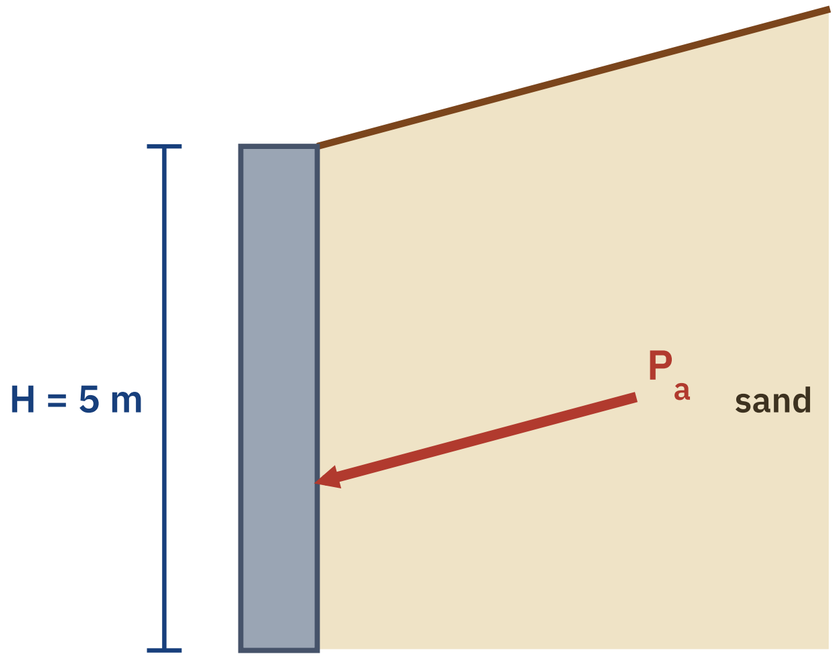
GeoTerva

Geotechnical lecture presentation · Lateral earth pressure

Rankine: inclined backfill and inclined wall

Lecture: Rankine solution when the fill surface is inclined or the wall face is inclined; pressure parallel to the surface, K coefficients, cohesive and layered fill; worked examples on every topic.

Sloping fill | Active and passive | Inclined wall · Chu (1991) | Cohesive fill | Layered fill



What will we learn in this lecture?

- That when the backfill surface is inclined the pressure is **parallel to the surface**, and why K_a and K_p vary with α
- The relation $K_a K_p = \cos^2\alpha$ and the limit **$\alpha \le \varphi'$**
- The **Chu (1991)** solution for an inclined wall + sloping fill: ψ, $K_{(R)}$, β and η
- Determining the **magnitude, direction (β) and location** of the force; separating its horizontal and vertical components
- For cohesive fill: the curved pressure diagram, the tension depth and, **in undrained clay**, the stability condition of the slope
- To use each layer's own K in layers parallel to the surface

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Forces are per unit length of wall (kN/m); α and θ in degrees, pressures in kPa. The wall is frictionless.

The pressure is parallel to the surface

THEORY

If the surface of the cohesionless fill behind the wall makes an angle α with the horizontal, the Rankine pressures are no longer horizontal: they act **parallel to the surface**, i.e. inclined at α, and still increase linearly with depth:

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\varphi'}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\varphi'}} \qquad P_a = \tfrac{1}{2} K_a \gamma H^2$$

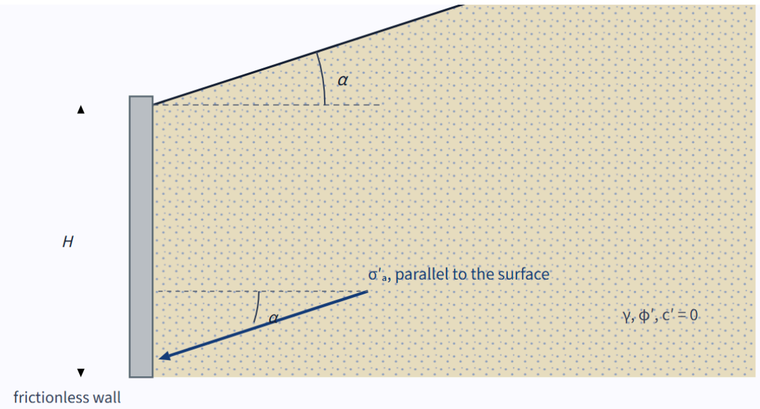
$$K_p = \cos\alpha \frac{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\varphi'}}{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\varphi'}} \qquad P_p = \tfrac{1}{2} K_p \gamma H^2$$

Both forces act at **H/3** above the base and make an angle α with the horizontal: $P_h = P \cos\alpha$, $P_v = P \sin\alpha$.

For α = 0 the equations reduce to $\tan^2(45° \mp \varphi'/2)$.

The product is $\mathbf{K_a K_p = \cos^2\alpha}$; at α = 0 it is 1.

SLOPING EMBANKMENT



K_a and K_p – α and φ′

THEORY

ACTIVE K_a

α \ φ′	28°	30°	32°	34°	36°	38°	40°
0°	0.3610	0.3333	0.3073	0.2827	0.2596	0.2379	0.2174
5°	0.3656	0.3372	0.3105	0.2855	0.2620	0.2399	0.2192
10°	0.3802	0.3495	0.3210	0.2944	0.2696	0.2464	0.2247
15°	0.4086	0.3729	0.3405	0.3108	0.2834	0.2581	0.2346
20°	0.4605	0.4142	0.3739	0.3381	0.3060	0.2769	0.2504
25°	0.5727	0.4936	0.4336	0.3847	0.3431	0.3070	0.2750

PASSIVE K_p

α \ φ′	28°	30°	32°	34°	36°	38°	40°
0°	2.770	3.000	3.255	3.537	3.852	4.204	4.599
5°	2.715	2.943	3.196	3.476	3.788	4.136	4.527
10°	2.551	2.775	3.022	3.295	3.598	3.936	4.316
15°	2.284	2.502	2.740	3.002	3.293	3.615	3.977
20°	1.918	2.132	2.362	2.612	2.886	3.189	3.526
25°	1.434	1.664	1.894	2.135	2.394	2.676	2.987

Problem statement

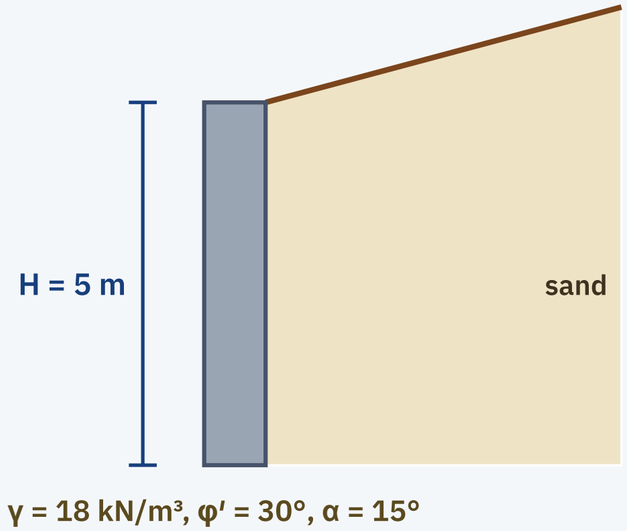
EXAMPLE

Given

Wall	$H = 5$ **m, vertical, frictionless**
Fill	**dry sand:** $\gamma = 18\ \mathrm{kN/m^3}$, $\varphi' = 30^\circ$**, surface slope** $\alpha = 15^\circ$

REQUIRED

K_a, P_a, height and direction of the line of action; horizontal and vertical components; comparison with horizontal backfill.



