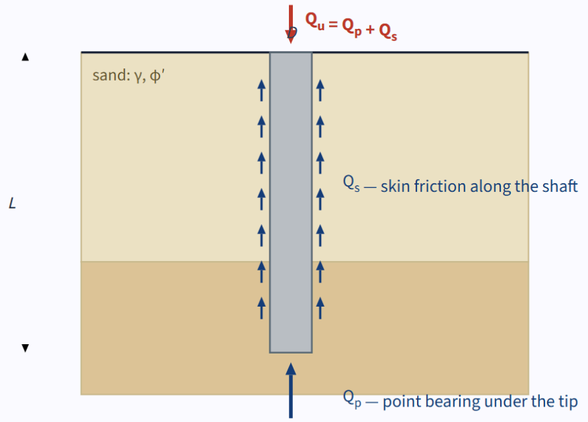
Geotechnical lecture presentation · Piles

Pile bearing capacity in sand

Lecture: end bearing, shaft friction and critical depth. The theory of each method, followed by a worked example.

Meyerhof (1976) · Vesic (1977) · Coyle and Castello (1981) · Worked examples



What will we learn in this lecture?

To be able to explain how a pile carries its load as **end bearing** and **shaft friction**

To be able to construct the **effective stress** profile in sand, including the water level

To be able to compute the end bearing by three methods: **Meyerhof**, **Vesic**, **Coyle and Castello**

To be able to interpret shaft friction and the role of the **critical depth** (L′)

To be able to reach the **allowable bearing capacity** and evaluate the results of the methods together

LECTURE FLOW

Each method is covered in **three steps**:

1 · **Theory**: how does the method work, why so?

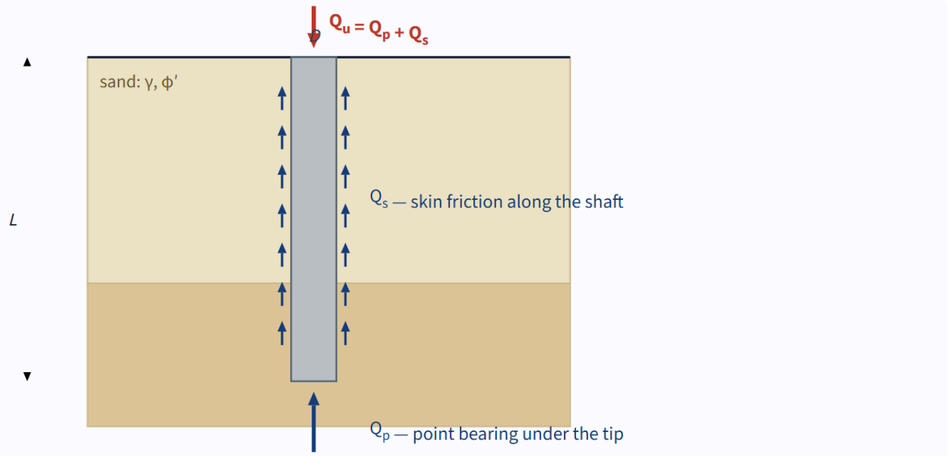
2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

All the examples use the same pile: thus the methods are compared side by side.

The load is transferred in two ways

THEORY



The pile transmits its load to the soil **by two paths at the same time**: the soil below the tip resists by **end bearing** (Q_p), and the sand along the shaft resists by **shaft friction** (Q_s).

$$Q_u = Q_p + Q_s \qquad Q_{all} = \frac{Q_u}{FS}$$

Shaft friction reaches its full value at a **small** displacement, whereas end bearing reaches it at a **much larger** displacement; the two components do not peak at the same time.

In this lecture: **single pile**, **sand** ($c' = 0$), driven or bored, constant section.

In sand, everything is proportional to effective stress

THEORY

In sand, both **end bearing** and **unit shaft friction** are proportional to the effective vertical stress. The calculation begins with determining the σ'_v values in the profile correctly.

$$\sigma'_v = \underbrace{\sum \gamma_i h_i}_{\text{above GWT}} + \underbrace{\sum (\gamma_{sat,i} - \gamma_w)\, h_i}_{\text{below GWT}}$$

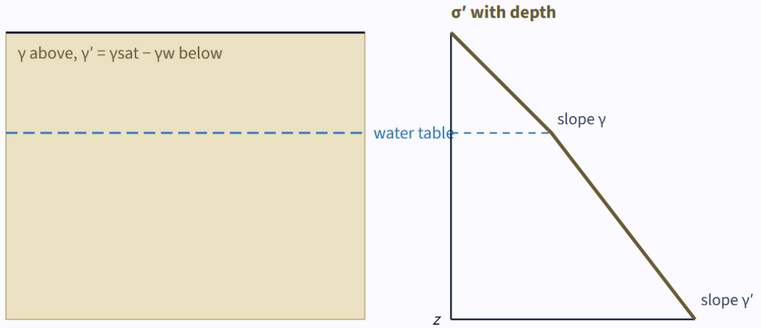
γ : moist unit weight (above the GWT)

γ_{sat} : saturated unit weight (below the GWT)

$\gamma_w = 9.81$ kN/m³ : unit weight of water

GWT: groundwater table

> Below the GWT the stress gradient **decreases**: γ′ = $\gamma_{sat} - \gamma_w$



Problem statement

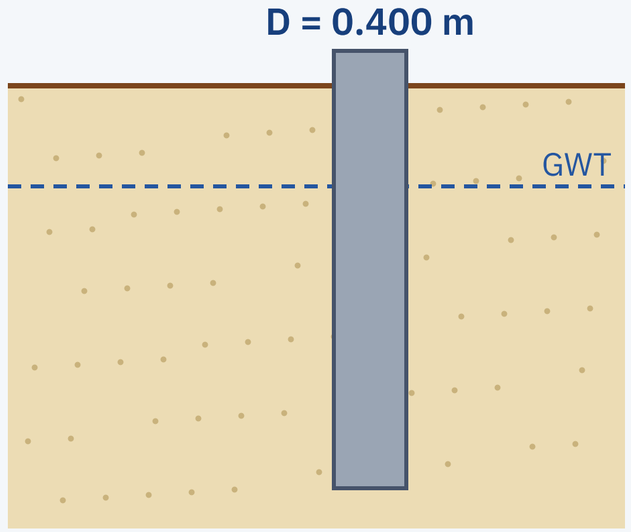
EXAMPLE

Given

Sand, above GWT	$\gamma = 17\ \mathrm{kN/m^3}$
Sand, below GWT	$\gamma_{sat} = 19.5\ \mathrm{kN/m^3}$
GWT depth	**2.0 m**
Unit weight of water	$\gamma_w = 9.81\ \mathrm{kN/m^3}$

REQUIRED

At **8 m depth**, determine the total vertical stress, the pore water pressure and the **effective vertical stress**.



