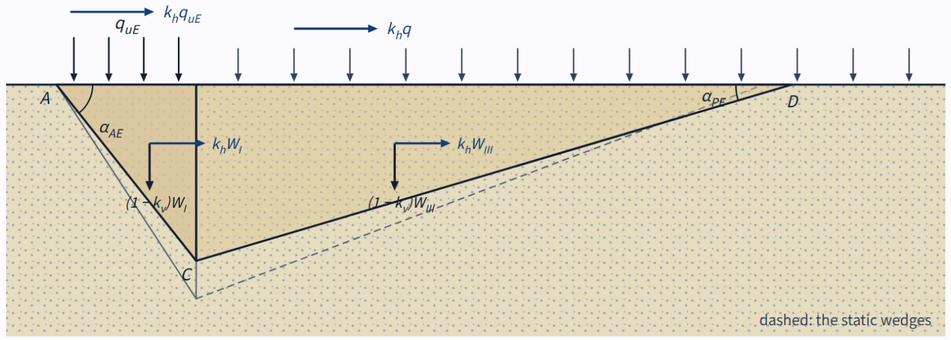
GeoTerva

presentation · Foundations

Bearing capacity of a strip footing during an earthquake

Lecture: from Richards, Elms and Budhu's two Coulomb wedges to static bearing capacity coefficients, to Mononobe–Okabe coefficients in earthquakes and how much the earthquake reduces them; a worked example on each topic.

- Two Coulomb wedges
- $K_A \cdot K_P \cdot \alpha_A$
- Mononobe–Okabe
- $\theta = \tan^{-1} k_h/(1-k_v)$
- Effect of the earthquake
- Worked examples



Based on the GeoTerva Theory page · geoterva.com

Introduction

What will we learn in this lecture?

- To set up the bearing capacity mechanism of a continuous footing **with two Coulomb wedges** (active ABC, passive BCD)
- To obtain the values of $\mathbf{K_A}$, $\mathbf{K_P}$, $\boldsymbol{\alpha_A}$, $\boldsymbol{\alpha_P}$ and from them the coefficients $\mathbf{N_q}$, $\mathbf{N_\gamma}$, $\mathbf{N_c}$ using closed-form expressions
- Treating the earthquake as a **pseudo-static** force: $\theta = \tan^{-1}[k_h/(1 - k_v)]$ and the **Mononobe–Okabe** coefficients K_{AE}, K_{PE}
- Computing the **earthquake bearing capacity $\mathbf{q_{uE}}$** and its ratio to the static value
- How to interpret the sensitivity of k_h and k_v and the **$\boldsymbol{\theta \geq \varphi}$ limit**
- To take the allowable pressure from q_{uE} and **check** the footing

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example foundation, the model opened in the module: a 1.5 m wide strip footing, D_f = 1 m, c = 30 kPa, φ' = 25°, γ = 17 kN/m³, k_h = 0.25.

Two Coulomb wedges instead of a log spiral

THEORY

Richards, Elms and Budhu (1993) proposed a theory for a continuous foundation that **can also be applied to earthquake loading**. Instead of Terzaghi's or Meyerhof's log spiral, it uses a simple failure surface made up of **two Coulomb wedges**; hence each coefficient is obtained **in closed form**.

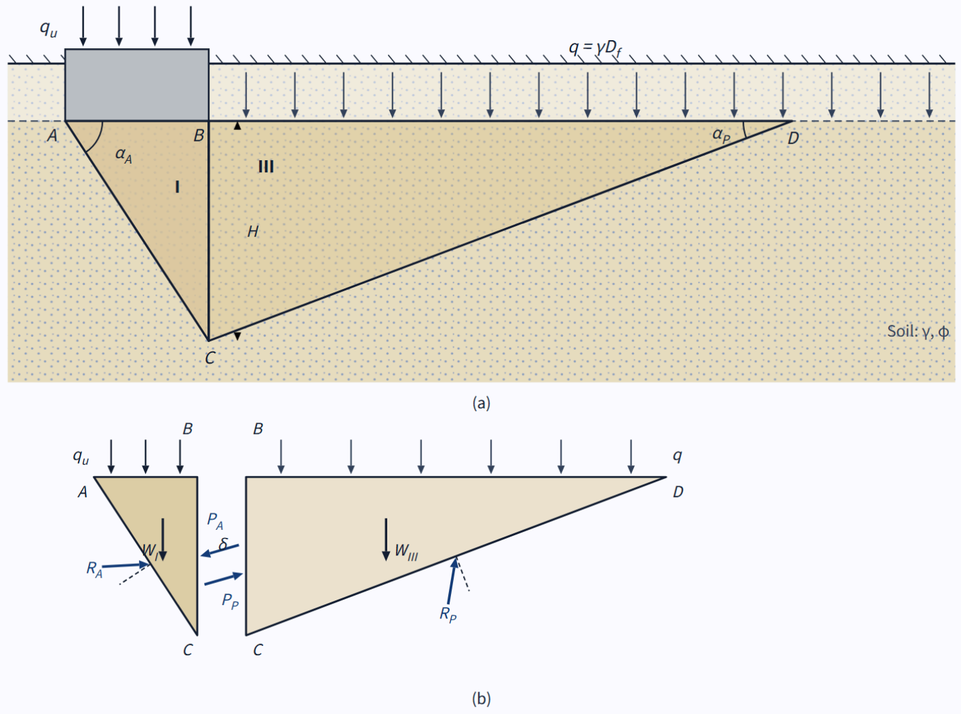
Continuous footing, width B, depth D_f; the soil above the base acts only as a surcharge $q = \gamma D_f$.

c–φ soil, $\varphi > 0$; no groundwater in the failure zone.