Solution

SOLUTION

1 Coefficient: cos 15° = 0.9659; √(cos²α − cos²φ′) = √(0.9330 − 0.75) = 0.4278

$$K_a = \cos\alpha \frac{\cos\alpha - r}{\cos\alpha + r} = 0.9659 \frac{0.5381}{1.3937} = \mathbf{0.3729}$$

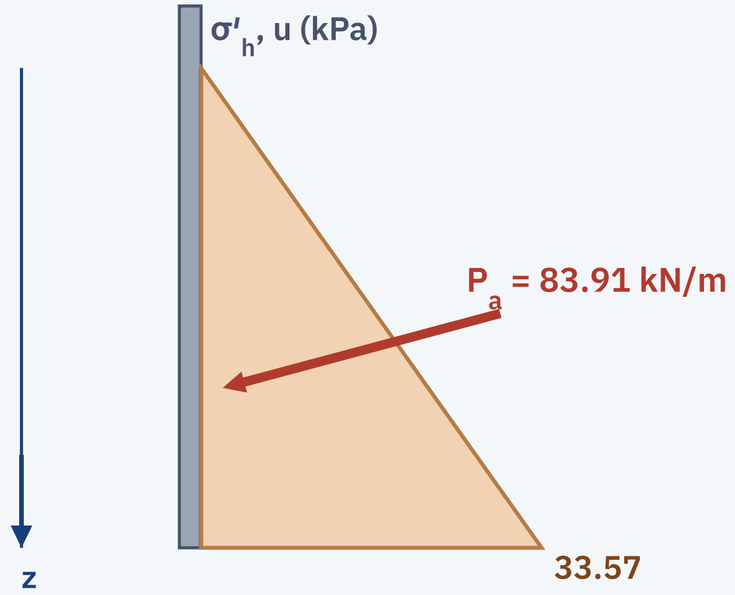
2 Pressure at the base (parallel to the surface)

$$\sigma'_a(5) = K_a \gamma H = 0.3729(18)(5) = \mathbf{33.57}\ \text{kPa}$$

3 Force, location and components

$$P_a = \tfrac{1}{2}(33.57)(5) = \mathbf{83.91}\ \text{kN/m} \qquad \bar{z} = \tfrac{5}{3} = 1.67\ \text{m}$$

$$P_h = P_a \cos 15^\circ = 81.05 \qquad P_v = P_a \sin 15^\circ = 21.72\ \text{kN/m}$$



P_a = **83.91 kN/m**, at 15° to the horizontal, 1.67 m above the base. For horizontal fill, P_a was 75.00 kN/m: the inclination increases the force by 12%.

Check the solution in GeoTerva ↗

Problem statement

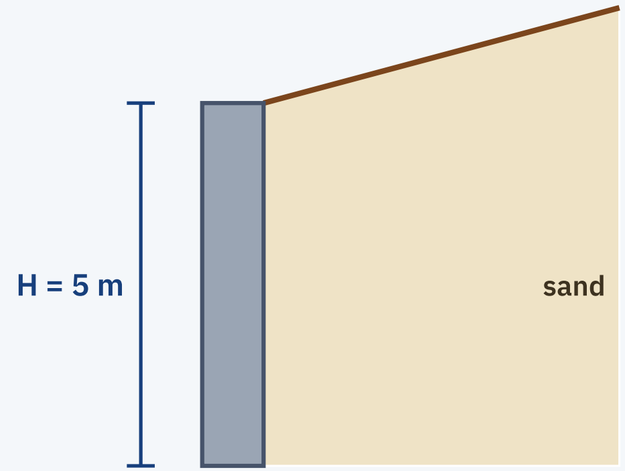
EXAMPLE

REQUIRED

K_p, P_p and their components; comparison with the passive force in horizontal fill.

Given

Wall and fill	**The wall and fill of Example 1: $H = 5$ m, $\gamma = 18\ \mathrm{kN/m^3}$, $\varphi' = 30°$, $\alpha = 15°$**



γ = 18 kN/m³, φ′ = 30°, α = 15° (passive)

Solution

SOLUTION

1 Coefficient (r = 0.4278)

$$K_p = \cos\alpha\,\frac{\cos\alpha + r}{\cos\alpha - r} = 0.9659\,\frac{1.3937}{0.5381} = \mathbf{2.5017}$$

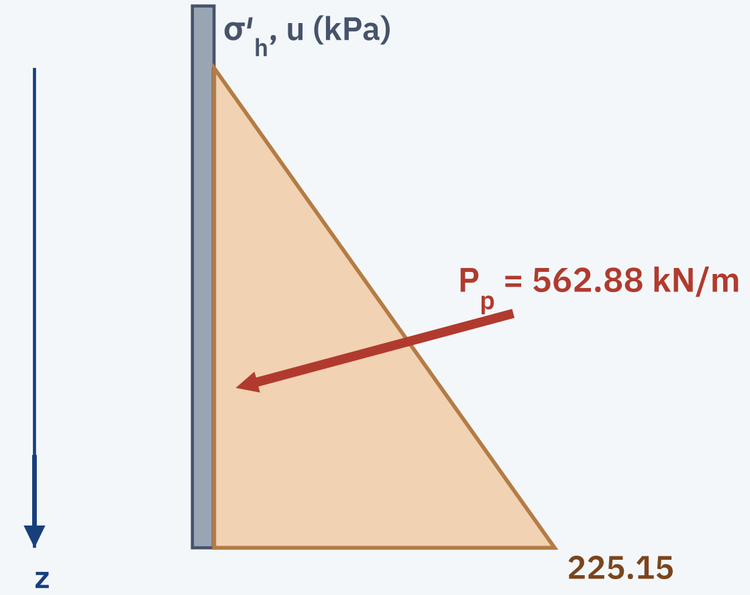
2 Pressure at the base and force

$$\sigma'_p(5) = K_p\gamma H = 2.5017(18)(5) = 225.15\ \text{kPa}$$

$$P_p = \tfrac{1}{2}(225.15)(5) = \mathbf{562.9}\ \text{kN/m} \qquad \bar{z} = 1.67\ \text{m}$$

3 Components (15° to the horizontal)

$$P_h = P_p\cos 15^\circ = 543.7 \qquad P_v = P_p\sin 15^\circ = 145.7\ \text{kN/m}$$



P_p = **562.9 kN/m**. For horizontal fill it was 675.0 kN/m: the slope **reduces** the passive resistance by 17% (while the active force increases). K_aK_p = 0.9330 = cos²15° check is satisfied.

