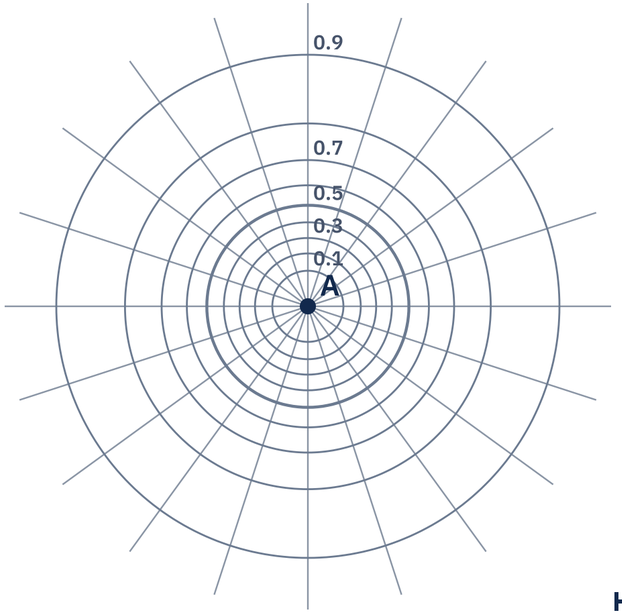
Geotechnical lecture presentation · Stresses in the soil mass

Newmark influence chart

Lecture: Newmark influence chart; the plan of the loaded area is placed on the chart, the covered elements are counted, and $\Delta\sigma_z$ = (IV) q M is obtained.

- Rings of equal stress increase
- Influence value IV = 0.005
- Element count
- Any plan
- Can be solved by hand



What will we learn in this lecture?

How the diagram is constructed: from the circular load solution, circles of radius $\mathbf{R/z = \sqrt{[(1 - \Delta\sigma_z/q)^{-2/3} - 1]}}$

10 rings × 20 sectors = 200 elements: each element has the same effect, **IV = 1/N = 0.005**

Use: the plan is drawn at scale **AB = z**, point A is placed at the center of the chart, **M** elements are counted; $\mathbf{\Delta\sigma_z = (IV)\ q\ M}$

Verifying the counting ring by ring by hand with the **rectangular corner solution**; depth, L-shaped plan and a point outside the area

That **a separate drawing** of the plan is required for each depth

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: formula and rationale

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example: 3 m × 3 m column footing, 660 kN (q = 73.33 kN/m²); point A is 3 m below the footing, beneath the midpoint of one edge.

Circles of equal stress increase

THEORY

Under the center of a uniformly loaded circular area, $\Delta\sigma_z/q = 1 - 1/[(R/z)^2 + 1]^{3/2}$. Inverting the relation gives the **radius** of the circular area that produces a selected stress ratio (both sides are dimensionless).

For $\Delta\sigma_z/q$ = 0.1, 0.2 ... 0.9, the circles drawn with these radii divide the plane into **rings**. Each ring loaded alone produces a stress increase of **0.1 q** at depth z below the centre; the last ring (between 0.9 and 1.0) extends to infinity

$$\frac{R}{z} = \sqrt{\left(1 - \frac{\Delta\sigma_z}{q}\right)^{-2/3} - 1}$$

$\Delta\sigma_z/q$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
R/z	0.0000	0.1865	0.2698	0.3383	0.4005	0.4598	0.5181	0.5768	0.6370	0.6998
$\Delta\sigma_z/q$	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
R/z	0.7664	0.8384	0.9176	1.0067	1.1097	1.2328	1.3871	1.5944	1.9083	2.5235