Earthquake **pseudo-static body force**: horizontal and vertical coefficients k_h, k_v act on the wedges and loads.

Wall friction δ = φ/2 on the vertical interface between the two wedges (the authors' recommendation).

The **active** wedge (ABC) below the footing and the **passive** wedge (BCD) alongside it; the two meet on face BC (height H).



Wedge angles and Coulomb coefficients

THEORY

The wedge inclinations αA and αP are the angles of the **critical Coulomb wedges** (measured from the horizontal):

$$\alpha_A = \varphi + \tan^{-1}\left\{\frac{[\tan\varphi(\tan\varphi+\cot\varphi)(1+\tan\delta\cot\varphi)]^{0.5}-\tan\varphi}{1+\tan\delta(\tan\varphi+\cot\varphi)}\right\}$$

$$\alpha_P = -\varphi + \tan^{-1}\left\{\frac{[\tan\varphi(\tan\varphi+\cot\varphi)(1+\tan\delta\cot\varphi)]^{0.5}+\tan\varphi}{1+\tan\delta(\tan\varphi+\cot\varphi)}\right\}$$

The horizontal components of the Coulomb forces are written with the following coefficients:

$$K_A = \frac{\cos^2\varphi}{\cos\delta\left[1+\sqrt{\frac{\sin(\varphi+\delta)\sin\varphi}{\cos\delta}}\right]^2} \qquad K_P = \frac{\cos^2\varphi}{\cos\delta\left[1-\sqrt{\frac{\sin(\varphi+\delta)\sin\varphi}{\cos\delta}}\right]^2}$$

NOTATIONS

δ = φ/2: wall friction between the wedges

α_A: angle of the inclined plane AC to the horizontal

α_P: angle of the inclined plane CD to the horizontal

P_A, P_P: Coulomb forces on the face BC

R_A, R_P: resultants on the inclined planes

W_I, W_{III}: wedge weights

Problem statement

EXAMPLE

Given

Soil	$\varphi' = 25^\circ$ **(the model the module opens with)**
Wall friction	$\delta = \varphi/2 = 12.5^\circ$

REQUIRED

Determine the coefficients **K_A, K_P** and the wedge angles **α_A, α_P**.

Solution (1/2)

SOLUTION

1 Common root term (sin 37.5° = 0.6088; sin 25° = 0.4226; cos 12.5° = 0.9763)

$$\sqrt{\frac{\sin(\varphi+\delta)\sin\varphi}{\cos\delta}} = \sqrt{\frac{0.6088\cdot 0.4226}{0.9763}} = \mathbf{0.5133}$$

2 K_A and K_P ($\cos^2 25° = 0.8214$)

$$K_A = \frac{0.8214}{0.9763(1+0.5133)^2} = \mathbf{0.3674} \qquad K_P = \frac{0.8214}{0.9763(1-0.5133)^2} = \mathbf{3.5524}$$

Solution (2/2)

SOLUTION

3 Radical term (tan φ = 0.4663; cot φ = 2.1445; tan δ = 0.2217)

$$[\tan\varphi(\tan\varphi+\cot\varphi)(1+\tan\delta\cot\varphi)]^{0.5}=[0.4663(2.6108)(1.4754)]^{0.5}=1.3402$$

$$1+\tan\delta(\tan\varphi+\cot\varphi)=1.5788$$

4 Wedge angles

$$\alpha_A=25^\circ+\tan^{-1}\frac{1.3402-0.4663}{1.5788}=25^\circ+28.97^\circ=\mathbf{53.97}^\circ$$

$$\alpha_P=-25^\circ+\tan^{-1}\frac{1.3402+0.4663}{1.5788}=-25^\circ+48.85^\circ=\mathbf{23.85}^\circ$$

The active wedge is steep (α_A = 53.97°), the passive wedge is flat (α_P = 23.85°). Wedge height H = B tan α_A = 1.5 × 1.375 = **2.06 m**.

Check the solution in GeoTerva ↗

Horizontal equilibrium at the BC face

THEORY

Consider only the surcharge (φ ≠ 0, γ = 0, q ≠ 0); let the foundation pressure be q'_u. On the BC face the horizontal forces are in equilibrium:

$$P_A \cos\delta = P_P \cos\delta$$

With uniform surcharge each force is pressure coefficient × surcharge × H: q'_u on the active side, q on the passive side:

$$P_A \cos\delta = q'_u K_A H \qquad P_P \cos\delta = q\, K_P H$$

When the two are equated, H cancels and **N_q** becomes the ratio of the two Coulomb coefficients:

$$q'_u = q\,\frac{K_P}{K_A} = q\,N_q \qquad N_q = \frac{K_P}{K_A}$$

NEDEN ORAN?

The active wedge beneath the footing pushes the adjacent passive wedge sideways. Both sides bear against each other at the same height H and with the same δ; therefore only the **ratio** of the two soil resistances determines equilibrium.