Check the solution in GeoTerva ↗

condition α ≤ φ′

THEORY

The radical expression is real only when **cos²α ≥ cos²φ′**, i.e. α ≤ φ′: a slope steeper than φ′ cannot stand in cohesionless soil. At the limit the radical becomes zero and both coefficients approach the same value:

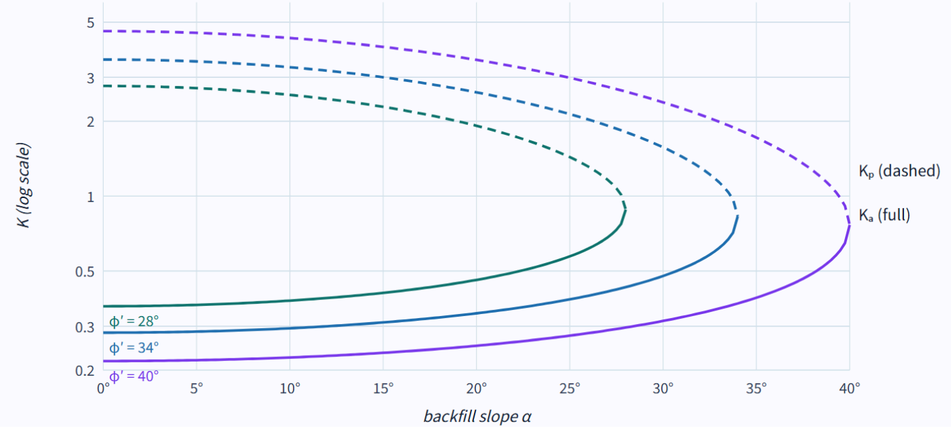
$$\alpha = \varphi' : \qquad K_a = K_p = \cos\varphi'$$

As α increases, K_a rises and K_p falls; the two merge at α = φ′ (the active and passive states become indistinguishable).

This is the state of the soil at its natural angle of slope (repose): the backfill is already at the verge of failure and the wall movement does not change the pressure.

For α > φ′ there is no solution; the module issues a warning.

K – α (log scale; φ′ = 28°, 34°, 40°)



Problem statement

EXAMPLE

Given

Soil	**cohesionless sand,** $\varphi' = 32^\circ$
Slopes	$\alpha = 0^\circ,\ 10^\circ,\ 20^\circ,\ 30^\circ,\ 32^\circ$ **(and** 33°**)**

REQUIRED

K_a, K_p and their products for each slope; the value at the limit; the result for α = 33°.

Solution

SOLUTION

1 Equations for each slope ($\varphi' = 32°$; $\cos 32° = 0.8480$)

α	K_a	K_p	K_aK_p	$\cos^2\alpha$
0°	0.3073	3.255	1.0000	1.0000
10°	0.3210	3.022	0.9698	0.9698
20°	0.3739	2.362	0.8830	0.8830
30°	0.5741	1.306	0.7500	0.7500
32°	0.8480	0.848	0.7192	0.7192

2 Limit: $\alpha = \varphi' = 32°$ and $\alpha = 33°$

$$\alpha = 32^\circ : \sqrt{\cos^2\alpha - \cos^2\varphi'} = 0 \Rightarrow K_a = K_p = \cos 32^\circ = \mathbf{0.8480}$$

$$\alpha = 33^\circ : \cos^2 33^\circ - \cos^2 32^\circ < 0 \Rightarrow \text{no real root}$$

As α increases, K_a goes from 0.307 → 0.574 and K_p from 3.25 → 1.306; at every α, $K_aK_p = \cos^2\alpha$. At $\alpha = \varphi' = 32°$ the two become equal to $\cos\varphi' = 0.848$; for $\alpha = 33°$ **there is no solution** (vertical slope).

Check the solution in GeoTerva ↗

Inclined wall face and sloping backfill: active

THEORY

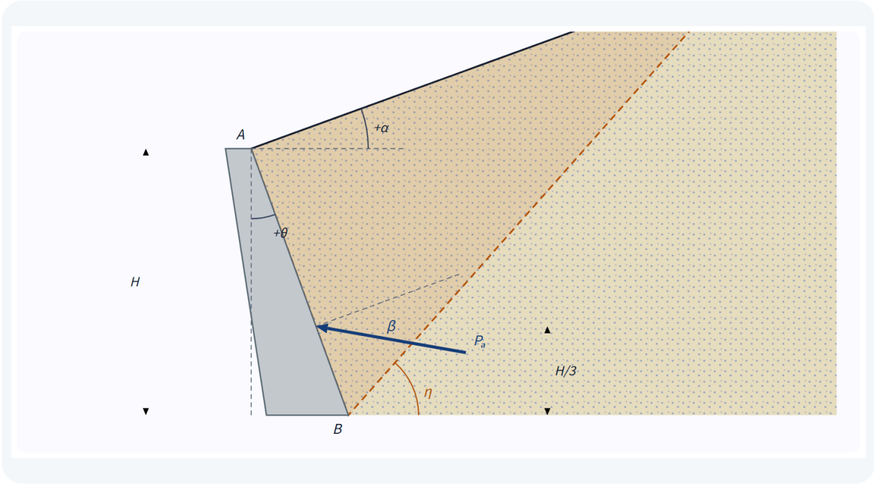
If the wall face is inclined at θ from the vertical and the backfill surface at α from the horizontal (both positive as in the figure; wall frictionless), the Chu (1991) solution gives the **magnitude and direction** of the force:

$$\psi_a = \sin^{-1}\frac{\sin\alpha}{\sin\varphi'} - \alpha + 2\theta$$

$$K_{a(R)} = \frac{\cos(\alpha-\theta)\sqrt{1+\sin^2\varphi' - 2\sin\varphi'\cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\varphi' - \sin^2\alpha}\right)}$$

$$P_a = \tfrac{1}{2}\gamma H^2 K_{a(R)} \qquad \beta = \tan^{-1}\frac{\sin\varphi'\sin\psi_a}{1-\sin\varphi'\cos\psi_a}$$

$$\eta = \frac{\pi}{4} + \frac{\varphi'}{2} + \frac{\alpha}{2} - \frac{1}{2}\sin^{-1}\frac{\sin\alpha}{\sin\varphi'}$$



β: the angle of the force with respect to the normal to the wall face; the force acts at **H/3** above the base. **η**: the angle of the base BC of the sliding wedge with the horizontal.

