GeoTerva

Geotechnical lecture presentation · Lateral earth pressure

Passive earth pressure under earthquake

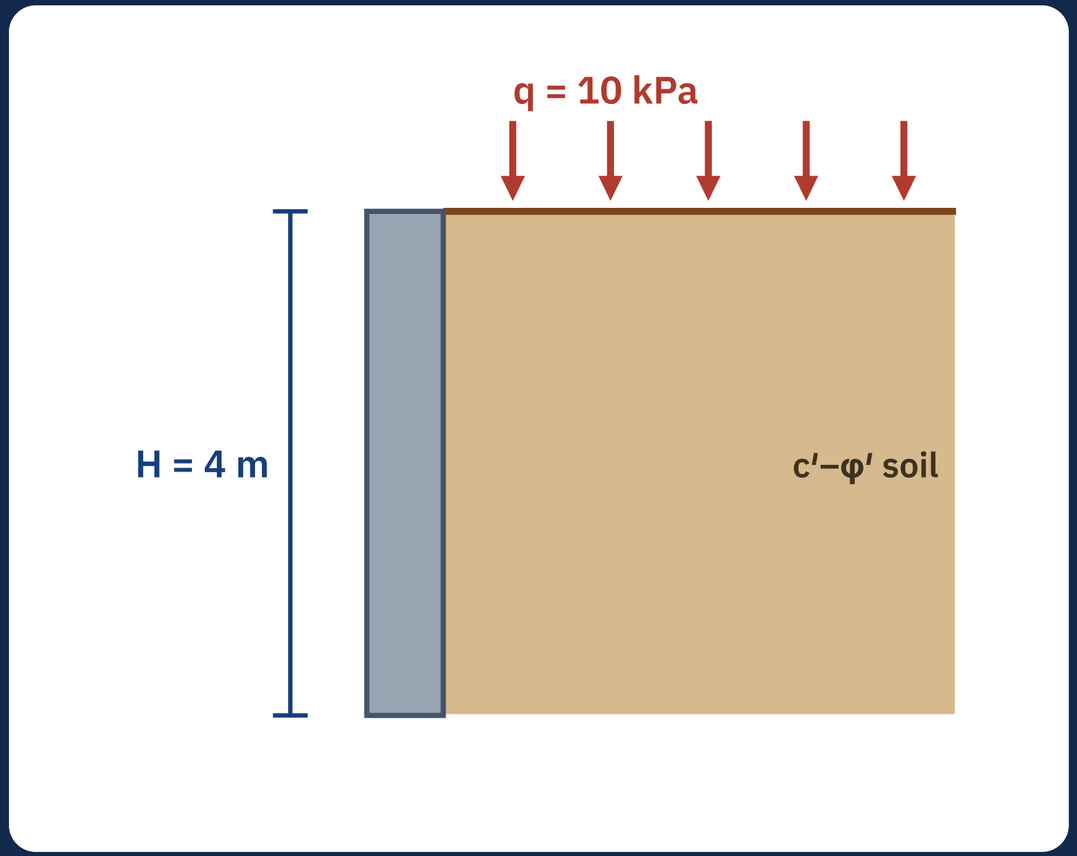
Lecture: pseudo-static analysis of the passive resistance of the soil in front of a wall during an earthquake (Subba Rao and Choudhury 2005): the coefficients $K_{p\gamma(e)}$, $K_{pq(e)}$, $K_{pc'(e)}$, k_v, layering, water and surcharge; a worked example for each topic.

$$P_{pe} = [\tfrac{1}{2}\gamma H^2 K_{p\gamma(e)} + qHK_{pq(e)} + 2c'HK_{pc'(e)}]/\cos\delta'$$

Subba Rao – Choudhury

k_h and k_v

Layer · water · surcharge



Introduction

What will we learn in this lecture?

- Why passive resistance **decreases** during an earthquake: the wedge inertial force helps move the soil away from the wall
- Computing the passive force with **three coefficients**: $P_{pe} = [\frac{1}{2}\gamma H^2 K_{p\gamma(e)} + qHK_{pq(e)} + 2c'HK_{pc'(e)}]/\cos\delta'$
- The **charts** of the coefficients ($\varphi' = 10–40°$, $\delta'/\varphi' = 0.5$ and 1, $k_v = 0$ and k_h) and how to read the intermediate values
- To find the location of the force (H/3 and H/2) and its direction (δ' from the normal)
- the effect of k_v, of **water** (confined or free), of the layers and of the surcharge
- The **k_h limit** where the charts end and why the module stops there

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

The forces are per unit length of wall (kN/m); vertical wall, horizontal surface; k_v positive upward, $0 \le k_v \le k_h$.