The weight of the wedges enters through H = B tan α_A

THEORY

Now consider only the weight (φ ≠ 0, γ ≠ 0, q = 0); the bearing pressure is q''_u. On the active side the weight is superimposed like a surcharge, on the passive side only the weight acts:

$$P_A \cos\delta = q''_u H K_A + \tfrac{1}{2}\gamma H^2 K_A \qquad P_P \cos\delta = \tfrac{1}{2}\gamma H^2 K_P$$

When equated:

$$q''_u = \tfrac{1}{2}\gamma H \left(\frac{K_P}{K_A} - 1\right)$$

The height of the face comes from the active wedge, H = B tan α_A; thus **N_γ** is obtained:

$$q''_u = \tfrac{1}{2}\gamma B\, N_\gamma \qquad N_\gamma = \tan\alpha_A \left(\frac{K_P}{K_A} - 1\right)$$

COMMENT

N_γ depends on the wedge depth: the factor **tan α_A** indicates how steep the wedge is. A steep wedge (large α_A) extends deeper and the self-weight term increases.

Superposition and N_c

THEORY

Surcharge and weight terms are summed (superposition):

$$q_u = q'_u + q''_u = q\,N_q + \tfrac{1}{2}\gamma B\,N_\gamma$$

For cohesion at φ = 0, Coulomb wedge analysis gives $N_c = 6$; for brevity, the authors use the known relationship:

$$N_c = (N_q - 1)\cot\varphi \qquad q_u = c\,N_c + q\,N_q + \tfrac{1}{2}\gamma B\,N_\gamma$$

This **static** value is the bearing capacity of these two wedges; it is **different** from the Prandtl-type coefficients of Terzaghi, Meyerhof, Hansen or Vesic (φ = 25°: N_q = 9.67; Prandtl–Reissner 10.66).

The seismic bearing capacity is found by **reducing** this value; the module presents both side by side.

STATIC COEFFICIENTS (δ = φ/2)

φ′	δ	α_A	N_q	N_γ	N_c
5	2.5	42.22	1.53	0.49	6.11
10	5.0	45.19	2.37	1.38	7.77
15	7.5	48.13	3.70	3.02	10.09
20	10.0	51.06	5.90	6.06	13.46
25	12.5	53.97	9.67	11.92	18.59
30	15.0	56.86	16.51	23.76	26.86
35	17.5	59.74	29.89	49.52	41.26
40	20.0	62.60	59.03	111.96	69.16
45	22.5	65.45	133.87	290.89	132.87

Problem statement

EXAMPLE

Given

Foundation	**strip (continuous), B = 1.5 m, base depth D_f = 1.0 m**
Soil	$c = 30\ \mathrm{kPa},\ \varphi' = 25^\circ$
Unit weight	$\gamma = 17\ \mathrm{kN/m^3}$**; GWT not in the failure zone**
Previous example	$K_A = 0.3674,\ K_P = 3.5524,\ \alpha_A = 53.97^\circ$

REQUIRED

Determine the values of **N_q, N_γ, N_c** and static **q_u**.

Solution

SOLUTION

1 Bearing capacity factors (tan 53.97° = 1.3748)

$$N_q = \frac{K_P}{K_A} = \frac{3.5524}{0.3674} = \mathbf{9.67} \qquad N_c = \frac{9.67 - 1}{0.4663} = \mathbf{18.59}$$

$$N_\gamma = 1.3748(9.67 - 1) = \mathbf{11.92}$$

2 Static bearing capacity ($q = \gamma D_f$ = 17 kPa)

$$q_u = 30(18.59) + 17(9.67) + \tfrac{1}{2}(17)(1.5)(11.92)$$

$$= 557.8 + 164.4 + 152.0 = \mathbf{874.14}\ \mathrm{kPa}$$

Static bearing capacity **874.14 kPa**; cohesion term 64%, surcharge 19%, self-weight 17%. The earthquake will reduce this value (next section).

Check the solution in GeoTerva ↗

Angle θ and Mononobe–Okabe coefficients

THEORY

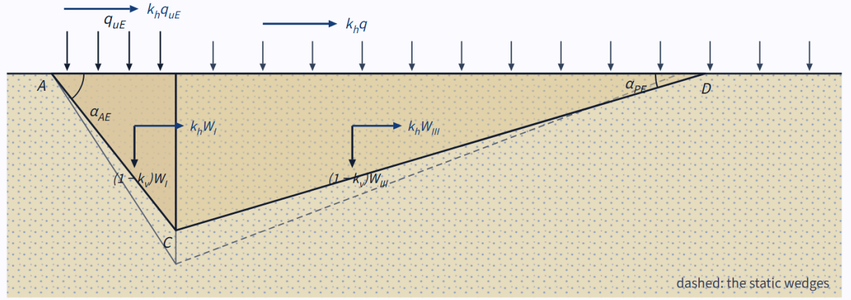
Under an earthquake the same two wedges are solved **with pseudo-static inertial forces**. The resultant of the weight and the inertial force inclines by θ from the vertical; the wedges are computed with α = φ − θ instead of φ:

$$\theta = \tan^{-1}\frac{k_h}{1-k_v} \qquad \alpha = \varphi - \theta$$

The Coulomb coefficients become **Mononobe–Okabe** coefficients:

$$K_{AE} = \frac{\cos^2(\varphi-\theta)}{\cos\theta\cos(\delta+\theta)\left[1+\sqrt{\dfrac{\sin(\varphi+\delta)\sin(\varphi-\theta)}{\cos(\delta+\theta)}}\right]^2}$$

$$K_{PE} = \frac{\cos^2(\varphi-\theta)}{\cos\theta\cos(\delta+\theta)\left[1-\sqrt{\dfrac{\sin(\varphi+\delta)\sin(\varphi-\theta)}{\cos(\delta+\theta)}}\right]^2}$$



When θ = 0, each earthquake expression reduces to its static form: $K_{AE} = K_A$, $K_{PE} = K_P$.