For θ = 0 it reduces to the inclined backfill formula for $K_{a(R)}$ (Eq. 1), and β reduces to α.

In the passive case the signs change

THEORY

In the passive case, the signs change for the same geometry:

$$\psi_p = \sin^{-1}\frac{\sin\alpha}{\sin\varphi'} + \alpha - 2\theta \qquad \beta = \tan^{-1}\frac{\sin\varphi'\sin\psi_p}{1+\sin\varphi'\cos\psi_p}$$

$$K_{p(R)} = \frac{\cos(\alpha-\theta)\sqrt{1+\sin^2\varphi' + 2\sin\varphi'\cos\psi_p}}{\cos^2\theta\left(\cos\alpha - \sqrt{\sin^2\varphi' - \sin^2\alpha}\right)}$$

$$\eta = \frac{\pi}{4} - \frac{\varphi'}{2} + \frac{\alpha}{2} + \frac{1}{2}\sin^{-1}\frac{\sin\alpha}{\sin\varphi'} \qquad P_p = \tfrac{1}{2}\gamma H^2 K_{p(R)}$$

The force acts at H/3 above the base. The wall face and the surface intersect at an angle of 90° − |α − θ|; if this angle closes, the wall extends along the slope and the effect on the face is the weight of the overlying soil; the module accepts |α − θ| ≤ 75°.

Problem statement

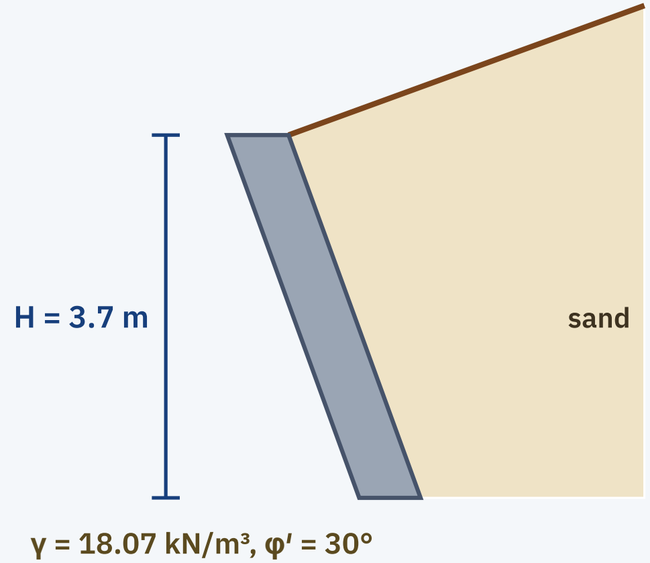
EXAMPLE

Given

Wall	**frictionless; face inclined at $\theta = +20^\circ$ from vertical; $H = 3.658$ m (12 ft)**
Fill	**dry sand: $\gamma = 18.07\ \mathrm{kN/m^3}$ (115 pcf), $\varphi' = 30^\circ$; surface slope $\alpha = +20^\circ$**

REQUIRED

ψ_a, $K_{a(R)}$, P_a, β, η and the height of action. The module opens with this wall.



Solution (1/2)

SOLUTION

1 Auxiliary angle and ψ_a

$$\sin^{-1}\frac{\sin 20^\circ}{\sin 30^\circ} = 43.16^\circ$$

$$\psi_a = 43.16^\circ - 20^\circ + 2(20^\circ) = \mathbf{63.16^\circ}$$

2 Coefficient $K_{a(R)}$

$$\sqrt{1+\sin^2 30^\circ - 2\sin 30^\circ \cos 63.16^\circ} = 0.8936 \qquad \cos^2 20^\circ = 0.8830$$

$$\sqrt{\sin^2 30^\circ - \sin^2 20^\circ} = 0.3647$$

$$K_{a(R)} = \frac{\cos 0^\circ\,(0.8936)}{0.8830\,(0.9397+0.3647)} = \mathbf{0.776}$$

Solution (2/2)

SOLUTION

3 Force and location

$$P_a = \tfrac{1}{2}\gamma H^2 K_{a(R)} = \tfrac{1}{2}(18.07)(3.658)^2(0.7758) = \mathbf{93.79}\ \mathrm{kN/m}$$

$$\bar{z} = \frac{H}{3} = \mathbf{1.219}\ \mathrm{m}$$

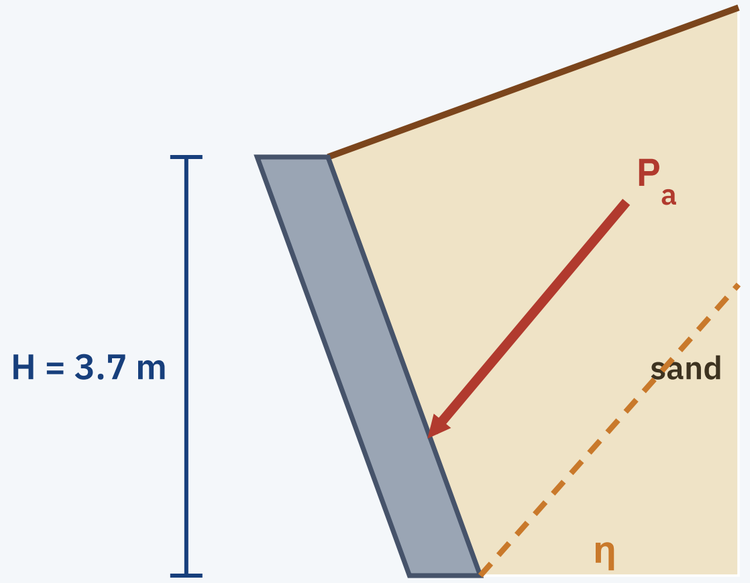
4 Strike β and wedge angle η

$$\beta = \tan^{-1}\frac{\sin 30^\circ \sin 63.16^\circ}{1 - \sin 30^\circ \cos 63.16^\circ} = \mathbf{29.95}^\circ\ (\approx 30^\circ)$$

$$\eta = 45^\circ + 15^\circ + 10^\circ - \tfrac{1}{2}(43.16^\circ) = \mathbf{48.42}^\circ$$

5 Components (perpendicular and parallel to the face)

$$N = P_a \cos\beta = 81.27 \qquad T = P_a \sin\beta = 46.83\ \mathrm{kN/m}$$



P_a = **93.79 kN/m** (6,424 lb/ft in US units), **30.0°** from the normal to the wall face, at H/3 = 1.22 m above the base.

