4c_u)$; quadratic equation for D (example: 0.678 m; 2.288 m with water-filled crack); $z' = P_1/\sigma_6$, $M_{max} = P_1(z' + \bar{z}_1) - \sigma_6 z'^2/2$

A **water-filled crack** greatly increases P_1, D and M (25 → 79.5 kN/m; 15.5 → 181.9 kN·m/m); assuming it dry is unsafe

Design depth: f = 1.4–1.6 (moment unchanged) or σ_p/FS_p; the larger moment of the two cases at the given depth

Section: EN 1993-1-1 η = 0.638; AISC LRFD/ASD 0.840 / 0.789. **Earthquake**: $D_E = 2.50$ m ≤ 3.43 m; by the modulus of subgrade reaction method, the peak ≈ 9 mm

Base heave: $FS = (5.14c_u(1 + 0.2B''/L_{exc}) + c_{side}H/B')/(\sigma_v(H) + q)$; in the example 4.23 ≥ 1.5

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Bowles, J.E. (1996). Foundation Analysis and Design, 5th ed. McGraw-Hill.

Jaky, J. (1944). The coefficient of earth pressure at rest. Journal of the Society of Hungarian Architects and Engineers, 7, 355–358.

Mononobe, N. and Matsuo, H. (1929). On the determination of earth pressure during earthquakes. World Engineering Congress, Tokyo. Okabe, S. (1926). General theory of earth pressure. Journal of the Japan Society of Civil Engineering, 12(1).

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NOTE

All of the numerical examples have been produced with the module's own calculation engine; the module gives the same values. The worked example on the module page is also this wall (L = 5 m, γ = 18 kN/m³, c_u = 30 / 70 kPa).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own wall height, layered clay profile, water, tension crack, sheet pile section and earthquake coefficients in GeoTerva; the module gives the pressure diagram, embedment depth, moment, section check, displacement and base heave together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com