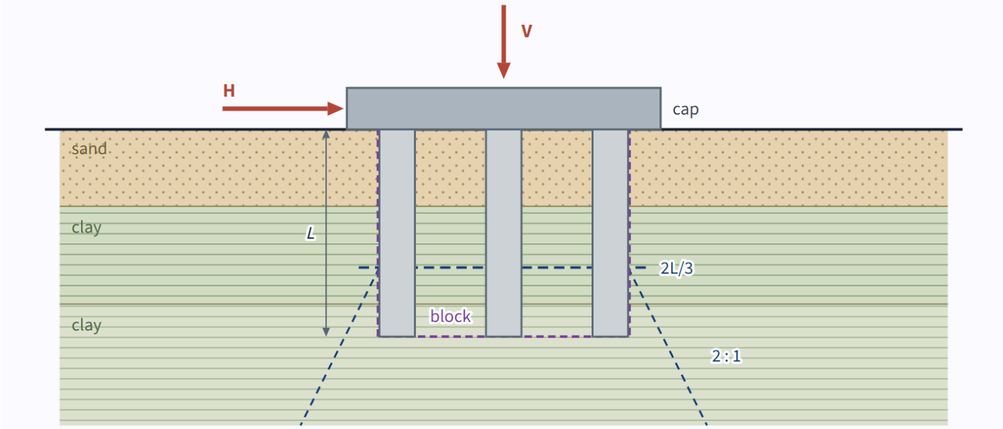
Geotechnical lecture presentation · Piles

Pile group analysis

Lecture: the load on each pile of a pile group under a cap, its capacity, its settlement and its behaviour under lateral load; a worked example for each topic.

- Rigid cap
- Group efficiency · block
- Settlement
- Lateral load: p-multipliers
- Worked examples



What will we learn in this lecture?

- Finding **the load on each pile** under a rigid cap and recognising the pile that tends to be in tension
- **Single pile** capacity built layer by layer as the basis of the group capacity
- Applying **group efficiency** (Converse–Labarre, Los Angeles, Seiler–Keeney; driven and bored rules in sand) layer by layer
- To check the group for **block failure** and select **the smaller**
- To compute elastic (Vesic) and **consolidation** settlement (2:1 spread)
- to interpret p-multipliers and the unequal shares of the rows under **lateral load**

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

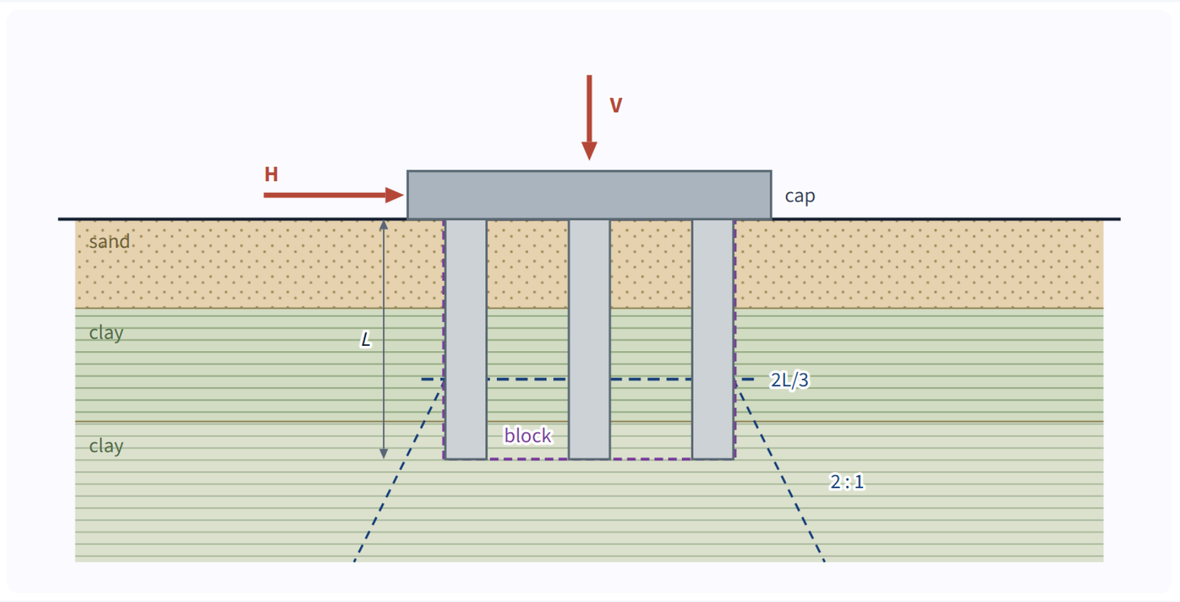
2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

This presentation builds on the **pile in layered soil** presentation: the layer-by-layer analysis of the single pile was covered there, and here we look at its place within the group.

What changes when the piles act together?

THEORY



Piles are arranged in rows under a **cap**; when they stand close together, the soil between and around them must serve all of them at once. Three things change, and the module treats each as a separate check:

Capacity: the soil between the piles is stretched from the shaft of each pile; each pile carries less than it would alone (efficiency $\eta < 1$). If the soil is weak and the piles are close together, the group and the soil between them may also fail as a single **block**.

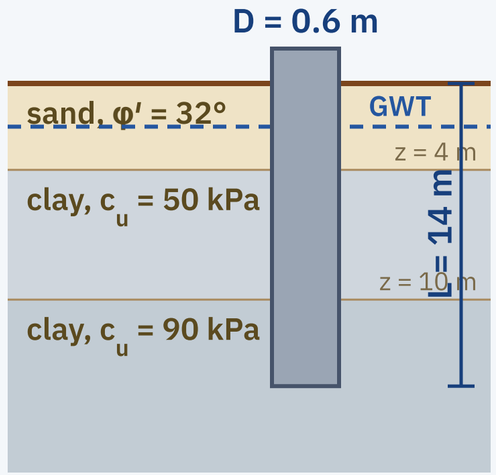
Settlement: the group load spreads over **the entire group area**, not that of a single pile, and reaches the clay layers far below the tips; the group settles far more than a single pile.

Lateral load: the rear rows work in soil pushed and strained by the preceding piles; the front row takes more load and the others less.

The group we will use throughout the lecture

Example group (model opened by the module)

Piles	**9 bored piles (diameter D = 0.60 m, length L = 14 m), 3 rows × 3 piles, cap base in the ground**
Spacing	$s_x = s_y = 1.80\ \text{m}\ (3.0\,D)$
Soil	**sand 0–4 m (φ′ = 32°), clay 4–10 m (c_u = 50), clay 10–18 m (c_u = 90 kPa); γ = 18, γ_{sat} = 19 kN/m³**
Groundwater	**at 2.0 m depth**
Loads	$V = 1\,600\ \text{kN},\ H = 900\ \text{kN},\ M_y = 900\ \text{kN}\cdot\text{m}$
Material, safety	$E_p = 30\,000\ \text{MPa},\ E_s = 30\ \text{MPa},\ \nu_s = 0.30;\ FS = 3$



Most of the examples use this group; the results are identical to the numbers in the **Model** tab of the module.

Rigid cap: the piles are identical springs

THEORY

The cap is assumed rigid and the piles identical vertical springs. A rigid cap moves as a **plane**: its settlement and rotations are the only unknowns. Therefore the load on each pile is a **linear** function of its position (bolt-group rule):

$$P_i = \frac{V}{n} + B\,x_i + C\,y_i$$

$$B = \frac{M_y}{\sum x^2} \qquad C = \frac{M_x}{\sum y^2} \qquad \left(\sum xy = 0\right)$$

$$\sum x^2 = n_y s_x^2 \,\frac{n_x(n_x^2 - 1)}{12}$$

If M_y is positive, it adds pressure to the **+x** side (the side pushed by the lateral load, the front side). Moments are taken **at the base of the cap**: a load H at height h above the base gives M_y = H h.

If the piles are arranged in a staggered (triangular) pattern, Σxy is not zero; the module takes the sums from the pile positions and solves the two equations together.

The largest load goes to the front piles and the smallest to the rear ones; **if the moment is large** the rear pile may tend to go into tension.

Problem statement

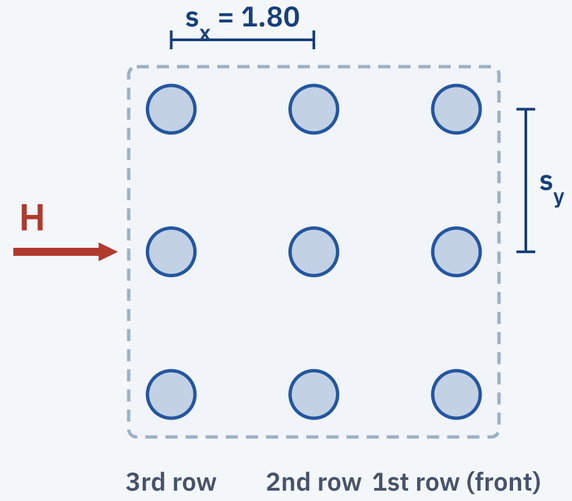
EXAMPLE

Given

Group	**9 bored piles, 3 rows × 3 piles, D = 0.60 m, L = 14 m, s = 1.80 m**
Soil	**sand 0–4 m (φ′ = 32°) · clay 4–10 m (c_u = 50) · clay 10–18 m (c_u = 90 kPa)**
Loads	$V = 1\,600$ kN, $M_y = 900$ kN·m **(at the base of the heading)**
Title	**rigid; piles identical**

REQUIRED

Find the load on **each pile**. The rows are numbered **from the front** (front = the side pushed by the lateral load, +x).



