GeoTerva

presentation · Lateral earth pressure

Mononobe–Okabe earthquake earth pressure

Lecture: the pseudo-static horizontal and vertical forces on the wedge behind the wall during an earthquake, the Mononobe–Okabe active coefficient, the equilibrium limit, the line of action; surcharge and groundwater; a worked example in every topic.

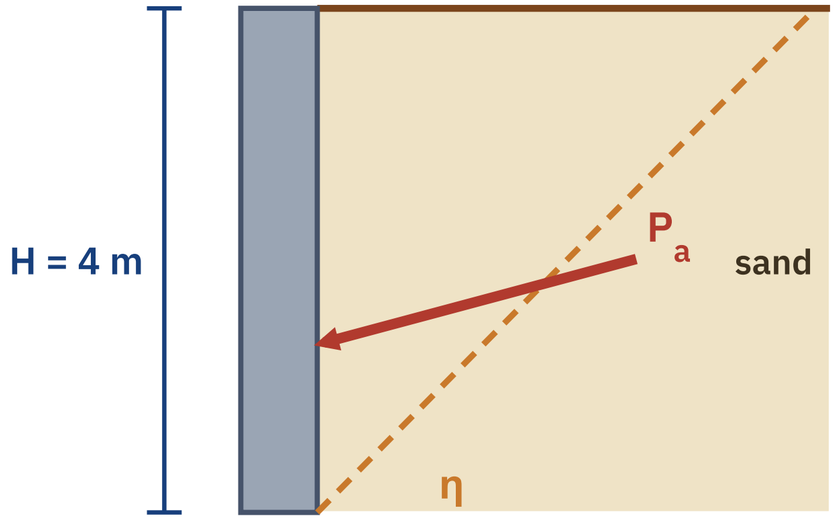
Pseudo-static wedge

$\psi = \tan^{-1} k_h/(1 - k_v)$

Seed – Whitman

Surcharge · water

Equilibrium limit



Introduction

What will we learn in this lecture?

- Representing the earthquake by **pseudo-static** horizontal (k_hW) and vertical (k_vW) forces
- **Seismic angle** ψ and Mononobe–Okabe coefficient $\mathbf{K'_a}$; $P_{ae} = ½\gamma H^2(1 - k_v)K'_a$
- **Equilibrium limit**: if $k_{h,cr} = (1 - k_v) \tan \varphi'$ is exceeded, no wedge remains in equilibrium
- Finding the **location** of the force with Seed–Whitman: $P_{ae} = P_a + \Delta P_{ae}$, with ΔP_{ae} at 0.6H
- Accounting for surcharge and **groundwater** (moving or not moving with the soil)
- to see how the force changes when k_h, k_v and the geometry change

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Forces are per unit length of wall (kN/m); k_h, k_v are the acceleration ratios to g; k_v is positive upward.

Okabe (1926) and Mononobe–Matsuo (1929)

THEORY

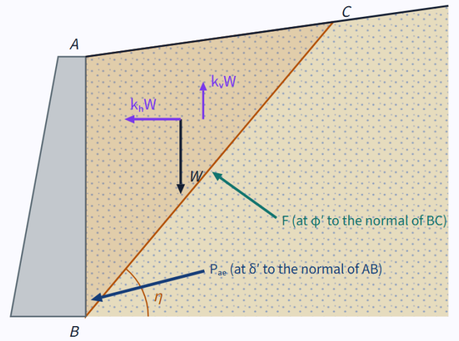
During an earthquake the wedge behind the wall accelerates. The pseudo-static method represents the shaking by two **rigid-body forces** acting on the Coulomb wedge:

- **$k_h W$**: horizontal, toward the wall (k_h = horizontal acceleration / g)
- **$k_v W$**: vertical (here upward; reduces the weight)
- W, F (φ′ inclined) and P_{ae} (δ′ inclined) are as in Coulomb.

