GeoTerva

Geotechnical lecture presentation · Lateral earth pressure

Passive earth pressure: logarithmic spiral

Lecture: calculation of the passive pressure acting on a rough wall with a curved failure surface (logarithmic spiral), the Terzaghi–Peck trial wedge, the Lancellotta solution, comparison with Coulomb; inclined back, sloping surface, layers and water; a worked example on every topic.

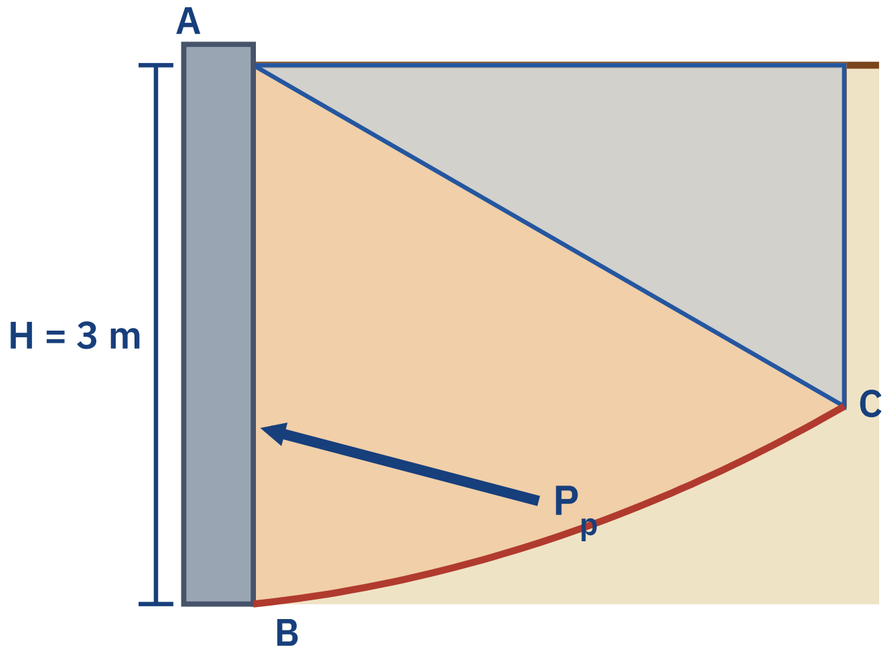
Wall friction

$r = r_0 e^{\omega \tan \varphi'}$

Terzaghi – Peck wedge

Lancellotta · Coulomb

Sloping backfill · layer · water



Introduction

What will we learn in this lecture?

- Why a **curved surface** is required instead of a **planar wedge** (Coulomb) for a rough wall
- The **logarithmic spiral**: $r = r_0 e^{\omega \tan \varphi'}$; every radius meets the curve at φ', and the resultant passes the spiral through the centre
- Calculation of P by moment equilibrium on the **Terzaghi–Peck** trial wedge; finding the force from the **smallest** trial wedge
- Comparing K_p with the **Lancellotta** closed-form solution and with Coulomb, Shields–Tolunay, Zhu–Qian
- The generalized wedge for inclined backfill, sloping surface, surcharge, **layers and water**
- How **unsafe** Coulomb becomes at large δ'

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Forces are per unit length of wall head (kN/m); $P_p = \tfrac{1}{2} K_p \gamma H^2$; the wall friction δ' acts in the direction in which the wall drags the soil downward.

Why does a planar surface not occur in the passive case?

THEORY

Coulomb's planar slip surface is a good approximation for **active** pressure; for the **passive** resistance of a rough wall, however, it gives more than the actual resistance, and increasingly more as wall friction increases: **on the unsafe side**.

When the wall is pushed into the soil, the soil rises relative to the wall; the wall moves the soil **downward** (positive wall friction) and the passive force is inclined δ′ **downward** from the normal to the wall face. In this case the failure surface is:

- near the toe (BC) **is a curve** (logarithmic spiral)
- near the surface (CD) is **straight**: it makes 45° – φ′/2 with the horizontal; the ACD zone is in the Rankine passive state

HOW MUCH MORE DOES COULOMB GIVE?

φ′ / δ′	Spiral	Coulomb
30° / 15°	4.61	4.98
30° / 30°	6.68	10.10
40° / 20°	9.67	11.77
40° / 40°	18.86	92.59

K_p coefficients; for φ′ = δ′ = 40° Coulomb gives 5 times more.