Solution (1/2)

SOLUTION

1 Equal spacing

$$\frac{V}{n} = \frac{1\,600}{9} = \mathbf{177.8}\ \mathrm{kN}$$

2 Σx² (symmetric group, closed form)

$$\sum x^2 = n_y s_x^2 \, \frac{n_x(n_x^2 - 1)}{12} = 3(1.8)^2 \, \frac{3(9-1)}{12} = \mathbf{19.44}\ \mathrm{m}^2$$

3 Moment coefficient (C = 0 since $M_x = 0$)

$$B = \frac{M_y}{\sum x^2} = \frac{900}{19.44} = \mathbf{46.30}\ \mathrm{kN/m}$$

Solution (2/2)

SOLUTION

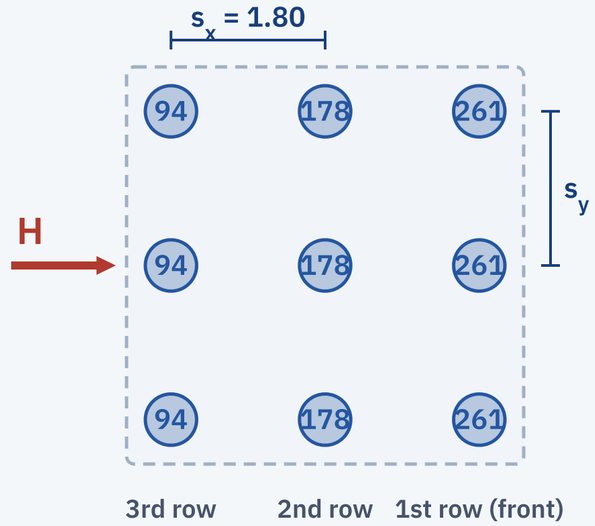
4 Row loads: P = V/n + B x (x = +1.8; 0; −1.8 m)

$$P_1 = 177.8 + 46.30(1.8) = \mathbf{261.1}\ \text{kN (front)}$$

$$P_2 = \mathbf{177.8}\ \text{kN}$$

$$P_3 = 177.8 - 46.30(1.8) = \mathbf{94.4}\ \text{kN (back)}$$

Total: 3(261.1 + 177.8 + 94.4) = **1,600 kN** = V. The largest load is on the front piles (261.1 kN), the smallest on the rear ones (94.4 kN); **no pile is in tension**.



Check the solution in GeoTerva ↗

If the moment is large, the rear piles are in tension

THEORY

If the moment is large relative to the vertical load, the piles on the opposite side of the moment **are unloaded** and may be subjected to tension. The pile resists tension **only through its shaft**: the clay layer with its full value, the sand layer with **three-quarters** of its compressive resistance; the base contributes nothing.

$$T_u = \sum_i \chi_i \, Q_{s,i} \qquad \chi = 1 \ (\text{clay}),\ \chi = 0.75 \ (\text{sand})$$

$$T_g = \min(T_{ind},\, T_{block}) \quad T_{ind} = n_T \sum_i \eta_i \chi_i Q_{s,i}$$

Single pile in tension: $T_{max} \le T_u/FS$. Piles in tension **together**: $\Sigma T \le T_g/FS$; the block is held by its own weight and the shear on its side faces.

CHECK IN COMPRESSION

$$P_{max} \le \frac{Q_u}{FS}$$

The largest pile load is compared with the allowable load of a **single pile**; group effects are not considered here. Under moment, the most heavily loaded pile receives more than V/n and may exceed its share even **when the group as a whole is within its capacity**.

Problem statement

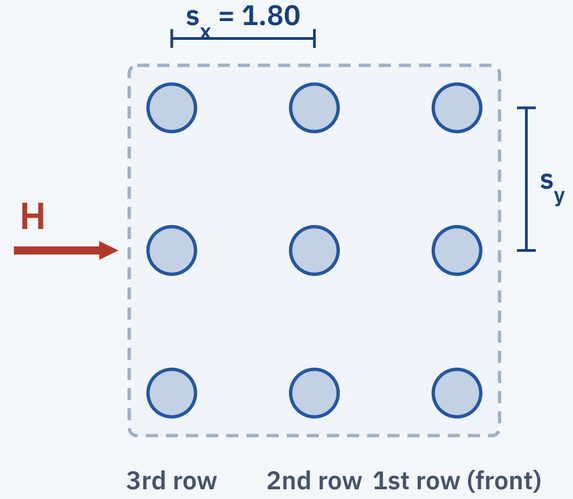
EXAMPLE

Given

Group	**9 bored piles, 3 rows × 3 piles, D = 0.60 m, L = 14 m, s = 1.80 m**
Soil	**sand 0–4 m (φ′ = 32°) · clay 4–10 m (c_u = 50) · clay 10–18 m (c_u = 90 kPa)**
Load	$V = 1\,600$ kN, **moment increased:** $M_y = 2\,400$ kN·m
Single pile	**Q_s contributions (Example 3): sand 49.8, clay 311.0, clay 317.2 kN**
Efficiency	**sand 0.767, clay 0.727 (Example 5–6)**

REQUIRED

Find the loads; check the capacity of the pile in tension and of the piles in tension **together** with FS = 3.



Solution (1/2)

SOLUTION

1 Loads ($B = M_y/\Sigma x^2$)

$$B = \frac{2\,400}{19.44} = 123.46\ \text{kN/m}$$

$$P_{front} = 177.8 + 123.46(1.8) = \mathbf{400.0}\ \text{kN}$$

$$P_{rear} = 177.8 - 123.46(1.8) = \mathbf{-44.4}\ \text{kN (tension)}$$

2 Capacity of a single pile in tension (sand × 0.75, clay × 1)

$$T_u = 0.75(49.8) + 311.0 + 317.2 = \mathbf{665.6}\ \text{kN} \quad \Rightarrow \quad \frac{T_u}{FS} = 221.9\ \text{kN}$$

Pile in tension: T_{max} = **44.4 kN** ≤ T_u/FS = 221.9 kN → **adequate**. Number of piles in tension is 3 (rear row), total tension ΣT = 133.3 kN; group check is in the next step.

Solution (2/2)

SOLUTION

3 Combined capacity of the 3 piles in tension: sum of effective shafts × 3

$$T_{ind} = 3\left[\eta_s \chi_s Q_{s1} + \eta_c Q_{s2} + \eta_c Q_{s3}\right] = \mathbf{1{,}456}\ \text{kN}$$

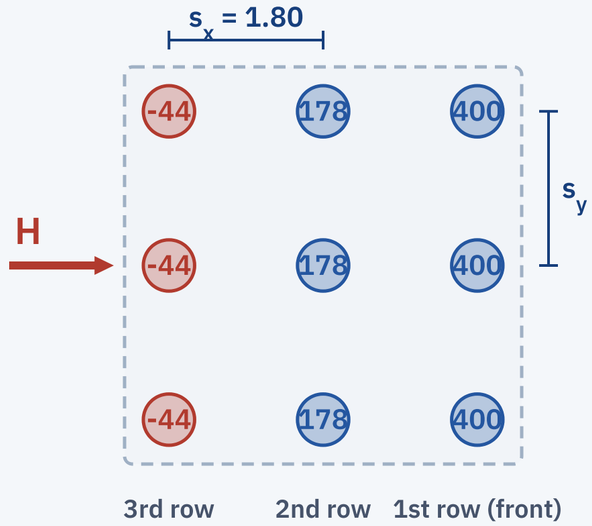
4 Tension block (3 piles, L_t = D = 0.6 m, B_t = 4.2 m): side shear + block weight

$$T_{block} = \sum \tau_i P_t \Delta L_i + W = 6{,}692 + 369 = \mathbf{7{,}061}\ \text{kN}$$

5 Check: minimum and FS

$$T_g = \min(1{,}456;\ 7{,}061) = 1{,}456\ \text{kN}$$

$$\sum T = 133.3 \le \frac{T_g}{3} = 485\ \text{kN} \ \Rightarrow\ \text{sufficient}$$



Check the solution in GeoTerva ↗

Group capacity is built on that of the single pile

THEORY

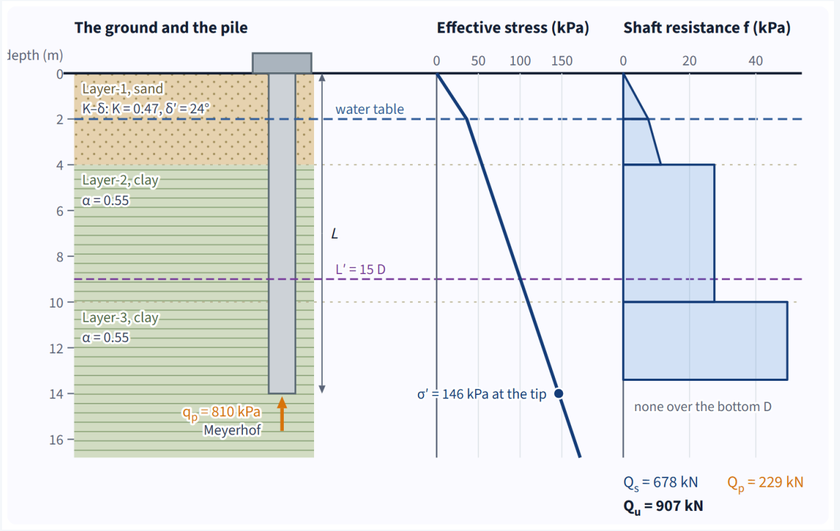
First, the **single** resistance of one pile in the group is computed: each layer by the rule for its own material, **no entered resistance**. The rules are as in the *pile in layered soil* presentation:

$$Q_u = Q_s + Q_p \qquad Q_s = p \sum_i f_i \, \Delta L_i \qquad Q_p = A_b \, q_p$$

Sand: f = Kσ′tanδ′ (critical depth L′ = 15D). For a bored pile **K = K₀ = 1 − sinφ′**, δ′ = 0.75φ′; for driven, K = 1.4K₀.

