Geotechnical lecture slide · Retaining walls

Mechanically stabilized earth (MSE) wall

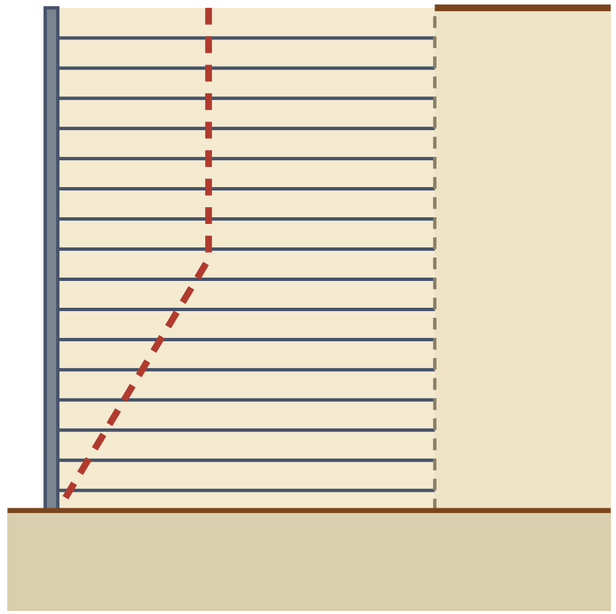
Lecture: internal stability (rupture, pullout), external stability (overturning, sliding, bearing capacity) and earthquake check of a reinforced soil wall; a worked example for each topic.

Reinforced soil

Rupture · pullout

Overturning · sliding · bearing capacity

Earthquake



What will we learn in this lecture?

The logic of reinforced soil: earth pressure causes tension in the reinforcement, and friction between them holds the two together; the facing merely retains the soil

Internal stability: tension at each level $T = \sigma'_h S_V S_H$, **rupture** (including corrosion allowance), **pullout** (effective length behind the failure surface)

The **failure surface** (bilinear or Rankine wedge) and the K_r/K_a ratio: in stiff reinforcement the lateral stress at the top is greater than Rankine

External stability: the reinforced block L × H acts as a gravity wall; overturning, eccentricity (L/6), sliding and bearing capacity (Meyerhof effective width)

Why **live surcharge** only loads, and the effect of the **earthquake** on the block, the thrust and the reinforcement levels

How to correct an inadequate design: reinforcement length, reinforcement type

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example wall: H = 10 m, galvanized steel strips 75 × 6 mm (f_y = 240 MPa), L = 7.5 m, S_V = 0.6 m, S_H = 1.0 m; fill γ = 16.5 kN/m³, φ′ = 36°; foundation φ′ = 28°, c′ = 50 kPa. Forces are per metre of wall (per strip).

What does it do, what is checked?

THEORY

The soil wall stands on its own only at the angle of repose. If thin **tensile elements** (galvanized steel strips, geotextile, geogrid) are placed in the fill, the soil carries compression and the reinforcement carries tension, and **friction** holds the two together. The facing (concrete panels, blocks or the wrapping of the reinforcement) retains the soil, and carries no other load.

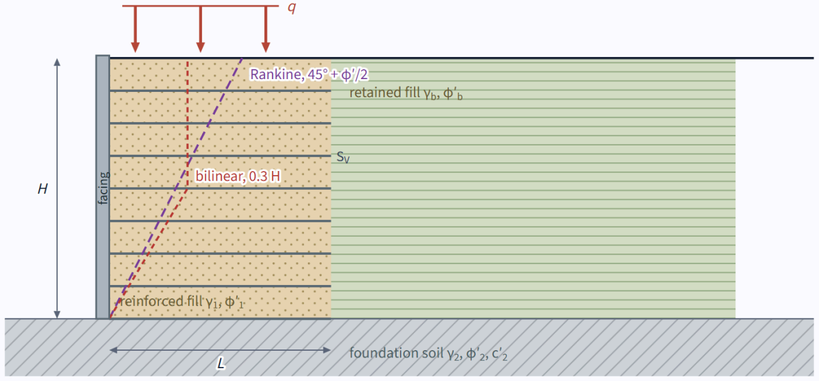
Internal stability: each level must carry the tension (1) without **rupture**, (2) without **pullout** from behind the failure surface, (3) without **rupture** at the facing (connection).

External stability: the reinforced block of width L and height H is a **gravity wall**: it must not overturn under the thrust of the backfill behind it, must not slide at its base, and the foundation soil must be able to carry it.

Earthquake: the two sets of checks again, with dynamic thrust + block and effective-zone inertia (required factors of safety are 75% of the static ones).

The **fill** must be freely draining granular material; **overall stability** (slip surface passing through the foundation or a composite slip surface) is outside the scope of this module.

Two methods: the source's **Rankine** approach and the application's (FHWA/AASHTO) **simplified method** (the method the module opens with).



Vertical and lateral stress, K_r/K_a ratio

THEORY

z is the depth from the top of the wall, γ_1, φ'_1 are the unit weight and friction angle of the reinforced fill. The fill is in the active state:

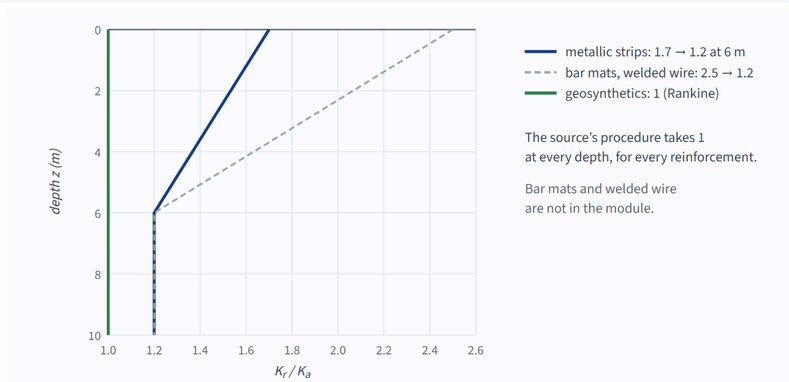
$$\sigma'_v = \gamma_1 z + q_D + q_L \qquad \sigma'_h = K_r\,(\gamma_1 z + q_D + q_L) \qquad K_a = \tan^2\!\left(45^\circ - \frac{\varphi'_1}{2}\right)$$

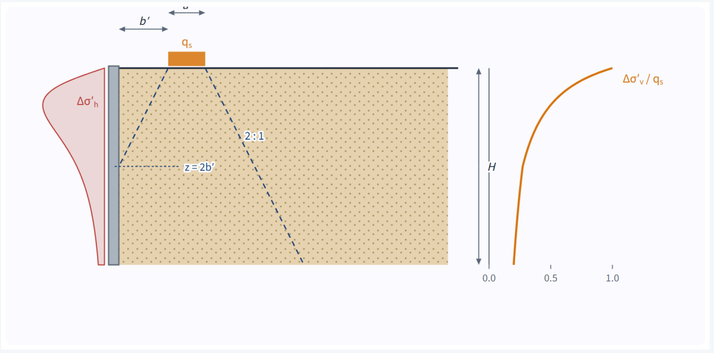
Stiff (inextensible) reinforcement — steel strips — holds the fill **more stiffly** at the top than the Rankine condition; in the simplified method K_r/K_a is taken according to the reinforcement type:

$$\text{slices: } \frac{K_r}{K_a} = 1.7 - 0.5\frac{z}{6}\ (z \le 6\ \text{m}),\ \ 1.2 \text{ under} \qquad \text{geosynthetic: } \frac{K_r}{K_a} = 1$$

Uniform surcharge increases the vertical stress by q at every depth, and also increases the lateral stress through K_r. A **live** surcharge (q_L) loads but **does not hold**: it is not included in the overturning/sliding weight nor in the vertical stress that resists pullout (conservative).

Strip load (Laba–Kennedy): the vertical stress spreads at 2:1, the lateral pressure is the elastic solution for a rigid wall (M = 1.4 → 1).