200 elements, influence value 0.005

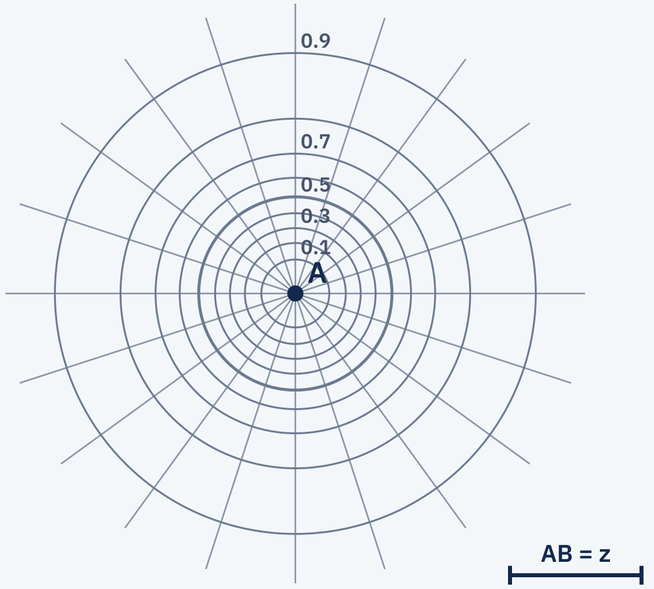
THEORY

Newmark (1942) divided the rings into sectors with equally spaced **radial lines**. Due to symmetry, every element in the same ring has the same influence

If the total number of elements is **N**, the contribution of each loaded element to the stress below its centre is **IV = 1/N** times q: the **influence value**. For 10 rings and 20 sectors, N = 200, IV = 0.005

Since the effect of each element is independent of its position in the chart, a loaded area of any shape can be measured **by counting the elements**

$$IV = \frac{1}{N} = \frac{1}{200} = 0.005$$



$\Delta\sigma_z = (IV)\, q\, M$

THEORY

1. The **depth z** below the loaded area at which the stress is required is selected

2. The plan of the loaded area is drawn to a scale such that the depth z equals the **unit length AB** of the diagram

3. The plan is placed on the diagram so that the point whose stress is required coincides with the **center** of the diagram

4. The number of **M** elements remaining within the plan is determined; elements partially inside are accounted for by estimation

A **separate drawing of the plan** is required for each depth

$$\Delta\sigma_z = (IV)\, q\, M$$

Number of covered elements M

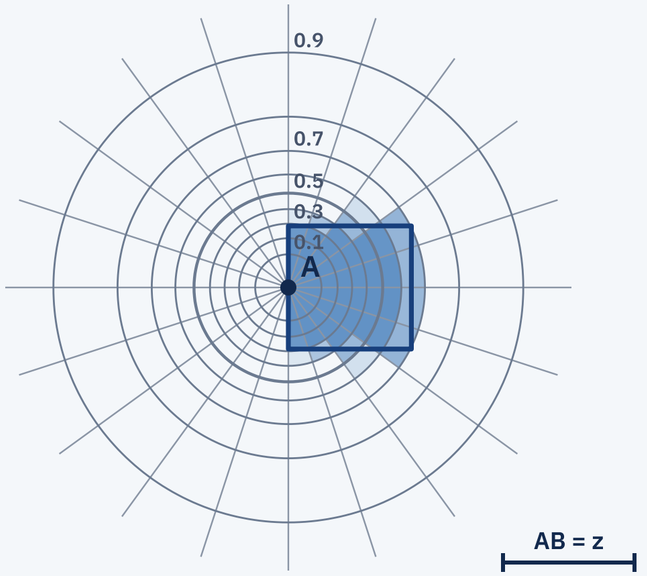
THEORY

The module draws for the entered depth: the covered elements are shaded according to **the parts remaining in plan**

For the count a **exact value** is used: the Boussinesq solution, as for the circle, is **integrated over the polygon** in polar coordinates about the point, giving **M = (Δσ$_z$/q)/IV**. This is the value an exact count would give

The plan may be any polygon with at most six vertices

$$M = \frac{\Delta\sigma_z / q}{IV}$$



Problem statement

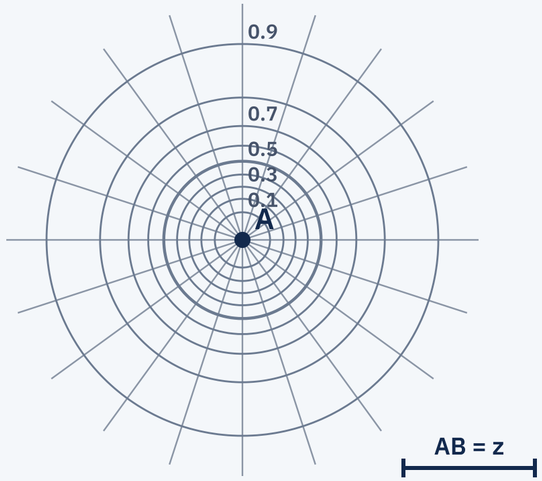
EXAMPLE

Given

Stress ratios	$\Delta\sigma_z/q = 0.1;\ 0.2 \ldots 0.9$ **(nine circles; the tenth ring extends to infinity)**
Sector	**20 equal radial slices**

REQUIRED

The **R/z** ratios of the circles; the number of elements **N**, the influence value **IV**; the number of elements covered by each circle.