The property that makes the spiral useful

THEORY

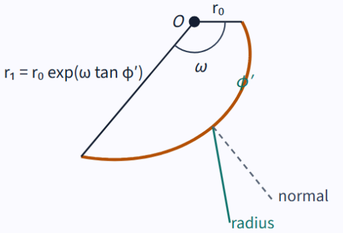
The curved BC part of the slip surface is taken as a logarithmic spiral centered at O:

$$r = r_0 \, e^{\omega \tan \varphi'}$$

r_0 is the radius at ω = 0. The property of the spiral: **every radius meets the curve at φ′**. In granular soil the resultant of the normal and shear forces on the slip surface also makes φ′ with the normal; that is, on the spiral the resultant lies along a radius and **passes through O**: it follows from the moment equation about O.

$$A_{OAB} = \frac{r_1^2 - r_0^2}{4 \tan \varphi'}$$

It is the area of the slice between r_0 and r_1; the module determines the wedge area and centroid numerically.



the resultant F of the normal and shear forces on the spiral (at ϕ′ to the normal) lies on a radius — it passes through O

Problem statement

EXAMPLE

Given

Spiral	**center O; initial radius** $r_0 = 1.00$ **m;** $\varphi' = 30^\circ$
Angle	$\omega = 60^\circ$ **and** $\omega = 90^\circ$

REQUIRED

$\mathbf{r_1}$, the angle of the radius to the curve, and the sector **area**.

Solution

SOLUTION

1 ω = 60° = 1.0472 rad

$$r_1 = r_0 e^{\omega \tan \varphi'} = 1.00\, e^{1.0472 \times 0.5774} = 1.00\, e^{0.6046}$$

$$r_1 = \mathbf{1.831}\ \mathrm{m}$$

2 Sector area

$$A = \frac{r_1^2 - r_0^2}{4 \tan \varphi'} = \frac{3.351 - 1.00}{4(0.5774)} = \mathbf{1.018}\ \mathrm{m^2}$$

3 for ω = 90°

$$r_1 = 1.00\, e^{1.5708 \times 0.5774} = \mathbf{2.477}\ \mathrm{m} \qquad A = \mathbf{2.223}\ \mathrm{m^2}$$

The radius increases by a factor of $e^{\tan \varphi'}$ = 1.781 per radian; at ω = 60°, r_1 = 1.831 m and the area is 1.018 m². The radius makes **φ′ = 30° at every point** with the curve; therefore the resultant passes through O.

Check the solution in GeoTerva ↗

Terzaghi and Peck (1967) method

THEORY

For the vertical wall AB of height H:

From A, the line AC is drawn below the surface at **45° − φ′/2**; a point C is chosen on it.

On the line AC, the center of the spiral with radius r_0 = OB passing through B and reaching C (r_1 = OC = $r_0 e^{\omega \tan \varphi'}$) is found as **O**.

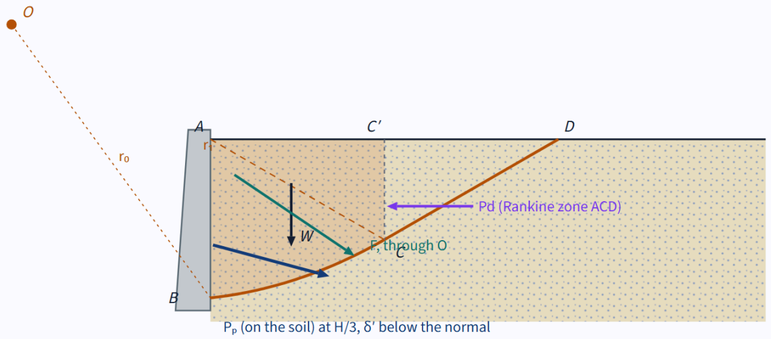
The face CC′ carries the Rankine passive force of zone ACD (horizontal, d/3 above C):

$$P_d = \tfrac{1}{2}\gamma d^2 \tan^2\left(45^\circ + \frac{\varphi'}{2}\right)$$

Moment about O for the weight W of wedge ABCC′, P_d and the wall force P (at H/3, δ′ below the normal) (the resultant F passes through O):

$$P = \frac{W\, l_W + P_d\, l_1}{l_P} \qquad P_p = \min_C P \qquad K_p = \frac{2P_p}{\gamma H^2}$$

The trial is repeated for many C values; the smallest force is the passive force. At δ′ = 0 it reduces to the spiral plane and $K_p = \tan^2(45° + \varphi'/2)$.



Problem statement

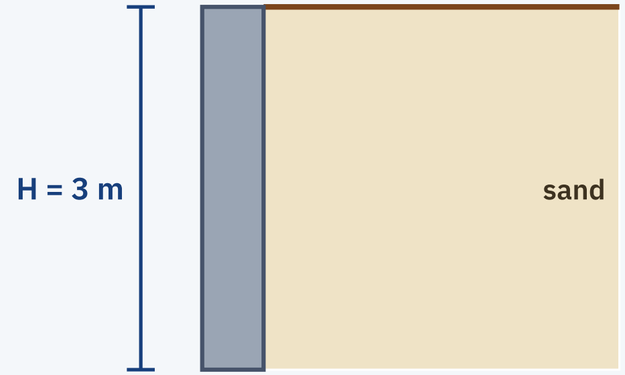
EXAMPLE

REQUIRED

Geometry of the smallest trial wedge and **P_p, K_p**; comparison with several trial wedges. The module opens with this wall.