Problem statement

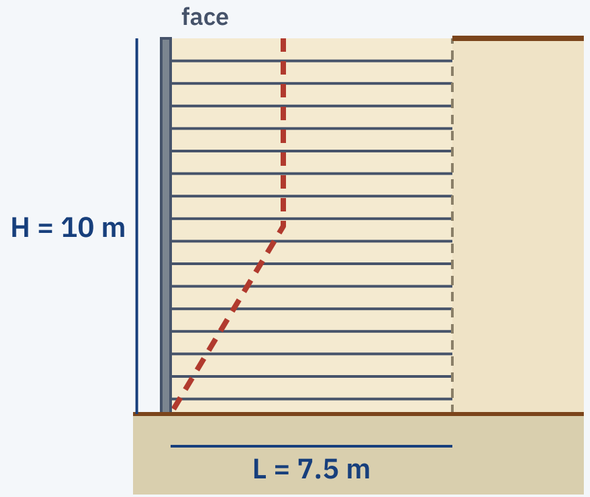
EXAMPLE

REQUIRED

K_a, K_r/K_a, σ'_v and σ'_h at every depth; where is the maximum lateral stress?

Given

Fill	$\gamma_1 = 16.5\ \mathrm{kN/m^3}$, $\varphi'_1 = 36^\circ$**; steel strips; simplified method; no surcharge**
Depths	$z = 0.6;\ 3.0;\ 6.0$ **and** 10.0 **m (base)**



Solution

SOLUTION

1 Active coefficient

$$K_a = \tan^2(45^\circ - 18^\circ) = \mathbf{0.2596}$$

2 At each depth ($K_r/K_a = 1.7 - 0.5z/6$, $z \le 6$ m; 1.2 beyond 6 m)

z (m)	$\sigma'_v = 16.5z$ (kPa)	K_r/K_a	K_r	$\sigma'_h = K_r\sigma'_v$ (kPa)
0.6	9.9	1.65	0.4284	4.24
3.0	49.5	1.45	0.3764	18.63
6.0	99.0	1.20	0.3115	30.84
10.0	165.0	1.20	0.3115	51.40

Base: σ'_v = **165 kPa**, $K_r = 1.2K_a = 0.3115$, σ'_h = **51.4 kPa**. At the upper level (z = 0.6 m), $K_r/K_a = 1.65$: 1.65 times Rankine's. σ'_h increases with depth; the maximum lateral stress (and tension) is at the base.

Check the solution in GeoTerva ↗

Effective length $L_e = L - L_a$

THEORY

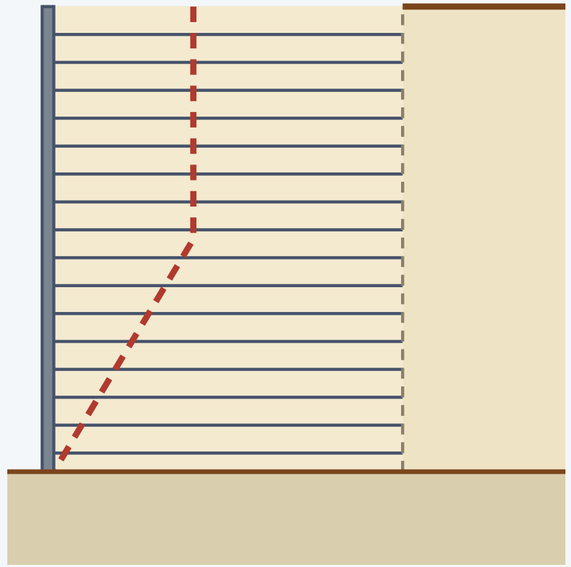
The reinforcement works where the fill would otherwise fail. The **part of each level within the failure zone** (between the facing and the failure surface) goes with the sliding soil and does not hold; only the **part behind the surface** anchors the level: active length L_a and effective length $L_e = L - L_a$.

Rankine: $L_a = \dfrac{H - z}{\tan(45^\circ + \varphi'_1/2)}$ bilinear: $L_a = 0.3H\ (z \le H/2),\ \ L_a = 0.6(H - z)\ (z > H/2)$

For each reinforcement, the **source** takes the Rankine wedge starting at the front end (45° + $\varphi'_1/2$ to the horizontal): at $\varphi' = 36°$, 5.1 m from the face at the top of a 10 m wall.

In stiff (steel strip) reinforcement, measurements show the maximum tension surface much closer to the facing: practice adopts a **bilinear** surface — vertical at 0.3H from the facing in the upper half, then straight to the heel. In geosynthetics, Rankine is retained.

In the module's simplified method, the steel strip takes the bilinear surface and the geosynthetic takes the Rankine surface; in both, the source method is Rankine.



Red dashed: bilinear failure surface (strip). The Rankine wedge starts 5.1 m from the face.

Problem statement

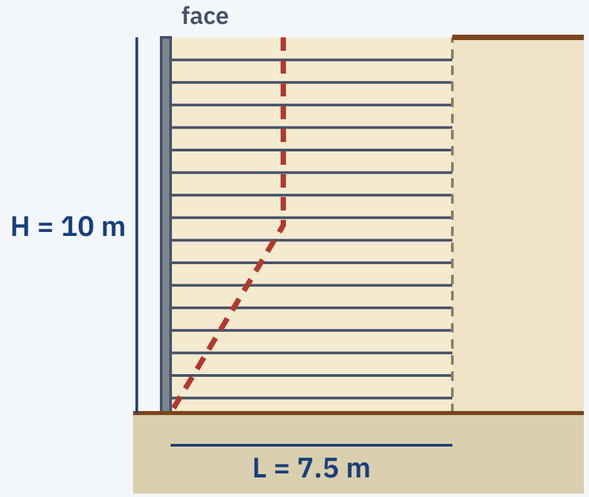
EXAMPLE

Given

Wall	$H = 10$ **m,** $L = 7.5$ **m, steel strips;** $\varphi'_1 = 36^\circ$
Depths	$z = 0;\ 0.6;\ 3;\ 5;\ 6;\ 9.4$ **m**

REQUIRED

At each depth, $\mathbf{L_a}$ (bilinear and Rankine) and $\mathbf{L_e}$ with the bilinear surface.



Solution

SOLUTION

1 Bilinear: for $z \le 5$ m, $0.3H = 3.00$ m; for $z > 5$ m, $0.6(H - z)$. Rankine: $(H - z)/\tan 63° = (H - z)/1.963$

z (m)	L_a bilinear (m)	$L_e = 7.5 - L_a$ (m)	L_a Rankine (m)	L_e Rankine (m)
0.0	3.00	4.50	5.10	2.40
0.6	3.00	4.50	4.79	2.71
3.0	3.00	4.50	3.57	3.93
5.0	3.00	4.50	2.55	4.95
6.0	2.40	5.10	2.04	5.46
9.4	0.36	7.14	0.31	7.19

The bilinear surface is closer to the face than Rankine at the upper levels ($z = 0.6$ m: L_a = 3.00 m, Rankine 4.79 m); therefore the **effective length is larger** (4.50 m, Rankine 2.71 m). The upper levels, in turn, carry K_r/K_a times more tension (Example 4).

Check the solution in GeoTerva ↗

Leveling of the level and bearing capacity of the reinforcement

THEORY

Each level carries the lateral stress of the fill within a limited **contributing height** (S_V) at the mid-heights of adjacent levels. The strip retains a fill width equal to the horizontal spacing S_H; since the layer and the grid are continuous, the tension is per metre of wall:

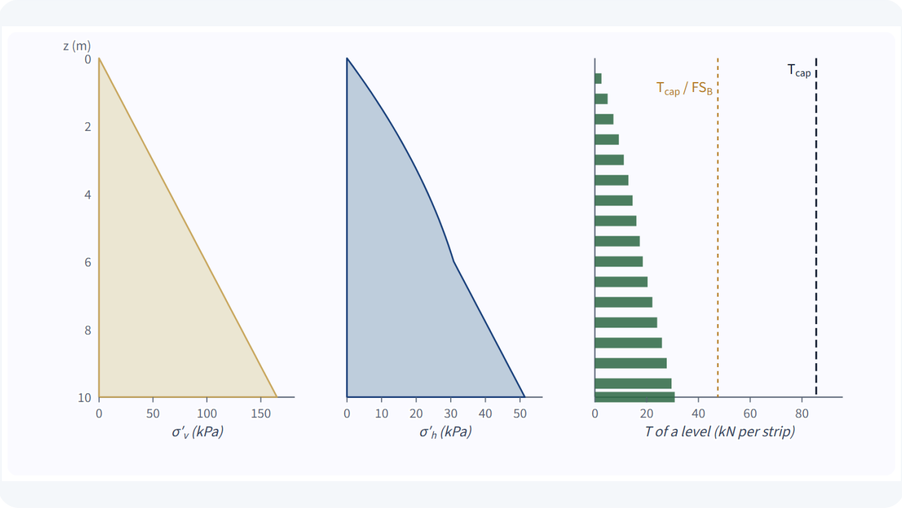
$$T = \sigma'_h S_V S_H \quad \text{(per strip)} \qquad T = \sigma'_h S_V \quad \text{(per metre of wall)}$$