Clay: α, λ or β. For a bored pile, **α* = 0.55** ($c_u/p_a \le 1.5$) and α* = 0 along the **top 1.5 m and the bottom 1D**.

Tip: the rule of the layer in which the tip lies: $q'N^*_q \le q_l$ in sand, $9c_u$ in clay (Reese–O'Neill may be selected for bored piles).



Problem statement

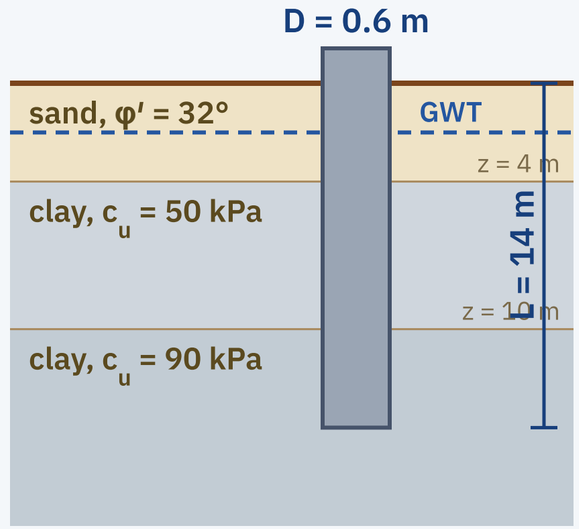
EXAMPLE

Given

Group	**9 bored piles, 3 rows × 3 piles, D = 0.60 m, L = 14 m, s = 1.80 m**
Soil	**sand 0–4 m (φ′ = 32°) · clay 4–10 m (c_u = 50) · clay 10–18 m (c_u = 90 kPa)**
Establishment	**bored pile: K = K_0 = 0.47, δ′ = 0.75φ′ = 24°, α* = 0.55; tip: Meyerhof**
Groundwater	**2.0 m**
Unit weight	$\gamma = 18,\ \gamma_{sat} = 19\ \mathrm{kN/m^3}$

REQUIRED

Determine the **shaft** resistance of a single pile layer by layer, its **tip** resistance and the value of $\mathbf{Q_u}$; check $P_{max} \le Q_u/FS$.



Solution (1/3)

SOLUTION

1. Sand (0–4 m): L′ = 15D = 9 m > 4 m, no cap; σ′(2) = 18(2) = 36.0 and σ′(4) = 36.0 + (19 − 9.81)(2) = 54.4 kPa

$$\bar{\sigma}' = \frac{126.4}{4} = 31.6\ \text{kPa}$$

$$f = K \tan\delta'\, \bar{\sigma}' = 0.47(0.4452)(31.6) = 6.61\ \text{kPa}$$

$$Q_{s1} = p\, f\, H = 1.885(6.61)(4) = \mathbf{49.8}\ \text{kN}$$

Solution (2/3)

SOLUTION

2 Clay (4–10 m): α* = 0.55 (c_u/p_a = 0.5 ≤ 1.5); the 1.5 window → L − D = 13.4 m fully covers the layer

$$f = 0.55(50) = 27.5\ \text{kPa} \qquad Q_{s2} = 1.885(27.5)(6) = \mathbf{311.0}\ \text{kN}$$

3 Clay (10–14 m): the window ends at 13.4 m → lower 1D = 0.6 m is disregarded, effective length 13.4 − 10 = 3.4 m

$$f = 0.55(90) = 49.5\ \text{kPa} \qquad Q_{s3} = 1.885(49.5)(3.4) = \mathbf{317.2}\ \text{kN}$$

Solution (3/3)

SOLUTION

4 Shaft sum and end (layer at the end: clay, c_u = 90 kPa, Meyerhof N^*_c = 9)

$$Q_s = 49.8 + 311.0 + 317.2 \approx \mathbf{678.1}\ \text{kN}$$

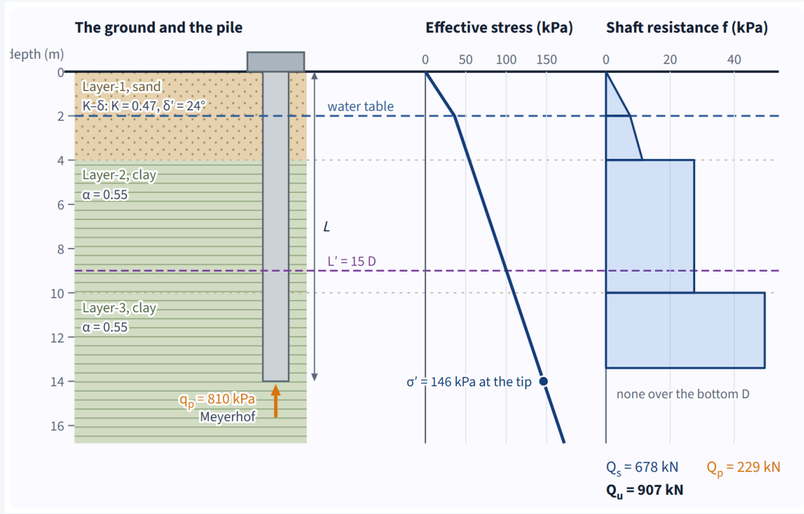
$$q_p = 9(90) = 810\ \text{kPa}$$

$$Q_p = A_b q_p = 0.2827(810) = \mathbf{229.0}\ \text{kN}$$

5 Capacity of a single pile and check

$$Q_u = 678.1 + 229.0 = \mathbf{907.1}\ \text{kN}$$

$$\frac{Q_u}{FS} = 302.4\ \text{kN} \geq P_{max} = 261.1\ \text{kN} \ \Rightarrow\ \text{sufficient}$$



Check the solution in GeoTerva ↗

Closely spaced piles carry less than they would individually

THEORY

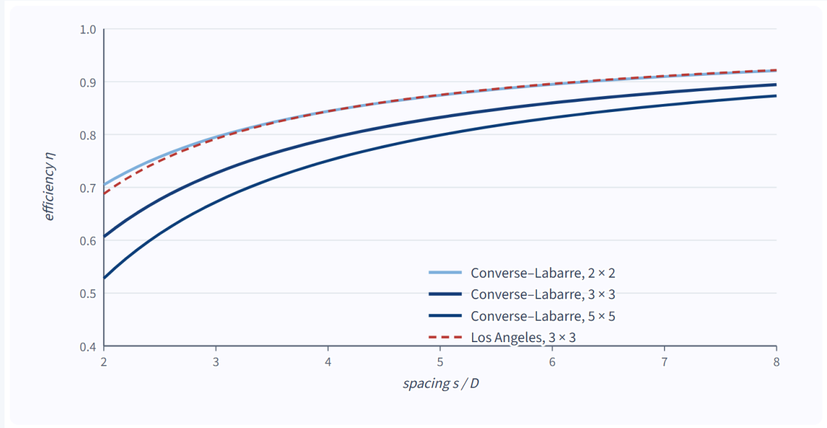
As the piles are brought closer together, the soils sheared and strained by each pile overlap. **Group efficiency** is the ratio of the group capacity to the sum of the single-pile capacities. The source gives three empirical equations for a group of $n_1 \times n_2$ piles of diameter D at spacing d:

$$\eta = \frac{Q_{g(u)}}{\sum Q_u}$$

$$\eta_{CL} = 1 - \frac{\theta}{90}\,\frac{(n_1 - 1)n_2 + (n_2 - 1)n_1}{n_1 n_2}, \quad \theta = \tan^{-1}\frac{D}{d}\ (\text{degree})$$

Converse–Labarre counts **pairs of adjacent piles**: $(n_1 - 1)n_2$ pairs in a row and $(n_2 - 1)n_1$ pairs side by side, each with an angle $\theta = \tan^{-1}(D/d)$ of its own spacing.

The module uses this equation for clay; it reduces the shaft of clay layers and the base of a pile whose tip is in clay with it.



As spacing increases, efficiency approaches 1; in a 3 × 3 group, about 0.7 – 0.8 at 3D.

The other two equations and limits

THEORY

The other two equations of the source are also for a rectangular group. The **Los Angeles** equation and the **Seiler–Keeney** equation (with d in **ft**):

$$\eta_{LA} = 1 - \frac{D}{\pi d\, n_1 n_2}\left[n_1(n_2-1) + n_2(n_1-1) + \sqrt{2}(n_1-1)(n_2-1)\right]$$

$$\eta_{SK} = \left[1 - \frac{11d}{7(d^2-1)}\,\frac{n_1+n_2-2}{n_1+n_2-1}\right] + \frac{0.3}{n_1+n_2}$$

Seiler–Keeney has a pole at 1 ft and is meaningless below 2 ft; the module reports it from **2 ft** onward.