The sum of W, $k_h W$ and $k_v W$ is a force rotated from the vertical by the seismic angle **ψ**; that is, the seismic wedge is a Coulomb wedge rotated by ψ (wall face θ + ψ, backfill α + ψ):

$$\psi = \tan^{-1}\frac{k_h}{1-k_v} \qquad W' = \frac{W(1-k_v)}{\cos\psi}$$

The active force is again the **largest** of the trial wedges.



force polygon — the resultant of W and the inertia forces tilts by ψ

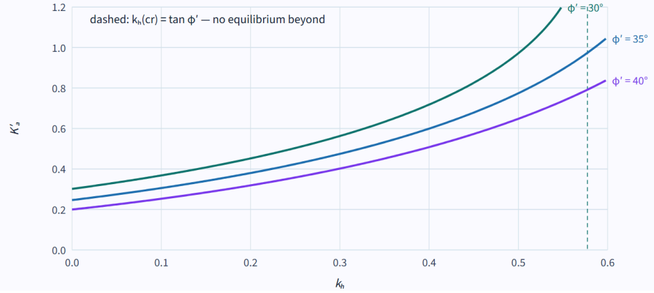
Dynamic active force

THEORY

$$P_{ae} = \tfrac{1}{2}\gamma H^2 (1 - k_v) K'_a$$

$$K'_a = \frac{\cos^2(\varphi' - \theta - \psi)}{\cos\psi \cos^2\theta \cos(\delta' + \theta + \psi)\left[1 + \sqrt{\dfrac{\sin(\varphi' + \delta')\sin(\varphi' - \alpha - \psi)}{\cos(\delta' + \theta + \psi)\cos(\theta - \alpha)}}\right]^2}$$

k_h	φ′ = 28°	30°	35°	40°	45°
0.1	0.396	0.368	0.306	0.253	0.207
0.2	0.485	0.452	0.380	0.319	0.267
0.3	0.604	0.563	0.474	0.402	0.340
0.4	0.778	0.718	0.599	0.508	0.433
0.5	1.115	0.972	0.774	0.648	0.552



for $k_h = k_v = 0$, ψ = 0 and K'_a is the Coulomb K_a. Table: θ = α = 0, δ′ = φ′/2.

K'_a grows rapidly with k_h; it increases without bound as it approaches the limit.

Problem statement

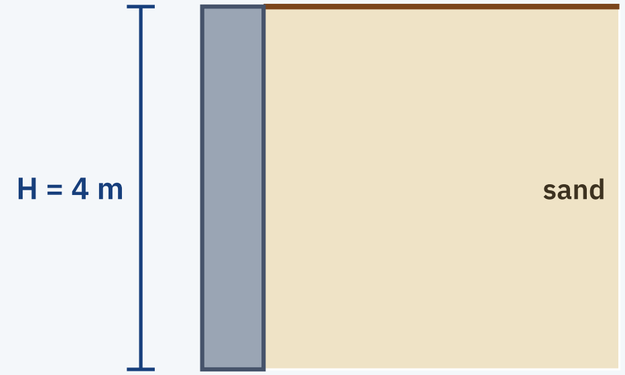
EXAMPLE

Given

Wall	$H = 4$ **m, vertical; horizontal fill; no surcharge**
Soil	**dry sand:** $\gamma = 15.5\ \mathrm{kN/m^3},\ \varphi' = 30^\circ,\ \delta' = 15^\circ$
Earthquake	$k_h = 0.2,\ k_v = 0$

REQUIRED

ψ, **K′a**, **Pae** and the critical wedge; comparison with static Coulomb. The module opens with this wall.



γ = 15.5 kN/m³, φ′ = 30°, δ′ = 15°, k_h = 0.2

Solution (1/2)

SOLUTION

1 Seismic angle

$$\psi = \tan^{-1}\frac{0.2}{1-0} = \mathbf{11.31^\circ}$$

2 Mononobe–Okabe coefficient (θ = α = 0)

$$\cos^2(30^\circ - 11.31^\circ) = 0.8973 \qquad \cos\psi = 0.9806 \qquad \cos(15^\circ + 11.31^\circ) = 0.8964$$

$$\sqrt{\frac{\sin 45^\circ \sin(30^\circ - 11.31^\circ)}{\cos(15^\circ + 11.31^\circ)}} = \sqrt{0.2528} = 0.5028$$

$$K'_a = \frac{0.8973}{0.9806(0.8964)(1+0.5028)^2} = \mathbf{0.4520}$$

Solution (2/2)