Subba Rao and Choudhury (2005)

THEORY

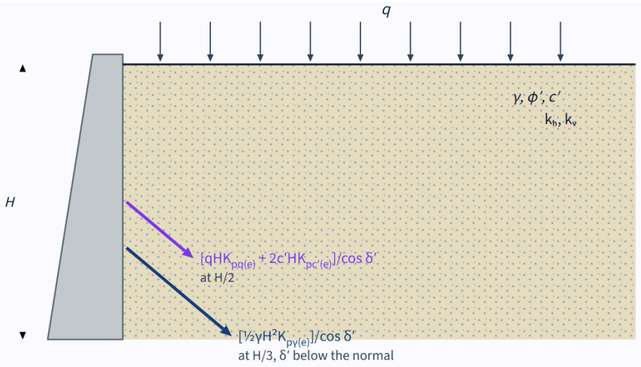
During an earthquake, the inertia of the wedge in front of the wall helps push the soil away from the wall; **the passive resistance decreases**. Subba Rao and Choudhury extended the limit equilibrium of the passive pressure with a curved surface (spiral + straight portion) using the forces $k_h W$ and $k_v W$ (vertical wall, horizontal c′–φ′ backfill, surcharge q, wall friction δ′, adhesion c'_a):

$$P_{pe} = \left[\tfrac{1}{2}\gamma H^2 K_{p\gamma(e)} + qHK_{pq(e)} + 2c'HK_{pc'(e)}\right]\frac{1}{\cos\delta'}$$

The three coefficients are components **perpendicular** to the wall; they are therefore divided by cos δ′. The γ part acts at H/3, the q and c′ part at H/2:

$$\bar{z} = \frac{P_\gamma\,(H/3) + (P_q + P_{c'})\,(H/2)}{P_{pe}}$$

> The force is inclined δ′ **downward** from the normal to the wall face.



Nomograms and reading of intermediate values

THEORY

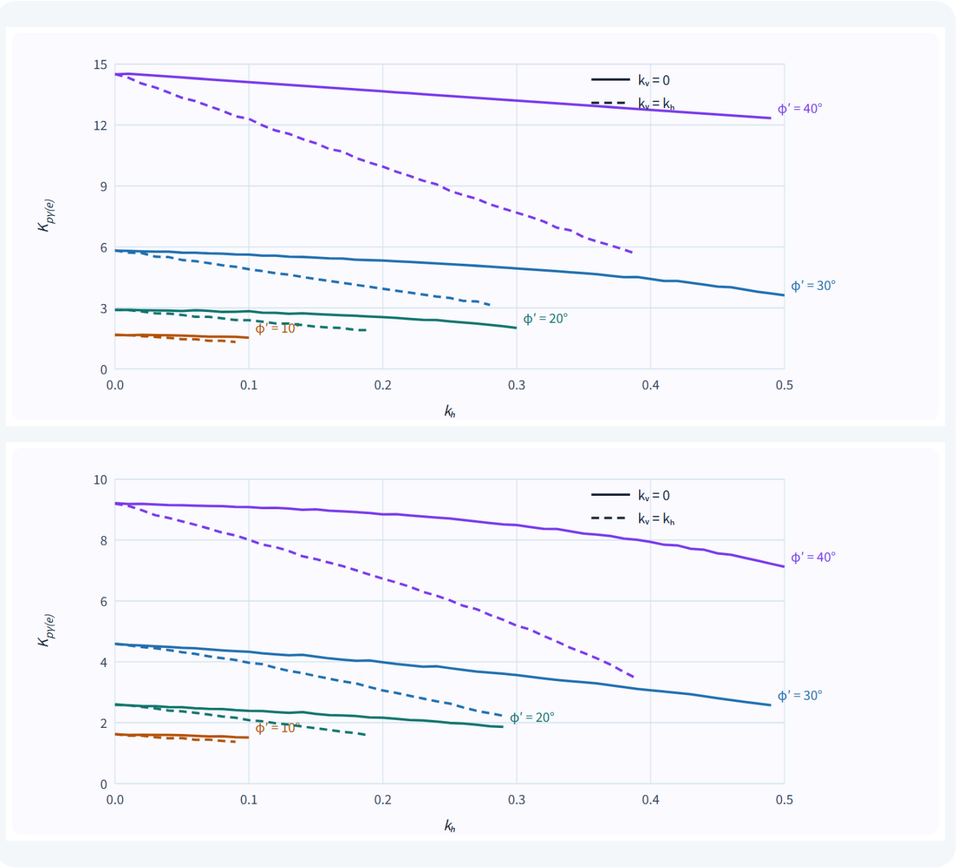
$K_{p\gamma(e)}$ and $K_{pq(e)}$ are given as **design charts** for $\varphi' = 10°, 20°, 30°, 40°$; $\delta'/\varphi' = 0.5$ and 1; $k_v = 0$ and $k_v = k_h$ (the module charts are tabulated in steps of $\Delta k_h = 0.01$). Intermediate values: linear in k_h and δ'/φ', **log-linear** in φ'; no extrapolation outside the range (φ' 10–40°, δ' $\varphi'/2 - \varphi'$).

As k_h increases the coefficients decrease; with **$k_v = k_h$ much faster**. Each curve ends at the k_h at which the passive resistance is exhausted; the module stops there.