Solution

SOLUTION

1 Total vertical stress (moist unit weight above the GWT, saturated unit weight below)

$$\sigma_v = 17(2) + 19.5(6) = 34 + 117 = \mathbf{151.0}\ \text{kPa}$$

2 Pore water pressure (6 m head of water below the GWT)

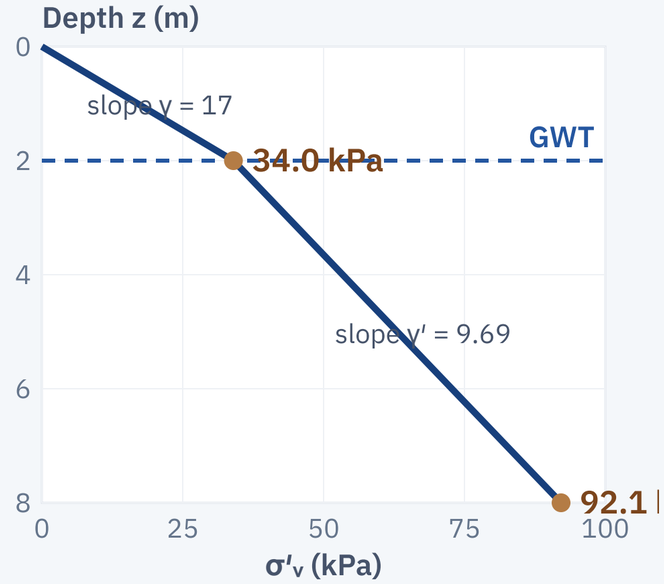
$$u = \gamma_w\, h_w = 9.81\,(6) = \mathbf{58.9}\ \text{kPa}$$

3 Effective vertical stress

$$\sigma'_v = \sigma_v - u = 151.0 - 58.9 = \mathbf{92.1}\ \text{kPa}$$

Same result: $\sigma'_V = 17(2) + (19.5 - 9.81)(6) = \mathbf{92.1\ kPa}$.

Below the GWT the effective unit weight is 9.69 kN/m³.



Check the solution in GeoTerva ↗

End resistance: general expression

THEORY

The sand below the tip fails by **bearing capacity failure** as beneath a foundation; however, at the depth of the tip the **overburden stress** confines the sand tightly.

Since c′ = 0 in sand, only the depth (overburden) term remains. Because the pile diameter is small, the width term ($0.5\gamma' D N_\gamma$) is neglected:

$$q_p = \sigma'_v \, N^*_q \qquad Q_p = A_p \, q_p$$

σ'_v : effective vertical stress at the depth of the tip

N^*_q : bearing capacity factor for the pile problem

A_p : area of the pile tip

WHERE DOES N^*_q COME FROM? THREE APPROACHES

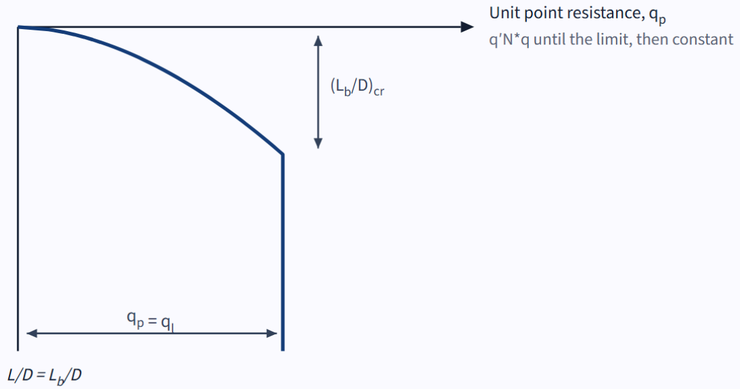
Meyerhof (1976): N^*_q depends only on φ′; q_p is capped by a **limit value**

Vesic (1977): cavity expansion theory; the **rigidity** of the sand is also taken into account

Coyle and Castello (1981): fitted to 24 full-scale load tests; N^*_q also depends on L/D

End bearing does not increase indefinitely

THEORY



In homogeneous sand, the unit end bearing increases with the embedment ratio and **peaks** at **$(L_b/D)_{cr}$**; beyond this it remains constant (q_l).

VESIC (1970) · OGEECHEE RIVER PILE TEST (D = 0.457 m)

L (m)	L/D	q_p (kN/m²)
3.02	6.61	3 304
6.12	13.39	9 365
8.87	19.40	11 472
12.00	26.26	11 587
15.00	32.82	13 971

Maximum value of q_p is about **12 000 kN/m²**

$(L/D)_{cr}$ approximately **16 – 18**

For an average $N_{60} \approx 30$, $4p_aN_{60} = 4(100)(30) = 12\,000$ kN/m²: **the correlation agrees with the measured plateau**

Meyerhof: N*q from φ′, with a limit

THEORY

Meyerhof takes N^*_q solely from the **angle of internal friction** (his own chart, below). The computed q_p cannot exceed a **limit value**:

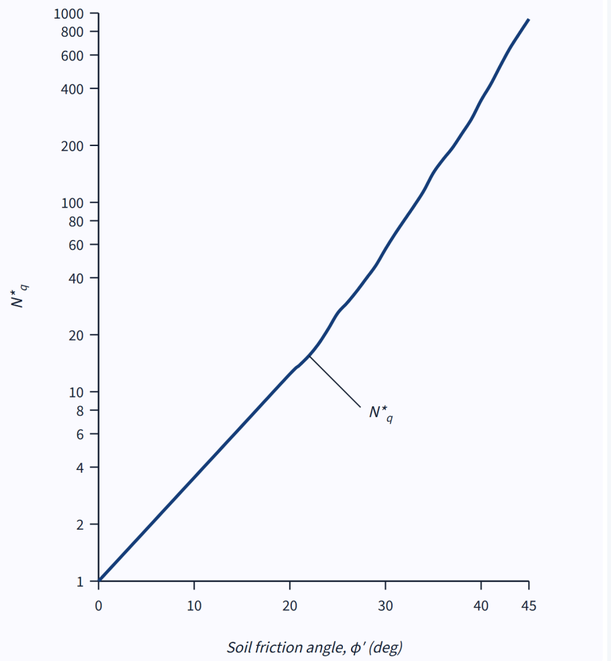
$$q_p = \sigma'_v N^*_q \;\leq\; q_l \qquad q_l = 0.5\, p_a\, N^*_q \tan\varphi'$$

p_a = 100 kN/m² : atmospheric pressure

φ′ : effective friction angle of the sand at the tip level

N^*_q VALUES (MEYERHOF CHART; THE SERIES READ BY THE MODULE)

φ′ (°)	20	25	30	32	34	35	36	38	40	42	45
N^*_q	12.4	26.0	56.7	81.0	115.0	143.0	168.0	231.0	346.0	525.0	930.0



For φ′ = 35°, N^*_q = 143.0 and q_l = 5,006 kPa; in sand with γ = 18 kN/m³ this limit is reached at a depth of about **1.9 m**