Solution (1/2)

SOLUTION

1 For $\Delta\sigma_z/q = 0.3$

$$\frac{R}{z} = \sqrt{(1-0.3)^{-2/3} - 1} = \sqrt{1.2684 - 1} = \sqrt{0.2684} = \mathbf{0.5181}$$

2 For $\Delta\sigma_z/q = 0.9$

$$\frac{R}{z} = \sqrt{(1-0.9)^{-2/3} - 1} = \sqrt{4.6416 - 1} = \sqrt{3.6416} = \mathbf{1.9083}$$

The third circle has a radius of **0.5181** times z, the ninth circle **1.9083** times; the radii grow rapidly as the stress ratio increases and go to infinity as the ratio approaches 1.

Solution (2/2)

SOLUTION

1 N = 10 rings × 20 sectors = 200 elements

$$IV = \frac{1}{N} = \frac{1}{200} = \mathbf{0.005} \qquad k.\ \text{circle}: \ 20\,k\ \text{element} \ \Rightarrow \ IV\,(20\,k) = 0.1\,k = \frac{\Delta\sigma_z}{q}$$

Circle	$\Delta\sigma_z/q$	R/z	Element it covers	Total influence = IV × element
1	0.1	0.2698	20 (= 20 × 1)	0.10
2	0.2	0.4005	40 (= 20 × 2)	0.20
3	0.3	0.5181	60 (= 20 × 3)	0.30
4	0.4	0.6370	80 (= 20 × 4)	0.40
5	0.5	0.7664	100 (= 20 × 5)	0.50
6	0.6	0.9176	120 (= 20 × 6)	0.60
7	0.7	1.1097	140 (= 20 × 7)	0.70
8	0.8	1.3871	160 (= 20 × 8)	0.80
9	0.9	1.9083	180 (= 20 × 9)	0.90
10th ring	1.0	∞	200	1.00

Each ring consists of 20 elements and alone contributes **0.1 q**: the effect of the 20 k elements within the k-th circle is the same as $\Delta\sigma_z/q = 0.1\ k$ given by the circular load solution.

Check the solution in GeoTerva ↗

Problem statement

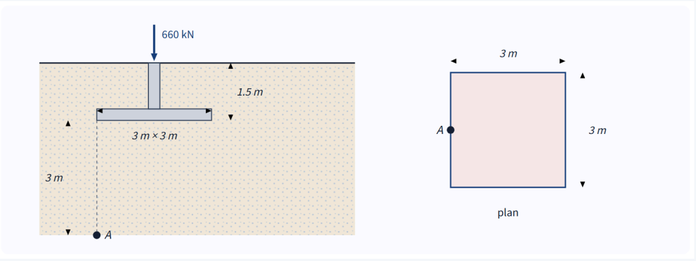
EXAMPLE

Given

Foundation	3×3 **m, column load** 660 **kN (**$q = 660/(3 \times 3)$**)**
Point A	$z = 3$ **m below the footing, below the midpoint of one edge**
Diagram	$IV = 0.005$ **(10 rings, 20 sectors)**

REQUIRED

q, plan scale, **M**, **Δσ$_z$**; checking M with the rectangular solution.