At $k_h = 0$ the charts give the static passive coefficient: for $\varphi' = \delta' = 40°$, $K_{p\gamma(e)} = 14.49$; in the log-spiral module $K_p \cos \delta' = 14.45$.

$0 \le k_v \le k_h$ (upward, unfavourable for passive): linear interpolation between the $k_v = 0$ and $k_v = k_h$ curves; tested with the Coulomb coefficient, error < 0.4% (in the low range).

$K_{pc'(e)}$ in the table: for $\varphi' = 40°$, $\delta'/\varphi' = 1$, 5.95 at $c'_a = 0$ and 5.97 with adhesion.



Problem statement

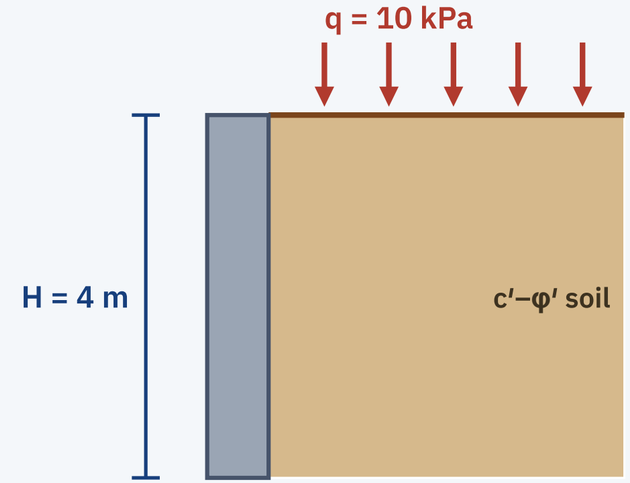
EXAMPLE

Given

Wall	$H = 4$ **m, vertical; horizontal surface;** $q = 10$ **kPa**
Soil	$\gamma = 17.5\ \mathrm{kN/m^3},\ \varphi' = 40°,\ c' = 10$ **kPa;** $\delta' = 40°, c'_a = 0$
Earthquake	$k_h = 0.3, k_v = 0$

REQUIRED

Chart coefficients, $\mathbf{P_{pe}}$, its direction and $\bar{\mathbf{z}}$. The module opens with this wall.



Solution (1/2)

SOLUTION

1 Coefficients (δ′/φ′ = 1, k_h = 0.3, k_v = 0; the module reads from the charts)

$$K_{p\gamma(e)} = \mathbf{13.20} \qquad K_{pq(e)} = \mathbf{11.00} \qquad K_{pc'(e)} = \mathbf{5.95}$$

$$(\text{static}, k_h = 0: \; K_{p\gamma} = 14.49, \; K_{pq} = 11.11)$$

Solution (2/2)

SOLUTION

2 Normal components and resultant

$$P_\gamma = \tfrac{1}{2}(17.5)(4)^2(13.20) = 1{,}848 \text{ kN/m}$$

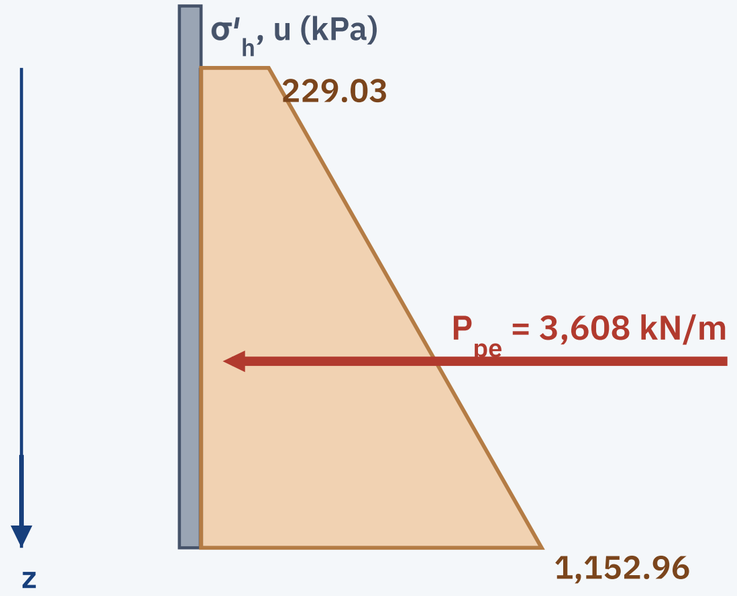
$$P_q = 10(4)(11.00) = 440 \qquad P_{c'} = 2(10)(4)(5.95) = 476$$

$$P_{pe} = \frac{1{,}848 + 440 + 476}{\cos 40^\circ} = \frac{2{,}764}{0.766} = \mathbf{3{,}608} \text{ kN/m}$$

3 Location and components

$$\bar{z} = \frac{2{,}412.2(4/3) + 1{,}195.9(2)}{3{,}608} = \mathbf{1.554} \text{ m}$$

$$P_h = P_{pe}\cos 40^\circ = 2{,}764 \qquad P_v = P_{pe}\sin 40^\circ = 2{,}319 \text{ kN/m}$$



P_{pe} ≈ **3,608 kN/m**, inclined 40° downward from the normal, at **1.554 m** above the base; the soil-weight contribution is 2,412 and the q and c′ contribution is 1,196 kN/m (when divided by the normal).