Strip (width w, thickness t, yield f_y): galvanized steel reinforcement corrodes at 0.025–0.050 mm per year; for a design life of n, the thickness decreases by r n:

$$t_{eff} = t - rn \qquad T_{diameter} = w\, t_{eff}\, f_y \qquad FS_B = \frac{T_{diameter}}{T} \qquad t_{req} = \frac{FS_B T_{max}}{w\, f_y} + rn$$

Geosynthetic: the long-term strength is the ultimate tensile strength divided by the damage, creep and degradation factors; the grid does not always cover the entire plan (C_r):

$$T_{all} = \frac{T_{ult}}{RF_{id}\, RF_{cr}\, RF_{cbd}} \qquad FS_B = \frac{T_{all} C_r}{T} \qquad S_{V,req} = \frac{T_{all} C_r}{\sigma'_h(H)\, FS_B}$$



Problem statement

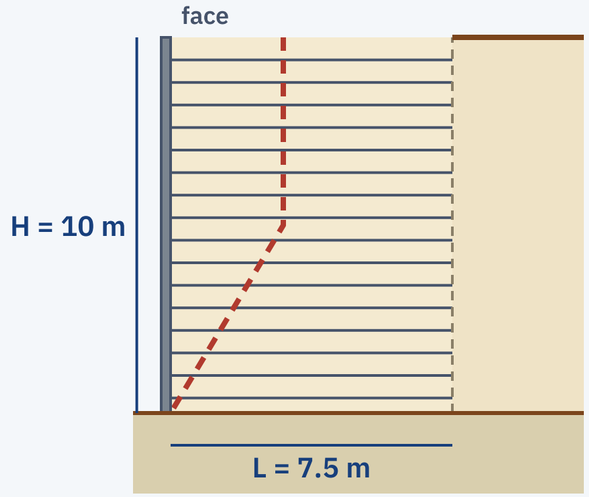
EXAMPLE

Given

Strip	**75 × 6 mm installed; $f_y = 240$ MPa; corrosion 0.025 mm/year, design life 50 years; $S_V = 0.6$ m, $S_H = 1.0$ m**
Required	$FS_B \geq 1.8$ **(application: 0.55 f_y in a corroded section)**

REQUIRED

Maximum tension T_{max} (per strip); T_{cap}, FS_B; required thickness t_{req}.



Solution

SOLUTION

1 The maximum tension is at the base level (z = 10 m; σ'_h = 51.40 kPa)

$$T_{max} = \sigma'_h S_V S_H = 51.40(0.6)(1.0) = \mathbf{30.84}\ \text{kN/strip}$$

2 Corroded section and bearing capacity

$$t_{eff} = 6 - 0.025(50) = 4.75\ \text{mm} \quad (1.25\ \text{mm loss})$$

$$T_{diameter} = w\, t_{eff} f_y = 0.075(0.00475)(240\,000) = \mathbf{85.5}\ \text{kN}$$

$$FS_B = 85.5/30.84 = \mathbf{2.772} \geq 1.80\ \checkmark$$

3 Required thickness (restoring the corrosion allowance)

$$t_{req} = \frac{1.8(30.84)}{0.075(240\,000)} + rn = 3.084 + 1.25 = \mathbf{4.33}\ \text{mm}$$

FS_B = **2.77** ≥ 1.80: a 6 mm strip is sufficient; the required thickness is **4.33 mm**. The critical level is the base (k = 17).

Check the solution in GeoTerva ↗

Friction behind the failure surface

THEORY

A level that does not rupture may still **pull out** of the fill. Behind the failure surface the fill holds the reinforcement by friction on both of its faces; the friction is the vertical stress above the level times the coefficient mobilised by the reinforcement:

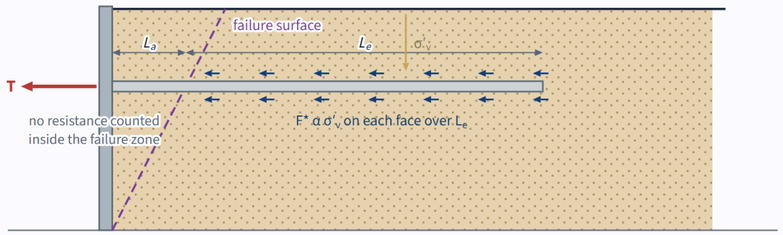
$$P_r = 2R_c\,F^*\alpha\,\sigma'_v\,L_e \qquad FS_P = \frac{P_r}{T} \qquad L_{req} = L_a + \frac{FS_P\,T}{2R_c F^*\alpha\,\sigma'_v}$$

F* pullout resistance coefficient: tan φ′ for strips (lower bound), ⅔ tan φ′ for geosynthetics (lower bound of the module; determined by pullout test). **α** scale effect: 1.0 for strips, 0.8 for geogrid, 0.6 for geotextile.

R_c coverage: w in the strip (per strip), C_r in the geogrid (per metre of wall), 1 in the cover. **σ'_v** is the vertical stress in the resistant zone: $\gamma_1 z$ + permanent surcharge; **live load is not included** (it does not hold).

In a strip, T and P_r grow with depth: in the source method the required effective length is the same at every level; in the simplified method the upper levels carry K_r/K_a times more tension, and the effective length increases by the same ratio.

Connection: the tension T_{max} is transferred to the facing; the strength is entered from connection tests and checked with the rupture coefficient.



Problem statement

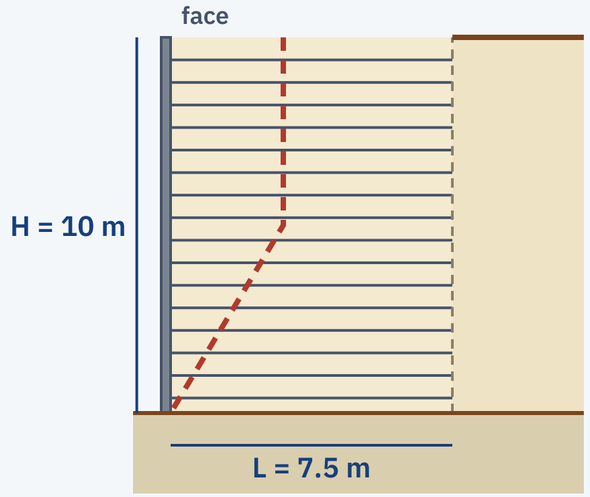
EXAMPLE

Given

Strip and soil	**The wall in Examples 1–3;** $F^* = \tan 36^\circ = 0.727$, $\alpha = 1$; $FS_P \geq 1.5$
Levels	**z = 0.6 m (1st level) and other levels**

REQUIRED

At level 1 **P_r**, **FS_P**, the required length **L_{req}**; summary of all levels.



Solution (1/2)

SOLUTION

1 Level 1 (z = 0.6 m): $K_r/K_a = 1.65$; $\sigma'_h = 4.24$ kPa

$$T = \sigma'_h S_V S_H = 4.24(0.6) = 2.54 \text{ kN} \qquad L_a = 0.3H = 3.00, \quad L_e = 4.50 \text{ m}$$

$$P_r = 2(0.075)(0.727)(1.0)(9.9)(4.50) = \mathbf{4.86} \text{ kN}$$

$$FS_P = \frac{4.86}{2.54} = \mathbf{1.908} \geq 1.5 \checkmark$$

2 Required length: $L_{req} = L_a + FS_P T/(F^* \alpha \sigma'_v \cdot 2w)$

$$L_{req} = 3.00 + \frac{1.5(2.54)}{0.727(1)(9.9)(2)(0.075)} = 3.00 + 3.54 = \mathbf{6.54} \text{ m}$$

Solution (2/2)

SOLUTION

3 Summary of all levels

Level	z (m)	T (kN)	L_a	L_e	P_r (kN)	FS_P	L_{req} (m)
1	0.6	2.54	3.00	4.50	4.86	1.91	6.54
2	1.2	4.93	3.00	4.50	9.71	1.97	6.43
5	3.0	11.18	3.00	4.50	24.28	2.17	6.11
10	6.0	18.51	2.40	5.10	55.02	2.97	4.97
17	10.0	30.84	0.00	7.50	134.86	4.37	2.57

The smallest pullout safety is **FS_P = 1.91 at level 1** (1.5 required); the longest required length is **6.54 m** (L = 7.5 m is sufficient). At the upper level the tension is small, but since σ'_V is also small, pullout is weakest there.