Meyerhof: step by step and the meaning of the limit

THEORY

CALCULATION STEPS

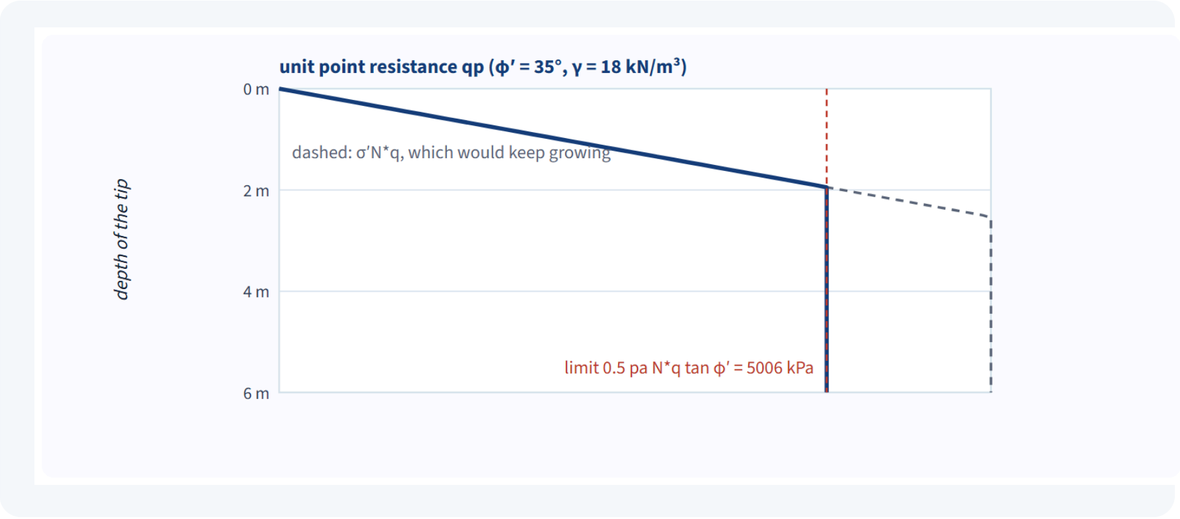
1. Calculate σ'_v at the tip level

2. Read the N^*_q value for φ' from the chart/table

3. $q_p = \sigma'_v N^*_q$

4. $q_l = 0.5\ p_a\ N^*_q \tan\varphi'$

5. $q_p = \min(\sigma'_v N^*_q, q_l)$ and $Q_p = A_p q_p$



The curve follows the $\sigma' N^*_q$ line **until the limiting value governs**; after that it remains horizontal. Without the limit, q_p would increase indefinitely with depth, which field tests do not show.

Problem statement

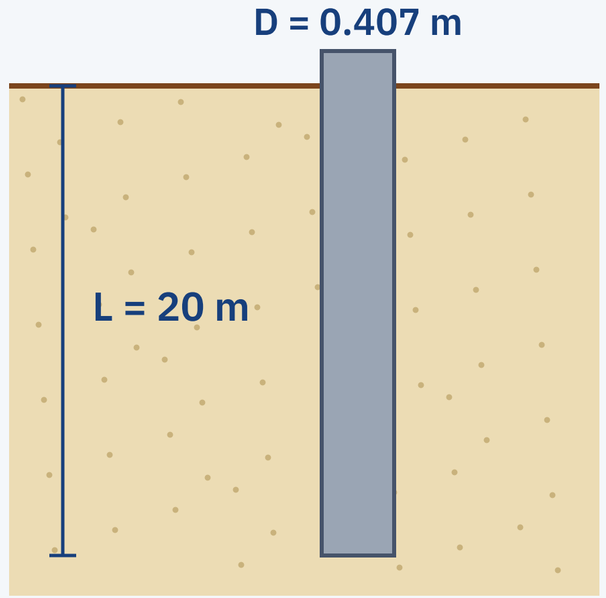
EXAMPLE

Given

Pile	**reinforced concrete, driven, square section**
Section size	**D = 0.407 m**
Length	**L = 20 m**
Sand (homogeneous)	$\gamma = 18\ \mathrm{kN/m^3}$, $\varphi' = 35^\circ$
Groundwater	**none**

REQUIRED

Determine the **tip resistance** Q_p of the pile according to the **Meyerhof (1976)** method.



Solution

SOLUTION

1 Cross-sectional area and effective stress at the tip

$$A_p = 0.407^2 = 0.1656\ \text{m}^2 \qquad \sigma'_v = \gamma L = 18(20) = 360\ \text{kPa}$$

2 N*q value (φ′ = 35°, Meyerhof table) and σ′N*q

$$N_q^* = 143.0 \qquad \sigma'_v N_q^* = 360(143.0) = 51{,}480\ \text{kPa}$$

3 Limit value

$$q_l = 0.5\, p_a N_q^* \tan\varphi' = 0.5(100)(143.0)\tan 35^\circ = 5{,}006\ \text{kPa}$$

4 Unit tip resistance: the smaller of the two; the limit governs

$$q_p = \min(51{,}480;\ 5{,}006) = \mathbf{5{,}006}\ \text{kPa}$$



$Q_p = A_p\, q_p = 0.1656 \times 5{,}006 =$ **829 kN**. Without the limit it would have been about 8,528 kN.

Check the solution in GeoTerva ↗

Vesic: the sand beneath the tip expands like a cavity

THEORY

Vesic derives the end resistance **from cavity expansion theory**: the tip expands a cavity in the surrounding soil; therefore the answer involves not only the **strength** of the sand but also its **stiffness**.

$$Q_p = A_p \, \bar{\sigma}'_o \, N^*_\sigma$$

Here $\bar{\sigma}'_o$ is not **vertical** but the **average effective normal stress** at the tip level:

$$\bar{\sigma}'_o = \frac{1 + 2K_o}{3} \, q' \qquad K_o = 1 - \sin \varphi'$$

K_o: coefficient of lateral earth pressure at rest

q': effective vertical stress at the tip level ($= \sigma'_v$)

RELATIONSHIP WITH MEYERHOF

$$N^*_\sigma = \frac{3 \, N^*_q}{1 + 2K_o}$$

The form is the same as Meyerhof's; the two methods differ in **what the coefficient is derived from**, not in the shape of the equation.

Vesic: reduced rigidity index

THEORY

N^*_σ depends on the **reduced rigidity index** I_{rr} of the sand and on φ'; I_{rr} also accounts for the volume change in the plastic zone:

$$I_{rr} = \frac{I_r}{1 + I_r\,\Delta} \qquad I_r = \frac{E_s}{2(1+\mu_s)\,q'\tan\varphi'} = \frac{G_s}{q'\tan\varphi'}$$

E_s : modulus of elasticity of the soil, μ_s : Poisson's ratio, G_s : shear modulus

Δ : average volumetric strain in the plastic zone below the tip

PRACTICAL APPROACHES (CHEN AND KULHAWY, 1994; 25° ≤ φ′ ≤ 45°)

$$\frac{E_s}{p_a} = m \qquad \mu_s = 0.1 + 0.3\,\frac{\varphi' - 25}{20} \qquad \Delta = 0.005\left(1 - \frac{\varphi' - 25}{20}\right)$$

SINGLE INPUT: m = E_s / p_a

Sand	m
loose	100 – 200
medium dense	200 – 500
dense	500 – 1000

The rigidity index is generally **75 – 150** (sand with relative density of 50 – 80%)

If CPT is available: I_r = 300/F_r(%) (mechanical), 170/F_r(%) (electric cone)