As the horizontal acceleration increases, α decreases: the wedges flatten, the mechanism becomes shallower.

Problem statement

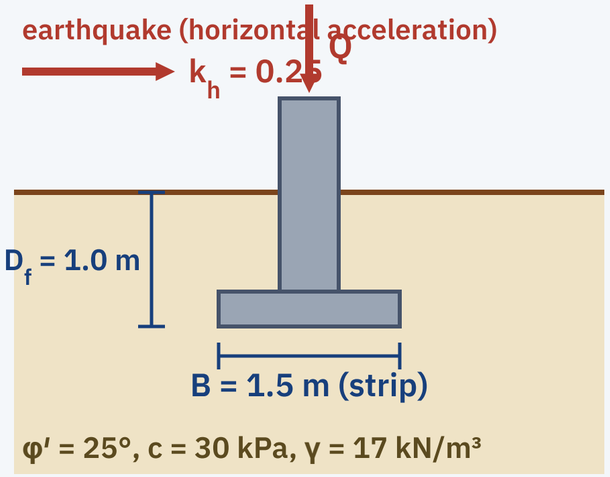
EXAMPLE

Given

Foundation	**strip (continuous), B = 1.5 m, base depth D_f = 1.0 m**
Soil	$c = 30\ \mathrm{kPa},\ \varphi' = 25^\circ$
Unit weight	$\gamma = 17\ \mathrm{kN/m^3}$**; GWT not in the failure zone**
Earthquake	**k_h = 0.25; k_v = 0**
Wall friction	$\delta = 12.5^\circ$

REQUIRED

Find **θ**, α = φ − θ and the values of **K_{AE}, K_{PE}**.



Solution

SOLUTION

1 The inclination angle and the root term (sin 37.5° = 0.6088; sin α = 0.1901; cos(δ + θ) = cos 26.54° = 0.8945)

$$\theta = \tan^{-1}\frac{0.25}{1-0} = \mathbf{14.04}^\circ \qquad \alpha = \varphi - \theta = 10.96^\circ$$

$$\sqrt{\frac{\sin(\varphi+\delta)\sin\alpha}{\cos(\delta+\theta)}} = \sqrt{\frac{0.6088\cdot 0.1902}{0.8947}} = \mathbf{0.3597}$$

2 K_{AE} and K_{PE} ($\cos^2\alpha = 0.9638$; $\cos\theta = 0.9701$)

$$K_{AE} = \frac{0.9638}{0.9701(0.8947)(1+0.3597)^2} = \mathbf{0.6006}$$

$$K_{PE} = \frac{0.9638}{0.9701(0.8947)(1-0.3597)^2} = \mathbf{2.7089}$$

The earthquake **increases** K_A from 0.367 → 0.601 and **reduces** K_P from 3.552 → 2.709.

Check the solution in GeoTerva ↗

Wedge angles, coefficients and q_{uE}

THEORY

$$\alpha_{AE} = \alpha + \tan^{-1}\left\{\frac{\sqrt{(1+\tan^2\alpha)\,[1+\tan(\delta+\theta)\cot\alpha]} - \tan\alpha}{1+\tan(\delta+\theta)(\tan\alpha+\cot\alpha)}\right\}$$

$$\alpha_{PE} = -\alpha + \tan^{-1}\left\{\frac{\sqrt{(1+\tan^2\alpha)\,[1+\tan(\delta-\theta)\cot\alpha]} + \tan\alpha}{1+\tan(\delta+\theta)(\tan\alpha+\cot\alpha)}\right\}$$

Since the equilibrium is the same, the coefficients keep the same form:

$$N_{qE} = \frac{K_{PE}}{K_{AE}} \qquad N_{\gamma E} = \tan\alpha_{AE}\left(\frac{K_{PE}}{K_{AE}} - 1\right) \qquad N_{cE} \approx (N_{qE} - 1)\cot\varphi$$

$$q_{uE} = c\,N_{cE} + q\,N_{qE} + \tfrac{1}{2}\gamma B\,N_{\gamma E}$$

NOTE

For N_{cE} the static relationship is **carried over** to the earthquake case (the authors write the theory for granular soil). When c > 0 the module reports this as an assumption and gives the sensitivity for c = 0.

Problem statement

EXAMPLE

Given

Foundation	**strip (continuous), B = 1.5 m, base depth D_f = 1.0 m**
Soil	$c = 30\ \mathrm{kPa},\ \varphi' = 25^\circ$
Unit weight	$\gamma = 17\ \mathrm{kN/m^3}$**; GWT not in the failure zone**
Earthquake	**k_h = 0.25; k_v = 0**
Previous example	$\theta = 14.04^\circ,\ K_{AE} = 0.6006,\ K_{PE} = 2.7089$

REQUIRED

Find **α_{AE}, N_{qE}, $N_{\gamma E}$, N_{cE}** and their ratios to the static coefficients.