Check the solution in GeoTerva ↗

Problem statement

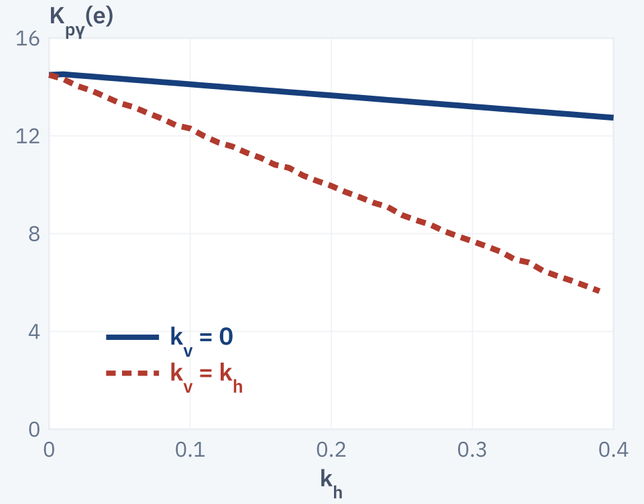
EXAMPLE

Given

Soil	$\varphi' = \delta' = 40^\circ$ ($\delta'/\varphi' = 1$); **dry sand, cohesionless**
Variable	$k_h = 0,\ 0.1,\ 0.2,\ 0.3,\ 0.35$ **and** $k_v = 0$ **or** $k_v = k_h$

REQUIRED

$K_{p\gamma(e)}$ values; reduction ratios at $k_h = 0.3$.



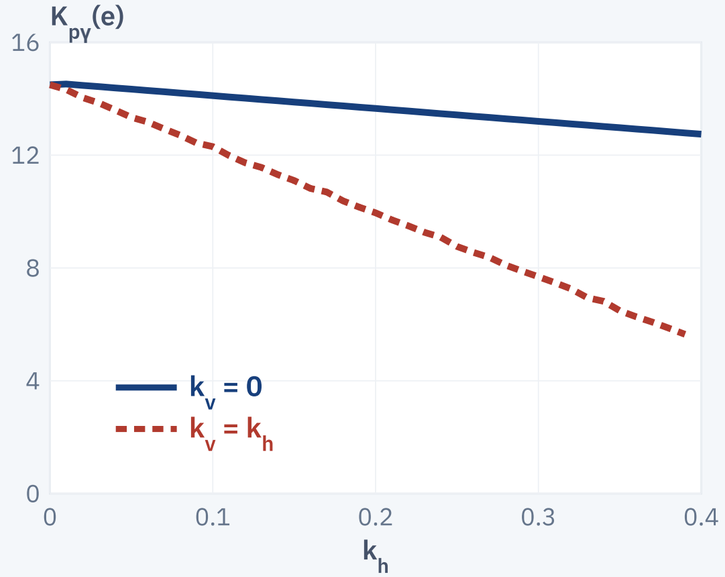
Solution

SOLUTION

1 Chart readings ($\delta'/\varphi' = 1$)

k_h	$K_{p\gamma(e)}$ ($k_v = 0$)	$K_{p\gamma(e)}$ ($k_v = k_h$)
0.00	14.49	14.49
0.10	14.11	12.30
0.20	13.65	9.95
0.30	13.20	7.69
0.35	12.97	6.48

k_h = 0 to 0.3, $K_{p\gamma(e)}$ 14.49 → 13.20 (9% decrease, k_v = 0); when $k_v = k_h$ it drops to 7.69 (47%). In the static case 14.49 is consistent with $K_p \cos \delta'$ = 14.45 in the log-spiral module.



Check the solution in GeoTerva ↗

Problem statement

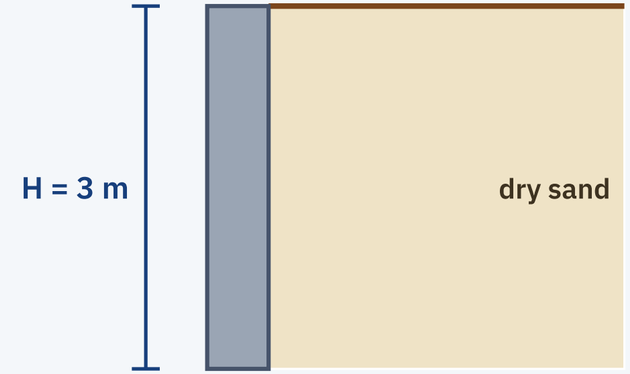
EXAMPLE

Given

Wall	$H = 3$ m, vertical; horizontal surface; no surcharge
Soil	dry sand: $\gamma = 16\ \mathrm{kN/m^3}$, $\varphi' = 30°$; $\delta' = 15°$ ($\delta'/\varphi' = 0.5$)
Earthquake	$k_h = 0.2$, $k_v = 0$

REQUIRED

P_{pe}, $\bar{z}$, components; comparison with the static passive force.