Vesic: step by step

THEORY

CALCULATION STEPS

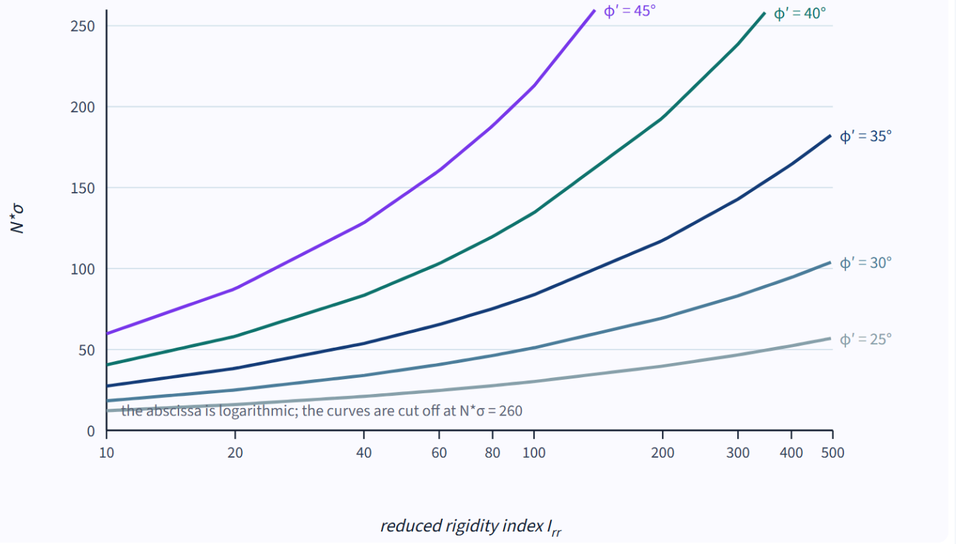
1. $K_o = 1 - \sin\varphi'$ and $\bar{\sigma}_o = (1 + 2K_o)q'/3$

2. $E_s = m \cdot p_a$, μ_s, I_r

3. Δ and $I_{rr} = I_r/(1 + I_r\Delta)$

4. N^*_σ from φ' and I_{rr} (from the table, logarithmic interpolation)

5. $q_p = \bar{\sigma}_o N^*_\sigma$ and $Q_p = A_p q_p$



One curve for each φ': **N^*_σ increases as stiffness increases**; dense and stiff sand gives much greater end bearing than loose sand.

Problem statement

EXAMPLE

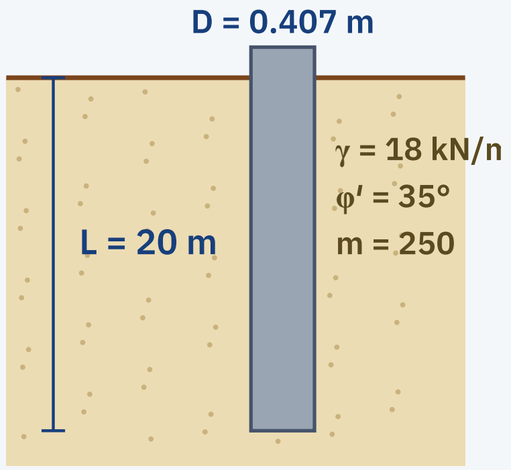
Given

Pile	**Same as Example 2: D = 0.407 m, L = 20 m**
Sand	$\gamma = 18\ \mathrm{kN/m^3}$, $\varphi' = 35^\circ$
Groundwater	**none**
Modulus of elasticity ratio	$m = E_s/p_a = 250$ **(medium dense – dense)**
Atmospheric pressure	$p_a = 100\ \mathrm{kPa}$

REQUIRED

Determine the **tip resistance** Q_p of the pile according to the **Vesic (1977)** method.

Effective stress at the tip: q′ = 18(20) = 360 kPa



Solution (1/3)

SOLUTION

1 At-rest coefficient and average effective normal stress

$$K_o = 1 - \sin 35^\circ = 0.426 \qquad \bar{\sigma}'_o = \frac{1 + 2(0.426)}{3}(360) = \mathbf{222.3}\ \text{kPa}$$

2 Soil properties

$$E_s = m\,p_a = 250(100) = 25{,}000\ \text{kPa} \quad \mu_s = 0.1 + 0.3\frac{35 - 25}{20} = 0.25$$

3 Rigidity index and volumetric unit strain

$$I_r = \frac{25{,}000}{2(1 + 0.25)(360)\tan 35^\circ} = \mathbf{39.7} \qquad \Delta = 0.005\left(1 - \frac{35 - 25}{20}\right)\frac{360}{100} = 0.0090$$

Solution (2/3)

SOLUTION

4 Reduced rigidity index

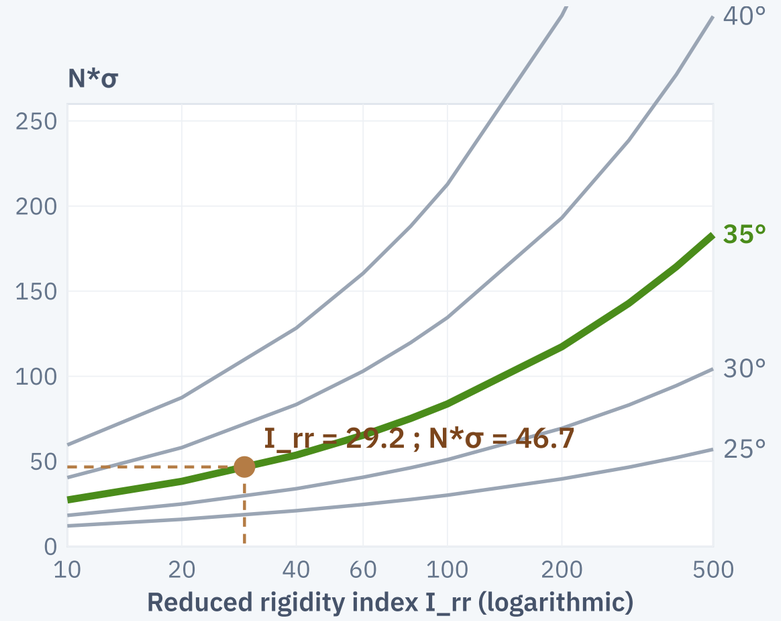
$$I_{rr} = \frac{I_r}{1 + I_r \Delta} = \frac{39.7}{1 + (39.7)(0.0090)} = \mathbf{29.2}$$

5 N*σ: in the table, logarithmic interpolation between 20 and 40 for the φ′ = 35° row and I_{rr} = 29.2

$$N_\sigma^* = 38.32 + \frac{\log 29.2 - \log 20}{\log 40 - \log 20}(53.67 - 38.32)$$

$$= \mathbf{46.7}$$

In the chart the green curve φ′ = 35°: at its intersection with the vertical line I_{rr} = 29.2, **$N^*_\sigma \approx 46.7$**.



Solution (3/3)

SOLUTION

6 Unit end bearing

$$q_p = \bar{\sigma}'_o N^*_\sigma = 222.3(46.7) = \mathbf{10{,}389}\ \mathrm{kPa}$$

7 Tip resistance

$$Q_p = A_p\, q_p = 0.1656(10{,}389) = \mathbf{1{,}721}\ \mathrm{kN}$$

For the same pile, **Meyerhof: 829 kN**, **Vesic: 1,721 kN**. Because Vesic accounts for the stiffness of the sand (m = 250), it gives about 2.1 times Meyerhof's limited value.