Solution (1/3)

SOLUTION

1 Foundation pressure

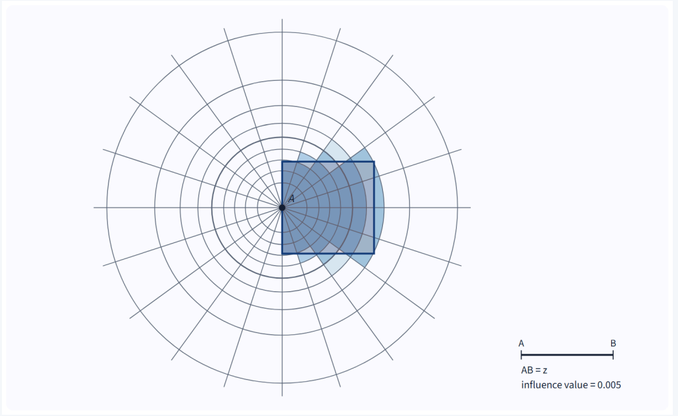
$$q = \frac{660}{3 \times 3} = \mathbf{73.33}\ \mathrm{kN/m^2}$$

2 Plan scale: AB = z = 3 m

$$\text{footing edge} = \frac{3\ \mathrm{m}}{AB} = \frac{3}{3} = 1\ AB$$

3 Placement: point A is set at the centre of the diagram so that the left edge of the footing passes through the centre

The footing is drawn as a square with side of length **AB**, and the midpoint of its left edge (A) is placed at the centre of the diagram (figure on the right).



Solution (2/3)

SOLUTION

4 A is at the middle of the edge: the foundation is divided into two rectangles with a corner at A (1.5 m × 3 m)

$$m = \frac{1.5}{3} = 0.5\,, \quad n = \frac{3}{3} = 1\,, \quad m^2 = 0.25\,, \quad n^2 = 1\,, \quad \sqrt{m^2+n^2+1} = 1.50$$

5 Boussinesq rectangular corner solution $I_3(0.5;\,1)$

$$I_3 = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\cdot\frac{m^2+n^2+2}{m^2+n^2+1} + \tan^{-1}\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1-m^2n^2}\right]$$

$$= \frac{1}{4\pi}\left[\frac{1.5}{2.5}\cdot\frac{3.25}{2.25} + \tan^{-1}\frac{1.5}{2.0}\right] = \frac{0.8667 + 0.6435}{4\pi} = \mathbf{0.12018}$$

The influence value of the 1.5 m × 3 m rectangle with a corner at A is **I_3 = 0.12018**; since the two rectangles are symmetric on either side of A, the total is 2 I_3.

Solution (3/3)

SOLUTION

6 Number of elements

$$M = \frac{\Delta\sigma_z/q}{IV} = \frac{2\,I_3}{IV} = \frac{2\,(0.12018)}{0.005} = \mathbf{48.07}$$

7 Stress increase

$$\Delta\sigma_z = (IV)\,q\,M = (0.005)\left(\frac{660}{3\times 3}\right)(48.07) = 0.005 \times 73.33 \times 48.07 = \mathbf{17.63}\ \mathrm{kN/m^2}$$

8 Directly with the Boussinesq rectangular solution

$$\Delta\sigma_z = 2\,q\,I_3 = 2\,(73.33)(0.12018) = \mathbf{17.63}\ \mathrm{kN/m^2}$$

At point A the diagram covers **M = 48.07 elements** (24.0% of the 200-element plan): **$\Delta\sigma_z$ = 17.63 kN/m²** (24.0% of q); the module gives the same value.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Plan	**Example 2: 3×3 m footing, A midpoint of the left edge, $z = 3$ m ($AB = 3$ m)**
Problems	**number of elements covered in each ring; total M; contribution of one element to the stress**

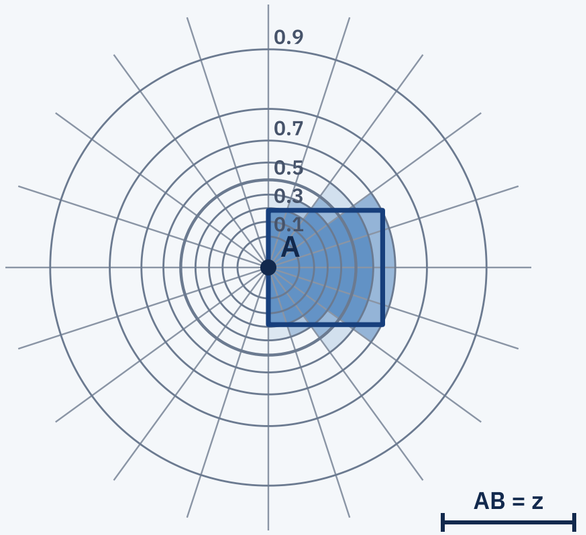
REQUIRED

Covered elements per ring (out of 20 elements) and the total; the effect of a **single-element** error in the count.