Solution (1/2)

SOLUTION

1 α = 10.96°: tan α = 0.1937, cot α = 5.1620, tan(δ + θ) = tan 26.54° = 0.4994

$$\sqrt{(1+\tan^2\alpha)(1+\tan(\delta+\theta)\cot\alpha)} = \sqrt{1.0375(3.5777)} = 1.9267$$

$$1+\tan(\delta+\theta)(\tan\alpha+\cot\alpha) = 3.6745$$

2 Earthquake wedge angle (static α_A = 53.97°)

$$\alpha_{AE} = 10.96^\circ + \tan^{-1}\frac{1.9267-0.1937}{3.6745} = 10.96^\circ + 25.25^\circ = \mathbf{36.21}^\circ$$

Solution (2/2)

SOLUTION

3 Seismic bearing capacity coefficients (tan α_{AE} = 0.7322)

$$N_{qE} = \frac{2.7089}{0.6006} = \mathbf{4.51} \qquad N_{\gamma E} = 0.7322(4.51 - 1) = \mathbf{2.57} \qquad N_{cE} = \frac{4.51 - 1}{0.4663} = \mathbf{7.53}$$

	N_q	N_γ	N_c
static	9.67	11.92	18.59
earthquake (k_h = 0.25)	4.51	2.57	7.53
ratio	**0.466**	**0.216**	**0.405**

The earthquake **most of all** reduces the self-weight term ($N_{\gamma E}$ is only 22% of the static value); N_{qE} and N_{cE} retain about half.

Check the solution in GeoTerva ↗

Problem statement

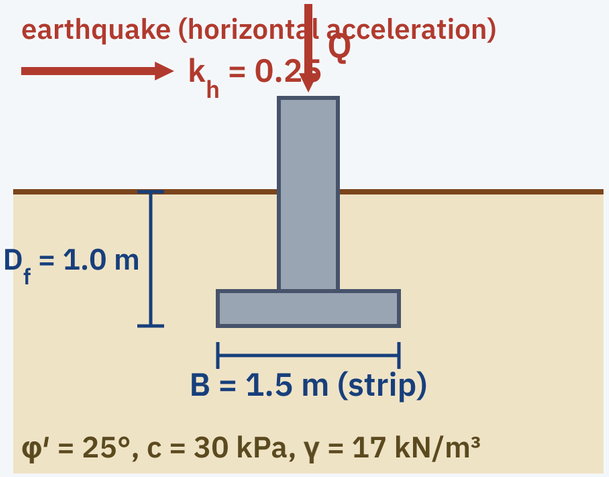
EXAMPLE

Given

Foundation	**strip (continuous), B = 1.5 m, base depth D_f = 1.0 m**
Soil	$c = 30\ \mathrm{kPa},\ \varphi' = 25^\circ$
Unit weight	$\gamma = 17\ \mathrm{kN/m^3}$**; GWT not in the failure zone**
Earthquake	**k_h = 0.25; k_v = 0**
Safety	**FS = 3**

REQUIRED

q_{uE}, its ratio to the static q_u, gross and net **q_{all}** and the allowable load **Q_{all}** (per unit length).



Solution

SOLUTION

1 Earthquake bearing capacity (q = 17 kPa)

$$q_{uE} = 30(7.53) + 17(4.51) + \tfrac{1}{2}(17)(1.5)(2.57)$$
$$= 225.83 + 76.67 + 32.77 = \mathbf{335.28}\ \mathrm{kPa}$$

2 Allowable pressure and load

$$q_{all,g} = \frac{335.28}{3} = \mathbf{111.76}\ \mathrm{kPa} \qquad q_{all,n} = \frac{335.28 - 17}{3} = 106.09\ \mathrm{kPa}$$

$$Q_{all} = q_{all,g}\, B = 111.76(1.5) = \mathbf{167.6}\ \mathrm{kN/m}$$

The earthquake reduces the bearing capacity from **874.14 kPa** to **335.28 kPa**, that is, to **38%** of the static value (the module gives the static q_u alongside). The allowable pressures are taken from q_{uE}.

Check the solution in GeoTerva ↗

How much does an earthquake reduce it, and where is the limit?