Check the solution in GeoTerva ↗

Problem statement

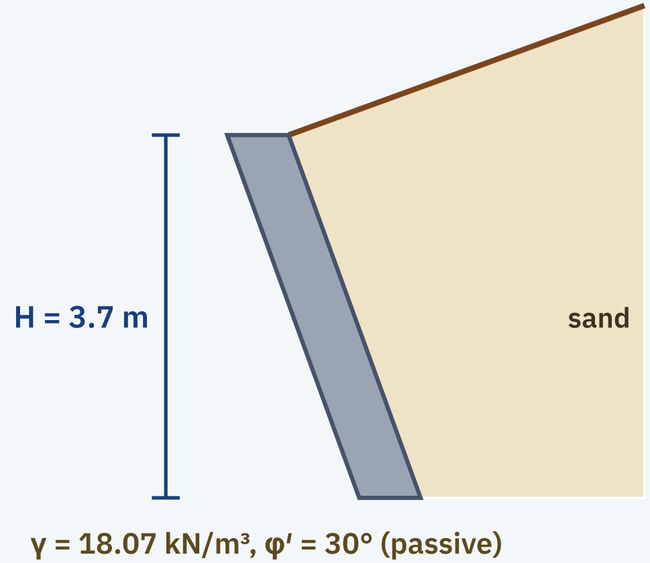
EXAMPLE

REQUIRED

ψ_p, **$K_{p(R)}$**, **P_p**, **β** and **η**; comparison with the active case.

Given

Wall and fill	**The wall and backfill in Example 4:** $\theta = +20^\circ,\ \alpha = +20^\circ,\ \varphi' = 30^\circ,\ \gamma = 18.07\ \mathrm{kN/m^3},\ H = 3.658$ **m**



Solution

SOLUTION

1 ψ_p and $K_{p(R)}$

$$\psi_p = 43.16^\circ + 20^\circ - 2(20^\circ) = \mathbf{23.16^\circ}$$

$$K_{p(R)} = \frac{\cos 0^\circ \sqrt{1 + \sin^2 30^\circ + 2 \sin 30^\circ \cos 23.16^\circ}}{\cos^2 20^\circ \left(\cos 20^\circ - \sqrt{\sin^2 30^\circ - \sin^2 20^\circ}\right)} = \mathbf{2.901}$$

2 Force, direction and wedge angle

$$P_p = \tfrac{1}{2}(18.07)(3.658)^2(2.9010) = \mathbf{350.7}\ \mathrm{kN/m}$$

$$\beta = \tan^{-1} \frac{\sin 30^\circ \sin 23.16^\circ}{1 + \sin 30^\circ \cos 23.16^\circ} = \mathbf{7.67^\circ} \qquad \eta = \mathbf{61.58^\circ}$$

P_p = **350.7 kN/m**, β = 7.7°; the active force for the same wall was 93.79 kN/m (ratio 3.7). For passive pressure the force is almost perpendicular to the face (β small), whereas for active pressure it is inclined at ~30°.

Check the solution in GeoTerva ↗

How does the wall face inclination θ change the force?

THEORY

If the fill slope is kept constant (α = 20°, φ′ = 30°) and only the wall face is rotated, $K_{a(R)}$ and β change:

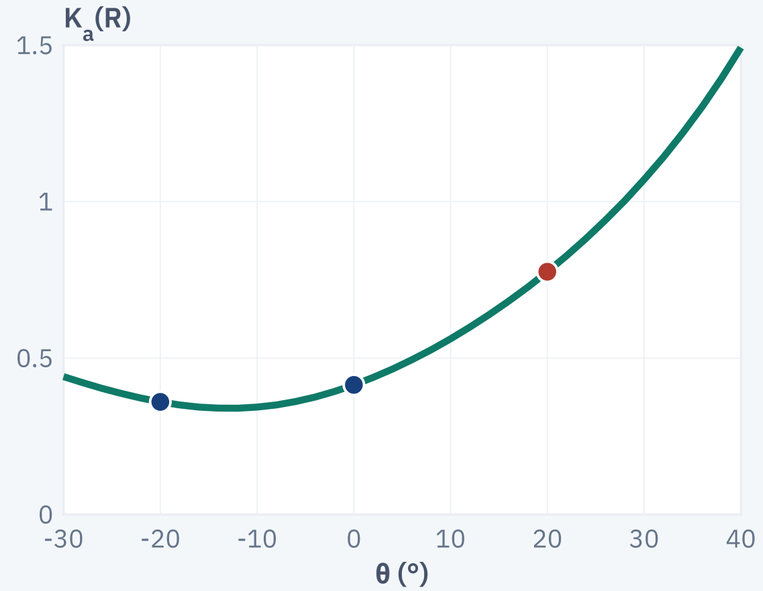
θ	$K_{a(R)}$	β
−20°	0.360	-15.5°
0°	0.414	20.0°
+20°	0.776	30.0°

If θ is positive (the face leans onto the top of the fill), the fill rests on the top of the wall face and the force increases; if negative, it decreases.

For θ = 0 and α = 20°, β = α = 20°: the force is parallel to the surface.

$K_{(R)}$ includes together the two components of the force **parallel and perpendicular** to the face; for the horizontal component alone, P cos(β + θ) is taken.

$K_{a(R)}$ – θ (α = 20°, φ′ = 30°)



Mazindrani and Ganjali (1997)

THEORY

For a c′–φ′ soil with surface inclined at α behind a vertical, frictionless wall, the active pressure at depth:

$$\sigma'_a = \gamma z\, K'_a \cos\alpha \qquad r = \frac{c'}{\gamma z}$$

$$K'_a = \frac{2\cos^2\alpha + 2r\cos\varphi'\sin\varphi' - \sqrt{D}}{\cos^2\varphi'} - 1$$

$$D = 4\cos^2\alpha\left(\cos^2\alpha - \cos^2\varphi'\right) + 4r^2\cos^2\varphi' + 8r\cos^2\alpha\sin\varphi'\cos\varphi'$$

$$z_c = \frac{2c'}{\gamma}\sqrt{\frac{1+\sin\varphi'}{1-\sin\varphi'}}$$

K'_a depends on depth through c′/γz: the pressure is **not linear**; it is negative above the ground surface, and the tension crack depth z_c is independent of the inclination.

In the passive case, the sign in front of the root becomes +. α = 0 → Rankine ($\sigma'_a = K_a\gamma z - 2c'\sqrt{K_a}$); c′ = 0 → Eq. 1.

In the cracked calculation, the part below z_c is integrated; in the uncracked case, the full height is integrated; valid only for θ = 0.