Solution

SOLUTION

Ring	R/z range	Covered element	Element in the ring
1	0.000 – 0.270	**10.00**	20
2	0.270 – 0.400	**10.00**	20
3	0.400 – 0.518	**9.82**	20
4	0.518 – 0.637	**6.78**	20
5	0.637 – 0.766	**5.09**	20
6	0.766 – 0.918	**4.08**	20
7	0.918 – 1.110	**2.29**	20
8	1.110 – 1.387	**0.00**	20
9	1.387 – 1.908	**0.00**	20
10	1.908 – ∞	**0.00**	20
Total		**48.07**	200



The first two rings are semicircles (**10 + 10**): side A lies on the boundary. Total **M = 48.07**. One element = IV q = 0.37 kN/m²: a difference of 1 element in the count changes $\Delta\sigma_z$ by 0.37 kN/m² (2.1%).

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Footing and point	**The footing in Example 2 and point A ($q = 73.33$ kN/m²)**
Depths	**$z = 1.2;\ 2.4;\ 3.6 \ldots 12$ m (the module's profile table)**

REQUIRED

The plan scale at each depth, **M** and **$\Delta\sigma_z$**; by hand for z = 6 m.

Solution (1/2)

SOLUTION

z (m)	Footing edge (AB unit)	M (element)	$\Delta\sigma_z$ = IV q M (kN/m²)
1.2	2.500	87.03	**31.91**
2.4	1.250	59.27	**21.73**
3.6	0.833	39.16	**14.36**
4.8	0.625	26.72	**9.80**
6.0	0.500	19.01	**6.97**
7.2	0.417	14.07	**5.16**
8.4	0.357	10.76	**3.95**
9.6	0.312	8.47	**3.11**
10.8	0.278	6.82	**2.50**
12.0	0.250	5.61	**2.06**

As depth increases, the plan diagram shrinks toward its center (footing edge 2.50 AB at z = 1.2 m, 0.25 AB at z = 12 m) and covers fewer elements: **M = 87.03 → 5.61**. The plan has been redrawn for each row.

Solution (2/2)

SOLUTION

1 Plan scale and m, n

$$AB = z = 6\ \text{m} \Rightarrow \text{footing edge} = \frac{3}{6} = 0.5\,AB\ ; \quad m = \frac{1.5}{6} = 0.25\ , \quad n = \frac{3}{6} = 0.5$$

2 Number of elements and stress

$$I_3(0.25; 0.5) = 0.04753 \ \Rightarrow \ M = \frac{2\,(0.04753)}{0.005} = \mathbf{19.01}$$

$$\Delta\sigma_z = 0.005 \times 73.33 \times 19.01 = \mathbf{6.97}\ \text{kN/m}^2$$

At z = 6 m it is very small relative to the centre of the footing diagram and covers only **19.01 elements**; $\Delta\sigma_z$ = **6.97 kN/m²** (17.63 kN/m² at z = 3 m).

Check the solution in GeoTerva ↗

Problem statement

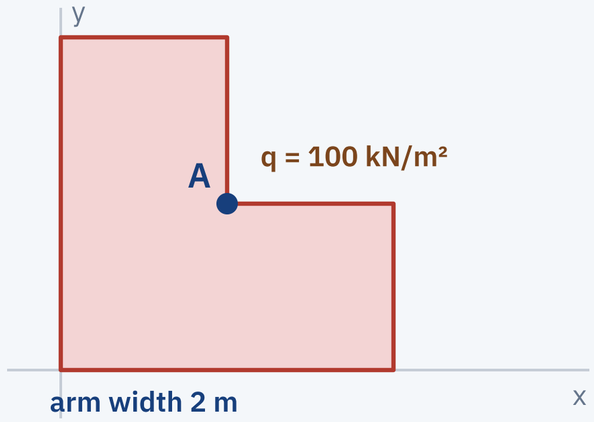
EXAMPLE

Given

Area	**L-shaped (6 corners):** $(0;0),\ (4;0),\ (4;2),\ (2;2),\ (2;4),\ (0;4)$ **m;** $q = 100$ **kN/m²**
Point A	**below the re-entrant corner,** $(2;\ 2)$**;** $z = 2$ **m**
Hint	**Three of the four 2 m × 2 m quarters around A are loaded**

REQUIRED

M and **$\Delta\sigma_z$**; ring-by-ring summation.