Given

Wall	$H = 3$ **m, vertical; horizontal fill**
Soil	**dry sand:** $\gamma = 15.7\ \mathrm{kN/m^3},\ \varphi' = 30^\circ$**; wall friction** $\delta' = 15^\circ$



γ = 15.7 kN/m³, φ′ = 30°, δ′ = 15° (passive)

Solution (1/2)

SOLUTION

1 Geometry (AC at 45° − 15° = 30° to the horizontal)

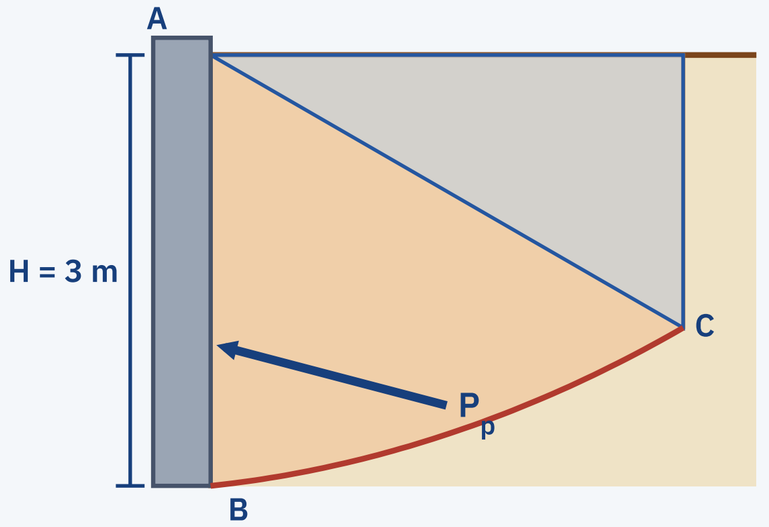
$$t = 3.80\ \text{m}: \quad C = (3.29;\ 1.10)\ \text{m}, \quad d = H - y_C = 1.90\ \text{m}$$

$$O = (-3.76;\ 5.17)\ \text{m}, \quad r_0 = OB = 6.39\ \text{m}, \quad r_1 = OC = 8.14\ \text{m}$$

2 Spiral angle and wedge weight

$$\omega = \ln(r_1/r_0)/\tan 30^\circ = 23.97^\circ$$

$$W = \gamma A_{ABCC'} = 15.7\,(8.480) = \mathbf{133.14}\ \text{kN/m}$$



Solution (2/2)

SOLUTION

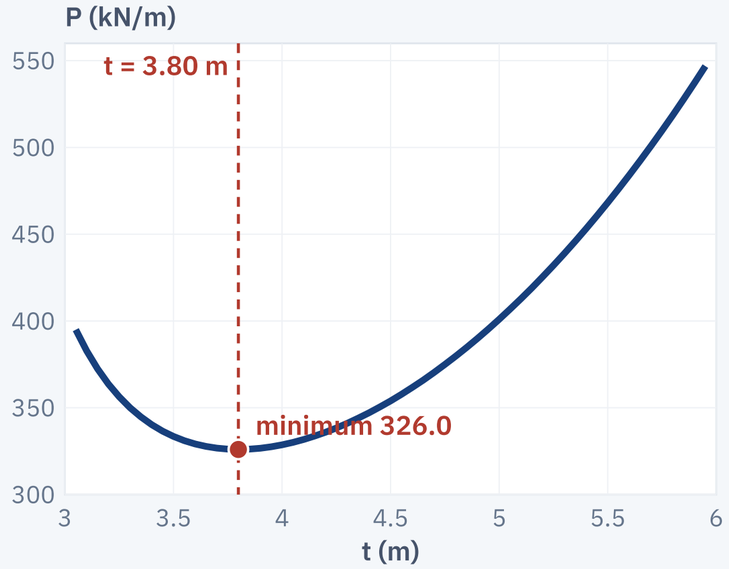
3 Rankine zone: $P_d = ½γd²\tan²60°$

$$P_d = \tfrac{1}{2}(15.7)(1.899)^2(3) = \mathbf{84.96}\ \mathrm{kN/m}$$

4 Moment about O (arms $l_W = 5.289$, $|l_1| = 3.438$, $l_P = 3.056$ m)

$$P = \frac{W\,l_W + P_d\,|l_1|}{l_P} = \frac{133.14(5.289) + 84.96(3.438)}{3.056} = \mathbf{326.03}\ \mathrm{kN/m}$$

The smallest trial force is **P_p = 326.0 kN/m** (t = 3.80 m); $K_p = 2P_p/(γH²)$ = **4.615**. At smaller or larger t the force increases (t = 3.2 m: 363.8; t = 4.5 m: 354.0 kN/m), so the passive force is the **smallest** trial wedge.