γ = 16 kN/m³, φ′ = 30°, δ′ = 15°, k_h = 0.2

Solution

SOLUTION

1 Coefficients (δ′/φ′ = 0.5)

$$k_h = 0: \; K_{p\gamma} = 4.592 \qquad k_h = 0.2: \; K_{p\gamma(e)} = \mathbf{3.987}$$

2 Force (c′ = q = 0)

$$P_{pe} = \frac{\frac{1}{2}(16)(3)^2(3.987)}{\cos 15^\circ} = \frac{287.1}{0.9659} = \mathbf{297.2}\ \mathrm{kN/m}$$

$$\text{static: } P_p = 342.3\ \mathrm{kN/m}\ (K_p = 4.592)$$

3 Location and components

$$\bar{z} = \frac{H}{3} = \mathbf{1.00}\ \mathrm{m} \qquad P_h = P_{pe}\cos 15^\circ = 287.1 \qquad P_v = P_{pe}\sin 15^\circ = 76.9\ \mathrm{kN/m}$$

P_{pe} = **297.2 kN/m**; the static value was 342.3 kN/m: k_h = 0.2 **reduced** the resistance by 13%.

Check the solution in GeoTerva ↗

Problem statement

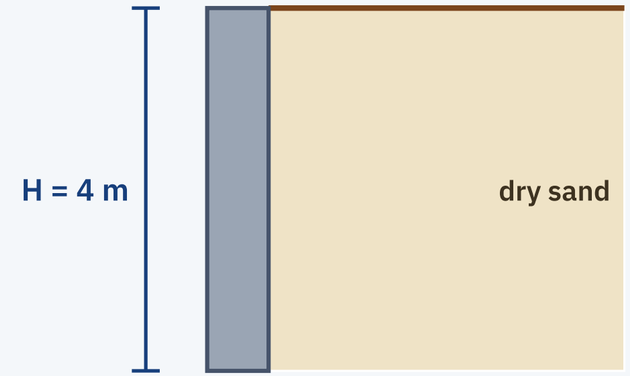
EXAMPLE

REQUIRED

For each k_v, $K_{p\gamma(e)}$ and P_{pe}; how is the intermediate value found?

Given

Wall and soil	$H = 4$ **m; dry sand:** $\gamma = 17.5\ \mathrm{kN/m^3},\ \varphi' = \delta' = 40^\circ$
Earthquake	$k_h = 0.3$**;** $k_v = 0,\ 0.15\ (= 0.5k_h),\ 0.30\ (= k_h)$



γ = 17.5, φ′ = δ′ = 40°, k_h = 0.3

Solution

SOLUTION

1 Interpolation: $K(k_v) = K(0) + [K(k_h) - K(0)]\cdot k_v/k_h$; $P_{pe} = ½(17.5)(16)K_{p\gamma(e)}/\cos 40° = 182.7\ K_{p\gamma(e)}$

k_v	k_v/k_h	$K_{p\gamma(e)}$	$K_{pq(e)}$	P_{pe} (kN/m)
0.00	0.0	13.20	11.00	2,412
0.15	0.5	10.44	9.29	1,909
0.30	1.0	7.69	7.58	1,405

At $k_v = 0.5k_h$ the force is 2,412 → 1,909 kN/m (21%), at $k_v = k_h$ it is 1,405 kN/m (42%). Since upward vertical acceleration reduces the weight, it is **unfavorable for passive**; $k_v = 0.5k_h$ is a common choice.

Check the solution in GeoTerva ↗

Layer extension of the module

THEORY

The charts are for a single soil, vertical wall and horizontal surface; the geometry remains as such. The module divides the soil into **horizontally bounded layers**; each layer takes its own coefficient, with the weight of the overlying layers (and q) acting as a surcharge through $K_{pq(e)}$, its own weight through $K_{p\gamma(e)}$, and its cohesion through $K_{pc'(e)}$:

$$\sigma_n = K_{pq(e)}\sigma'_{v,top} + K_{p\gamma(e)}\left(\sigma'_v - \sigma'_{v,top}\right) + 2c'K_{pc'(e)} \qquad \text{along the face: } \sigma_n \tan\delta'$$

Below the water level $\gamma' = \gamma_{sat} - \gamma_w$, u is perpendicular to the wall. In an earthquake, pore water moves with the soil (**restricted**) or does not (**free**); the charts are read at a larger k_h:

$$k_{h,w} = \frac{\gamma_{sat}}{\gamma_{sat} - \gamma_w}k_h \ \text{(restrained)} \qquad k_{h,w} = \frac{\gamma_d}{\gamma_{sat} - \gamma_w}k_h \ \text{(free)}$$

In free water, the Westergaard hydrodynamic pressure $(7/8)k_h\gamma_w\sqrt{(d_w z')}$ is **subtracted from the resistance** (unfavourable phase for passive). Under water, k_v is read with the ratio $k_v/k_{h,w}$.