SOLUTION

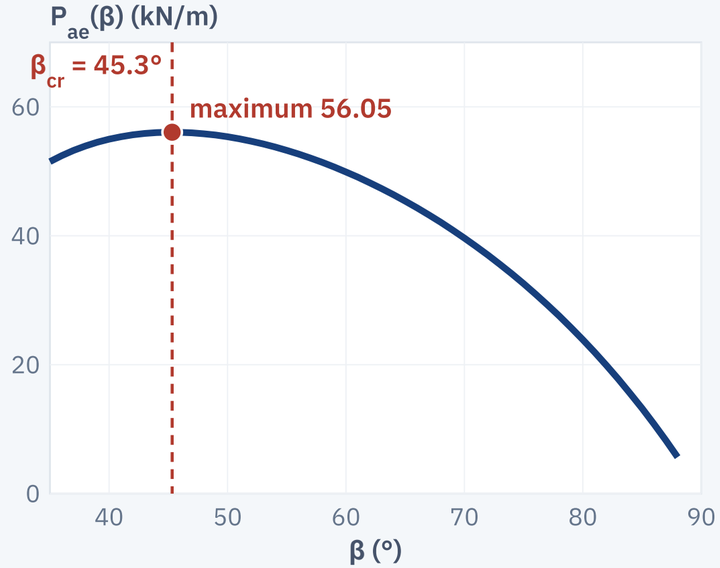
3 Dynamic force

$$P_{ae} = \tfrac{1}{2}\gamma H^2(1-k_v)K'_a$$
$$= \tfrac{1}{2}(15.5)(4)^2(1-0)(0.4520) = \mathbf{56.05}\ \mathrm{kN/m}$$

4 Critical wedge (apex of trial wedges)

$$\beta_{cr} = \mathbf{45.3}^\circ\ (\text{with horizontal})$$

P_{ae} = **56.05 kN/m**; the static Coulomb force (K_a = 0.3014) was 37.38 kN/m: the earthquake force **increased it by 50%**. The critical wedge makes 45.3° with the horizontal.



Check the solution in GeoTerva ↗

Critical horizontal acceleration

THEORY

The sin(φ′ − α − ψ) under the square root is not real when φ′ − α − ψ < 0: **no wedge can be in equilibrium**, regardless of the wall's behaviour. The fill is stable only under the following condition:

$$\alpha \leq \varphi' - \psi$$

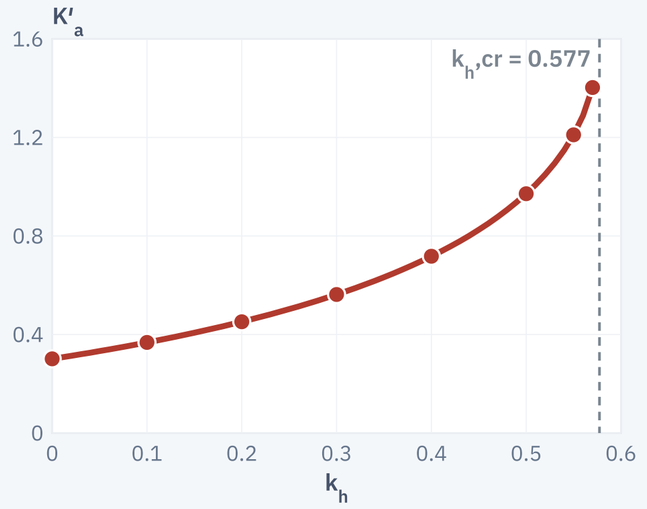
For a horizontal fill this requires ψ ≤ φ′, i.e. the horizontal acceleration cannot exceed:

$$k_{h(cr)} = (1 - k_v)\tan\varphi' \qquad (\alpha \neq 0 :\ \tan(\varphi' - \alpha))$$

As the limit is approached K'_a increases very rapidly ($k_{h,cr}$ = 0.577 for φ′ = 30°).

If the modulus k_h is above the limit, it stops the analysis with a message.

K'_a – k_h (φ′ = 30°, δ′ = 15°)



Problem statement

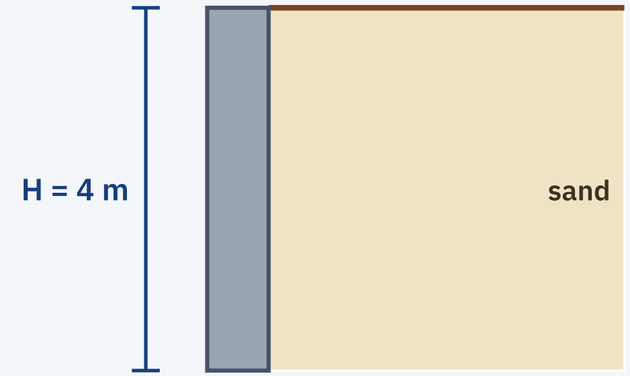
EXAMPLE

REQUIRED

For each k_h: **ψ**, **K'_a** and **P_{ae}**; the critical k_h; the result for $k_h = 0.6$.