Check the solution in GeoTerva ↗

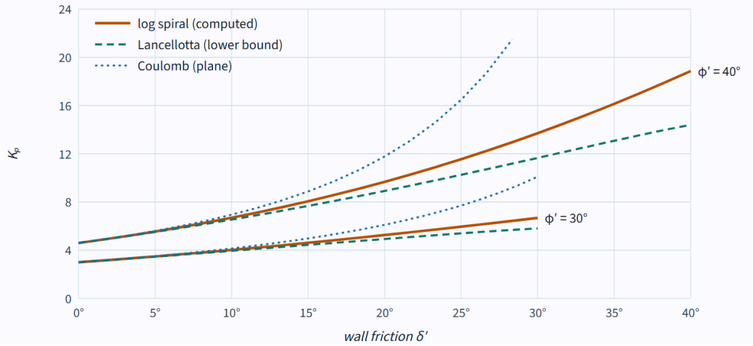
Spiral and other solutions

THEORY

Vertical wall, horizontal fill: other solutions (all $P_p = ½K_p\gamma H^2$): Shields–Tolunay (slice method, table), Sokolowskiĭ (characteristics, table), Zhu–Qian (triangular slice, table) and the **Lancellotta (2002)** closed-form solution (lower bound):

$$K_p = \frac{\cos\delta'}{1-\sin\varphi'}\left(\cos\delta' + \sqrt{\sin^2\varphi' - \sin^2\delta'}\right) e^{2\theta\tan\varphi'}\frac{1}{\cos\delta'} \qquad 2\theta = \sin^{-1}\frac{\sin\delta'}{\sin\varphi'} + \delta'$$

φ′	δ′	Log. spiral	Lancellotta	Coulomb
20°	10°	2.56	2.52	2.64
20°	20°	3.07	2.87	3.53
30°	15°	4.61	4.44	4.98
30°	30°	6.68	5.80	10.10
40°	20°	9.67	8.92	11.77
40°	40°	18.86	14.39	92.59



Lancellotta is the lower bound, the logarithmic spiral is the upper bound; the method of characteristics lies between the two, very close to the spiral.

Coulomb overpredicts at large δ′ (φ′ = δ′ = 40°: 92.6).

Problem statement

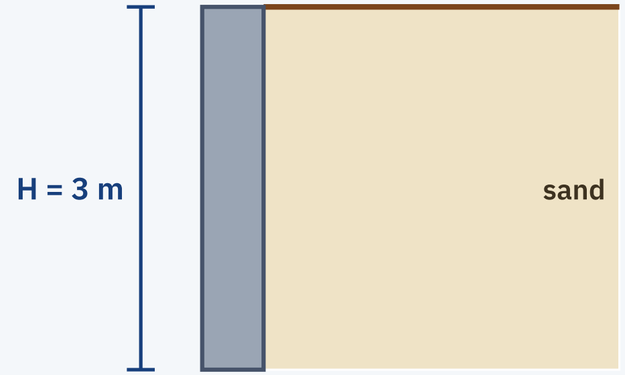
EXAMPLE

REQUIRED

Lancellotta's K_p step by step; the P_p values of the five solutions and their ranking.

Given

Wall and soil	**Wall in Example 2:** $H = 3$ **m,** $\gamma = 15.7\ \mathrm{kN/m^3}$, $\varphi' = 30°$, $\delta' = 15°$



γ = 15.7 kN/m³, φ′ = 30°, δ′ = 15° (passive)

Solution (1/2)

SOLUTION

1 Lancellotta (lower bound): 2θ and K_p

$$2\theta = \sin^{-1}\frac{\sin 15^\circ}{\sin 30^\circ} + 15^\circ = 31.17^\circ + 15^\circ = 0.8059\ \text{rad}$$

$$K_p = \frac{\cos 15^\circ}{1-\sin 30^\circ}\left(\cos 15^\circ + 0.4278\right) e^{0.8059\tan 30^\circ}\frac{1}{\cos 15^\circ} = \mathbf{4.439}$$

Solution (2/2)