PRESSURE PARALLEL TO SURFACE

As in the cohesionless case, the pressure acts **parallel** to the surface, i.e. at an angle α to the horizontal; the component of the force perpendicular to the wall face is P cos α.

Problem statement

EXAMPLE

Given

Wall	$H = 5$ **m, vertical, frictionless**
Fill	$\gamma = 18\ \mathrm{kN/m^3},\ \varphi' = 25^\circ,\ c' = 10$ **kPa; surface slope** $\alpha = 10^\circ$

REQUIRED

z_c, the pressure at the base, P_a with and without a crack, and the height of action; comparison with horizontal backfill.

Solution (1/2)

SOLUTION

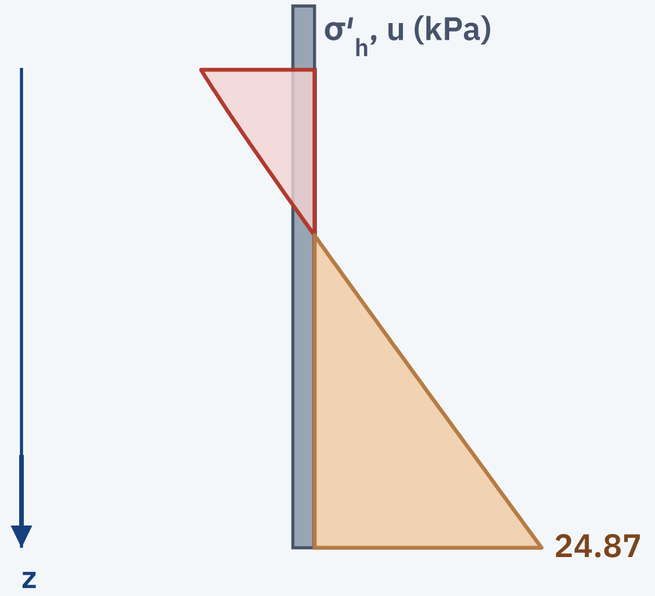
1 Tension depth (independent of slope)

$$z_c = \frac{2(10)}{18}\sqrt{\frac{1+\sin 25^\circ}{1-\sin 25^\circ}} = 1.111(1.5697) = \mathbf{1.744}\ \text{m}$$

2 At the base (z = 5 m): r = c′/γz = 10/(18·5) = 0.1111

$$D = 0.9467 \qquad K'_a = 0.2806$$

$$\sigma'_a(5) = \gamma z K'_a \cos\alpha = 18(5)(0.2806)\cos 10^\circ = \mathbf{24.87}\ \text{kPa}$$



Solution (2/2)

SOLUTION

3 Cracked force (area of the curved diagram from z_c to H, numerical integration)

$$P_a = \int_{z_c}^{H} \sigma'_a \, dz = \mathbf{40.33}\ \mathrm{kN/m}\ (\text{parallel to surface, } 10^\circ)$$

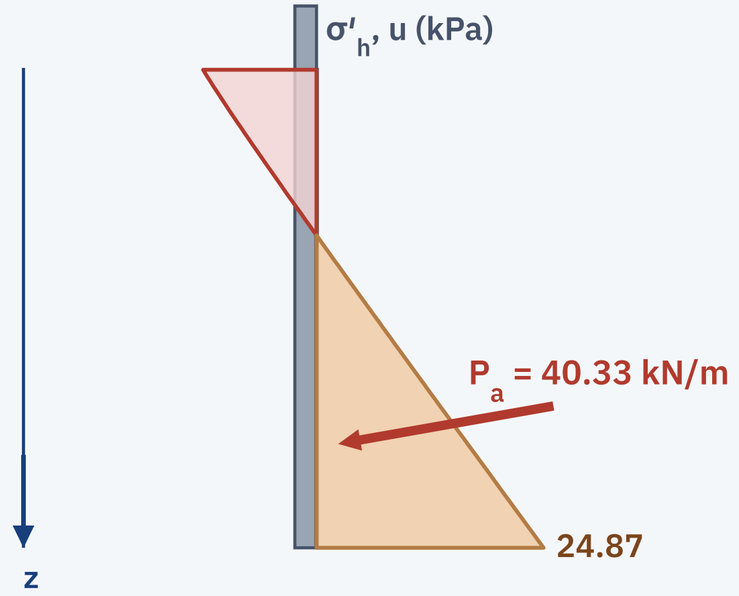
$$\bar{z} = 1.08\ \mathrm{m}\ (\text{from the base})$$

4 Comparison

$$\text{without crack: } P_a = 29.16\ \mathrm{kN/m}$$

$$\text{horizontal fill: } P_a = \tfrac{1}{2}(23.79)(3.256) = 38.72\ \mathrm{kN/m}$$

The force with the crack is **40.33 kN/m**, 38% larger than without a crack; compared with the horizontal backfill (38.72 kN/m), the slope increased the force by 4%. The value with the crack is the design value.



Check the solution in GeoTerva ↗

Layers parallel to the surface

THEORY

If the fill consists of layers **parallel to the surface** (a vertical borehole behind the wall intersects the same thicknesses everywhere), each layer is evaluated with its own γ, φ′ and c′ under the weight of the soil above, $\sigma'_v = \Sigma \gamma_i h_i$:

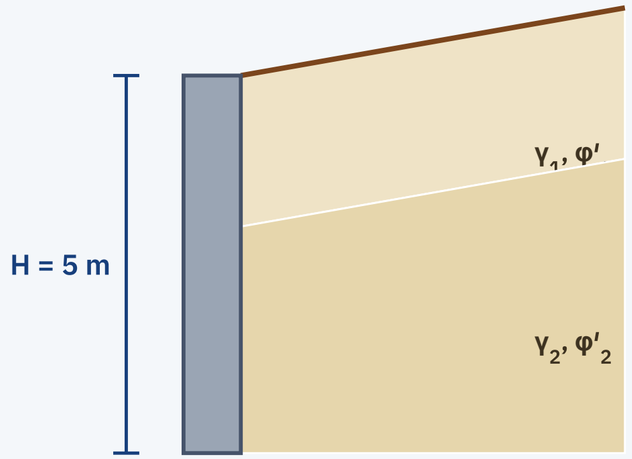
$$\sigma' = \sigma'_v \frac{K_{(R)} \cos\theta}{1 + \tan\theta \tan\alpha} \qquad \theta = 0 : \ \sigma' = \sigma'_v K_{(R)}$$

Since K changes at a layer boundary, the pressure **jumps**; the force of each layer is resolved into components N_i perpendicular and T_i parallel to the wall face; $P = \sqrt{(N^2 + T^2)}$, $\beta = \tan^{-1}(T/N)$.