Given

Wall and soil	**Wall in Example 1:** $H = 4$ **m,** $\gamma = 15.5\ \mathrm{kN/m^3},\ \varphi' = 30^\circ,\ \delta' = 15^\circ,\ k_v = 0$
Variable	$k_h = 0;\ 0.1;\ 0.2;\ 0.3;\ 0.4;\ 0.5;\ 0.55;\ 0.57$

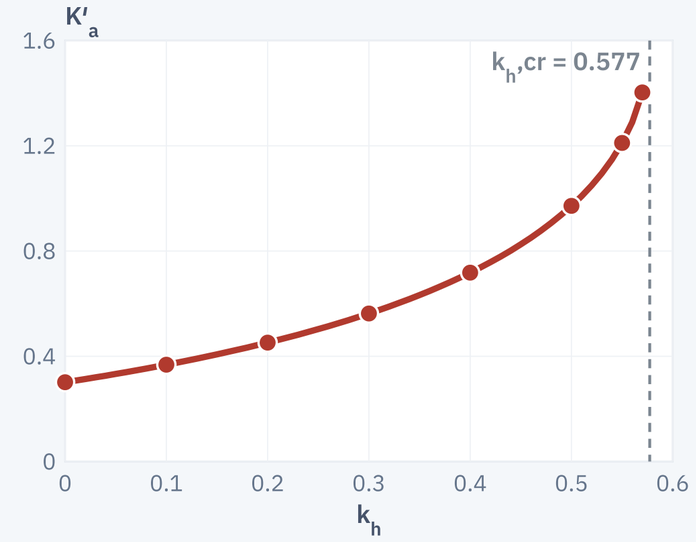


Solution

SOLUTION

1 $\psi = \tan^{-1} k_h$; $k_{h,cr} = \tan 30° = 0.577$

k_h	ψ (°)	K'_a	P_{ae} (kN/m)
0.00	0.0	0.301	37.4
0.10	5.7	0.368	45.6
0.20	11.3	0.452	56.1
0.30	16.7	0.563	69.8
0.40	21.8	0.718	89.0
0.50	26.6	0.972	120.5
0.55	28.8	1.211	150.1
0.57	29.7	1.403	174.0



From k_h = 0 to 0.2, P_{ae} goes from 37.4 to 56.1 kN/m (50%), at 0.5 it is 120.5 kN/m (3.2 times); as k_h approaches 0.57, K'_a increases without bound. **k_h = 0.6 > 0.577**: the square root is not real, no wedge is in equilibrium: the module stops the calculation.

Check the solution in GeoTerva ↗

Problem statement

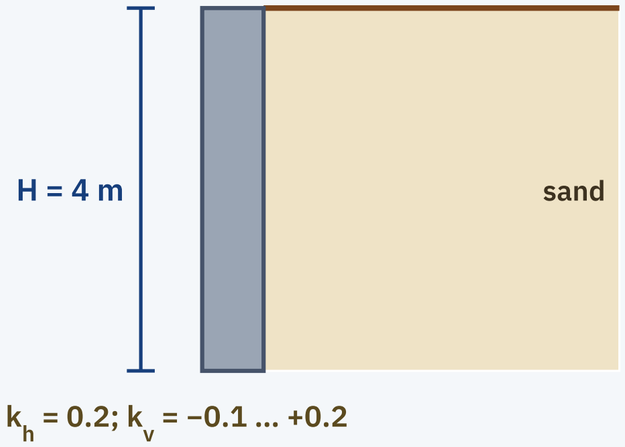
EXAMPLE

Given

Wall and soil	**Wall and soil in Example 1;** $k_h = 0.2$
Variable	$k_v = -0.1$ **(downward),** 0**,** $+0.1$**,** $+0.2$ **(upward)**

REQUIRED

For each k_V: **ψ**, **K'_{ae}**, **$(1 - k_V)$** and **P_{ae}**; which direction is more unfavourable?