Check the solution in GeoTerva ↗

The block is a gravity wall

THEORY

The reinforced block that provides internal stability is **a soil block of size L × H** that retains the fill behind it. The retained fill (γ_b, φ'_b) pushes on the vertical rear face of the block with Rankine active thrust; surcharge adds extra thrust:

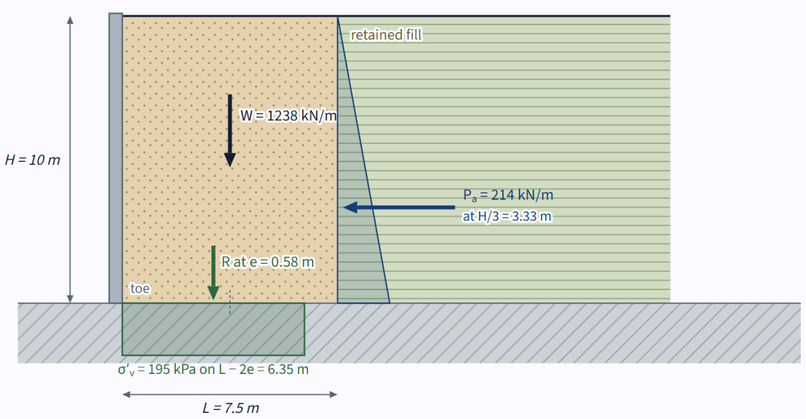
$$P_a = \tfrac{1}{2} K_a \gamma_b H^2 \;\; (H/3) \qquad P_q = K_a \, q \, H \;\; (H/2) \qquad W = \gamma_1 H L \;\; (L/2)$$

$$FS_{ot} = \frac{M_R}{M_o} = \frac{W \, L/2}{P_a H/3 + P_q H/2} \qquad x_R = \frac{M_R - M_o}{\Sigma V} \qquad e = \frac{L}{2} - x_R$$

Resisting: block weight and **permanent** surcharge. **Live load does not resist** when the thrust acts, as it may not be present (loads only).

The same check is read in two ways: **moment ratio** ($FS_{ot} \geq 2$) and the location of the resultant: **e ≤ L/6** so that the base remains entirely under compression (L/3 under earthquake).

The facing and the embedded part in front of it are not included in the resisting weight (conservative).



Problem statement

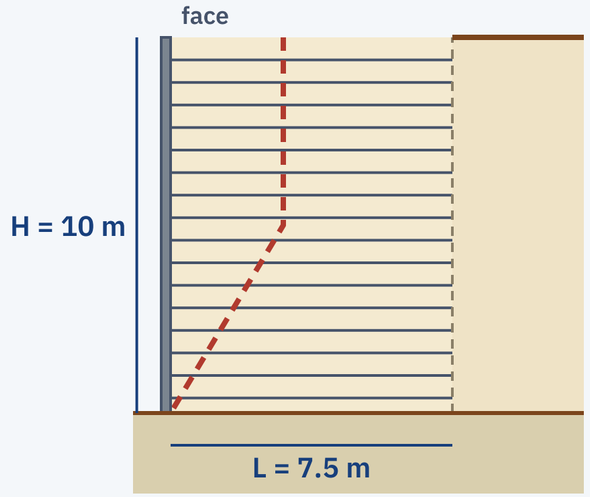
EXAMPLE

Given

Block	$H = 10$ **m,** $L = 7.5$ **m; reinforced fill** $\gamma_1 = 16.5$ **kN/m³**
Retained fill	**same soil (**$\gamma_b = 16.5$**,** $\varphi'_b = 36°$**); no surcharge**

REQUIRED

P_a and M_O; **W** and M_R; **FS_{ot}**; location of the resultant x_R and **e** (limit L/6).



Solution (1/2)

SOLUTION

1 Active thrust (K_a = 0.2596) and overturning moment

$$P_a = \tfrac{1}{2} K_a \gamma_b H^2 = \tfrac{1}{2}(0.2596)(16.5)(10)^2 = \mathbf{214.18}\ \mathrm{kN/m}$$

$$M_o = P_a\, H/3 = 214.18(3.333) = \mathbf{713.9}\ \mathrm{kN\cdot m/m}$$

2 Block weight and resisting moment

$$W = \gamma_1 H L = 16.5(10)(7.5) = \mathbf{1{,}237.5}\ \mathrm{kN/m}$$

$$M_R = W\, L/2 = 1{,}237.5(3.75) = \mathbf{4{,}640.6}\ \mathrm{kN\cdot m/m}$$

Solution (2/2)

SOLUTION

3 Overturning and location of the resultant

$$FS_{ot} = M_R / M_o = 4{,}640.6 / 713.9 = \mathbf{6.50} \;\geq 2.0 \;\checkmark$$

$$x_R = (M_R - M_o)/\Sigma V = (4{,}640.6 - 713.9)/1{,}237.5 = 3.173 \text{ m}$$

$$e = L/2 - x_R = 3.75 - 3.173 = \mathbf{0.577} \text{ m} \;\leq L/6 = 1.25 \;\checkmark$$

Overturning safety **6.50** (≥ 2.0); the resultant has shifted only **0.58 m** from the middle of the base toward the toe: the base is entirely in compression (e ≤ L/6 = 1.25 m).

Check the solution in GeoTerva ↗

Base friction

THEORY

If the thrust exceeds the base friction, the block slides on its base. The **source** takes friction as k ≈ ⅔, as in a concrete wall resting on soil; at the base of a reinforced block, however, the **soil rests on soil**: practice is based on the friction angle of the **weaker** of the reinforced fill and the foundation soil. The module retains the ⅔ coefficient, selects the weaker soil and disregards the cohesion of the foundation soil:

$$FS_{sl} = \frac{\Sigma V \, \tan\left(\frac{2}{3}\varphi'_{min}\right)}{\Sigma H} \qquad \varphi'_{min} = \min\left(\varphi'_1, \varphi'_2\right)$$

ΣH: thrust of the retained fill and (if present) thrust due to surcharge; ΣV: weight of the block and permanent surcharge. **Live load enters the thrust but not ΣV.**

The source path (fill only, φ'_1) is also calculated; if the foundation soil is weak, the module's value is lower and is **the governing one**.

Sliding is solved by widening the block (ΣV increases linearly with L); the length also improves pull-out and eccentricity.

REQUIRED FACTORS OF SAFETY

Check	Static (required)	Earthquake (required, 75%)
Rupture FS_B	1.80	1.35
Pullout FS_P	1.50	1,125
Overturning FS_{ot}	2.00	1.50
Eccentricity e	≤ L/6	≤ L/3
Sliding FS_{sl}	1.50	1,125
Bearing capacity FS_{bc}	2.50	1,875

The earthquake values are 75% of the static ones (eccentricity L/6 → L/3). The live load is retained in both cases.

Problem statement

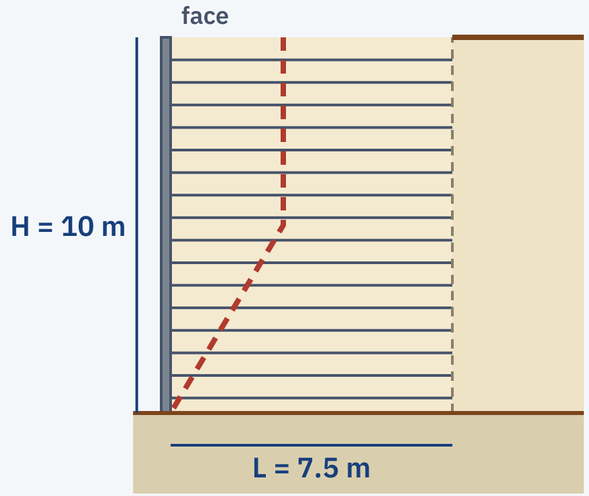
EXAMPLE

Given

Block	**The block in Example 5 ($\Sigma V = 1\,237.5$ kN/m, $\Sigma H = P_a = 214.18$ kN/m)**
Soils	**reinforced fill $\varphi_1' = 36^\circ$; foundation soil $\varphi_2' = 28^\circ$ ($c_2' = 50$ kPa, cohesion neglected)**

REQUIRED

FS_{sl} from base friction; comparison with the source's approach (fill only).