Check the solution in GeoTerva ↗

Coyle and Castello: empirical N*q

THEORY

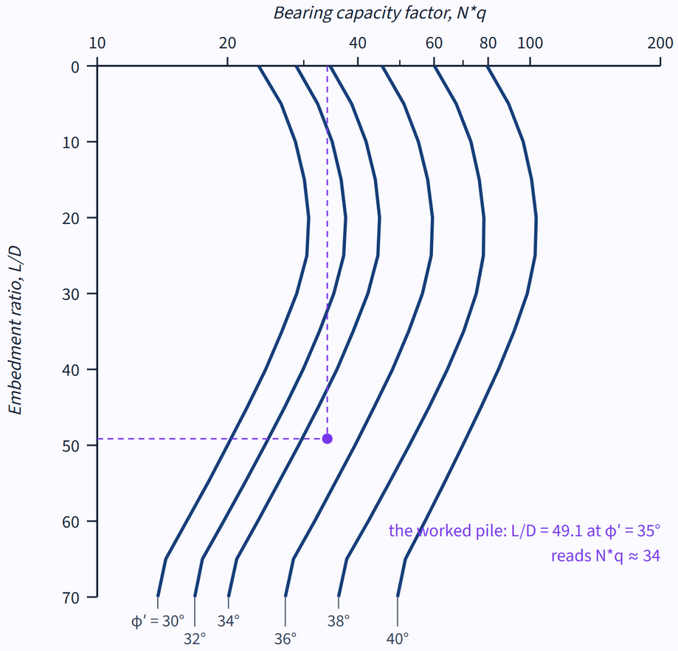
Coyle and Castello examined **24 large-scale field load tests on driven piles** and proposed the following relation for sand:

$$Q_p = q' \, N_q^* \, A_p$$

q′ is the effective vertical stress at the tip; N^*_q, besides φ′, also depends on **the embedment ratio L/D** (chart, at the side).

The coefficient is **much smaller** than Meyerhof's: about **34** for φ′ = 35° and a deep pile, versus **143** in Meyerhof. This is because it is fitted to **measured capacities** rather than to a failure mechanism.

Once the embedment ratio exceeds about 20, the coefficient **decreases**: a longer pile of the same diameter takes a smaller N^*_q.



Problem statement

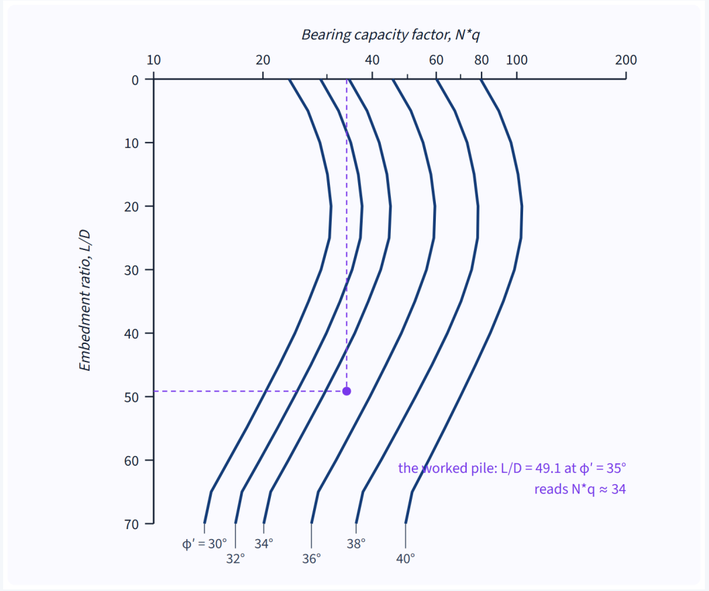
EXAMPLE

Given

Pile and sand	**Same as Example 2: L = 20 m, D = 0.407 m, φ′ = 35°, γ = 18 kN/m³**
Embedment ratio	$L/D = 20/0.407 = 49.1$
Coefficient read from the chart	$N_q^* \approx 34$ **(Figure 7; φ′ = 35°, L/D ≈ 49)**

REQUIRED

Using the **Coyle and Castello (1981)** method, determine the **end bearing** Q_p of the pile.



Solution

SOLUTION

1 Effective stress at the tip and coefficient (from the chart, approximate reading)

$$q' = \gamma L = 18(20) = 360\ \text{kPa} \qquad N_q^* \approx 34$$

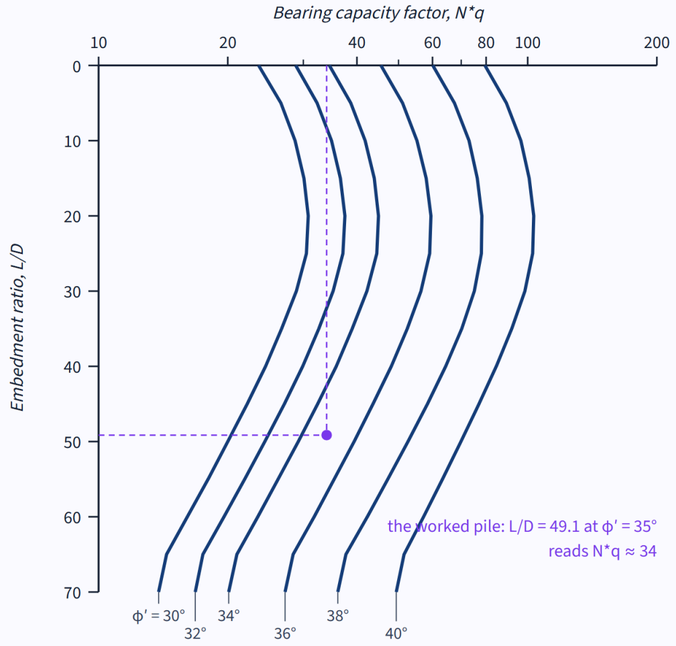
2 Unit end bearing

$$q_p = q' N_q^* = 360(34) = 12{,}240\ \text{kPa}$$

3 Tip resistance

$$Q_p = A_p q_p = 0.1656(12{,}240) = \mathbf{2{,}028}\ \text{kN}$$

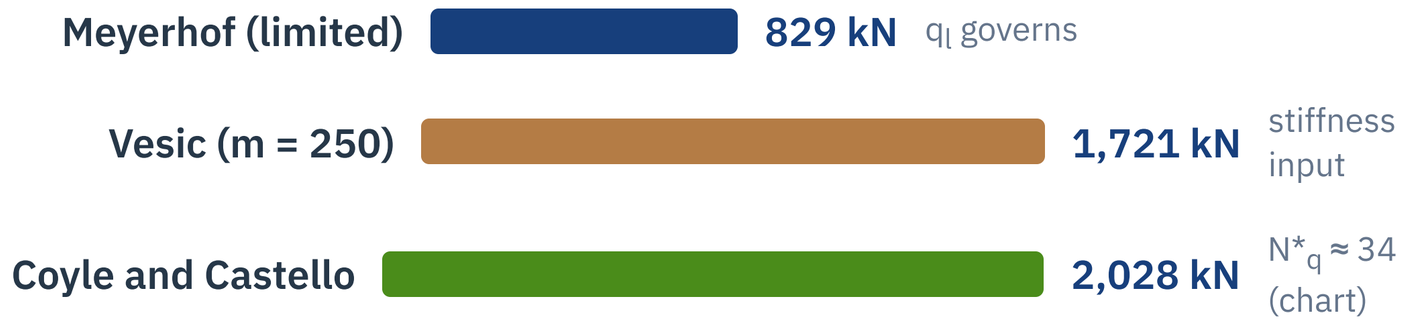
The N^*_q read from the chart is an approximate value; it may vary by ±1 – 2 units depending on φ′ and L/D. The result is the largest of the three methods: **2,028 kN**.