Solution (1/2)

SOLUTION

1 Plan scale and quadrants

$$AB = z = 2\,\text{m} \Rightarrow \text{arm width} = 1\,AB\,; \quad \text{each quarter: } m = n = \frac{2}{2} = 1$$

2 Number of elements in one quadrant and the total

$$I_3(1;1) = 0.17522 \;\Rightarrow\; M_{\text{quadrant}} = \frac{0.17522}{0.005} = 35.04$$

$$M = 3\,M_{\text{quadrant}} = 3\,(35.04) = \mathbf{105.13}$$

3 Stress

$$\Delta\sigma_z = 0.005 \times 100 \times 105.13 = \mathbf{52.57}\ \text{kN/m}^2 \qquad \big(= 3\,q\,I_3 = 3\,(100)(0.17522)\big)$$

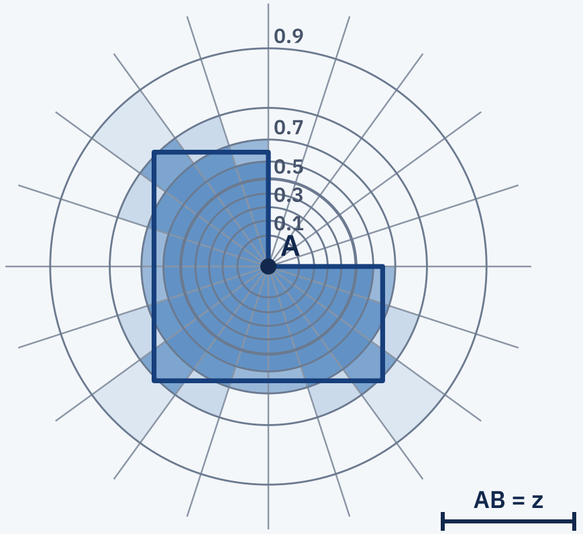
Under the re-entrant corner the L-shaped load covers **105.13 elements**: three of the four quadrants. If all four quadrants were loaded, M would be 140.18.

Solution (2/2)

SOLUTION

Ring	Covered element	Element in the ring
1	**15.00**	20
2	**15.00**	20
3	**15.00**	20
4	**15.00**	20
5	**15.00**	20
6	**15.00**	20
7	**11.94**	20
8	**3.18**	20
9	**0.01**	20
10	**0.00**	20
Total	**105.13**	200

In rings 1–6, **15 elements** of each ring (three quarters of 20) are covered; in the outer rings only the corners of the plan are effective (ring 7: 11.94 elements, ring 8: 3.18 elements). Total **M = 105.13**.



Check the solution in GeoTerva ↗

Problem statement

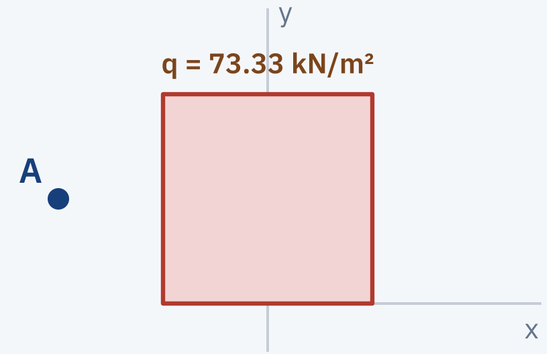
EXAMPLE

Given

Plan and load	**The 3×3 m footing in Example 2 ($q = 73.33$ kN/m²)**
Point A	**outside the foundation: $x = -3$ m, $y = 1.5$ m (1.5 m from the left edge); $z = 3$ m**

REQUIRED

M and **$\Delta\sigma_z$**; the meaning of counting at an exterior point.