Solution

SOLUTION

1 $\psi = \tan^{-1}[k_h/(1 - k_v)]$; $P_{ae} = \tfrac{1}{2}\gamma H^2(1 - k_v)K'_a = 124\,(1 - k_v)\,K'_a$

k_v	ψ (°)	K'_a	$1 - k_v$	P_{ae} (kN/m)
-0.1	10.30	0.4351	1.1	59.35
0.0	11.31	0.4520	1.0	56.05
0.1	12.53	0.4739	0.9	52.89
0.2	14.04	0.5032	0.8	49.92

Downward k_v (−0.1) increases the force to 59.3 kN/m (+6%); upward k_v = 0.2 reduces it to 49.9 kN/m. Although ψ increases with upward k_v, the $(1 - k_v)$ factor dominates. In practice, $k_v = \pm 0.5\,k_h$ is used with the direction that gives the larger force.

Check the solution in GeoTerva ↗

Seed and Whitman (1970)

THEORY

The Mononobe–Okabe analysis gives the magnitude and direction of the force, **but not where it acts**. Seed and Whitman split the force into the static Coulomb force and the seismic increment:

$$P_{ae} = P_a + \Delta P_{ae}$$

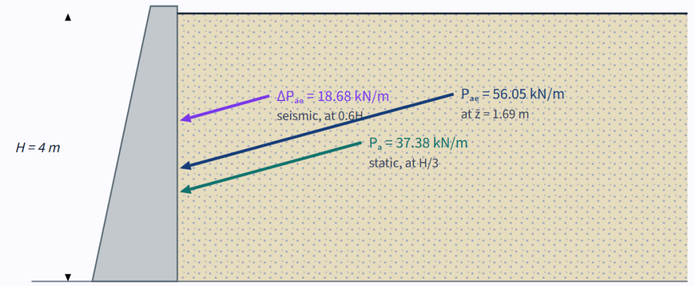
$$\bar{z} = \frac{P_a \dfrac{H}{3} + \Delta P_{ae}\,(0.6H)}{P_{ae}}$$

P_a (with the same δ′, θ, α as ½K_aγH^2) acts at **H/3** above the base; ΔP_{ae} acts at **0.6H** (the triangular diagram is an inverted triangle narrowing toward the base).

The resultant is again inclined at δ′ from the normal to the wall face.

The increment is sometimes taken at mid-height; 0.6H gives a larger overturning moment and is adopted.

STATIC + SEISMIC INCREMENT



Problem statement

EXAMPLE

Given

Wall and earthquake

Wall in Example 1: $H = 4$ **m,** $\gamma = 15.5\ \mathrm{kN/m^3}$, $\varphi' = 30°$, $\delta' = 15°$, $k_h = 0.2$

REQUIRED

P_a, **ΔP_{ae}** and **$\bar{z}$**; $\bar{z}$ if ΔP_{ae} is taken at half height.

Solution

SOLUTION

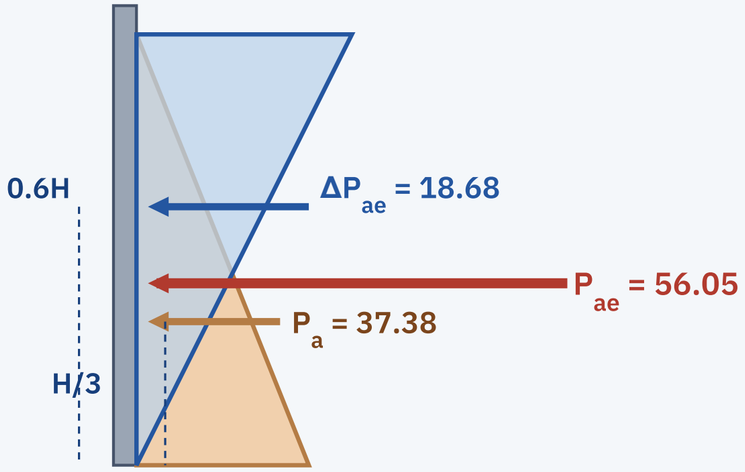
1 Static force (K_a = 0.3014) and seismic increment

$$P_a = \tfrac{1}{2}(15.5)(4)^2(0.3014) = 37.38\ \text{kN/m}$$

$$\Delta P_{ae} = P_{ae} - P_a = 56.05 - 37.38 = \mathbf{18.68}\ \text{kN/m}$$

2 Height of application (from the base)

$$\bar{z} = \frac{37.38(4/3) + 18.68(2.4)}{56.05} = \mathbf{1.69}\ \text{m}$$



$\bar{z}$ = **1.69 m** (it was 1.33 m in the static case). Had the increment been taken at half the height (2.0 m), $\bar{z}$ would come out as 1.56 m: the 0.6H assumption moves the force upward and **increases the overturning moment**.