Surcharge q is added to σ'_v as a layer of weight q/cos α at the surface; below the **water** level γ′ is used, and u acts perpendicular to the wall.

This is the module's extension of the Chu and Mazindrani–Ganjali solutions to layered soils (no closed-form layered solution exists in the literature); for a single layer it reproduces the closed-form solution to within 10^{-8}.

LAYERS PARALLEL TO THE SURFACE



Problem statement

EXAMPLE

Given

Wall	$H = 5$ **m, vertical, frictionless**
Fill	**ground surface slope $\alpha = 10°$; two layers parallel to the surface (vertical thickness)**
Layer 1 (2 m)	$\gamma = 17\ \mathrm{kN/m^3},\ \varphi' = 30°$
Layer 2 (3 m)	$\gamma = 19\ \mathrm{kN/m^3},\ \varphi' = 35°$

REQUIRED

In each layer, pressures, **P**, **N**, **T** and the height of action.

Solution (1/2)

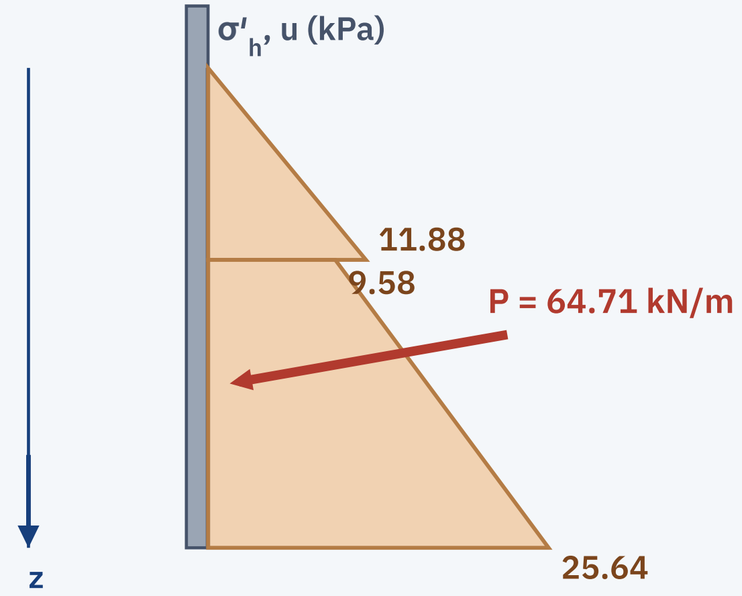
SOLUTION

1 Coefficients (α = 10°)

$$K_1(30^\circ) = \mathbf{0.3495}$$
$$K_2(35^\circ) = \mathbf{0.2818}$$

2 Vertical stresses: at the boundary 17(2) = 34; at the base 34 + 19(3) = 91 kPa

$$\sigma_1' = 0.3495(34) = \mathbf{11.88}\ \text{kPa (top)}$$
$$\sigma_2' = 0.2818(34) = \mathbf{9.58}\ \text{kPa (bottom)}$$
$$\sigma'(5) = 0.2818(91) = \mathbf{25.64}\ \text{kPa}$$



Solution (2/2)

SOLUTION

3 Forces (parallel to the surface): upper triangle · lower rectangle · lower triangle

$$P = \tfrac{1}{2}(2)(11.88) + 3(9.58) + \tfrac{1}{2}(3)(16.06) = 11.88 + 28.74 + 24.09 = \mathbf{64.71}\ \mathrm{kN/m}$$

4 Components and height of action

$$N = P\cos 10^\circ = 63.73 \qquad T = P\sin 10^\circ = 11.24\ \mathrm{kN/m}$$

$$\beta = \tan^{-1}(T/N) = 10^\circ \qquad \bar{z} = \mathbf{1.71}\ \mathrm{m}$$

P = **64.71 kN/m**, N = 63.73 kN/m perpendicular to the wall face; 1.71 m above the base. At the boundary the pressure **drops** from 11.88 → 9.58 kPa, because the φ′ value of the underlying layer is larger; σ'_v, however, is continuous.

Check the solution in GeoTerva ↗

Stability condition of the slope

THEORY

Undrained clay ($\varphi_u = 0$, c_u, total stress) is the same relationship; take $\varphi' = 0$ and $c' = c_u$:

$$K'_a = 2\cos^2\alpha - 2\sqrt{r^2 - \sin^2\alpha\cos^2\alpha} - 1 \qquad r = \frac{c_u}{\gamma z}$$

A solution exists only **as long as the slope stands**: the shear stress on planes parallel to the surface cannot exceed the cohesion:

$$\tau = \gamma z \sin\alpha\cos\alpha \le c_u \;\Rightarrow\; z_{max} = \frac{c_u}{\gamma\sin\alpha\cos\alpha} \qquad z_c = \frac{2c_u}{\gamma}$$

The module checks this condition down to the base of the wall; if the wall is higher than z_{max}, the calculation stops.

For $\alpha = 0$, $\sigma_a = \gamma z - 2c_u$, tension crack at $2c_u/\gamma$.

WHY IS THERE A LIMIT?

Beneath an inclined surface, the shear stress on every plane parallel to the surface increases with depth; the clay can carry this stress only up to c_u. At greater depth the slope slides under its own weight.

Problem statement

EXAMPLE

Given

Fill	**saturated clay, undrained:** $\gamma = 18\ \mathrm{kN/m^3}$, $c_u = 20$ **kPa; surface slope** $\alpha = 15^\circ$
Wall	$H = 4$ **m (and 5 m), vertical, frictionless**

REQUIRED

z_{max}, z_c, the pressure at the base and the cracked P_a; what happens at H = 5 m?

Solution (1/2)

SOLUTION

1 Failure limit and depth of tension

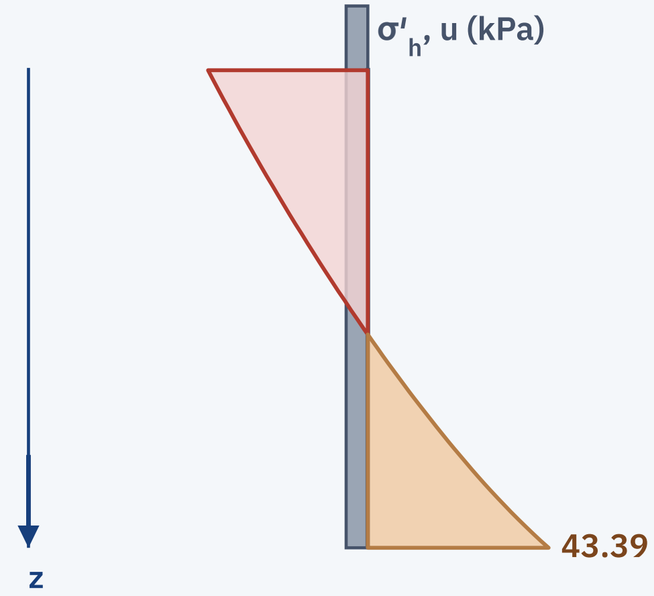
$$z_{max} = \frac{20}{18 \sin 15^\circ \cos 15^\circ} = \mathbf{4.44}\ \text{m}$$