A spacing smaller than 2.5D is **flagged**: this is the smallest spacing the source gives for friction piles, and the equations were written for wider spacings.

For a triangular arrangement, Converse–Labarre is written by counting adjacent pairs; this is an **extension of the module** and gives a lower efficiency than the mesh at the same spacing.

Problem statement

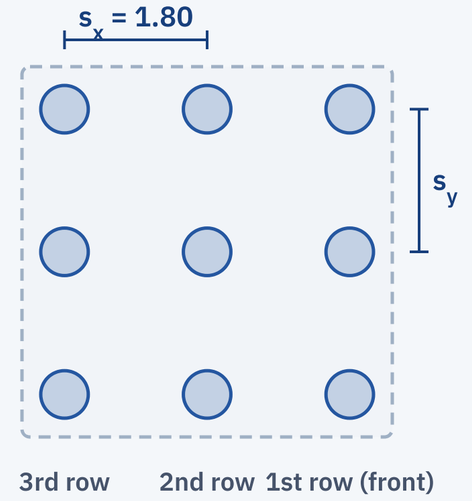
EXAMPLE

REQUIRED

Determine the group efficiency using the **Converse–Labarre**, **Los Angeles** and **Seiler–Keeney** equations.

Given

Group	**3 × 3 piles, D = 0.60 m, spacing d = 1.80 m (in both directions)**
Seiler–Keeney	**d in feet: 1.80 m = 5.906 ft**



Solution

SOLUTION

1 Converse–Labarre: θ = tan⁻¹(D/d); adjacent pairs: (2)(3) + (2)(3) = 12

$$\theta = \tan^{-1}\frac{0.6}{1.8} = 18.43^\circ\ ;\quad \eta_{CL} = 1 - \frac{18.43}{90}\cdot\frac{12}{9} = \mathbf{0.727}$$

2 Los Angeles

$$\eta_{LA} = 1 - \frac{0.6}{\pi(1.8)(9)}\left[3(2) + 3(2) + \sqrt{2}(2)(2)\right] = \mathbf{0.792}$$

3 Seiler–Keeney (d = 5.906 ft)

$$\eta_{SK} = \left[1 - \frac{11(5.906)}{7(5.906^2 - 1)}\cdot\frac{4}{5}\right] + \frac{0.3}{6} = \mathbf{0.831}$$

Three equations in the range 0.727 – 0.831. In clay the module uses **Converse–Labarre** (0.727).

Check the solution in GeoTerva ↗

Efficiency in sand: a rule, not an equation

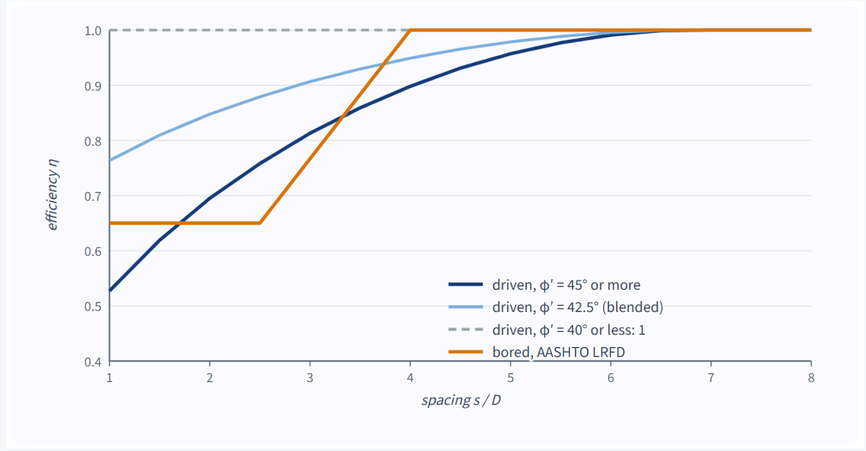
THEORY

A driven pile **compacts** loose or medium-dense sand; in this sand the group carries as much as the sum of the piles (in tests even more). Only in dense sand (φ′ > about 40°) does close spacing reduce capacity. For a driven pile the module takes the source's lower bound and **never raises the efficiency above 1**:

- φ′ ≤ 40°: **η = 1**
- 40° < φ′ < 45°: **linear interpolation** between 1 and the 45° curve
- φ′ ≥ 45°: the 45° curve (1.0 at s/D = 7; 0.53 at s/D = 1)

Bored pile does not compact the soil, it may even loosen it. AASHTO LRFD multiplies the resistance of each bored pile in the group by **0.65** at s/D = 2.5 and by **1.0** at 4:

$$\eta = 0.65 + 0.35\,\frac{s/D - 2.5}{1.5} \qquad 2.5 \le s/D \le 4$$



The capacity of the sand layer is applied to the shaft of that layer and to the tip resistance of the pile whose base is in the sand. Spacing: **the smallest distance between two piles**.

Problem statement

EXAMPLE

Given

Spacing	**s = 1.80 m, D = 0.60 m: s/D = 3.0**
Bored pile	**AASHTO rule in sand**
Driven pile	**sand, $\varphi' = 42^\circ$ (45° curve at s/D = 3: $\eta_{45} = 0.813$)**

REQUIRED

Determine the efficiency of the sand layer for a **fore** pile and for a **driven** pile in sand with φ′ = 42°; what happens for φ′ = 35°?

Solution

SOLUTION

1 Bored pile (AASHTO): s/D = 3.0

$$\eta = 0.65 + 0.35\,\frac{3.0 - 2.5}{1.5} = \mathbf{0.767}$$

2 Driven pile, φ′ = 42°: weight w = (φ′ − 40)/5 and the 45° curve

$$w = \frac{42 - 40}{5} = 0.4 \qquad \eta = 1 - w(1 - \eta_{45}) = 1 - 0.4(1 - 0.813) = \mathbf{0.925}$$

3 Driven pile, φ′ = 35°

$$\varphi' \le 40^\circ \;\Rightarrow\; \eta = \mathbf{1.000}$$

In the same range, a **bored** pile takes 0.767 in sand, while a **driven** pile (in loose–medium sand) takes 1.00: driving densifies, drilling does not.

Check the solution in GeoTerva ↗

The modulus of each layer is the modulus of that soil

THEORY

The pile shaft is cut at layer boundaries and each segment is calculated by its own method. A group effect related to the soil should also be applied **segment by segment**: a layer's capacity is that soil's capacity, while the **base** takes the capacity of the soil it rests on:

$$Q_{ind} = n \left(\sum_i \eta_i \, Q_{s,i} + \eta_b \, Q_p \right)$$

$$\eta_{group} = \frac{Q_{ind}}{\sum Q_u}$$

Clay layers and the base of a pile with its tip in clay: Converse–Labarre.

Sand layer: chart for driven piles, AASHTO rule for bored piles.

The group efficiency is **not an input value** but a **weighted average** of the layer contributions: a pile taking most of its load from sand reflects that sand's group effect, while a pile hanging in clay reflects the clay's group effect.

Problem statement

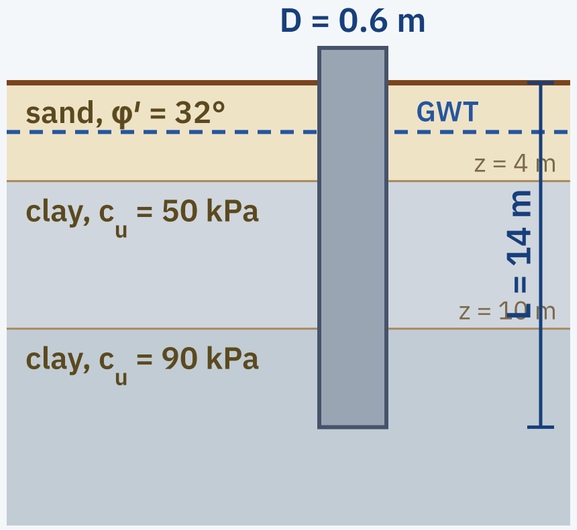
EXAMPLE

Given

Single pile (Example 3)	**Q_{s1} = 49.8 (sand), Q_{s2} = 311.0, Q_{s3} = 317.2 (clay), Q_p = 229.0 kN**
Efficiencies	**sand (bored, s/D = 3): 0.767; clay: 0.727; base (clay): 0.727**
Group	**9 piles**

REQUIRED

What are Q_{ind} and the **group efficiency** $\eta = Q_{ind}/\Sigma Q_u$?



Solution

SOLUTION

1 Efficient contribution of each part (η Q): sand shaft, clay shaft 1, clay shaft 2, base

$$0.767(49.8) = 38.2\ ; \quad 0.727(311.0) = 226.1$$
$$0.727(317.2) = 230.6\ ; \quad 0.727(229.0) = 166.5$$

2 Single-pile working capacity and group

$$\sum \eta Q = 38.2 + 226.1 + 230.6 + 166.5 = \mathbf{661.4}\ \text{kN}$$
$$Q_{ind} = 9(661.4) \approx \mathbf{5{,}952}\ \text{kN}$$

3 Group efficiency

$$\eta = \frac{Q_{ind}}{n\,Q_u} = \frac{5{,}952}{9(907.1)} = \mathbf{0.729}$$

The group carries **73%** of the sum of the piles acting individually (sand η = 0.767, clay η = 0.727).