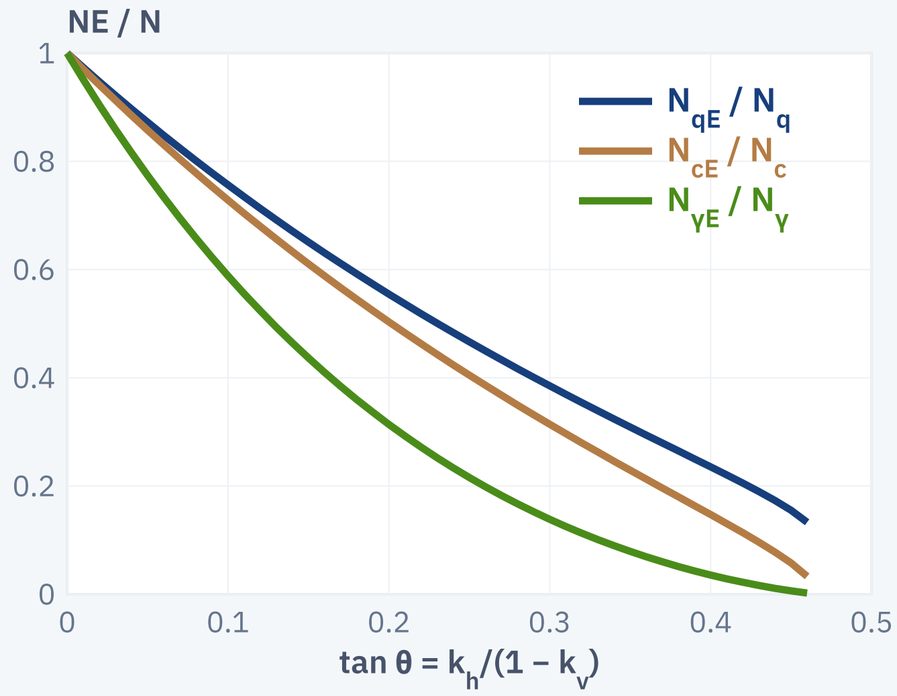
THEORY

The source gives the reductions with the charts of Richards et al. (1993); the page plots them from the closed-form expressions (at the side). **The self-weight term decreases the most**: at tan θ = 0.2, $N_{\gamma E}$ is about one-third of the static value; N_{qE} and N_{cE} remain about half.

As θ → φ, α = φ − θ → 0: the active wedge flattens and all three coefficients approach zero; the frictional soil has no reserve of resistance against horizontal acceleration.

The theory is valid only when $\mathbf{k_h/(1 - k_v) < \tan \varphi}$; the module **stops** once θ ≥ φ. This ratio is also the ratio at which the theory predicts that seismic settlement will begin.

When θ approaches φ′ (θ > 0.85φ), the module warns: this is not a design capacity but the limit of the method.



Drawn from the closed-form expressions for φ′ = 25°, δ = φ/2.

Problem statement

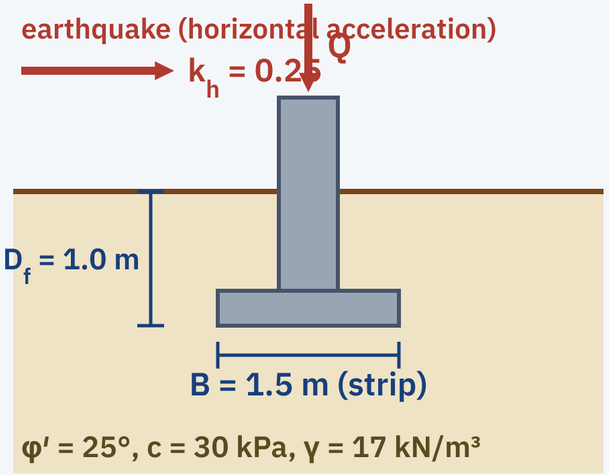
EXAMPLE

Given

Foundation	**strip (continuous), B = 1.5 m, base depth D_f = 1.0 m**
Soil	$c = 30\ \mathrm{kPa},\ \varphi' = 25^\circ$
Unit weight	$\gamma = 17\ \mathrm{kN/m^3}$**; GWT not in the failure zone**
Earthquake	**k_v = 0; k_h = 0, 0.05, … 0.45**

REQUIRED

For each k_h, find **θ, N_{qE}, $N_{\gamma E}$, N_{cE}, q_{uE}** and q_{uE}/q_u; compute the k_h limit at which the module stops.



Solution (1/2)

SOLUTION

k_h	θ (°)	N_{qE}	$N_{\gamma E}$	N_{cE}	q_{uE} (kPa)	q_{uE}/q_u
0.00	0.0	9.67	11.92	18.59	**874**	%100
0.05	2.9	8.44	9.23	15.94	**739**	%85
0.10	5.7	7.31	7.01	13.54	**620**	%71
0.15	8.5	6.29	5.20	11.35	**514**	%59
0.20	11.3	5.36	3.74	9.36	**420**	%48
0.25	14.0	4.51	2.57	7.53	**335**	%38
0.30	16.7	3.72	1.65	5.84	**260**	%30
0.35	19.3	2.99	0.94	4.26	**191**	%22
0.40	21.8	2.28	0.42	2.74	**126**	%14
0.45	24.2	1.50	0.07	1.07	**59**	%7

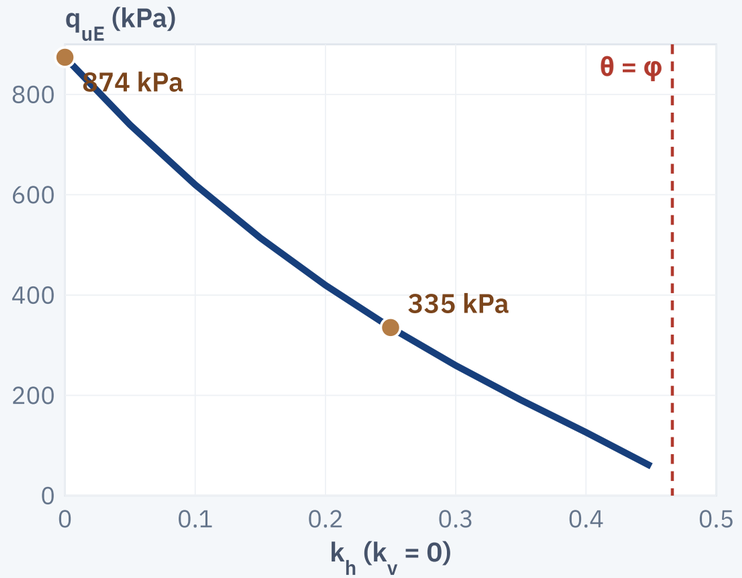
Solution (2/2)