SOLUTION

2 Five solutions ($P_p = ½K_p(15.7)(9) = 70.65\ K_p$)

Solution	K_p	P_p (kN/m)
Coulomb (1776), planar surface	4.977	351.6
Terzaghi & Peck (logarithmic spiral)	4.615	326.0
Shields & Tolunay (1973)	4.280	302.4
Zhu & Qian (2000)	4.575	323.2
Lancellotta (2002)	4.439	313.6

The planar Coulomb solution gives the largest value (351.6 kN/m); curved-surface solutions cluster in the range **302–326 kN/m**. Lancellotta (lower bound) gives 313.6, the logarithmic spiral (upper bound) 326.0 kN/m.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Soil	**sand, $\varphi' = 30^\circ$; vertical wall, horizontal fill**
Variable	$\delta' = 0^\circ,\ 10^\circ,\ 15^\circ,\ 20^\circ,\ 30^\circ$

REQUIRED

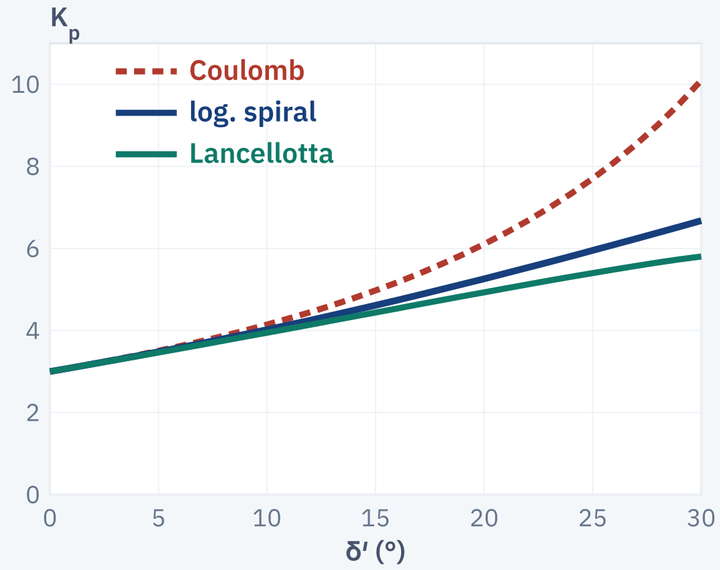
For each δ′, **K_p** (log. spiral, Lancellotta, Coulomb) and the excess over Coulomb.

Solution

SOLUTION

1 Four methods for the same φ′ = 30°

δ′	Log. spiral	Lancellotta	Coulomb	Coulomb / spiral
0°	3.000	3.000	3.000	1.00
10°	4.020	3.946	4.143	1.03
15°	4.615	4.439	4.977	1.08
20°	5.260	4.930	6.105	1.16
30°	6.677	5.804	10.095	1.51



At δ′ = 0 all three equal Rankine (3.00); as δ′ increases the Coulomb spiral rapidly exceeds it: 8% higher at δ′ = 15°, 51% higher at δ′ = 30° (unsafe). In practice, for passive resistance either the logarithmic spiral or δ′ ≤ φ′/2 is used.

Check the solution in GeoTerva ↗

Inclined backfill, sloping surface, surcharge, layer, and water

THEORY

The module extends the Terzaghi–Peck wedge to an inclined crest at θ and an inclined face at α: the region between AC and the surface is in the Rankine passive state of the inclined surface; the spiral on AC is drawn from O; the spiral can degenerate into a plane (the Coulomb plane is also a mechanism, and the smaller of the two is taken).

$$P_p = \tfrac{1}{2}\gamma H^2 K_{p\gamma} + q_e H K_{pq} \qquad q_e = q\frac{\cos\theta}{\cos(\theta-\alpha)}$$

The soil weight (H/3) and surcharge (H/2) contributions are found from their own smallest trial wedges. In a **layered** fill, each layer takes its own $K_{p\gamma}$, K_{pq} value; the weight of the overlying soil acts like a surcharge; **under water**, γ′ and u are added perpendicular to the back face:

$$\sigma'_p = \left[K_{pq}\sigma'_{v,top} + K_{p\gamma}\left(\sigma'_v - \sigma'_{v,top}\right)\right]\cos\theta$$

Case (δ′ = φ′ = 30°)	K_p charts (Caquot–Kerisel)	K_p module	Difference
θ = 20°	3.84	3.82	−0.6 %
θ = 30°	2.76	2.80	+1.6 %
α/φ′ = 0.4 (α = 12°)	10.60	10.22	−3.6 %
α/φ′ = 0.8 (α = 24°)	15.10	14.51	−3.9 %

$K_p = K_{p\gamma}\cos^2\theta$ (in Caquot–Kerisel, the back length $H_1 = H/\cos\theta$ is taken instead of H). This layered extension is the engineering extension of the module.