$$z_c = \frac{2c_u}{\gamma} = \frac{40}{18} = \mathbf{2.22}\ \text{m}$$

2 H = 4 m: τ(4) = 18(4) sin15° cos15° = 18.0 ≤ 20 kPa: the slope stands; at the base r = 20/(18·4) = 0.2778

$$K'_a = 2\cos^2 15^\circ - 2\sqrt{r^2 - \sin^2 15^\circ \cos^2 15^\circ} - 1 = 0.6239$$

$$\sigma_a(4) = \gamma z K'_a \cos\alpha = 18(4)(0.6239)\cos 15^\circ = \mathbf{43.39}\ \text{kPa}$$



Solution (2/2)

SOLUTION

3 Force with crack (from z_c to 4 m)

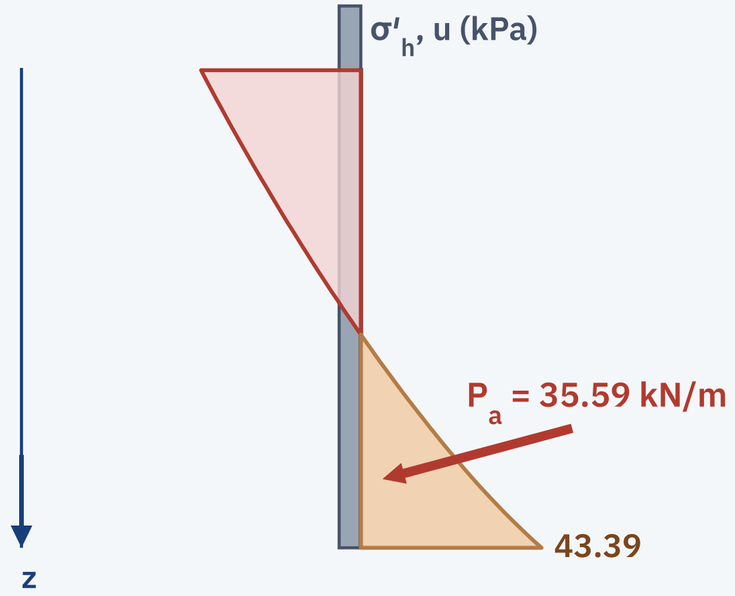
$$P_a = \int_{z_c}^{H} \sigma_a \, dz = \mathbf{35.59}\ \mathrm{kN/m}\ (15^\circ \text{ with the horizontal})$$

$$\bar{z} = 0.57\ \mathrm{m} \qquad \text{without crack: } P_a = -9.37\ \mathrm{kN/m}$$

4 H = 5 m: τ(5) = 22.5 kPa > c_u = 20 kPa

$$\tau(5) = 18(5)\sin 15^\circ \cos 15^\circ = 22.5 > c_u \ \Rightarrow\ \text{slope fails}$$

For H = 4 m, the cracked P_a = **35.59 kN/m**; the uncracked force is negative (–9.37 kN/m), so the tension-crack assumption is decisive. Since z_{max} = 4.44 m, there is no solution for a 5 m wall.



Check the solution in GeoTerva ↗

Scope of the solution

THEORY

The wall is frictionless; in the Chu solution the pressure is given at an angle β to the wall normal (β derives only from the soil's own stress state, not from wall–soil friction). For a wall with friction, see the Coulomb module.

The **embankment** is a plane surface with a single uniform slope; the layers are parallel to the surface. Changes in surface slope, benches or broken surfaces are not included in this solution.

Cohesive fill is given by Mazindrani–Ganjali only for a vertical wall ($\theta = 0$); a tension crack is assumed up to z_c, and a water-filled crack must be considered separately.

Geometry: the wall face and the surface must intersect at at least 15° ($|\alpha - \theta| \leq 75°$); otherwise the pressure acting on the face is not lateral earth pressure but the weight of the overlying soil.

Earthquake and **wall friction** are outside the scope of this module (Mononobe–Okabe, Coulomb).

Summary

Key points

Sloping fill ($\alpha \le \varphi'$): the pressure is **parallel** to the surface, $K_a = \cos\alpha\,(\cos\alpha - r)/(\cos\alpha + r)$, K_p with the opposite sign; $P = \tfrac{1}{2}K\gamma H^2$ at $H/3$ from the base, at angle α to the horizontal

$K_aK_p = \cos^2\alpha$; as α increases, K_a increases and K_p decreases; at $\alpha = \varphi'$ both become $\cos\varphi'$, and for $\alpha > \varphi'$ there is no solution

Chu (1991): $\psi_a = \sin^{-1}(\sin\alpha/\sin\varphi') - \alpha + 2\theta$; $K_{a(R)}$, β and η in closed form; the force is inclined at β from the face normal

Wall inclination θ: if positive the force increases, if negative it decreases; for passive, $\psi_p = \sin^{-1}(\ldots) + \alpha - 2\theta$

In a **cohesive** fill K'_a depends on depth, and the diagram is curved; $z_c = (2c'/\gamma)\sqrt{((1+\sin\varphi')/(1-\sin\varphi'))}$ is independent of the slope angle; the force with a crack is essential

Layered fill: $\sigma' = \sigma'_v K_{(R)}$; jump at the boundary, $P = \sqrt{(N^2 + T^2)}$

Undrained clay: if $z_{max} = c_u/(\gamma \sin\alpha \cos\alpha)$ is exceeded, the slope fails under its own weight and no solution exists

References

Das, B.M. (2018). Principles of Geotechnical Engineering, 9th ed. Cengage Learning.

Chu, S.C. (1991). Rankine analysis of active and passive pressures on dry sands. Soils and Foundations, 31(4), 115–120.

Das, B.M. (2016). Principles of Foundation Engineering, 8th ed. Cengage Learning.

Mazindrani, Z.H. and Ganjali, M.H. (1997). Lateral earth pressure problem of cohesive backfill with inclined surface. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 123(2), 110–112.

NOTE

All numerical examples are produced with the module's own calculation engine; the module gives the same values. The module opens with the wall from Example 4 (H = 3.658 m, γ = 18.07 kN/m³).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter the inclination of your own wall, the backfill slope, the layers and the groundwater in GeoTerva; the module gives the force, its direction and its height of application together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com