Check the solution in GeoTerva ↗

Three methods, same pile

THEORY



Because the methods rest on **separate assumptions**, the results also diverge. The range is an honest indication of how well the tip resistance can be known.

Meyerhof: **conservative** due to the limit value

Vesic: explicitly inputs sand density through **m**

Coyle–Castello: based on measured driven piles; chart reading is approximate

In practice: at least two methods are computed, the range is examined, and in the absence of a **load test** or local experience a value near the cautious end is selected.

Shaft friction: f = K σ′ tanδ′

THEORY

Shaft friction is the sum of unit friction along the shaft surface:

$$Q_s = \sum p\,\Delta L\,f$$

Unit friction f is written as a function of depth as follows (L′: critical depth):

$$f = K\,\sigma'_o \tan\delta' \quad (0 \le z \le L')$$
$$f = f_{z=L'} \quad (L' \le z \le L)$$

p : perimeter of the pile cross-section, ΔL : length over which p and f are taken constant

K : coefficient of effective lateral earth pressure, σ'_o : effective vertical stress at that depth

δ′ : soil–pile friction angle (usually 0.5φ′ – 0.8φ′)

K VALUES (BY PILE TYPE)

Pile type	K
Bored or jetted	≈ Ko = 1 – sin φ′
Low-displacement driven	≈ Ko to 1.4Ko = 1.4(1 – sin φ′)
High-displacement driven	≈ Ko to 1.8Ko = 1.8(1 – sin φ′)

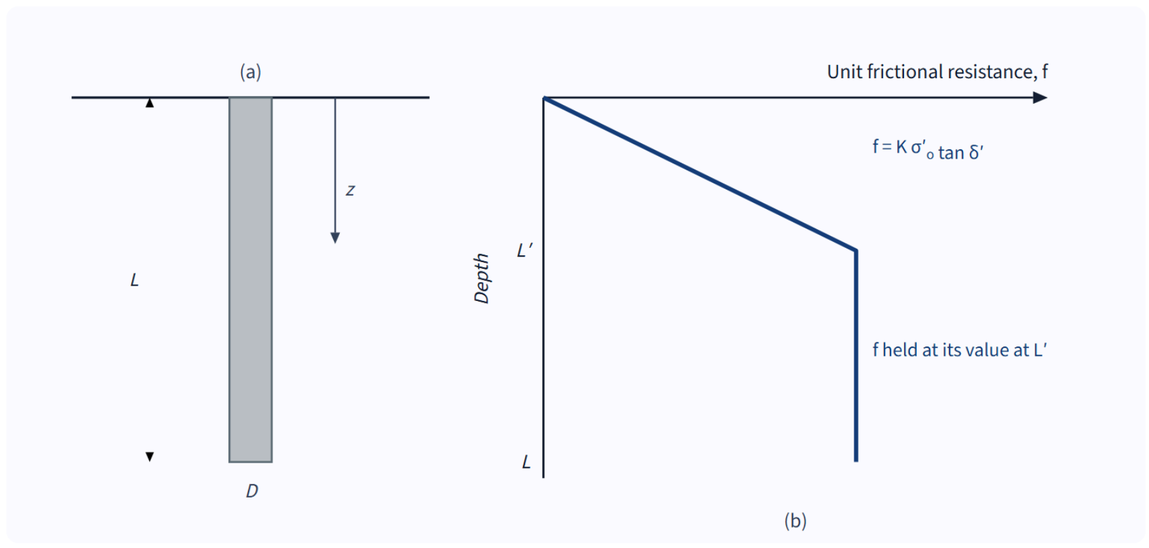
Driving compacts the sand around it (up to about 2.5D)

A high-displacement pile takes more friction than a low-displacement pile

Fore/jet piles give lower friction than driven piles

Critical depth: why does friction not increase?

THEORY



The measured friction does not increase with depth **indefinitely** as does the effective stress; it increases up to an approximate **critical depth** and then remains constant.

$$L' \approx 15\,D$$

The usual explanation is **arching**: the sand does not transfer the full overburden stress around the shaft to the pile. The module holds the effective stress below L′ **at its value at depth L′**; the unit friction remains constant down to the tip.

> For L′, the source gives 15D – 20D; conservative value **15D**

Problem statement

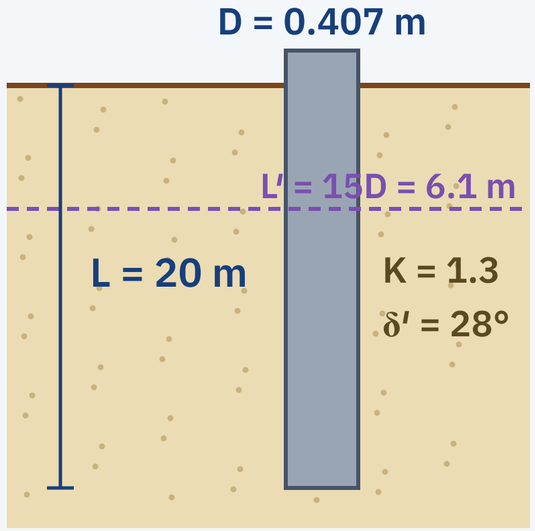
EXAMPLE

Given

Pile and sand	**Same as Example 2: L = 20 m, D = 0.407 m, γ = 18 kN/m³**
Perimeter	$p = 4D = 4(0.407) = 1.628$ m
Coefficients	$K = 1.3$ **(high-displacement driven),** $\delta' = 28^\circ$ **(**$\approx 0.8\varphi'$**)**

REQUIRED

Taking the critical depth (L′ = 15D) into account, determine the **shaft friction** Q_s of the pile.



Solution

SOLUTION

1 Critical depth and the stress at that depth

$$L' = 15D = 15(0.407) = \mathbf{6.105}\ \text{m} \qquad \sigma'(L') = 18(6.105) = 109.9\ \text{kPa}$$

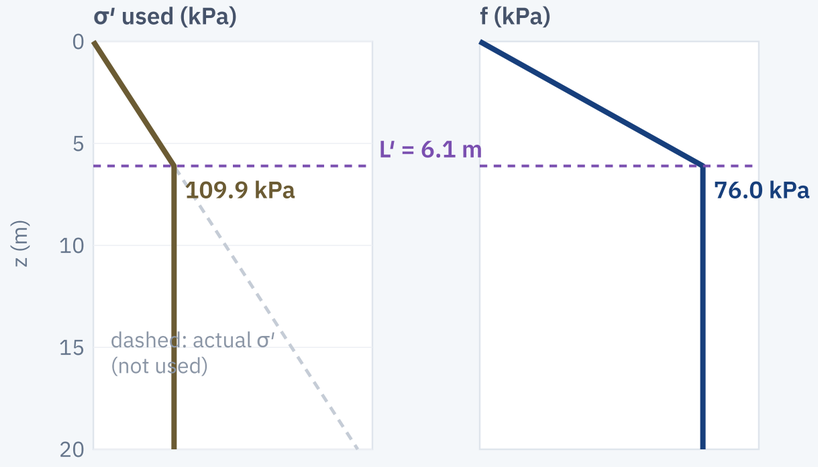
2 L′ and unit friction at the tip (constant below L′)

$$f_{L'} = K\,\sigma'(L')\tan\delta' = 1.3(109.9)\tan 28^\circ = \mathbf{76.0}\ \text{kPa}$$