SOLUTION

1 Validity limit: θ < φ

$$\frac{k_h}{1-k_v} < \tan\varphi \;\Rightarrow\; k_{h,\max} = (1-k_v)\tan 25^\circ = \mathbf{0.4663}$$

At k_h = 0.25, q_{uE} is 38% of the static value; **at k_h = 0.45, only 7%**. If k_h = 0.47 is entered, the module stops (θ ≥ φ). The curve drops steeply at the start: **even small accelerations** cause a large loss (k_h = 0.05 → 85%).



Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Foundation	**strip (continuous), B = 1.5 m, base depth D_f = 1.0 m**
Soil	$c = 30\ \mathrm{kPa},\ \varphi' = 25^\circ$
Unit weight	$\gamma = 17\ \mathrm{kN/m^3}$**; GWT not in the failure zone**
Earthquake	**k_h = 0.25; k_v = 0, 0.1, 0.2, 0.3**

REQUIRED

For each k_v, find the values of **θ** and **q_{uE}**.

Solution

SOLUTION

1 $\theta = \tan^{-1}[k_h/(1 - k_v)]$; example $k_v = 0.2$

$$\theta = \tan^{-1}\frac{0.25}{1-0.2} = \tan^{-1}(0.3125) = \mathbf{17.35}^{\circ}$$

k_v	θ (°)	N_{qE}	$N_{\gamma E}$	N_{cE}	q_{uE} (kPa)	q_{uE}/q_u
0.0	14.04	4.51	2.57	7.53	**335**	%38
0.1	15.52	4.07	2.03	6.57	**292**	%33
0.2	17.35	3.54	1.45	5.44	**242**	%28
0.3	19.65	2.88	0.86	4.04	**181**	%21

As k_v increases, $(1 - k_v)$ decreases, θ increases and q_{uE} decreases: $k_v = 0.2$ reduces q_{uE} from 335 to 242 kPa (28 %). Neglecting vertical acceleration remains **on the unsafe side**.

Check the solution in GeoTerva ↗

Allowable pressure from earthquake bearing capacity

THEORY

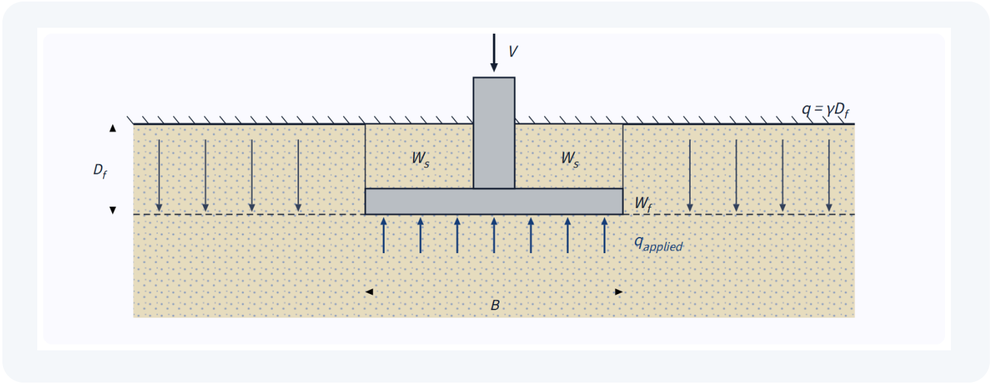
The module takes the allowable pressures from the **earthquake bearing capacity q_{uE}**; it keeps gross and net pressure separate and gives the static q_u alongside:

$$q_{u,net} = q_{uE} - q \qquad q_{all,g} = \frac{q_{uE}}{FS} \qquad q_{all,n} = \frac{q_{u,net}}{FS}$$

The applied pressure q_{app} is the **vertical gross pressure at the base** under the earthquake load case (V/B; column load + footing + soil weight). The footing is acceptable when:

$$q_{apply} \le q_{all,g} \quad (FS_{\text{actual}} = q_{uE}/q_{apply}) \qquad q_{apply} - q \le q_{all,n}$$

Allowable load per unit length $Q_{all} = q_{all,g}\, B$. The earthquake horizontal inertia is already accounted for by k_h; an independent horizontal force, moment or eccentricity is **beyond the scope** of this calculation.