Check the solution in GeoTerva ↗

Problem statement

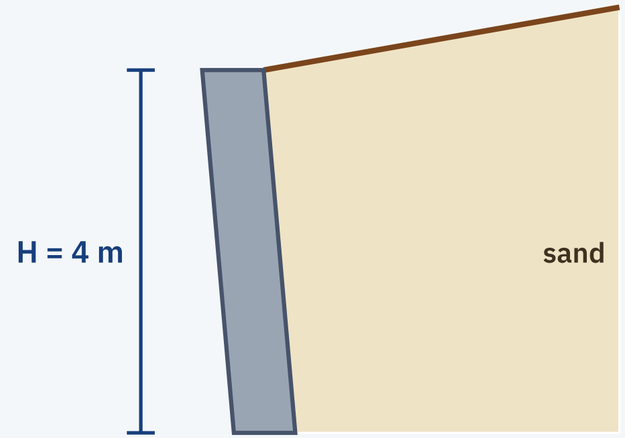
EXAMPLE

Given

Wall	$H = 4$ m; face inclined at $\theta = 5^\circ$ from vertical
Fill	surface slope $\alpha = 10^\circ$; $\gamma = 15\ \mathrm{kN/m^3}$, $\varphi' = 30^\circ$, $\delta' = 15^\circ$
Earthquake	$k_h = 0.15$, $k_v = 0$

REQUIRED

ψ, **K′$_a$**, **P$_{ae}$**, ΔP$_{ae}$, **z̄**; equilibrium-limit check.



γ = 15, φ′ = 30°, δ′ = 15°, k$_h$ = 0.15

Solution

SOLUTION

1 Seismic angle and equilibrium limit

$$\psi = \tan^{-1} 0.15 = \mathbf{8.53}^\circ \qquad k_{h(cr)} = \tan 20^\circ = 0.364 > 0.15$$

2 Coefficient and force (static K_a = 0.3872)

$$K'_a(\theta = 5^\circ, \alpha = 10^\circ, \psi = 8.53^\circ) = \mathbf{0.5435}$$

$$P_{ae} = \tfrac{1}{2}(15)(4)^2(0.5435) = \mathbf{65.22}\ \text{kN/m} \qquad P_a = 46.46\ \text{kN/m}$$

3 Height of load application

$$\Delta P_{ae} = 18.76\ \text{kN/m} \qquad \bar{z} = \frac{46.46(1.33) + 18.76(2.4)}{65.22} = \mathbf{1.64}\ \text{m}$$

P_{ae} = **65.22 kN/m**, 1.64 m above the base; critical wedge 44.2°; the earthquake force increased it by 40%.

Check the solution in GeoTerva ↗

Module extension

THEORY

The **surcharge** q is also added to the inertia of each wedge together with its weight; the critical wedge does not change:

$$q_e = q\frac{\cos\theta}{\cos(\theta-\alpha)} \qquad P_{ae} = (1-k_v)K'_a\left(\tfrac{1}{2}\gamma H^2 + q_e H\right)$$

In groundwater, the pore water moves with the soil (**constrained**, fine-grained fill) or does not move (**free**, k > 5·10⁻⁴ m/s). Inertia is carried by the saturated or dry mass, stress by the submerged weight; underwater the seismic angle increases:

$$\tan\psi_w = \frac{\gamma_{sat}}{\gamma_{sat}-\gamma_w}\frac{k_h}{1-k_v}\ \text{(restrained)} \qquad \tan\psi_w = \frac{\gamma_d}{\gamma_{sat}-\gamma_w}\frac{k_h}{1-k_v}\ \text{(free)}$$

$$p_{wd} = \tfrac{7}{8}k_h\gamma_w\sqrt{d_w z'} \qquad P_{wd} = \tfrac{7}{12}k_h\gamma_w d_w^2\ (0{,}4\,d_w\ \text{at height, Westergaard})$$

Westergaard hydrodynamic pressure is added for free water. In layered fill, each layer is taken with its own K'_a as $\sigma' = (1 - k_V)K'_{a,i}\sigma'_V\cos\theta$; the static part and ΔP_{ae} (0.6H) are summed separately.

WHY IS WATER IMPORTANT?

Water enlarges the seismic angle ($\psi_W > \psi$) and, in addition to the hydrostatic pressure, introduces **hydrodynamic pressure (in the free state)**.