CAQUOT – KERISEL R′

The Caquot–Kerisel charts are for δ′ = φ′; for other δ′, $K_p = R' K_{p(\delta'=\varphi')}$. For φ′ = 30°, δ′/φ′ = 0.5, R′ = 0.746.

Problem statement

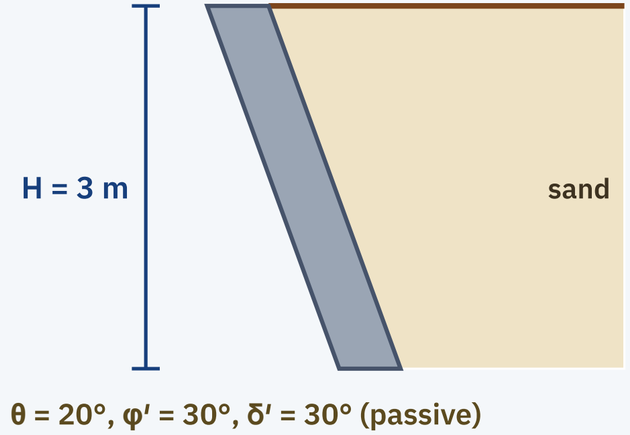
EXAMPLE

Given

Wall	$H = 3$ **m; back inclined at** $\theta = 20^\circ$ **to the vertical; horizontal fill**
Soil	**dry sand:** $\gamma = 15.7\ \mathrm{kN/m^3},\ \varphi' = 30^\circ$**;** $\delta' = 30^\circ = \varphi'$

REQUIRED

$K_{p\gamma}$, P_p; comparison with K_p from the chart; comparison with a wall having a vertical back.



Solution

SOLUTION

1 Spiral coefficient (module; δ′ = φ′ = 30°, θ = 20°)

$$K_{p\gamma} = \mathbf{4.321} \qquad K_{pq} = 4.127$$

2 Compare with force and chart

$$P_p = \tfrac{1}{2} K_{p\gamma} \gamma H^2 = \tfrac{1}{2}(4.321)(15.7)(9) = \mathbf{305.3}\ \mathrm{kN/m}$$

$$K_p = K_{p\gamma} \cos^2 20^\circ = 4.321(0.8830) = \mathbf{3.82}\ (\text{chart } 3.84)$$

P_p = **305.3 kN/m**; for a wall with a vertical back in the same soil (θ = 0, δ′ = 30°) the value would be 471.8 kN/m: a back inclined at 20° **reduces** the passive resistance by 35%. Difference from the chart: 0.6%.

Check the solution in GeoTerva ↗

Problem statement

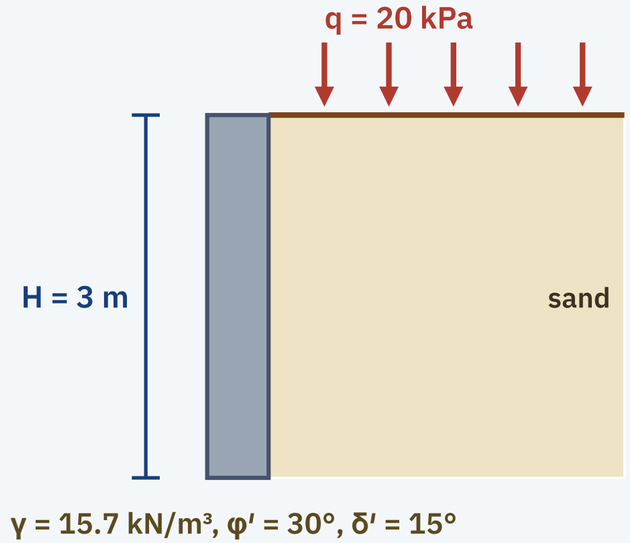
EXAMPLE

Given

Wall and soil	**Wall in Example 2:** $H = 3$ **m,** $\gamma = 15.7\ \mathrm{kN/m^3},\ \varphi' = 30^\circ,\ \delta' = 15^\circ$
Surcharge	$q = 20$ **kPa (on horizontal surface)**

REQUIRED

Soil and surcharge contributions, total $\mathbf{P_p}$ and $\bar{\mathbf{z}}$.