Check the solution in GeoTerva ↗

The group may fail like a deep foundation

THEORY

If the piles are close together, the group and the soil between them may move down **together**: a deep foundation of size $L_g \times B_g$ and depth L. Its capacity is the **shear** on the four sides plus the **bearing** below the base:

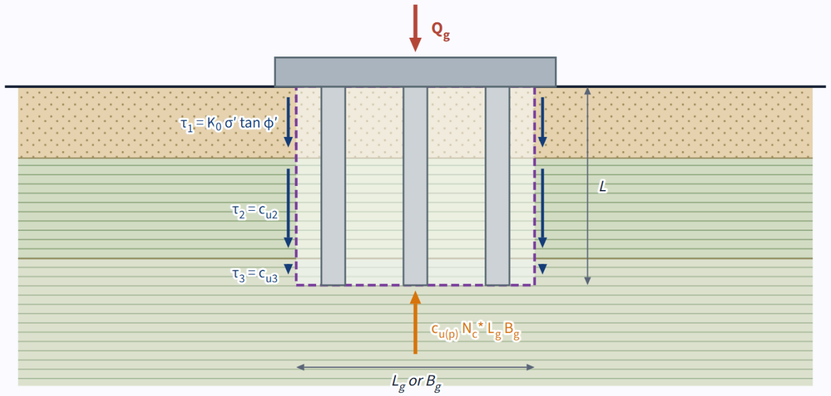
$$Q_{block} = \sum_i \tau_i \, P_b \, \Delta L_i + q_b \, L_g B_g \qquad P_b = 2(L_g + B_g)$$

Clay side face: $\tau = c_u$ (soil slides against soil, not reduced by α).

Sand side face (extension of the module from the source): $\tau = K_0 \bar{\sigma}' \tan\varphi'$, $K_0 = 1 - \sin\varphi'$.

Clay base: $q_b = c_u N^*_c$; N^*_c from the chart, according to L/B_g and L_g/B_g. **Sand base**: nQ_p.

$$Q_{g(u)} = \min(Q_{ind},\, Q_{block}) \qquad Q_{all} = \frac{Q_{g(u)}}{FS}$$



Problem statement

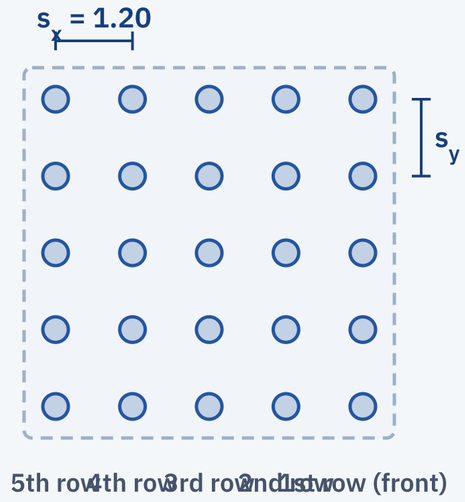
EXAMPLE

Given

Group	9 bored piles, 3 rows × 3 piles, D = 0.60 m, L = 14 m, s = 1.80 m
Soil	sand 0–4 m ($\varphi' = 32°$) · clay 4–10 m (c_u = 50) · clay 10–18 m (c_u = 90 kPa)
Block	$L_g = B_g = (3 - 1)(1.8) + 0.6$
Side faces	sand: $\tau = K_0 \bar{\sigma}' \tan\varphi'$ ($\bar{\sigma}'$ = 31.6 kPa); clay: $\tau = c_u$
Base	clay, c_u = 90 kPa; N^*_c from the chart (L_g/B_g = 1)
Individual	Q_{ind} = 5,952 kN (Example 6); V = 1,600 kN, FS = 3

REQUIRED

Find the value of Q_{block}, the capacity of the group and its allowable load.



Solution (1/2)

SOLUTION

1 Block dimensions and perimeter

$$L_g = B_g = 4.2\ \text{m} \qquad P_b = 2(L_g + B_g) = \mathbf{16.8}\ \text{m}$$

2 Shear on side faces ($\tau\ P_b\ \Delta L$)

$$\text{sand}: \tau = K_0 \bar{\sigma}' \tan\varphi' = 0.470(31.6)(0.6249) = 9.28 \ \Rightarrow\ 624\ \text{kN}$$

$$\text{clay 1}: \tau = 50 \ \Rightarrow\ 50(16.8)(6) = 5{,}040\ \text{kN}$$

$$\text{clay 2}: \tau = 90 \ \Rightarrow\ 90(16.8)(4) = 6{,}048\ \text{kN}$$

Solution (2/2)

SOLUTION

3 Base: N^*_c from the chart for $L/B_g = 14/4.2 = 3.33$ and $L_g/B_g = 1$

$$q_b L_g B_g = c_u N_c^* \, L_g B_g = 90(8.90)(17.64) = \mathbf{14{,}126}\ \text{kN}$$

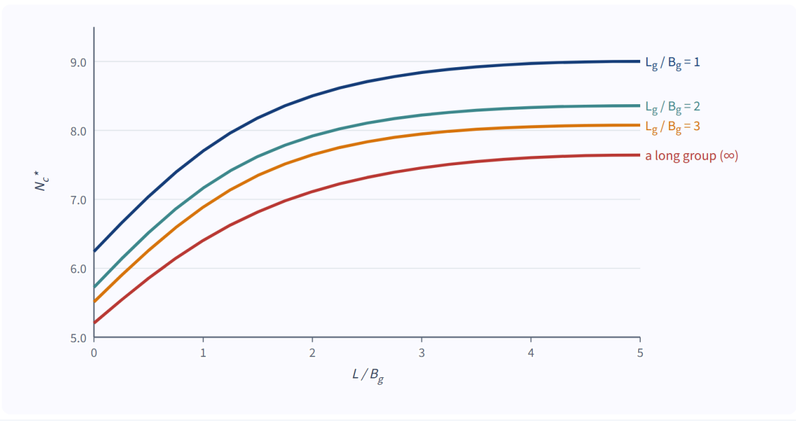
4 Block capacity, capacity of the group and allowable load

$$Q_{block} = 11{,}712 + 14{,}126 = 25{,}838\ \text{kN}$$

$$Q_{g(u)} = \min(5{,}952;\ 25{,}838) = \mathbf{5{,}952}\ \text{kN}$$

$$Q_{all} = \frac{5{,}952}{3} = 1{,}984\ \text{kN} \geq V = 1\,600\ \text{kN}$$

The block capacity is **more than 4 times** the individual sum: in this group **the individual piles govern** (individual). The block comes into play in closely spaced groups with many piles; see the chart on the side.



Check the solution in GeoTerva ↗

Problem statement

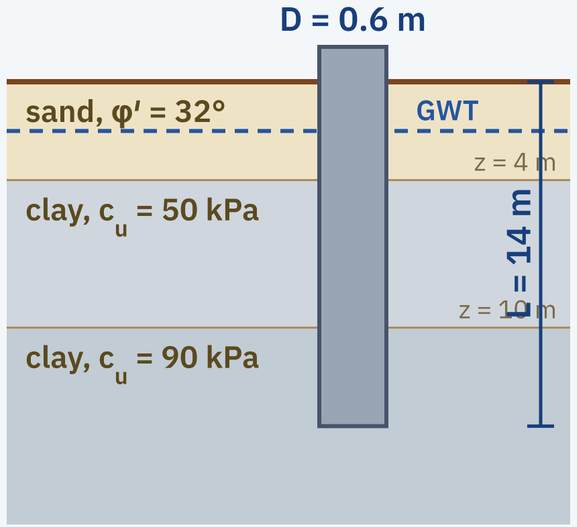
EXAMPLE

Given

Group	5 × 5 driven pile, D = 0.40 m, s = 1.20 m (3D), L = 12 m
Soil	single sand, φ′ = 32°, γ = 18, γ_{sat} = 19 kN/m³, groundwater at 3 m
Single pile	Q_s = 285.7, Q_p = 318.0 kN (driven, K–δ and Meyerhof)
Efficiency	driven pile, in sand with φ′ < 40°: η = 1

REQUIRED

Compare the Q_{ind} and Q_{blok} values and find the capacity of the group.



Solution (1/2)

SOLUTION

1 Individual piles (η = 1)

$$Q_{ind} = n\,Q_u = 25(603.7) = \mathbf{15{,}094}\ \text{kN}$$

2 Block: $L_g = B_g = 4(1.2) + 0.4 = 5.2$ m; $\sigma'(3) = 54.0$, $\sigma'(12) = 136.7$ kPa

$$P_b = 2(5.2 + 5.2) = 20.8\ \text{m} \qquad \bar{\sigma}' = \frac{939.2}{12} = 78.3\ \text{kPa}$$

$$\tau = K_0 \bar{\sigma}' \tan\varphi' = 0.470(78.3)(0.6249) = 22.99\ \text{kPa}$$

$$\text{lateral sliding} = 22.99(20.8)(12) = \mathbf{5{,}738}\ \text{kN}$$

Solution (2/2)

SOLUTION

3 Block base in sand: the base is the sum of the end resistances of the piles

$$\text{base} = nQ_p = 25(318.0) = \mathbf{7{,}950}\ \text{kN} \qquad Q_{block} = 5{,}738 + 7{,}950 = \mathbf{13{,}689}\ \text{kN}$$

4 Capacity and efficiency of the group

$$Q_{block} = 13{,}689 \ < \ Q_{ind} = 15{,}094$$

$$Q_{g(u)} = \mathbf{13{,}689}\ \text{kN} \qquad \eta_{group} = \frac{13{,}689}{15{,}094} = \mathbf{0.907}$$

Here the **block governs**: in a large group the sum of the pile shafts exceeds the slip capacity of the group's outer perimeter. Although the piles individually show η = 1.00, the capacity of the group is **91%** of the individual sum.