Problem statement

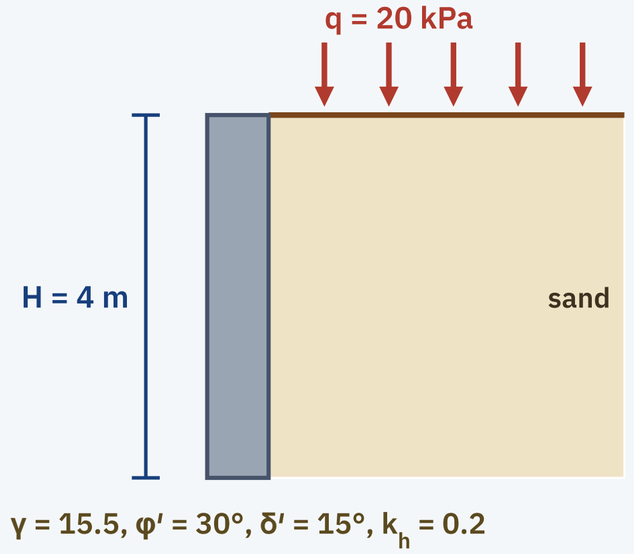
EXAMPLE

Given

Wall and soil	**Wall and soil in Example 1 ($k_h = 0.2$)**
Surcharge	**$q = 20$ kPa (on horizontal surface)**

REQUIRED

Total **P_{ae}**, static part, seismic increment and **$\bar{z}$**.



Solution

SOLUTION

1 Equivalent stress and forces ($K'_a = 0.4520$)

$$q_e = q\cos 0^\circ / \cos 0^\circ = 20\ \text{kPa}$$

$$P_{ae} = K'_a\left(\tfrac{1}{2}\gamma H^2 + q_e H\right) = 0.4520(124 + 80) = 0.4520(204) = \mathbf{92.21}\ \text{kN/m}$$

2 Static component (Coulomb $K_a = 0.3014$) and increment

$$P_a = K_a(204) = 61.49\ \text{kN/m} \qquad \Delta P_{ae} = 30.73\ \text{kN/m}$$

$$\bar{z} = \frac{(37.38)(1.33) + (24.11)(2.0) + (30.73)(2.4)}{92.21} = \mathbf{1.86}\ \text{m}$$

P_{ae} = **92.21 kN/m** (56.05 without surcharge); $\bar{z}$ = **1.86 m**. The static share of the surcharge acts at H/2, its seismic increment at 0.6H.

Check the solution in GeoTerva ↗

Problem statement

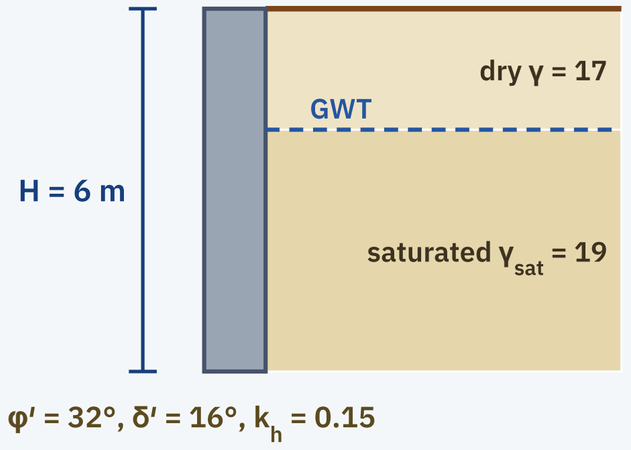
EXAMPLE

Given

Wall	$H = 6$ **m, vertical; horizontal fill; no surcharge**
Soil	$\gamma = 17\ \mathrm{kN/m^3}$ **(dry, 0–2 m),** $\gamma_{sat} = 19\ \mathrm{kN/m^3}$ **(2–6 m);** $\varphi' = 32^\circ,\ \delta' = 16^\circ$
Earthquake	$k_h = 0.15,\ k_v = 0$**; groundwater level 2 m (dw = 4 m)**

REQUIRED

The total **P** and **z̄** for the case where the water moves with the soil (**restrained**) and where it does not (**free**); comparison with the static value.