This layered format is the engineering extension of the module; the module stops where the chart curve ends.

BUILD-UP

Layer: K_{pq} (surcharge above) + $K_{p\gamma}$ (self-weight) + $K_{pc'}$ (cohesion); water: γ' and u; earthquake: $k_{h,w}$.

Problem statement

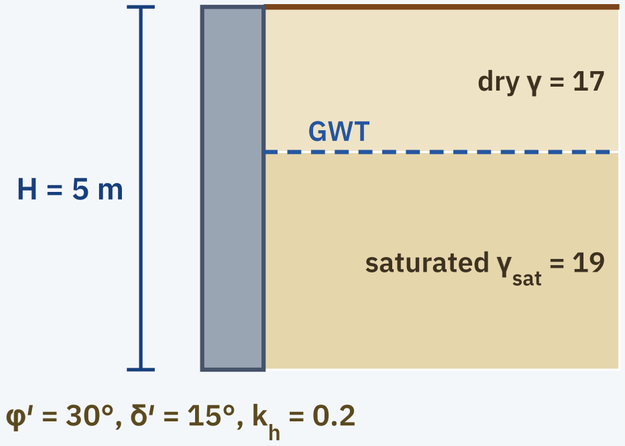
EXAMPLE

Given

Wall	$H = 5$ **m, vertical; horizontal surface; no surcharge**
Soil	$\gamma = 17\ \mathrm{kN/m^3}$ **(dry),** $\gamma_{sat} = 19$**,** $\gamma_d = 17\ \mathrm{kN/m^3}$**;** $\varphi' = 30^\circ,\ \delta' = 15^\circ$
Earthquake and water	$k_h = 0.2$**,** $k_v = 0$**; groundwater level 2 m (d_w = 3 m)**

REQUIRED

P_{pe} and $\bar{z}$ for the two cases of restricted and free water; comparison with dry and static values.



Solution (1/2)

SOLUTION

1 k_h read below the water

$$\text{restrained: } k_{h,w} = \frac{19}{19 - 9.81}(0.2) = \mathbf{0.413} \qquad \text{free: } k_{h,w} = \frac{17}{19 - 9.81}(0.2) = \mathbf{0.370}$$

Solution (2/2)

SOLUTION

2 Coefficients and forces ($\sigma_n = K_{p\gamma(e)}\sigma'_v$, no load at the top of the layer; below the water $u = 9.81(z - 2)$; $P_{wd} = (7/12)(0.2)(9.81)(3)^2 = 10.30$ kN/m is subtracted from the resistance)

	k_h read	$K_{p\gamma(e)}$	Soil N_s	Water	Hydrodynamic	P (kN/m)	$\bar{z}$ (m)
Above water (0–2 m)	0.200	3.987					
Confined underwater	0.413	3.008	566.8	44.1	–	629.5	1.84
Unconfined underwater	0.370	3.230	598.6	44.1	10.3	652.5	1.83

With confined water P = **629.5 kN/m**, with free water **652.5 kN/m**; in soil without groundwater 877.1, with static ($k_h = 0$) and water 885.9 kN/m. Since the $k_{h,w}$ read under water is large, $K_{p\gamma(e)}$ drops from 3.99 to 3.01 and the resistance **decreases**.

Check the solution in GeoTerva ↗

Problem statement

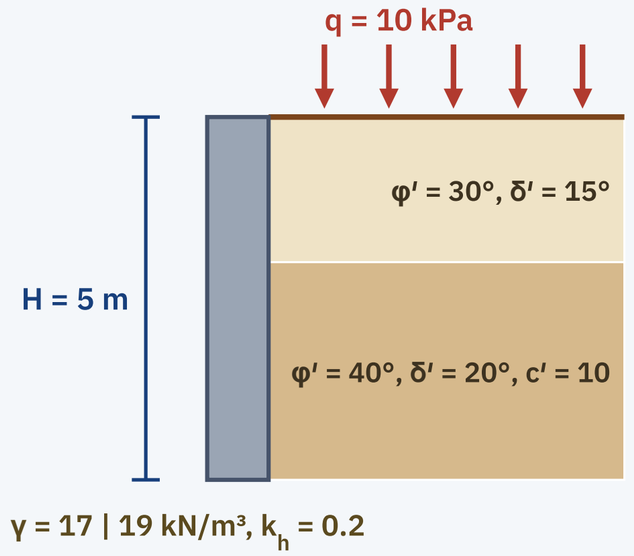
EXAMPLE

Given

Wall	$H = 5$ **m, vertical; horizontal surface;** $q = 10$ **kPa; no water**
Layer 1 (0–2 m)	$\gamma = 17\ \mathrm{kN/m^3},\ \varphi' = 30^\circ,\ \delta' = 15^\circ,\ c' = 0$
Layer 2 (2–5 m)	$\gamma = 19\ \mathrm{kN/m^3},\ \varphi' = 40^\circ,\ \delta' = 20^\circ,\ c' = 10$ **kPa**
Earthquake	$k_h = 0.2,\ k_v = 0$

REQUIRED

Pressure jump at the boundary, layer forces, **P**, β and **z̄**.