Check the solution in GeoTerva ↗

Vesic: three parts, then the group

THEORY

A single pile settles through the shortening of its own length, the compression of the soil beneath the base, and the compression of the soil loaded along the shaft. Vesic (1969) sums these three components for the working load Q_w (Q_{ws} is the shaft share, Q_{wp} the base share):

$$s_e = s_{e1} + s_{e2} + s_{e3}$$

$$s_{e1} = \frac{(Q_{wp} + \xi Q_{ws})L}{A_p E_p} \qquad s_{e2} = \frac{q_{wp} D}{E_s}(1 - \mu_s^2) I_{wp}$$

$$s_{e3} = \frac{Q_{ws}}{pL}\frac{D}{E_s}(1 - \mu_s^2) I_{ws} \qquad I_{ws} = 2 + 0.35\sqrt{L/D}$$

$$S_{g(e)} = \sqrt{\frac{B_g}{D}}\, s_e$$

The working load $Q_w = V/n$; the module divides it between the shaft and the base in the ratio **$Q_s : Q_p$**.

The group settles **$\sqrt{(B_g/D)}$** times more than a single pile under the same pile load (Vesic's empirical relation for piles in sand).

ξ: distribution of shaft resistance, between 0.5 (uniform) and 0.67 (increasing with depth); I_{wp} = 0.85.

Problem statement

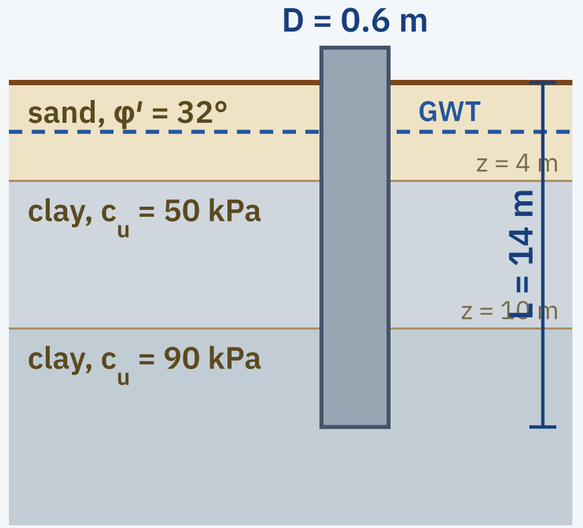
EXAMPLE

Given

Group	**9 bored piles, 3 rows × 3 piles, D = 0.60 m, L = 14 m, s = 1.80 m**
Soil	**sand 0–4 m (φ′ = 32°) · clay 4–10 m (c_u = 50) · clay 10–18 m (c_u = 90 kPa)**
Working load	**Q_w = V/n = 177.8 kN; Q_s : Q_p = 678.1 : 229.0 (Example 3)**
Material	$E_p = 30\,000$ MPa, $E_s = 30$ MPa, $\nu_s = 0.30$, $\xi = 0.6$, $I_{wp} = 0.85$

REQUIRED

Find the elastic settlement of a single pile in three parts, and the elastic settlement of the **group**.



Solution (1/2)

SOLUTION

1 Distribution of the working load

$$Q_{ws} = Q_w \frac{Q_s}{Q_u} = 177.8 \frac{678.1}{907.1} = 132.9\ \text{kN} \qquad Q_{wp} = 177.8 - 132.9 = 44.9\ \text{kN}$$

2 Pile shortening

$$s_{e1} = \frac{(44.9 + 0.6(132.9))(14)}{(0.2827)(30\,000\,000)} = 0.21\ \text{mm}$$

3 Base

$$q_{wp} = \frac{Q_{wp}}{A_b} = 158.7\ \text{kPa} \quad s_{e2} = \frac{158.7(0.6)}{30\,000}(1 - 0.3^2)(0.85) = 2.46\ \text{mm}$$

Solution (2/2)

SOLUTION

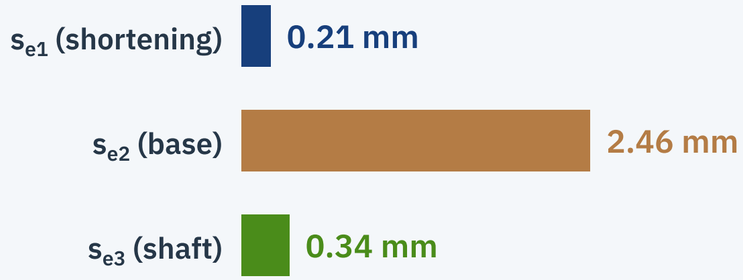
4 Shaft: $I_{ws} = 2 + 0.35\sqrt{(14/0.6)} = 3.69$

$$s_{e3} = \frac{132.9}{(1.885)(14)} \cdot \frac{0.6}{30\,000}(0.91)(3.69) = 0.34 \text{ mm}$$

5 Single pile and group

$$s_e = 0.21 + 2.46 + 0.34 \approx 3.00 \text{ mm}$$

$$S_{g(e)} = \sqrt{\frac{B_g}{D}}\, s_e = \sqrt{\frac{4.2}{0.6}}(3.00) = \mathbf{7.9} \text{ mm}$$



82% of the settlement comes from the base component (s_{e2}); the total elastic settlement is **7.9 mm**. The main settlement of the clay layers comes in the next step: consolidation.

Check the solution in GeoTerva ↗

Slow and large settlement of clay layers

THEORY

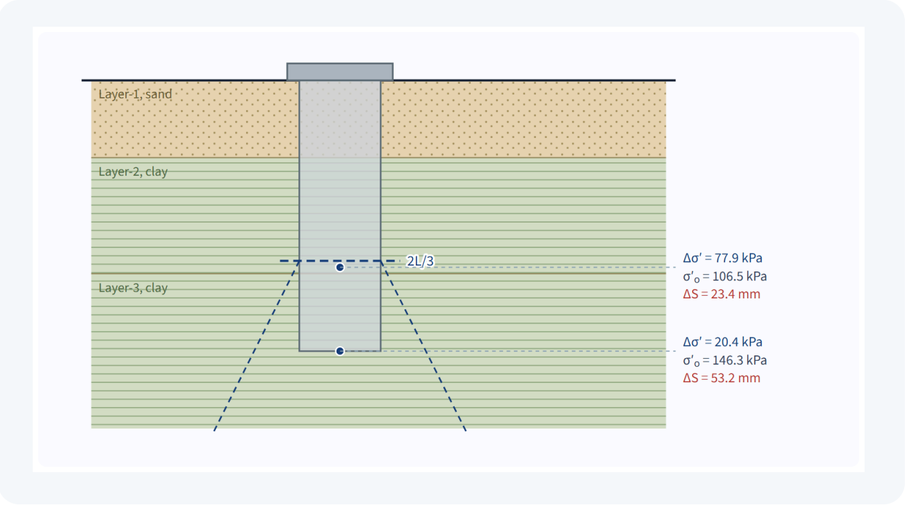
The clays beneath the cap settle slowly and mostly by **consolidation**. The source applies the group load from the cap level to an equivalent foundation **2L/3 below** it (the piles have transferred part of the load to the soil above) and spreads it below at **2 vertical : 1 horizontal**. At the middle of a clay layer, z below the 2L/3 level, the added stress:

$$\Delta\sigma'_i = \frac{Q_g}{(B_g + z_i)(L_g + z_i)}$$

$$\Delta S_i = \frac{C_c H_i}{1 + e_0} \log \frac{\sigma'_o + \Delta\sigma'_i}{\sigma'_o} \quad \text{(NC)}$$

In overconsolidated clay, C_s (C_c/5 if not given) is used up to the preconsolidation stress, and C_c beyond it.

The part of the clay layer **below** 2L/3 is included in the calculation; σ'_o is taken at the middle of this part, according to the layer and the water level. $S_c = \Sigma\Delta S_i$.



Problem statement

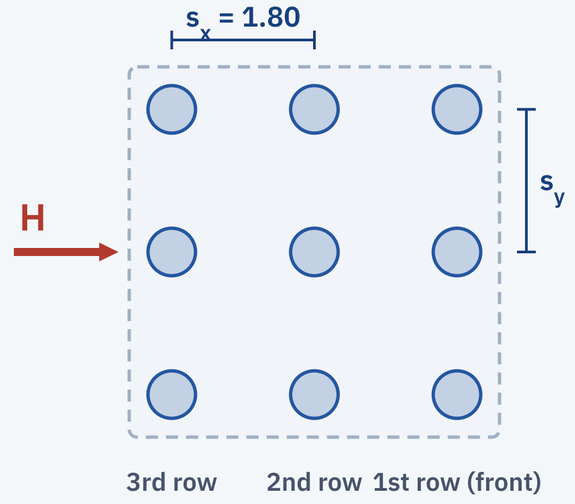
EXAMPLE

Given

Group	**9 bored piles, 3 rows × 3 piles, D = 0.60 m, L = 14 m, s = 1.80 m**
Soil	**sand 0–4 m (φ′ = 32°) · clay 4–10 m (c_u = 50) · clay 10–18 m (c_u = 90 kPa)**
Load	$Q_g = V = 1\,600$ kN; **block** $L_g = B_g = 4.2$ m
Clay 1 (4–10 m)	$e_0 = 0.90,\ C_c = 0.28$ **(NC)**
Clay 2 (10–18 m)	$e_0 = 0.70,\ C_c = 0.20$ **(NC)**

REQUIRED

Determine the 2L/3 level and calculate the settlement of **each clay layer** (the portion below 2L/3) and the total consolidation settlement.