Solution

SOLUTION

1 Weak soil: φ'_{min} = min(36°, 28°) = 28° (foundation soil)

$$\tan\left(\tfrac{2}{3}\cdot 28^\circ\right) = \tan 18.67^\circ = 0.3378$$

$$R = \Sigma V \tan(\ldots) = 1{,}237.5(0.3378) = 418.1\ \text{kN/m}$$

$$FS_{sl} = \frac{R}{\Sigma H} = \frac{418.1}{214.18} = \mathbf{1.952}\ \geq 1.5\ \checkmark$$

2 Path of the source (fill only, φ'_1 = 36°)

$$\tan\left(\tfrac{2}{3}\cdot 36^\circ\right) = \tan 24^\circ = 0.4452$$

$$FS_{sl} = \frac{1{,}237.5(0.4452)}{214.18} = \mathbf{2.572}$$

Since the foundation soil is weaker, the sliding safety is **1.95** (2.57 by the source's route): the module takes the **smaller** one as governing; it is above the required 1.5.

Check the solution in GeoTerva ↗

The foundation soil block carries like a strip footing

THEORY

The source compares the bearing capacity of a footing of width L at the ground surface with the vertical stress at the base, without a depth term. General bearing capacity factors (Vesic): $N_q = e^{\pi \tan \varphi'} \tan^2(45° + \varphi'/2)$, $N_c = (N_q - 1)\cot \varphi'$, $N_\gamma = 2(N_q + 1)\tan \varphi'$:

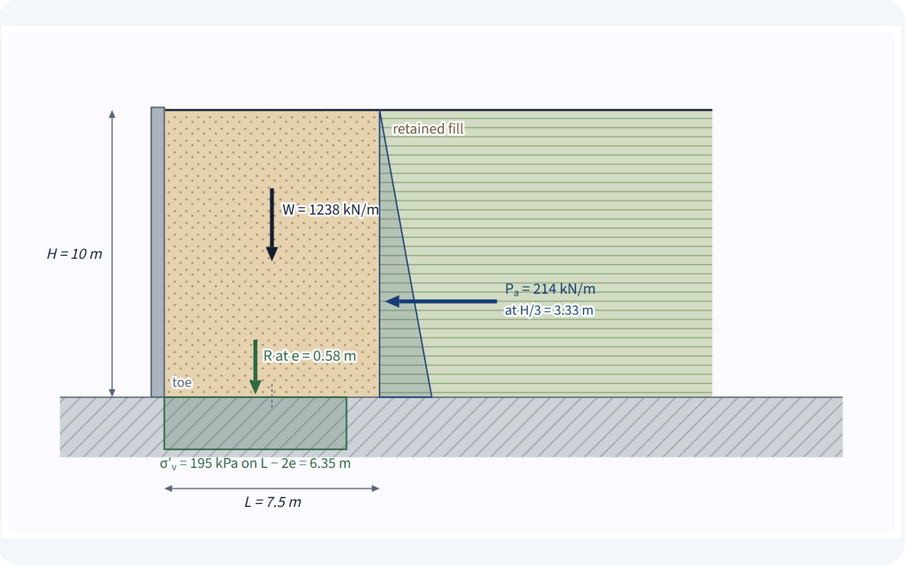
$$q_u = c'_2 N_c + \tfrac{1}{2}\gamma_2 L N_\gamma \qquad FS_{bc} = \frac{q_u}{\sigma'_o}, \quad \sigma'_o = \gamma_1 H + q_D$$

Application (FHWA/AASHTO): the resultant is not at the center (e), and the pressure at the toe is greater than the average; therefore the entire vertical load is spread uniformly over the **Meyerhof effective width** L′ = L − 2e, and the bearing capacity of a footing of this width is calculated:

$$L' = L - 2e \qquad \sigma'_v = \frac{\Sigma V}{L'} \qquad q_u = c'_2 N_c + \tfrac{1}{2}\gamma_2 L' N_\gamma \qquad FS_{bc} = \frac{q_u}{\sigma'_v}$$

The pressure is higher and the footing narrower: the Meyerhof factor of safety is lower; the module **calculates both and takes the smaller** as the governing value.

This page is a short-term check (no pore water pressure); general stability and settlement are outside the module.



Problem statement

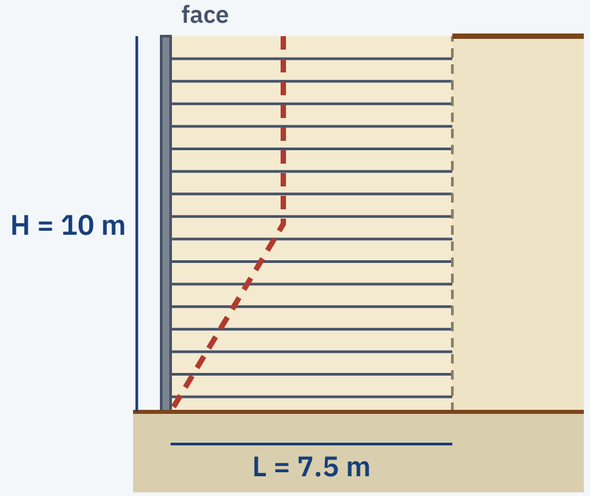
EXAMPLE

Given

Block	**Block in Example 5:** $\Sigma V = 1\,237.5$ **kN/m,** $e = 0.577$ **m**
Foundation soil	$\varphi'_2 = 28°$**,** $c'_2 = 50$ **kPa,** $\gamma_2 = 17.3$ **kN/m³**

REQUIRED

FS_{bc} by the source's approach, **FS_{bc}** with Meyerhof's effective width; which one governs.



Solution

SOLUTION

1 Coefficients (Vesic, $\varphi'_2 = 28°$): $N_q = e^{\pi \tan \varphi'} \tan^2(45° + \varphi'/2)$, $N_c = (N_q - 1)\cot \varphi'$, $N_\gamma = 2(N_q + 1)\tan \varphi'$

$$N_q = 14.72 \qquad N_c = 25.80 \qquad N_\gamma = 16.72$$

2 Path of the source (width $L = 7.5$ m, $\sigma'_o = \gamma_1 H = 165$ kPa)

$$q_u = 50(25.80) + \tfrac{1}{2}(17.3)(7.5)(16.72) = 2{,}375 \text{ kPa} \qquad FS_{bc} = 2{,}375/165 = \mathbf{14.39}$$

3 Meyerhof effective width ($e = 0.577$ m)

$$L' = 7.5 - 2(0.577) = 6.346 \text{ m} \qquad \sigma'_v = \Sigma V / L' = 1{,}237.5/6.346 = 195 \text{ kPa}$$

$$q_u = 50(25.80) + \tfrac{1}{2}(17.3)(6.346)(16.72) = 2{,}208 \text{ kPa}$$

$$FS_{bc} = 2{,}208/195 = \mathbf{11.32} \ \geq 2.5 \ \checkmark$$

The source's path gives **14.39**, the effective width gives **11.32**: since eccentricity increases the pressure and narrows the footing, **the Meyerhof value governs**; both are well above 2.5.

Check the solution in GeoTerva ↗

Problem statement

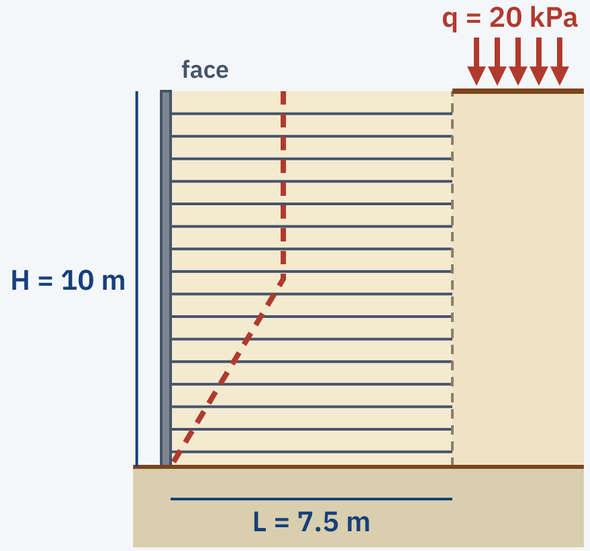
EXAMPLE

Given

Wall	**Wall from Example 1–7 (H = 10 m, L = 7.5 m, strips)**
Load	**live distributed surcharge $q_L = 20$ kPa on top of the wall (over and behind the block)**

REQUIRED

At level 1, **FS_P**; at the base, **T_{max}**, FS_B; external checks (thrust, overturning, sliding, bearing capacity).