Solution (1/2)

SOLUTION

1 Distances from A to the footing (z = 3 m)

$$x:\ 1.5\ \text{with m}\ 4.5\ \text{m};\quad y:\ \pm 1.5\ \text{m}\ \Rightarrow\ m=\frac{1.5}{3}=0.5\ \text{and}\ \frac{4.5}{3}=1.5,\ \ n=0.5$$

2 Two hatched rectangles (4.5 × 3 m minus 1.5 × 3 m), symmetric: twice that

$$\frac{\Delta\sigma_z}{q}=2\left[I_3(1.5;0.5)-I_3(0.5;0.5)\right]=2\left[0.13136-0.08403\right]=0.09466$$

$$M=\frac{0.09466}{0.005}=\mathbf{18.93}$$

$$\Delta\sigma_z=0.005\times 73.33\times 18.93=\mathbf{6.94}\ \text{kN/m}^2$$

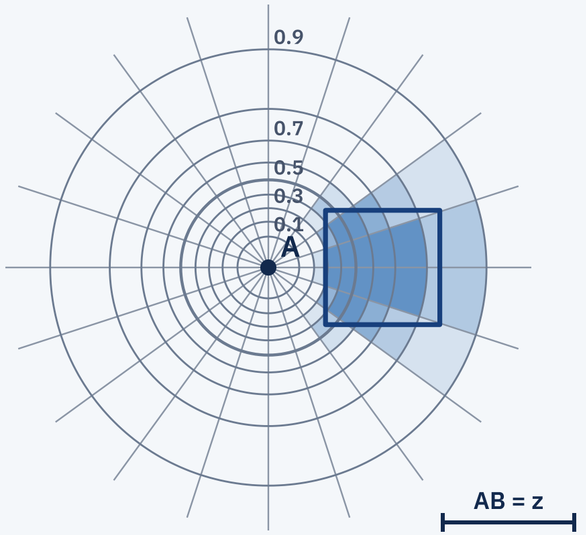
An external point also receives stress: **element M = 18.93**, **$\Delta\sigma_z$ = 6.94 kN/m²**; the plan does not cover the centre of the diagram.

Solution (2/2)

SOLUTION

Ring	Covered element	Element in the ring
1	**0.00**	20
2	**0.00**	20
3	**0.18**	20
4	**3.22**	20
5	**4.69**	20
6	**4.08**	20
7	**3.31**	20
8	**2.66**	20
9	**0.80**	20
10	**0.00**	20
Total	**18.93**	200

The plan enters the diagram from the 3rd ring onward and is most effective at the 5th ring (4.69 elements). Total **M = 18.93**; for the same foundation under the center it would be M = 67.22.



Check the solution in GeoTerva ↗

Summary

Key points

Diagram: nine circles of radius $R/z = \sqrt{[(1 - \Delta\sigma_z/q)^{-2/3} - 1]}$ and 20 radial lines; **200 elements**, the influence of each element being **IV = 1/N = 0.005**

Use: plan AB = z drawn to scale, with point A at the center; number of elements covered M; **$\Delta\sigma_z$ = (IV) q M**; a separate plan for each depth

Example (3 m × 3 m, 660 kN, z = 3 m, middle of edge): q = 73.33 kN/m²; M = 48.07; $\Delta\sigma_z$ = **17.63 kN/m²**; at z = 6 m, M = 19.01, $\Delta\sigma_z$ = 6.97 kN/m²

Every shape: L-shaped plan, re-entrant corner (M = 105.13, $\Delta\sigma_z$ = 52.57 kN/m²); point outside the area (M = 18.93, $\Delta\sigma_z$ = 6.94 kN/m²)

Accuracy: the contribution of one element is IV q = 0.37 kN/m²; it gives the modulus M by exact integral (faultless counting)

References

Das, B.M. (2018). Principles of Geotechnical Engineering, 9th ed. Cengage Learning.

Newmark, N.M. (1942). Influence charts for computation of stresses in elastic foundations. University of Illinois Engineering Experiment Station, Bulletin No. 338.

NOTE

All numerical examples have been generated with the module's own calculation engine; the module gives the same values. The module's opening example: 3 m × 3 m footing, 660 kN, 3 m below the middle of edge A.

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own plan (a polygon with at most six vertices), the load and point A in GeoTerva; the module draws the diagram for the entered depth, shades the covered elements, and provides M, the $\Delta\sigma_z$ value, the depth profile, the step-by-step solution and the report together.

Open the module → | **Theory page →**

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com