Solution (1/2)

SOLUTION

1 Coefficients ($k_h = 0.2$)

$$\text{layer 1: } K_{p\gamma} = 3.987,\ K_{pq} = 4.096$$

$$\text{layer 2: } K_{p\gamma} = 8.844,\ K_{pq} = 8.449,\ K_{pc'} = 4.33$$

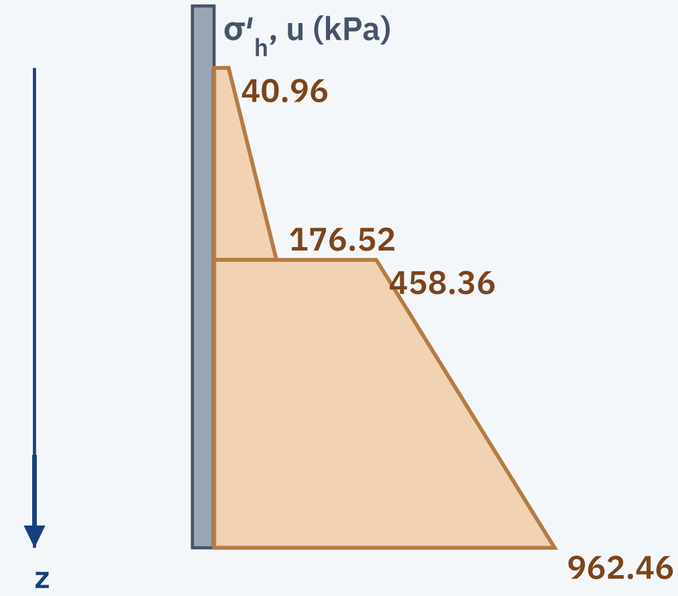
2 Normal stress σ_n (σ'_v = 10; 44; 101 kPa: top, boundary, base)

$$\sigma_n(0) = K_{pq} q = 4.096(10) = 40.96$$

$$\sigma_n(2^-) = 40.96 + 3.987(34) = 176.52$$

$$\sigma_n(2^+) = 8.449(44) + 2(10)(4.33) = \mathbf{458.36}$$

$$\sigma_n(5) = 458.36 + 8.844(57) = \mathbf{962.46}\ \text{kPa}$$



Solution (2/2)

SOLUTION

3 Components normal and along the face (with δ′ of the layer; arms from the base)

$$N_1 = 217.5 \ (\text{arm } 3.79 \text{ m})$$

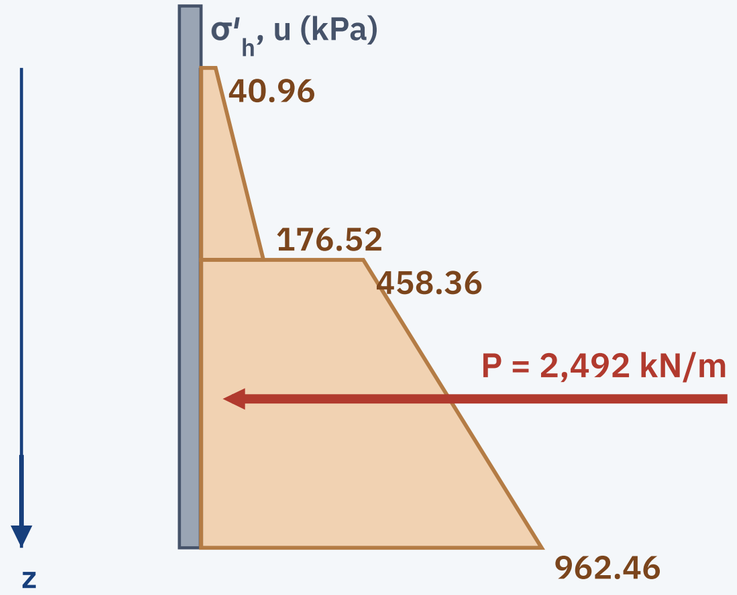
$$N_2 = 2{,}131.2 \ (\text{arm } 1.32 \text{ m})$$

$$N = 2{,}348.7 \qquad T = 834.0 \text{ kN/m}$$

4 Resultant and location

$$P = \sqrt{N^2 + T^2} = \mathbf{2{,}492.4} \text{ kN/m} \qquad \beta = \tan^{-1}\frac{T}{N} = 19.55^\circ$$

$$\bar{z} = \frac{217.5(3.79) + 2{,}131.2(1.32)}{2{,}348.7} = \mathbf{1.55} \text{ m}$$



P = **2,492 kN/m**, β = 19.5° (δ′ between 15° and 20°), 1.55 m above the base. At the boundary σ_n **jumps** from 176.5 to 458.4 kPa (φ′, δ′ and c′ change).