Solution (1/2)

SOLUTION

1 Level 1 (z = 0.6 m): the surcharge increases the lateral stress but not the pullout resistance

$$\sigma'_h = K_r\,(\gamma_1 z + q_L) = 0.4284(9.9 + 20) = 12.81\ \text{kPa} \quad T = 12.81(0.6) = 7.68\ \text{kN}$$

$$P_r = 4.86\ \text{kN} \quad (\sigma'_v = \gamma_1 z = 9.9\ \text{kPa; live load does not count})$$

$$FS_P = \frac{4.86}{7.68} = \mathbf{0.632} \ < 1.5 \ \text{✗}$$

2 Base: maximum tension and rupture

$$\sigma'_h = 0.3115(165 + 20) = 57.63\ \text{kPa} \quad T_{max} = 57.63(0.6) = 34.58\ \text{kN}$$

$$FS_B = \frac{85.5}{34.58} = \mathbf{2.47} \ \geq 1.8 \ \text{✓} \qquad L_{req} = 13.68\ \text{m} \ > 7.5 \ \text{✗}$$

At Level 1, as the tension increases from 2.54 to **7.68 kN**, the pullout resistance **stays the same**: FS_P 1.91 → **0.63**. The rupture is adequate (2.47), but the required length becomes 13.68 m: the live load **loads, it does not hold**.

Solution (2/2)

SOLUTION

3 External checks: the thrust increases; it is not included in the weight

$$P_q = K_a q_L H = 0.2596(20)(10) = 51.92\ \text{kN/m}\ \ (H/2) \qquad P_a = 266.11\ \text{kN/m}$$

$$M_o = 713.9 + 51.92(5) = 973.6\ \text{kN·m/m} \qquad W = 1{,}237.5\ \text{kN/m (no live load)}$$

$$FS_{ot} = 4{,}640.6/973.6 = \mathbf{4.77} \qquad e = \mathbf{0.787}\ \text{m}\ \le 1.25\ \checkmark$$

4 Sliding and bearing capacity (in bearing capacity the live load also loads the base)

$$FS_{sl} = 1{,}237.5(0.3378)/266.11 = \mathbf{1.57}\ \ge 1.5\ \checkmark$$

$$V = 1\,237.5 + 20(7.5) = 1{,}387.5\ \Rightarrow\ e_b = 0.702,\ \ L' = 6.10\ \text{m},\ \ \sigma'_v = 227.6\ \text{kPa}$$

$$q_u = 2{,}172\ \text{kPa} \qquad FS_{bc} = \mathbf{9.54}\ \ge 2.5\ \checkmark$$

External stability is adequate (overturning 4.77, sliding 1.57, bearing capacity 9.54), but **pullout fails** (FS_P = 0.63): for a wall with live load, the length of the upper levels governs.

Check the solution in GeoTerva ↗

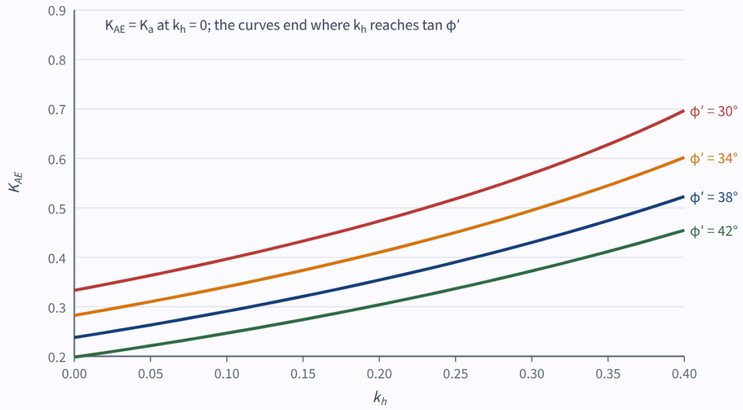
Dynamic thrust, block inertia and inertia of the active zone

THEORY

The earthquake (pseudo-static, k_h; $k_v = 0$) adds three forces: the dynamic thrust of the retained fill (Mononobe–Okabe), the inertia of the reinforced block, and the inertia of the **active zone** retained by the reinforcement:

$$\psi = \tan^{-1} k_h \qquad K_{AE} = \frac{\cos^2(\varphi_b' - \psi)}{\cos^2\psi \left[1 + \sqrt{\dfrac{\sin\varphi_b' \sin(\varphi_b' - \psi)}{\cos\psi}}\right]^2}$$

$$\Delta P_{AE} = \tfrac{1}{2}\gamma_b H^2 (K_{AE} - K_a) \;\; (0.6H) \qquad P_{IR} = \tfrac{1}{2} k_h \gamma_1 H^2 \;\; (H/2) \qquad P_i = k_h W_a \;\Rightarrow\; T_{md} = P_i \frac{L_e}{\Sigma L_e}$$



Half of ΔP_{AE} is taken (the thrust and the inertia do not reach their maximum values at the same instant); P_{IR} only for the block of width 0.5H behind the facing. Surcharge does not generate inertia force.

External checks: $\Sigma H = P_a + \tfrac{1}{2}\Delta P_{AE} + P_{IR}$; $e \le L/3$; factors of safety are 75% of the static ones.

Internal checks: each level is rechecked against rupture with $T + T_{md}$ and against pullout with **80%** of F*; the live load is kept as in the static case.

Problem statement

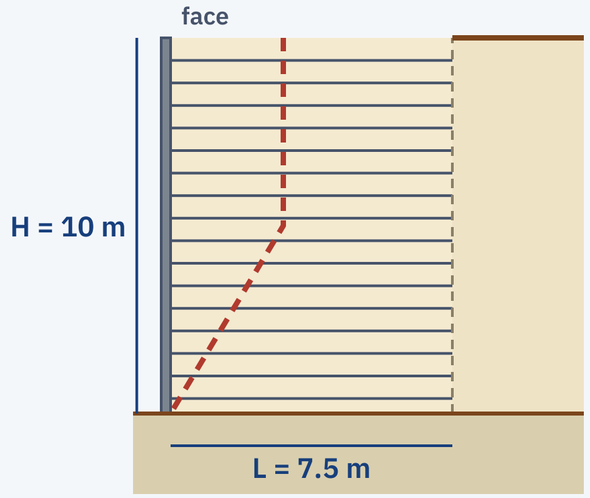
EXAMPLE

REQUIRED

K_{AE}, ΔP_{AE}, P_{IR}; external checks; active-zone inertia and rupture and pullout check of each level with **T + T_{md}**.

Given

Wall	**Wall in Examples 1–7; no surcharge**
Earthquake	$k_h = 0.15$ ($k_v = 0$); **required FS is 75% of the static value;** $e \leq L/3$**; at pullout** F^* **is 80% of the static value**



Solution (1/4)

SOLUTION

1 Seismic angle and coefficient

$$\psi = \tan^{-1} 0.15 = 8.53^\circ \qquad \sqrt{\sin 36^\circ \sin(27.47^\circ)/\cos 8.53^\circ} = 0.5236$$

$$K_{AE} = \cos^2(27.47^\circ)/\left[\cos^2 8.53^\circ (1+0.5236)^2\right] = \mathbf{0.3468} \quad (K_a = 0.2596)$$

2 Dynamic thrust increment, block inertia and total horizontal force

$$\Delta P_{AE} = \tfrac{1}{2}(16.5)(10)^2(0.3468 - 0.2596) = \mathbf{71.89}\ \text{kN/m} \ \ (6.0\ \text{m})$$

$$P_{IR} = \tfrac{1}{2}(0.15)(16.5)(10)^2 = \mathbf{123.75}\ \text{kN/m} \ \ (5.0\ \text{m})$$

$$P_s = 214.18 + \tfrac{1}{2}(71.89) + 123.75 = \mathbf{373.88}\ \text{kN/m}$$

3 Overturning moment

$$M_{o,E} = 214.18(3.333) + 35.94(6.0) + 123.75(5.0)$$

$$= 713.9 + 215.7 + 618.8 = \mathbf{1{,}548.4}\ \text{kN·m/m}$$

Solution (2/4)