Solution

SOLUTION

1 Coefficients (module, δ′ = 15°)

$$K_{p\gamma} = \mathbf{4.615} \qquad K_{pq} = \mathbf{4.532} \qquad q_e = 20\ \mathrm{kPa}$$

2 Forces: soil (moment arm H/3 = 1.00 m) + surcharge (moment arm H/2 = 1.50 m)

$$P_\gamma = \tfrac{1}{2} K_{p\gamma} \gamma H^2 = 326.03\ \mathrm{kN/m}$$

$$P_q = K_{pq} q_e H = 4.532(20)(3) = 271.90\ \mathrm{kN/m}$$

3 Total and location

$$P_p = 326.03 + 271.90 = \mathbf{597.94}\ \mathrm{kN/m}$$

$$\bar{z} = \frac{326.03(1.00) + 271.90(1.50)}{597.94} = \mathbf{1.23}\ \mathrm{m}$$

P_p = **597.9 kN/m**, $\bar{z}$ = **1.23 m** (1.00 m without surcharge); the surcharge share is 83% of the soil.

Check the solution in GeoTerva ↗

Problem statement

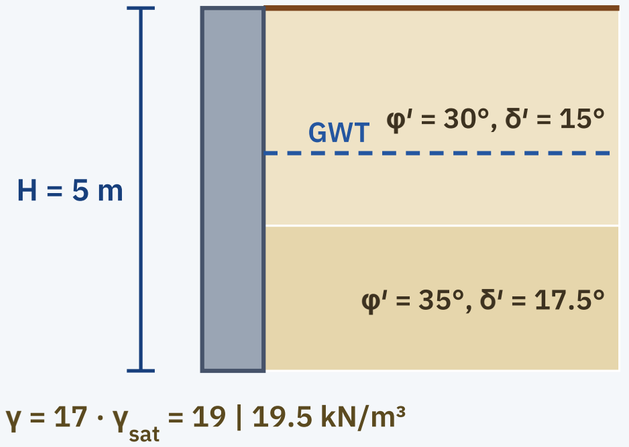
EXAMPLE

Given

Wall	$H = 5$ **m, vertical; horizontal fill; no surcharge**
Layer 1 (0–3 m)	$\gamma = 17,\ \gamma_{sat} = 19\ \mathrm{kN/m^3},\ \varphi' = 30^\circ,\ \delta' = 15^\circ$
Layer 2 (3–5 m)	$\gamma = 17,\ \gamma_{sat} = 19.5\ \mathrm{kN/m^3},\ \varphi' = 35^\circ,\ \delta' = 17.5^\circ$
Groundwater	**at 2 m depth**

REQUIRED

Pressure jump at the layer interface, components, **P**, β and **z̄**.



Solution (1/2)

SOLUTION

1 Coefficients (module)

$$\text{layer 1: } K_{p\gamma} = 4.615$$

$$\text{layer 2: } K_{p\gamma} = 6.522,\ K_{pq} = 6.351$$

2 Stresses: σ′$_v$(2) = 34; σ′$_v$(3) = 43.19; σ′$_v$(5) = 62.57 kPa

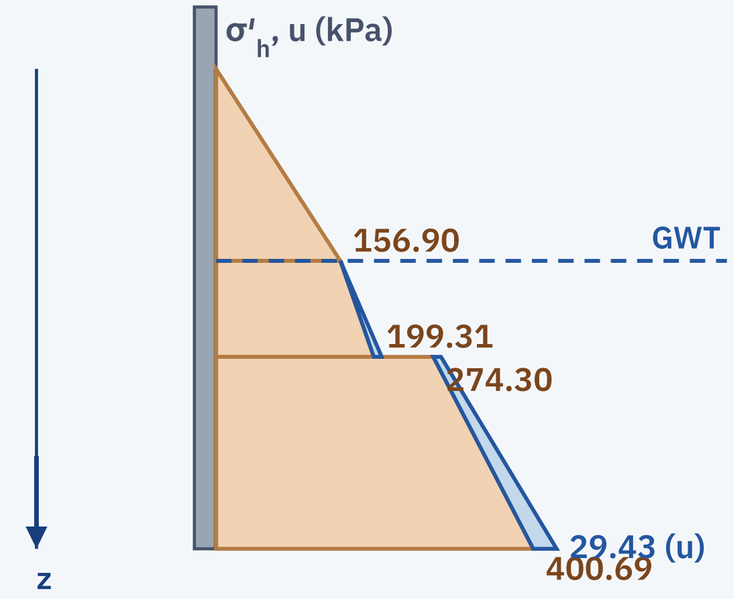
$$p(2) = 4.615(34) = 156.90$$

$$p(3^-) = 4.615(43.19) = 199.31$$

$$p(3^+) = K_{pq}\sigma'_v(3) = 6.351(43.19) = \mathbf{274.30}$$

$$p(5) = 274.30 + 6.522(19.38) = \mathbf{400.69}\ \text{kPa}$$

$$u(5) = \mathbf{29.43}\ \text{kPa}$$



Solution (2/2)