3 Q_s: triangle (0 – L′) + rectangle (L′ – L) area × perimeter

$$Q_s = p\left[\tfrac{1}{2} f_{L'} L' + f_{L'}(L - L')\right]$$
$$= 1.628\left[\tfrac{1}{2}(76.0)(6.10) + (76.0)(13.89)\right]$$

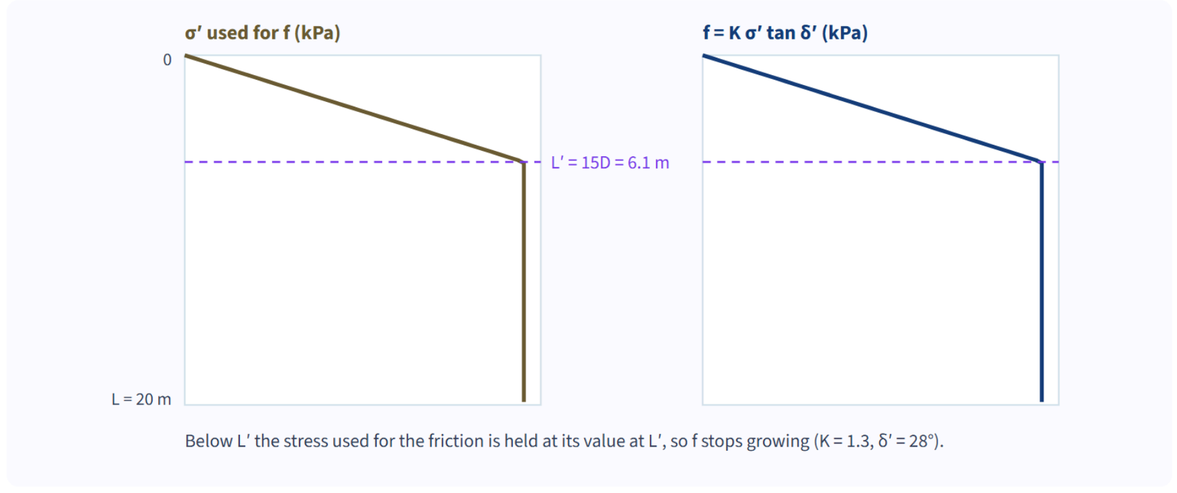
Q_S = 1,628 [231.9 + 1,055.4] = **2,096 kN**.



Check the solution in GeoTerva ↗

What happens if we forget the critical depth?

THEORY



Let us compute the same pile **without considering L′**, assuming the stress increases linearly up to the tip:

$$f_L = 1.3(360)\tan 28^\circ = 248.8\ \text{kPa}$$
$$Q_s = p\,\tfrac{1}{2} f_L L = 4{,}051\ \text{kN}$$

2,096 kN instead of **4,051 kN**: about **93% in excess**. Neglecting the critical depth in a long pile produces a large error on the unsafe side.

Coyle and Castello: with a single average stress

THEORY

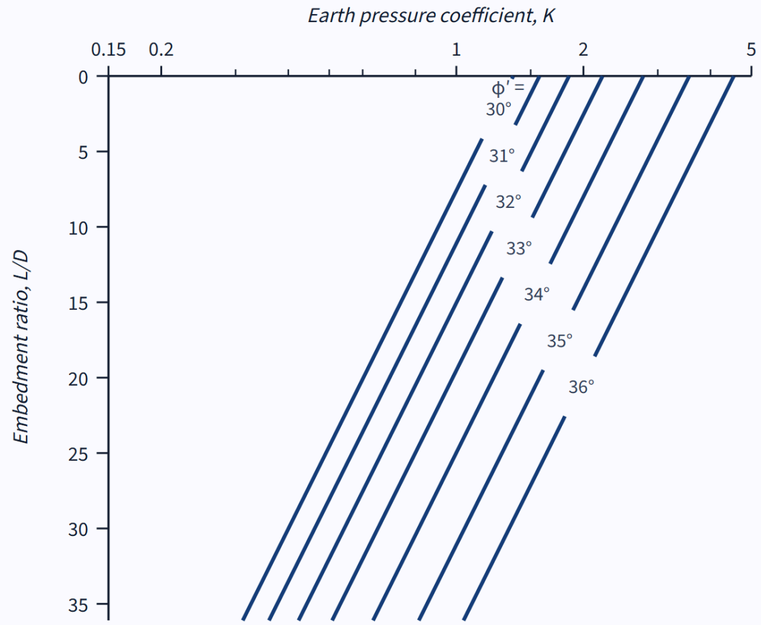
Coyle and Castello proposed computing the shaft friction **in a single step using the average effective stress over the embedded length**:

$$Q_s = f_{av}\, p\, L = (K\, \bar{\sigma}'_o \tan \delta')\, p\, L$$
$$\delta' = 0.8\varphi'$$

K is determined from tests and is read from the chart (at the side) according to **L/D** and φ′.

There is no separate critical depth: averaging takes care of it

Therefore the two perimeter methods can give **large differences for long piles**



Problem statement

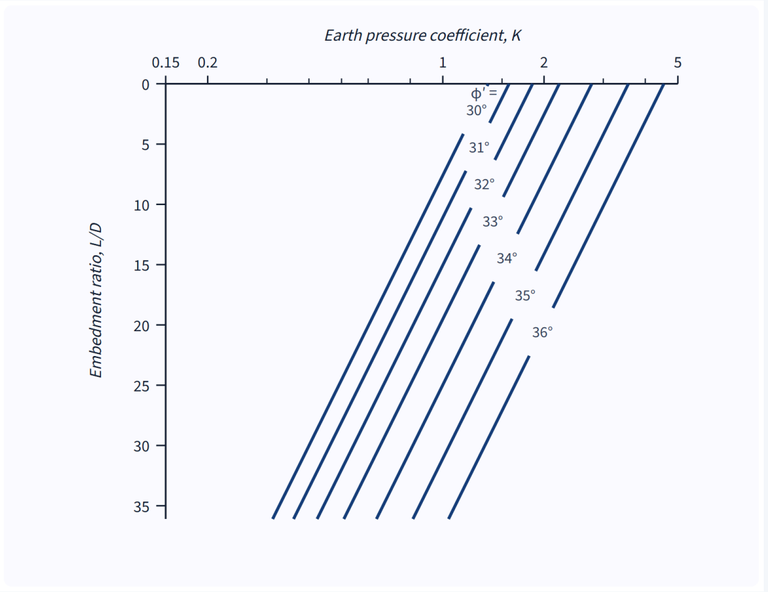
EXAMPLE

Given

Pile and sand	**Same as Example 2: L = 20 m, D = 0.407 m, φ′ = 35°, γ = 18 kN/m³**
Coefficient	$K \approx 0.41$ **(Figure 10; the chart is drawn up to L/D = 36, for L/D ≈ 49 the curve is read by extrapolation; enter this value in the K box of the module, the module fills in 0.48 from its own chart reading)**
Friction angle	$\delta' = 0.8\varphi' = 0.8(35^\circ) = 28^\circ$

REQUIRED

Using the **Coyle and Castello (1981)** method, determine the **shaft friction** Q_s of the pile.