Solution (1/2)

SOLUTION

1 Equivalent k_h and ψ_w under water

$$\text{restrained: } k_{h,w} = \frac{19}{19-9.81}(0.15) = 0.3101 \;\Rightarrow\; \psi_w = 17.23^\circ$$

$$\text{free: } k_{h,w} = \frac{17}{19-9.81}(0.15) = 0.2775 \;\Rightarrow\; \psi_w = 15.51^\circ$$

2 Coefficients (dry zone $\psi = 8.53°$)

$$K'_a(\text{dry}) = 0.3792 \qquad K'_a(\text{confined, submerged}) = 0.5372 \qquad K'_a(\text{free, submerged}) = 0.4991$$

Solution (2/2)

SOLUTION

3 Forces (kN/m); $P_{wd} = (7/12)(0.15)(9.81)(4)^2 = 13.73$ kN/m, $0.4 \cdot d_w = 1.6$ m above

	Soil P_{ae}	Hydrostatic water	Hydrodynamic (Westergaard)	Total P	$\bar{z}$ (m)
Static ($k_h = 0$)	67.74	78.48	–	144.80	1.71
Confined water	125.46	78.48	–	202.06	2.24
Unconfined water	117.47	78.48	13.73	207.67	2.15

The total earthquake force increased by **40%** (restrained) and **43%** (free) relative to the static case; $\bar{z}$ rose from 1.71 to 2.24 m. In the restrained case the soil force is larger (ψ_w large), while in the free case hydrodynamic pressure is added; the totals are very close (202.1 and 207.7 kN/m).

Check the solution in GeoTerva ↗

Scope of the solution

THEORY

Pseudo-static: the earthquake is represented by constant k_h, k_v; the dependence of acceleration on time and on wall movement (Newmark sliding, dynamic analysis) is absent.

Plane wedge, cohesionless dry fill (Mononobe–Okabe); the layered case is an extension of the module (there is no closed-form layered solution); in a single layer it reduces to the closed-form solution, and at $k_h = k_v = 0$ to layered Coulomb.

Water: confined (fine-grained) or free ($k > 5 \cdot 10^{-4}$ m/s); for free water the Westergaard hydrodynamic pressure is added.

Location of action Seed–Whitman: P_a at H/3, ΔP_{ae} at 0.6H; the water acts with its own arm.

For **passive** pressure see the Passive in earthquake (Subba Rao–Choudhury) module.

Summary

Key points

Pseudo-static: $k_h W$, $k_v W$; $\psi = \tan^{-1}[k_h/(1 - k_v)]$; the seismic wedge is the Coulomb wedge rotated by ψ

$P_{ae} = \frac{1}{2}\gamma H^2(1 - k_v)K'_a$; K'_a increases rapidly with k_h; at $k_h = k_v = 0$ it reduces to Coulomb

Equilibrium limit: $k_{h,cr} = (1 - k_v) \tan \varphi'$ ($\tan(\varphi' - \alpha)$ on a slope); if exceeded, no solution exists

k_v: increases the downward force; in practice the worse of $\pm 0.5 k_h$ is chosen

Location: $P_{ae} = P_a + \Delta P_{ae}$; P_a at $H/3$, ΔP_{ae} at $0.6H$; $\bar{z} = [P_a(H/3) + \Delta P_{ae}(0.6H)]/P_{ae}$

Surcharge $q_e = q \cos\theta/\cos(\theta - \alpha)$ is added; under **water** ψ_w increases (confined: γ_{sat}/γ', free: γ_d/γ') and in free water the hydrodynamic pressure $(7/12)k_h \gamma_w d_w^2$ is added

References

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Mononobe, N. and Matsuo, H. (1929). On the determination of earth pressure during earthquakes. Proceedings, World Engineering Congress, Tokyo, 9, 177–185.

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Seed, H.B. and Whitman, R.V. (1970). Design of earth retaining structures for dynamic loads. Specialty Conference on Lateral Stresses in the Ground and Design of Earth Retaining Structures, ASCE, 103–147.

Das, B.M. (2016). Principles of Foundation Engineering, 8th ed. Cengage Learning.

Westergaard, H.M. (1933). Water pressures on dams during earthquakes. Transactions ASCE, 98, 418–433.

NOTE

All numerical examples were generated with the module's own calculation engine; the module yields the same values. The module opens with the wall from Example 1 (k_h = 0.2, δ′ = 15°).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own wall, earthquake coefficients k_h, k_v, surcharge, layers, and groundwater in GeoTerva; the module gives P_{ae}, the point of application height, and the critical wedge together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com