SOLUTION

4 Overturning and eccentricity (limit L/3 = 2.50 m)

$$FS_{ot,E} = 4{,}640.6/1{,}548.4 = \mathbf{3.00} \ \geq 1.5 \ \checkmark$$

$$x_R = (4{,}640.6 - 1{,}548.4)/1{,}237.5 = 2.499 \text{ m} \qquad e = 3.75 - 2.499 = \mathbf{1.251} \text{ m} \ \leq 2.50 \ \checkmark$$

5 Sliding (no frontal resistance)

$$FS_{sl,E} = 1{,}237.5(0.3378)/373.88 = 418.1/373.88 = \mathbf{1.118} \ < 1.125 \ \times$$

6 Bearing capacity (Meyerhof)

$$L' = 7.5 - 2(1.251) = 4.998 \text{ m} \qquad \sigma'_v = 1{,}237.5/4.998 = 247.6 \text{ kPa}$$

$$q_u = 50(25.80) + \tfrac{1}{2}(17.3)(4.998)(16.72) = 2{,}013 \text{ kPa} \quad FS_{bc,E} = \mathbf{8.13} \ \geq 1.875 \ \checkmark$$

Solution (3/4)

SOLUTION

7 Inertia of the active zone and its distribution among levels

$$W_a = \gamma_1 A_a = 16.5(22.50) = 371.25\ \text{kN/m} \qquad P_i = 0.15(371.25) = 55.69\ \text{kN/m}$$

$$T_{md} = P_i L_e / \Sigma L_e \qquad \Sigma L_e = 91.5\ \text{m} \quad (17\ \text{level})$$

8 Level 1 (z = 0.6 m, L_e = 4.50 m): rupture force and pullout

$$T_{md} = 55.69(4.50)/91.5 = 2.739\ \text{kN} \qquad T + T_{md} = 2.54 + 2.739 = 5.283\ \text{kN}$$

$$P_{r,E} = 0.8(4.855) = 3.884\ \text{kN} \quad (F^*\ 80\%\ \text{of static})$$

$$FS_{P,E} = 3.884/5.283 = \mathbf{0.735} \ < 1.125 \quad ✗$$

9 Level 17 (base, L_e = 7.50 m): rupture

$$T_{md} = 55.69(7.50)/91.5 = 4.565\ \text{kN} \qquad FS_{B,E} = 85.5/(30.84 + 4.565) = \mathbf{2.415} \ \geq 1.35\ ✓$$

Solution (4/4)

SOLUTION

10 Summary of results (k_h = 0.15)

Check	Static	Earthquake	Required (static / earthquake)	Decision (earthquake)
Rupture FS_B (smallest)	2.77	2.41	1.80 / 1.35	Satisfied
Pullout FS_P (minimum)	1.91	**0.74**	1.50 / 1,125	**Not satisfied** (level 1)
Overturning FS_{ot}	6.50	3.00	2.00 / 1.50	Satisfied
Eccentricity e (m)	0.58	1.25	≤ 1.25 / ≤ 2.50	Satisfied
Sliding FS_{sl}	1.95	**1.118**	1.50 / 1,125	**Not satisfied** (at the limit)
Bearing capacity FS_{bc}	11.32	8.13	2.50 / 1,875	Satisfied

The earthquake reduces **pullout ($FS_{P,E}$ = 0.74) and sliding ($FS_{sl,E}$ = 1.12)** below the required value; overturning, eccentricity, bearing capacity and rupture are satisfied. At the upper levels the tension is small, but since σ'_v is also small, **pullout is the weakest link**: the reinforcement length must be increased (Example 10).

Check the solution in GeoTerva ↗

Problem statement

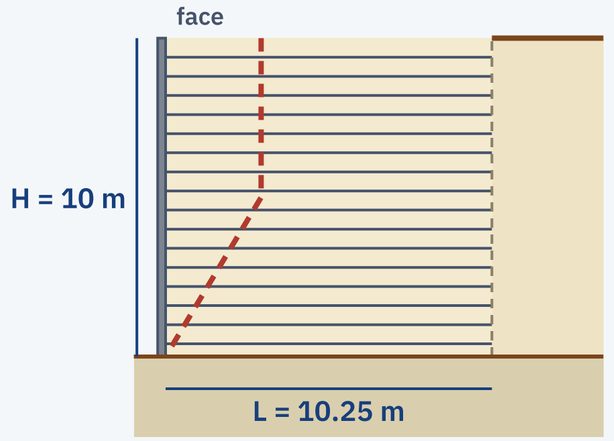
EXAMPLE

Given

Initial	**Example 9: pullout ($FS_{P,E} = 0.74$) and sliding ($FS_{sl,E} = 1.12$) insufficient for the earthquake**
Variable	**strip length L (increased in 0.25 m steps); S_V, S_H, the strip section and the fill remain the same**

REQUIRED

Minimum L: seismic checks ($FS_{P,E} \geq 1.125$; $FS_{sl,E} \geq 1.125$; $e \leq L/3$) must be satisfied; **L table**.



Solution (1/2)

SOLUTION

1 Factors of safety as the length increases (thick: below the required value)

L (m)	$FS_{P,E}$ (≥ 1,125)	$FS_{sl,E}$ (≥ 1,125)	e_E / (L/3) (m)	FS_P static	FS_{sl} static	FS_{bc} static
7.5	**0.735**	**1.118**	1.25 / 2.50	1.91	1.95	11.32
8.5	**0.884**	1.267	1.10 / 2.83	2.33	2.21	12.65
9.5	**1.033**	1.416	0.99 / 3.17	2.76	2.47	13.87
10.5	1.182	1.565	0.89 / 3.50	3.18	2.73	15.02
11.5	1.331	1.715	0.82 / 3.83	3.60	2.99	16.11
12.5	1.480	1.864	0.75 / 4.17	4.03	3.25	17.16

At L = 7.5 m, pullout (0.74) and sliding (1.12) are insufficient; **pullout governs**: even at L = 9.5 m it is 1.03 < 1.125. As the length increases, both the effective length L_e (pullout) and the block weight (sliding, eccentricity) increase.

Solution (2/2)

SOLUTION

2 Smallest length with 0.25 m steps: L = 10.25 m

$$FS_{P,E} = 1.145 \geq 1.125 \checkmark \qquad FS_{sl,E} = 1.528 \geq 1.125 \checkmark$$

$$e_E = 0.916 \leq \frac{L}{3} = 3.42\ \text{m} \checkmark \qquad FS_{ot,E} = 5.60 \quad FS_{bc,E} = 12.48$$

3 Static case with this length (check)

$$FS_P = 3.07 \quad FS_{sl} = 2.67 \quad FS_{ot} = 12.14 \quad e = 0.422\ \text{m} \quad FS_{bc} = 14.74$$

For earthquake, **L = 10.25 m** (≈ 1.02 H) is required; the static factors of safety are very high with this length. The same result can also be sought by densifying the reinforcement (S_V, S_H) or with wider strip/coated reinforcement; length is the most direct factor.

Check the solution in GeoTerva ↗

Problem statement

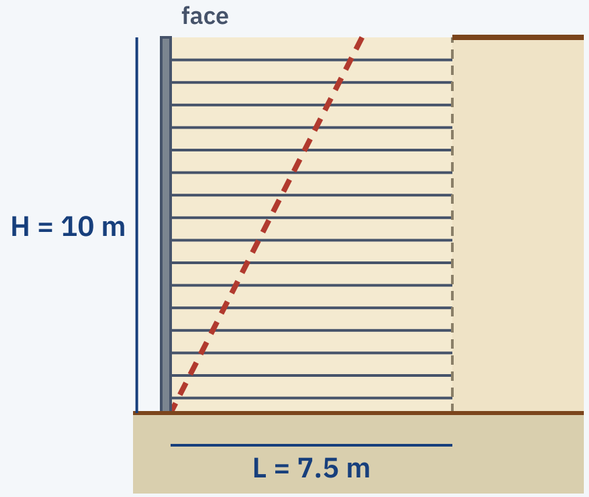
EXAMPLE

Given

Geogrid	$T_{ult} = 150$ **kN/m;** $RF_{id} = 1.2$**,** $RF_{cr} = 2.5$**,** $RF_{cbd} = 1.25$**; surcharge** $C_r = 0.8$**;** $S_V = 0.6$ **m**
Pullout	$F^* = \frac{2}{3}\tan 36^\circ$**,** $\alpha = 0.8$**; Rankine failure surface (**$K_r = K_a$**);** $FS_B \geq 1.8$**,** $FS_P \geq 1.5$

REQUIRED

T_{all}, T_{cap}, T_{max} (per meter of wall), **FS_B**, required S_V; at level 1, **FS_P** and L_{req}.