Check the solution in GeoTerva ↗

Where does the module stand?

THEORY

Each chart curve ends at the k_h value at which the passive resistance is fully mobilized; the module stops there (no extrapolation). The limit depends on φ', δ'/φ' and k_v:

φ'	$k_v = 0$, $\delta'/\varphi' = 0.5$	$k_v = 0$, $\delta'/\varphi' = 1$	$k_v = k_h$, $\delta'/\varphi' = 0.5$	$k_v = k_h$, $\delta'/\varphi' = 1$
10°	0.108	0.098	0.085	0.079
20°	0.292	0.279	0.185	0.198
30°	0.494	0.463	0.288	0.282
40°	0.500	0.463	0.393	0.397

$k_v = k_h$ curves end much earlier ($\varphi' = 30°$, $\delta' = \varphi'/2$: 0.29).

Measured under water $k_{h,w} = k_h \gamma_{sat}/\gamma' \approx 2k_h$ easily exceeds this limit.

Problem statement

EXAMPLE

Given

Soil	$\varphi' = 30^\circ$, $\delta' = 15^\circ$; $\gamma_{sat} = 19\ \mathrm{kN/m^3}$; **groundwater at the top of the wall (z_w = 0)**
Earthquake	$k_h = 0.3$, $k_v = 0$; **pore water restricted**

REQUIRED

$k_{h,w}$ to be read under water; chart limit; what does the module do?

Solution

SOLUTION

1 Read k_h and chart limit

$$k_{h,w} = \frac{19}{19 - 9.81}(0.3) = \mathbf{0.620}$$

$$k_{h,lim}(30^\circ,\ \delta'/\varphi' = 0.5,\ k_v = 0) = 0.494$$

$k_{h,w}$ = 0.620 > 0.494: the chart does not exist at this value (the region where passive resistance is exhausted), **the module stops the calculation**. Solution: lower the groundwater level or reduce k_h; for k_h = 0.2, $k_{h,w}$ = 0.41 is within the limit (Example 5).

Check the solution in GeoTerva ↗

Scope of the solution

THEORY

Pseudo-static: constant k_h, k_v; no time dependence and no permanent displacement.

Geometry: vertical wall, horizontal surface; charts for $\varphi' = 10–40°$, $\delta'/\varphi' = 0.5–1$; k_v upward only (unfavourable for passive), $0 \le k_v \le k_h$.

Cohesion: $K_{pc'(e)}$ table; adhesion is either zero or $c'_a/c' = \tan\delta'/\tan\varphi'$.

Layer, water and surcharge is the engineering extension of the module (no closed-form solution exists in the literature); water may be restricted or free, and in the free case the Westergaard resistance is removed.

Static passive: logarithmic spiral and Coulomb; **active earthquake**: Mononobe–Okabe modules.

Summary

Key points

$P_{pe} = [½\gamma H^2 K_{p\gamma(e)} + qHK_{pq(e)} + 2c'HK_{pc'(e)}] / \cos \delta'$; the γ part at H/3, the q and c′ parts at H/2; δ′ below the face normal

The **coefficients** are read from charts (φ′ 10–40°, δ′/φ′ 0.5–1, $k_v = 0$ and k_h): k_h is linear in δ′/φ′ and log-linear in φ′; the coefficients decrease as k_h increases

k_v (upward) is unfavorable for passive: at $k_v = k_h$, φ′ = δ′ = 40°, for $k_h = 0.3$, $K_{p\gamma(e)}$ 13.2 → 7.7

In the layers $\sigma_n = K_{pq}\sigma'_{v,top} + K_{p\gamma}(\sigma'_v - \sigma'_{v,top}) + 2c'K_{pc'}$; jump at the boundary; along the face $\sigma_n \tan \delta'$

Water: γ′ and u; the chart is read with $k_{h,w} = k_h\gamma_{sat}/\gamma'$ (restricted) or γ_d/γ' (free); in the free case the hydrodynamic pressure is subtracted from the resistance

Limit: if the chart curve ends (large $k_{h,w}$, $k_v = k_h$), the module stops

References

Das, B.M. (2018). Principles of Geotechnical Engineering, 9th ed. Cengage Learning.

Subba Rao, K.S. and Choudhury, D. (2005). Seismic passive earth pressures in soils. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 131(1), 131–135.

Westergaard, H.M. (1933). Water pressures on dams during earthquakes. Transactions ASCE, 98, 418–433.

NOTE

All numerical examples are taken from the module's own calculation code; the module gives the same values. The module opens with the wall from Example 1 ($H = 4$ m, $\varphi' = \delta' = 40°$, $k_h = 0.3$).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own wall's layers, cohesion, surcharge, groundwater and earthquake coefficients in GeoTerva; the module gives P_{pe}, its direction and its height of action together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com