Solution (1/2)

SOLUTION

1 2L/3 level (from the base of the cap)

$$z_0 = \frac{2L}{3} = \frac{2(14)}{3} = \mathbf{9.33}\ \text{m} \ \Rightarrow\ \text{clay 1 (9.33–10 m) and all of clay 2 count}$$

2 Clay 1 (4–10 m): 9.33–10 m, H = 0.67 m; midpoint 9.67 m, z = 0.33 m

$$\Delta\sigma' = \frac{1\,600}{(4.2+0.33)(4.2+0.33)} = 77.9\ \text{kPa} \qquad \sigma'_o = 106.5\ \text{kPa}$$

$$\Delta S = \frac{0.28(0.67)}{1+0.90}\log\frac{106.5+77.9}{106.5} = \mathbf{23.4}\ \text{mm}$$

Solution (2/2)

SOLUTION

3 Clay 2 (10–18 m): 10.00–18 m, H = 8.00 m; midpoint 14.00 m, z = 4.67 m

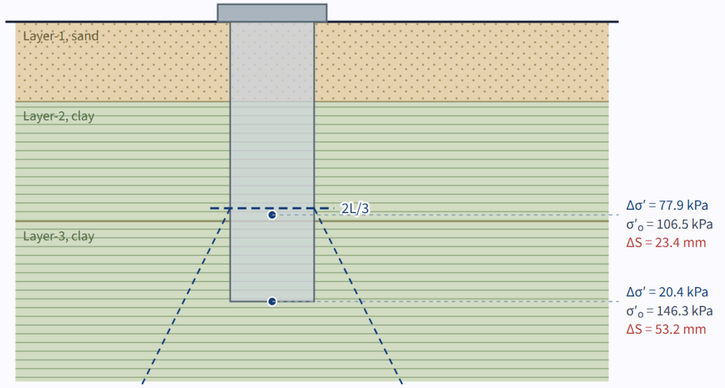
$$\Delta\sigma' = \frac{1\,600}{(4.2+4.67)(4.2+4.67)} = 20.4\ \text{kPa} \qquad \sigma'_o = 146.3\ \text{kPa}$$

$$\Delta S = \frac{0.20(8.00)}{1+0.70}\log\frac{146.3+20.4}{146.3} = \mathbf{53.2}\ \text{mm}$$

4 Total consolidation settlement and its combination with the elastic component

$$S_c = 23.4 + 53.2 = \mathbf{76.7}\ \text{mm}$$

$$S = S_{g(e)} + S_c = 7.9 + 76.7 = \mathbf{84.6}\ \text{mm}$$



91% of the settlement is from consolidation, and 69% of that is from the 10–18 m clays: the group settlement is governed by the thick clay **well below** the tips; the settlement of a single pile does not show this.

Check the solution in GeoTerva ↗

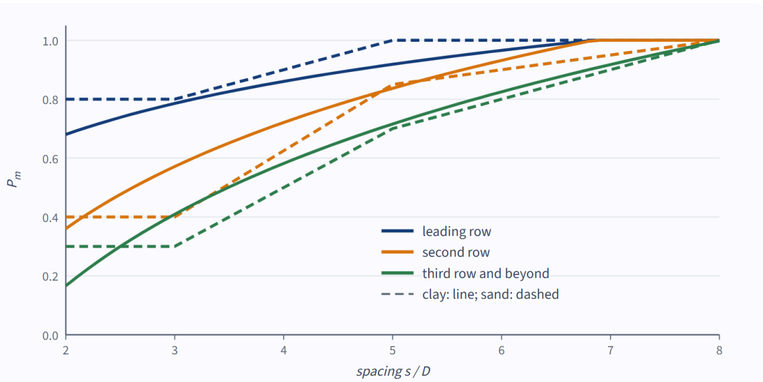
The rear rows are in the shadow of those in front

THEORY

Under lateral load, a pile pushes the soil and the soil reacts along the pile length (the p–y curve in the lateral modulus). When piles stand **in a row** in the direction of loading, each one works in soil that is strained and pushed by the pile ahead of it. Tests (Brown et al. 1988; McVay et al. 1995; Rollins et al. 2006) show that the p–y curve of the group keeps its shape but is reduced by a **p-multiplier P_m**; the multiplier depends on the row and the spacing:

$$p_{group}(y) = P_m \; p_{single}(y)$$

$$\text{clay (Rollins):} \quad \begin{aligned} P_{m1} &= 0.26 \ln \tfrac{s}{D} + 0.5 \\ P_{m2} &= 0.52 \ln \tfrac{s}{D} \\ P_{m3} &= 0.60 \ln \tfrac{s}{D} - 0.25 \end{aligned}$$



Sand: AASHTO LRFD multipliers (at 3D and 5D: 0.8·0.4·0.3 and 1.0·0.85·0.7; linear in between).

In layered soil, the p–y curve of **each layer** is scaled by the multiplier belonging to that layer's soil type. If the load **reverses**, each row takes the average of the group's row multipliers.

Problem statement

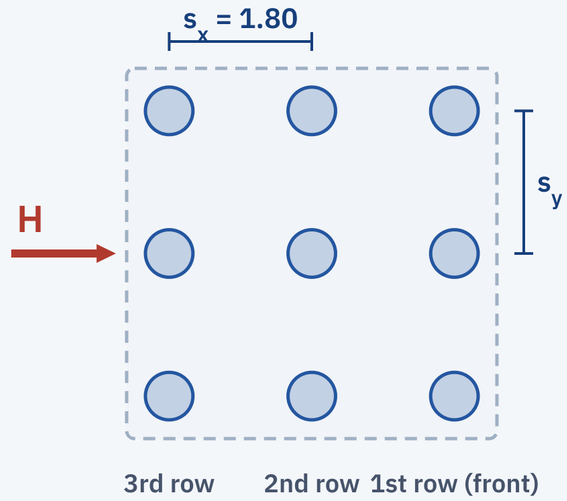
EXAMPLE

Given

Spacing (in the load direction)	s_x = 1.80 m, D = 0.60 m: s/D = 3
Clay	Rollins et al. (2006) equations
Sand	AASHTO LRFD: in 3D, $P_m = 0.8 \cdot 0.4 \cdot 0.3$

REQUIRED

Find the **p-multipliers** of the three rows for the clay layer; if the load **is reversed**, what is the average multiplier of the rows for clay and sand?



Solution

SOLUTION

1 Clay: ln(s/D) = ln 3 = 1.099

$$P_{m1} = 0.26(1.099) + 0.5 = \mathbf{0.786}$$

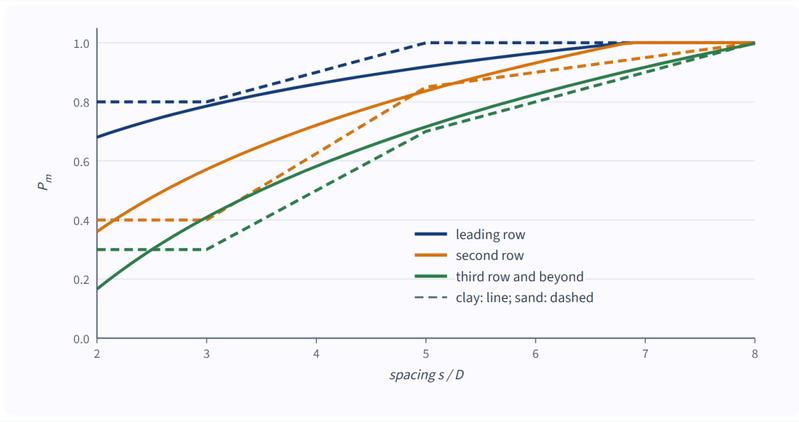
$$P_{m2} = 0.52(1.099) = \mathbf{0.571}$$

$$P_{m3} = 0.60(1.099) - 0.25 = \mathbf{0.409}$$

2 Reversible load: average of the rows

$$\bar{P}_{m,clay} = \frac{0.786 + 0.571 + 0.409}{3} = \mathbf{0.589}$$

$$\bar{P}_{m,sand} = \frac{0.8 + 0.4 + 0.3}{3} = \mathbf{0.500}$$



The front row retains about 79% of the curve, while the third and subsequent rows retain only 41% (in clay); in sand 80% and 30%. As the spacing increases the multipliers approach 1 (at 5D, sand 1.0 · 0.85 · 0.7).