Solution (1/2)

SOLUTION

1 Long-term strength and capacity

$$RF = 1.2(2.5)(1.25) = 3.75 \qquad T_{all} = \frac{T_{ult}}{RF} = \frac{150}{3.75} = 40.0\ \text{kN/m}$$

$$T_{diameter} = T_{all} C_r = 40.0(0.8) = 32.0\ \text{kN/m}$$

2 Tension and rupture ($K_r = K_a$)

$$\sigma'_h(H) = K_a(165) = 42.84\ \text{kPa} \qquad T_{max} = 42.84(0.6) = 25.70\ \text{kN/m}$$

$$FS_B = \frac{32.0}{25.70} = \mathbf{1.245} < 1.8 \ \text{✗} \qquad S_{V,req} = \frac{T_{all} C_r}{\sigma'_h FS_B} = \frac{32.0}{42.84(1.8)} = \mathbf{0.415}\ \text{m}$$

The installed spacing (0.6 m) is excessive: the rupture factor of safety is 1.25 < 1.8; the required spacing is **$S_V \le 0.415$ m** (or a stronger geogrid).

Solution (2/2)

SOLUTION

3 Level 1 (z = 0.6 m): Rankine surface

$$L_a = (H - z)/\tan 63^\circ = 4.79\ \text{m} \qquad L_e = 7.5 - 4.79 = 2.71\ \text{m}$$

$$\sigma'_h = K_a(9.9) = 2.57\ \text{kPa} \qquad T = 2.57(0.6) = 1.542\ \text{kN/m}$$

4 Pullout (per metre of wall, $R_c = C_r = 0.8$)

$$F^* = \tfrac{2}{3}\tan 36^\circ = 0.4844$$

$$P_r = 2C_rF^*\alpha\sigma'_vL_e = 2(0.8)(0.4844)(0.8)(9.9)(2.71) = 16.64\ \text{kN/m}$$

$$FS_P = 16.64/1.542 = \mathbf{10.79}\ \geq 1.5\ \checkmark \qquad L_{req} = L_a + \frac{1.5\,T}{2C_rF^*\alpha\sigma'_v} = \mathbf{5.17}\ \text{m}$$

In the geosynthetic, pullout is amply provided (FS_P = 10.79); the required length follows the Rankine wedge: **L_{req} = 5.17 m** (< 7.5 m). The governing mode is **rupture**: it is adjusted by the design strength and spacing.

Check the solution in GeoTerva ↗

Problem statement

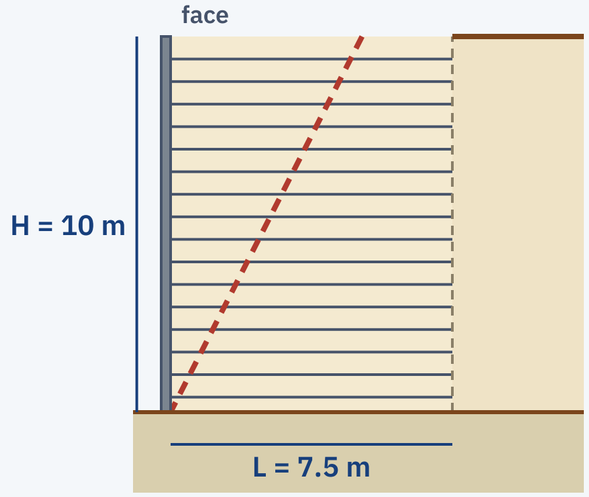
EXAMPLE

Given

Wall	**H = 10 m, L = 7.5 m, $S_V = 0.6$ m; fill $\varphi_1' = 36°$; same foundation soil**
Cases	**simplified method, source Rankine approach, live load, strip load, geogrid, short reinforcement**

REQUIRED

Write the **factors of safety** of the seven cases side by side and interpret the differences.



Solution

SOLUTION

1 The same wall, seven cases (static)

Condition	FS_B	FS_P	L_{req} (m)	FS_{ot}	e (m)	FS_{sl}	FS_{bc}
Opening: simplified, slices, L = 7.5 m	2.77	1.91	6.54	6.50	0.58	1.95	11.32
Rankine approach of the source (same strips)	3.33	1.90	6.93	6.50	0.58	1.95	11.32
Rankine approach, L = 13.5 m	3.33	6.09	6.93	21.06	0.32	3.51	18.18
Live surcharge q_L = 20 kPa	2.47	0.63	13.68	4.77	0.79	1.57	9.54
Strip load 60 kPa (a′ = 2 m, b′ = 1 m)	2.74	1.32	8.13	3.66	1.14	1.55	7.87
Geogrid 150 kN/m (Rankine, α = 0.8)	1.25	10.79	5.17	6.50	0.58	1.95	11.32
Strips L = 7 m (0.7H)	2.77	1.70	6.54	5.66	0.62	1.82	10.60

Rankine approach uses K_a at the base: tension is small (T_{max} 25.7 kN), rupture is high (3.33), but the length required at the top is 6.93 m. **Live load** reduces every factor of safety (FS_P 0.63: the loads do not hold); **strip load** reduces it less (FS_P 1.32), because it acts as weight on top of the block. In the geogrid, pullout is ample, and **rupture is governing** (FS_B 1.25). With L = 7 m the strips are still sufficient (FS_P 1.70, e = 0.62 m).

Check the solution in GeoTerva ↗

Scope of the solution

THEORY

The **fill** is freely draining granular material (no pore water pressure); there is **no** groundwater in or behind the wall and no inclined backfill above the wall.

Levels are at depths S_V, $2S_V$, …; a single reinforcement length, a trapezoidal section, and levels of different lengths are not modelled. Bar/wire mesh ($K_r/K_a \geq 2.5$) and the depth-varying F* value of ribbed strips are outside the scope.

Loads: distributed surcharge (permanent/live) and strip loads (Laba–Kennedy; superposed). Point and line loads are not modeled.

The **facing** carries no load; the connection is checked with a rupture factor of safety only when a strength is entered; the embedded portion in front of it is not counted.

Not checked: global and compound stability (slope stability analysis), settlement and differential settlement, drainage, facing, construction stages.

The checks are in **allowable stress** form (global factors of safety); AASHTO LRFD/GEC 11 load and resistance factors are not used. For an important wall, product data, global stability and settlement should additionally be performed.

Summary

Key points

Reinforced soil: the soil pressure is carried by reinforcement tension and joined by friction. Internal stability: rupture, pullout, (connection); external stability: overturning, sliding, bearing capacity for the L × H block

Stress: $\sigma'_h = K_r\sigma'_v$; in the strip $K_r/K_a = 1.7 \rightarrow 1.2$ (down to 6 m), and 1 in the geosynthetic. **Tension** $T = \sigma'_h S_V S_H$; the largest tension is at the base

Rupture: $T_{cap} = w(t - rn)f_y$ (strip) or $T_{all}C_r$ (geosynthetic); **pullout**: $P_r = 2R_cF^*\alpha\sigma'_vL_e$, with L_e behind the failure surface (bilinear / Rankine)

External checks: FS_{ot}, $e \le L/6$, $FS_{sl} = \Sigma V \tan(\frac{2}{3}\varphi'_{min})/\Sigma H$, bearing capacity (source and Meyerhof $L' = L - 2e$, the smaller)

Live load loads, it does not resist: it increases pull and thrust, and does not enter into pullout or weight; in the example FS_P 1.91 → 0.63

Earthquake: ΔP_{AE} (half of it, 0.6H), P_{IR}, $P_i \rightarrow T_{md}$; required FS is 75% of the static one; the example wall **fails in pullout (0.74) and sliding (1.12)**, and is satisfied with L = 10.25 m

Geogrid: $T_{all} = T_{ult}/(RF_{id}RF_{cr}RF_{cbd})$; rupture is governing ($S_{V,req}$ = 0.415 m), pullout is abundant

References

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NOTE

All numerical examples have been generated with the module's own calculation engine; the module returns the same values. The step-by-step example on the module page is also this wall (H = 10 m, galvanized strip 75 × 6 mm, L = 7.5 m, S_V = 0.6 m, S_H = 1.0 m).

GeoTerva

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From theory to practice

Enter your wall height, reinforcement (strip, geogrid, geotextile), fill, surcharge and earthquake coefficient in GeoTerva; the module reports the pullout and rupture safety of each level, the external checks of the block and the required length together.

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