Problem statement

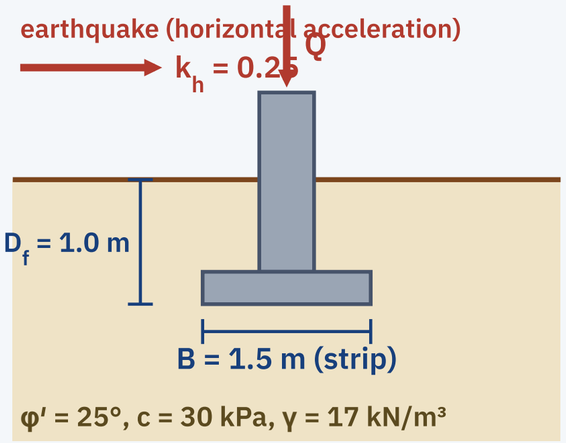
EXAMPLE

REQUIRED

In every case check the three conditions (gross, FS_{actual}, net).

Given

Result (Example 5)	**q_{uE} = 335.28 kPa; q = 17 kPa; FS = 3**
Applied pressure	**in the earthquake load case (a) q_{app} = 100 kPa · (b) q_{app} = 150 kPa**



Solution

SOLUTION

1 (a) q_{appl} = 100 kPa

$$100 \leq q_{all,g} = 111.76 \;\Rightarrow\; \textbf{valid} \qquad FS_{\text{actual}} = \frac{335.28}{100} = \mathbf{3.35} \geq 3$$

$$100 - 17 = 83 \leq q_{all,n} = 106.09 \;\Rightarrow\; \textbf{valid}$$

2 (b) q_{appl} = 150 kPa

$$150 > q_{all,g} = 111.76 \;\Rightarrow\; \textbf{does not pass} \qquad FS_{\text{actual}} = \frac{335.28}{150} = \mathbf{2.24} < 3$$

$$150 - 17 = 133 > q_{all,n} = 106.09 \;\Rightarrow\; \textbf{does not pass}$$

Under earthquake loading, only a foundation with **q_{app} ≤ 111.8 kPa** is acceptable. The same foundation would carry up to 291 kPa under static conditions ($q_{all,g}$ = 291.4).

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Foundation	**strip (continuous), B = 1.5 m, base depth D_f = 1.0 m**
Soil	$c = 30\ \mathrm{kPa},\ \varphi' = 25^\circ$
Unit weight	$\gamma = 17\ \mathrm{kN/m^3}$**; GWT not in the failure zone**
Earthquake	**k_h = 0.25; k_v = 0**
Comparison	**c = 30 kPa and c = 0**

REQUIRED

Determine the static and earthquake bearing capacity for c = 30 and c = 0; interpret the share of the cohesion terms.

Solution

SOLUTION

	$c\,N_c$	$q\,N_q$	$\frac{1}{2}\gamma B\,N_\gamma$	total
static, c = 30	557.8	164.4	152.0	**874.1**
earthquake, c = 30	225.8	76.7	32.8	**335.3**
static, c = 0	0	164.4	152.0	**316.4**
earthquake, c = 0	0	76.7	32.8	**109.4**

The cohesion term is **67%** of q_{uE} in the earthquake case; the assumption $N_{cE} = (N_{qE} - 1)\cot\varphi$ has a marked effect on the result. As a conservative sensitivity, for c = 0, q_{uE} = **109.4 kPa** (335.3 kPa with c = 30).

Check the solution in GeoTerva ↗

Summary

Key points

Two Coulomb wedges (active ABC, passive BCD), with wall friction $\delta = \varphi/2$ between them; $H = B \tan \alpha_A$; each coefficient in closed form

Static: $N_q = K_P/K_A$, $N_\gamma = \tan \alpha_A(K_P/K_A - 1)$, $N_c = (N_q - 1) \cot \varphi$; $q_u = cN_c + qN_q + \tfrac{1}{2}\gamma B N_\gamma$ (example: 874.14 kPa)

Earthquake: $\theta = \tan^{-1}[k_h/(1 - k_v)]$, $\alpha = \varphi - \theta$; K_{AE}, K_{PE} by Mononobe–Okabe; N_{qE}, $N_{\gamma E}$, N_{cE} in the same form; q_{uE} = 335.28 kPa (38% of the static value)

The self-weight term decreases the most ($N_{\gamma E}/N_\gamma = 0.22$); N_{qE} and N_{cE} retain about half

Limit: $k_h/(1 - k_v) < \tan \varphi$; when $\theta \geq \varphi$ the module stops, $\theta > 0.85\varphi$ gives a warning; if k_v is neglected, the result remains on the unsafe side

Check: allowable pressure from q_{uE}; $q_{appl} \leq q_{all,g}$, FS_{actual} and net; $Q_{all} = q_{all,g} B$

References

Richards, R., Elms, D.G. and Budhu, M. (1993). Seismic bearing capacity and settlements of foundations. Journal of Geotechnical Engineering, ASCE, 119(4), 662–674.

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Mononobe, N. and Matsuo, H. (1929). On the determination of earth pressures during earthquakes. Proc. World Engineering Congress, Tokyo, 9, 177–185.

Okabe, S. (1926). General theory of earth pressure. Journal of the Japanese Society of Civil Engineers, 12(1).

The figures are drawn to scale from the equations of this page.

NOTE

Some of the figures are taken from the GeoTerva Theory page; all numerical examples have been generated with the module's own calculation library, and the module gives the same values.

GeoTerva

Online geotechnical analysis

From theory to practice

Compute the example foundation in this lecture step by step in GeoTerva: the module opens with this model.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com