SOLUTION

3 Forces and components along the back (with layer δ′)

$$P_s = 335.0 + 675.0 = 1{,}010.0 \text{ kN/m}$$

$$N_s = 967.3 \qquad T = 289.7 \text{ kN/m}$$

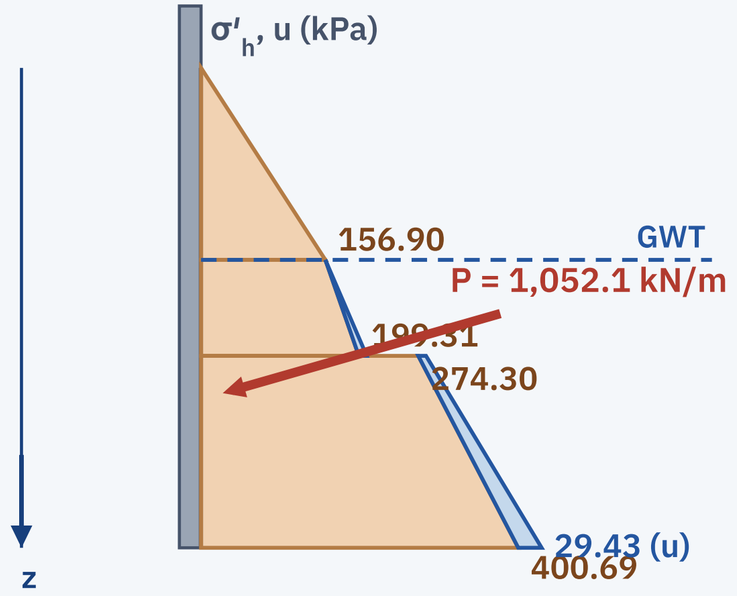
$$N_w = \tfrac{1}{2}(29.43)(3) = 44.15 \text{ kN/m (perpendicular to the ridge)}$$

4 Resultant and location

$$N = N_s + N_w = 1{,}011.5 \qquad P = \sqrt{N^2 + T^2} = \mathbf{1{,}052.1} \text{ kN/m}$$

$$\beta = \tan^{-1}\frac{T}{N} = 15.98^\circ \qquad \bar{z} = \mathbf{1.61} \text{ m}$$

P = **1,052.1 kN/m**, β = 16.0°, 1.61 m above the base. The water contribution is only 44.1 kN/m (4%): in the passive case the soil resistance is very large compared with the water; at the limit p jumps from 199.3 to 274.3 kPa.



Check the solution in GeoTerva ↗

Scope of the solution

THEORY

Granular soil ($c' = 0$); for cohesive passive pressure see the Rankine and Coulomb modules.

The logarithmic spiral is the upper bound, Lancellotta the lower bound; the actual resistance lies between the two. Planar Coulomb is unsafe at large δ'.

The **generalized wedge** (inclined back, inclined surface, layer, water, surcharge) is the engineering extension of the module; there is no closed-form layered solution; the soil weight and surcharge shares are found from their own minimum wedges.

Validity: $\alpha < \varphi'$; the back and the surface must intersect at a sufficient angle; if the spiral emerges from the soil or rotates in the wrong direction, it is not a mechanism (the plane is the limiting case).

For **earthquake**, the seismic passive module is used.

Summary

Key points

A planar wedge is unsafe in the passive case: with wall friction the failure surface is curved (spiral) at the heel and planar near the surface (Rankine zone)

Spiral: $r = r_0 e^{\omega \tan \varphi'}$; every radius meets the curve at φ', the resultant passes through O; area $(r_1^2 - r_0^2)/(4 \tan \varphi')$

Terzaghi–Peck: $P = (W\, l_W + P_d\, l_1)/l_P$; the P in the **smallest** trial wedge is the passive force; $K_p = 2P_p/(\gamma H^2)$

Lancellotta (lower bound, closed-form solution), spiral (upper bound); Coulomb gives 5 times more at $\varphi' = \delta' = 40°$

Generalized wedge: $P = \tfrac{1}{2}\gamma H^2 K_{p\gamma} + q_e H K_{pq}$; $K_p = K_{p\gamma} \cos^2\theta$ (chart definition); layer by layer $\sigma' = [K_{pq}\sigma'_{v,top} + K_{p\gamma}(\sigma'_v - \sigma'_{v,top})]\cos\theta$

Example wall ($H = 3$ m, $\varphi' = 30°$, $\delta' = 15°$): Coulomb 351.6 · spiral 326.0 · Lancellotta 313.6 kN/m

References

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NOTE

All numerical examples have been generated with the module's own calculation engine; the module gives the same values. The module opens with the wall in Example 2-3 (H = 3 m, δ′ = 15°).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own wall's backface inclination, backfill slope, layers, surcharge and groundwater in GeoTerva; the module provides the critical spiral, P_p and the height of the resultant together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com