Solution

SOLUTION

1 Average effective stress along the shaft (homogeneous sand, no GWT)

$$\bar{\sigma}'_o = \frac{\gamma L}{2} = \frac{18(20)}{2} = \mathbf{180}\ \text{kPa}$$

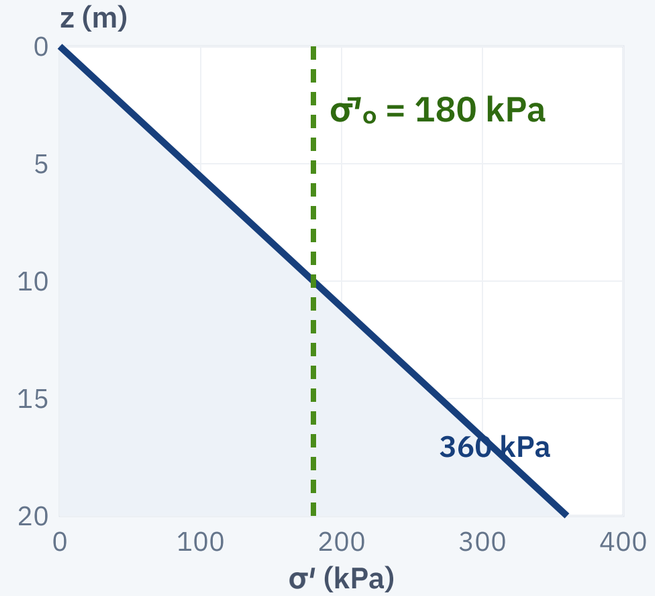
2 Average unit skin friction

$$f_{av} = K\,\bar{\sigma}'_o \tan\delta' = 0.41(180)\tan 28^\circ = \mathbf{39.2}\ \text{kPa}$$

3 Shaft friction

$$Q_s = f_{av}\,p\,L = 39.2(1.628)(20) = \mathbf{1{,}278}\ \text{kN}$$

The K–δ′ method (with L′) gives **2,096 kN**, Coyle–Castello gives **1,278 kN**: for this long pile there is about a 64% difference between the two.



Check the solution in GeoTerva ↗

Total with a single factor of safety

THEORY

The ultimate capacity is the sum of the two components; the allowable capacity is found by applying **a single factor of safety to the entire sum**:

$$Q_{all} = \frac{Q_p + Q_s}{FS}$$

Usual **FS = 2.5 – 4** for a single pile

The upper end is selected if the soil is known only from a few boreholes or if the capacity is mainly carried by the **tip**

The end and shaft methods can be selected **independently** of each other; the results are interpreted together

Problem statement

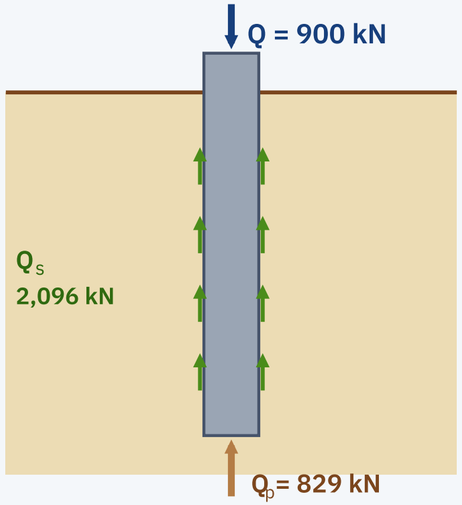
EXAMPLE

Given

Tip resistance	Q_p = 829 kN (Meyerhof, Example 2)
Shaft friction	Q_s = 2,096 kN (K–δ′, Example 5)
Factor of safety	FS = 3
Working load on the pile	Q = 900 kN

REQUIRED

What is the **allowable bearing capacity** Q_{all} of the pile? Is the working load **sufficient**?



Solution

SOLUTION

1 Ultimate bearing capacity

$$Q_u = Q_p + Q_s = 829 + 2{,}096 = \mathbf{2{,}925}\ \text{kN}$$

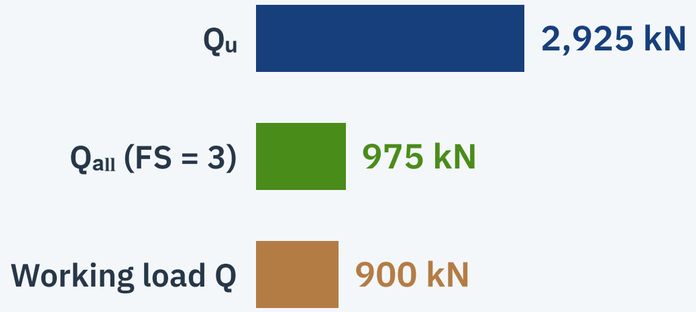
2 Allowable bearing capacity

$$Q_{all} = \frac{Q_u}{FS} = \frac{2{,}925}{3} = \mathbf{975}\ \text{kN}$$

3 Check

$$Q = 900\ \text{kN} \ \leq\ Q_{all} = 975\ \text{kN} \quad \Rightarrow \quad \text{sufficient}$$

Utilization ratio Q / Q_{all} = 900 / 975 = **0.92**.



Check the solution in GeoTerva ↗

Same pile: Q_u and Q_{all} (FS = 3)

THEORY

Q_u · Q_{all} (kN)

	Q_s: K–δ′ (with L′) **2,096 kN**	Q_s: Coyle–Castello **1,278 kN**
Q_p: Meyerhof 829 kN	2,925 · **975**	2,107 · **702**
Q_p: Vesic 1,721 kN	3,817 · **1,272**	2,999 · **1,000**
Q_p: Coyle–Castello 2,028 kN	4,123 · **1,374**	3,305 · **1,102**

For a twenty-metre pile, two shaft and three end-bearing methods give six different Q_u values: **2,107 – 4,123 kN**. Which combination to choose is an **engineering decision**.

Interpret the range, do not rely on a single number

Shaft friction is the major part of the total: the **critical depth** decision is decisive

Verify with a **pile load test** if possible

Summary

Key points

$Q_u = Q_p + Q_s$; in sand both components depend on **effective stress**

Meyerhof: N^*_q from φ', q_p limited ($q_l = 0.5 p_a N^*_q \tan\varphi'$)

Vesic: $\bar{\sigma}'_o N^*_\sigma$; the stiffness of the sand (m, I_{rr}) is taken into account

Coyle and Castello: $N^*_q(\varphi', L/D)$ and $K(\varphi', L/D)$ fitted to measured capacities

Shaft friction: $f = K\sigma'\tan\delta'$, constant below $\mathbf{L' \approx 15D}$

$Q_{all} = (Q_p + Q_s)/FS$, FS = 2.5 – 4

References

Das, B.M. (2016). Principles of Foundation Engineering, 8th ed. Cengage Learning; this page follows it.

Meyerhof, G.G. (1976). Bearing capacity and settlement of pile foundations. Journal of the Geotechnical Engineering Division, ASCE, 102(GT3), 197–228.

Vesic, A.S. (1977). Design of Pile Foundations. NCHRP Synthesis of Highway Practice 42, Transportation Research Board.

Coyle, H.M. and Castello, R.R. (1981). New design correlations for piles in sand. Journal of the Geotechnical Engineering Division, ASCE, 107(GT7), 965–986.

NOTE

The figures in this presentation are taken from the GeoTerva Theory page; all numerical examples were produced with the module's own calculation library, and the module gives the same values.

GeoTerva

Online geotechnical analysis

From theory to practice

Calculate the example pile in this presentation step by step in GeoTerva: the numbers are the same.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com