Check the solution in GeoTerva ↗

The cap undergoes a single displacement

THEORY

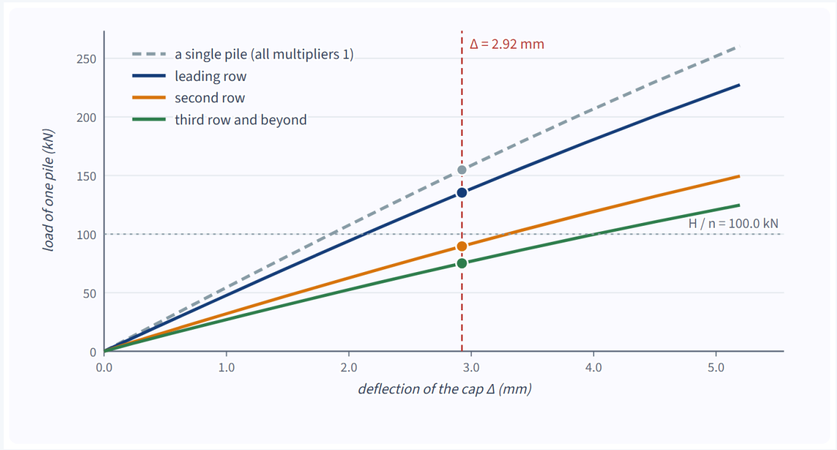
The cap is rigid and its rotation is restrained: **every pile head displaces by the same Δ**; but since the p–y curves are scaled differently, the rows carry different loads. The module finds the Δ at which the sum of the row loads equals the load on the cap:

$$\sum_{k=1}^{n_x} n_y \, H_k(\Delta) = H$$

The pile in row k is the single pile of the **lateral pile module** (finite elements + Newton); the curves are scaled by that row's multipliers.

The displacement is **controlled**: the pile head is held at Δ and its reaction is read; thus the peak of the softening (cyclic clay) curve can also be traced.

The maximum load the group can carry is the **first peak** of the row sum; if the required load is greater than this, the module reports it.



Problem statement

EXAMPLE

Given

Solution (module)	**Δ = 2.92 mm; load per pile: 1st row 135.3, 2nd row 89.6, 3rd row 75.1 kN**
Single pile	**Carries 154.8 kN at Δ; displaces 1.85 mm under 100 kN**
Load	**H = 900 kN, n = 9: equal share H/n = 100 kN**

REQUIRED

Check equilibrium; find the ratio of the cap displacement to that of a single pile and the **group efficiency under lateral load**.

Solution

SOLUTION

1 Equilibrium: $\Sigma n_\gamma H_k = H$

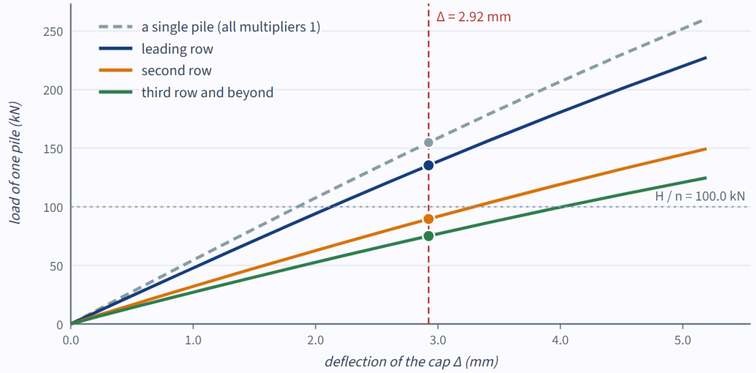
$$3(135.3 + 89.6 + 75.1) = \mathbf{900}\ \text{kN} = H\ \checkmark$$

2 Ratio of cap displacement to that of a single pile

$$\frac{\Delta}{y_{single}} = \frac{2.92}{1.85} = \mathbf{1.58}$$

3 Group efficiency under lateral load: average pile load / load on a single pile at the same Δ

$$\eta_{lateral} = \frac{H/n}{H_{single}(\Delta)} = \frac{100.0}{154.8} = \mathbf{0.65}$$



The front row carries **1.35 times** the equal share, the third row 0.75 times; the cap displaces **1.58 times** more than for a single pile. The maximum bending moment is at the pile head: 207 kN·m.

Check the solution in GeoTerva ↗

Which value goes to which check?

THEORY

Pile loads: $P_{max} \le Q_u/FS$ (single pile, no group effect); the pile in tension $T_{max} \le T_u/FS$, and the piles in tension together $\Sigma T \le T_g/FS$

Group capacity: $V \le Q_{all} = Q_{g(u)}/FS$; which one (individual or block) governs

Settlement: elastic + consolidation, compared with the entered allowable value

Lateral: pile-cap displacement, front-row load, maximum bending moment and lateral efficiency; compared with the allowable displacement

The load per pile and the bending moment are the loads for the **structural design** of the pile section

EXAMPLE GROUP: CHECKS

Check	Result
$P_{max} \le Q_u/FS$	261 ≤ 302 kN ✓
$V \le Q_{all}$	1 600 ≤ 1,984 kN ✓
Settlement	84.6 mm
Δ (lateral)	2.92 mm

Problem statement

EXAMPLE

Given

Group	**9 bored piles, 3 rows × 3 piles, D = 0.60 m, L = 14 m, s = 1.80 m**
Soil	**sand 0–4 m (φ′ = 32°) · clay 4–10 m (c_u = 50) · clay 10–18 m (c_u = 90 kPa)**
Varying	**pile spacing s = 2.5D · 3D · 4D · 5D · 6D (in both directions)**
Loads	**V = 1 600, H = 900 kN, M_y = 900 kN·m**

REQUIRED

As the spacing increases, how do the group capacity, efficiency and cap displacement change?

Solution (1/2)

SOLUTION

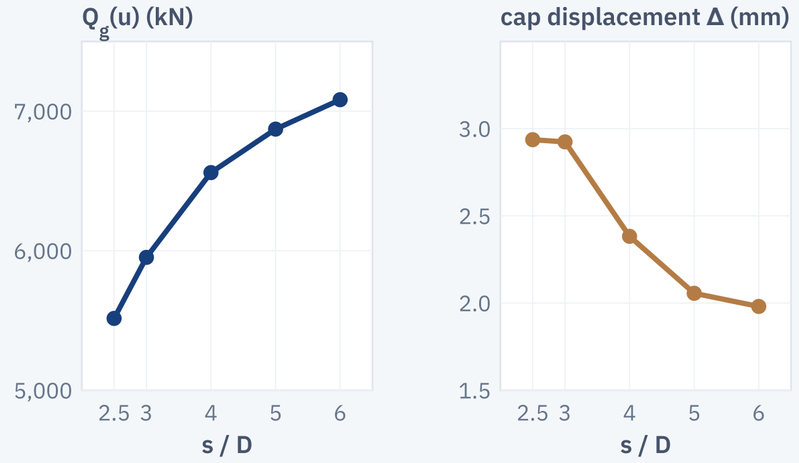
s / D	s (m)	η sand	η clay	$Q_{g(u)}$ (kN)	η group	Δ (mm)
2.5	1.50	0.65	0.677	5,515	**0.676**	2.94
3.0	1.80	0.77	0.727	5,952	**0.729**	2.92
4.0	2.40	1.00	0.792	6,560	**0.803**	2.38
5.0	3.00	1.00	0.832	6,871	**0.842**	2.06
6.0	3.60	1.00	0.860	7,083	**0.868**	1.98

s = 2.5D → 6D: capacity **5,515 → 7,083 kN**, group efficiency 0.68 → 0.87, cap displacement **2.94 → 1.98 mm**. As the spacing opens up, both efficiency and p-multipliers approach 1; 2.5D is the smallest spacing given by the source.

Solution (2/2)

SOLUTION

Comment. Increasing the spacing from 2.5D to 6D raises the capacity by 28% and reduces the cap displacement by 33%; but the cap grows and the piles spread farther apart. In practice a compromise between 3D – 3.5D and 5D. In the figure, $Q_{g(u)}$ on the left and Δ on the right.



Check the solution in GeoTerva ↗

The assumptions behind the numbers

THEORY

Group: vertical piles in a rectangular grid or triangular arrangement, of a single section and length, under a rigid cap. The cap transfers load but adds no resistance (passive pressure, the friction beneath it and bearing capacity are left on the safe side).

Single pile: layered soil rules, no resistance is entered.

Efficiency: empirical equations for driven piles in clay; in sand a single spacing (the minimum distance). For a triangular arrangement, Converse–Labarre is an **extension**.

Block: the foundation of the source in clay; the slip in sand and the sand base are an **extension of the module**.

Loads: V includes the weight of the cap; lateral load and moments act at the base of the cap. In uplift, only the shaft is considered, sand ¾.

Settlement: it is an option; the elastic part with a single E_s and v_s, consolidation with 2:1 spread, at the middle of each layer.

Lateral: p-multipliers come from tests on specific soils and spacings; the 3D ones are marked.

Summary

Key points

Rigid cap: **P = V/n + Bx + Cy**; the largest load is on the front pile, the smallest on the rear pile; if the moment is large, the rear pile goes into **tension**

The **single pile** is computed layer by layer; the group capacity is built on it

Efficiency is applied layer by layer: Converse–Labarre in clay, the chart for driving in sand ($\eta \leq 1$) and the AASHTO rule for bored piles

Capacity = **min(individual, block)**; the block governs in closely spaced groups with many piles

Settlement: elastic (Vesic, with $\sqrt{(B_g/D)}$) + consolidation (2:1 spread from 2L/3); the group settlement is governed by the thick clay

Lateral: p-multipliers decrease from the front row to the back; the cap undergoes a single Δ, the rows carry unequal loads

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