Geotechnical lecture presentation · Foundations

Meyerhof bearing capacity: general equation

Lecture: from Meyerhof's slip-line network to bearing capacity factors, from shape, depth and inclination corrections to groundwater; a worked example on every topic.

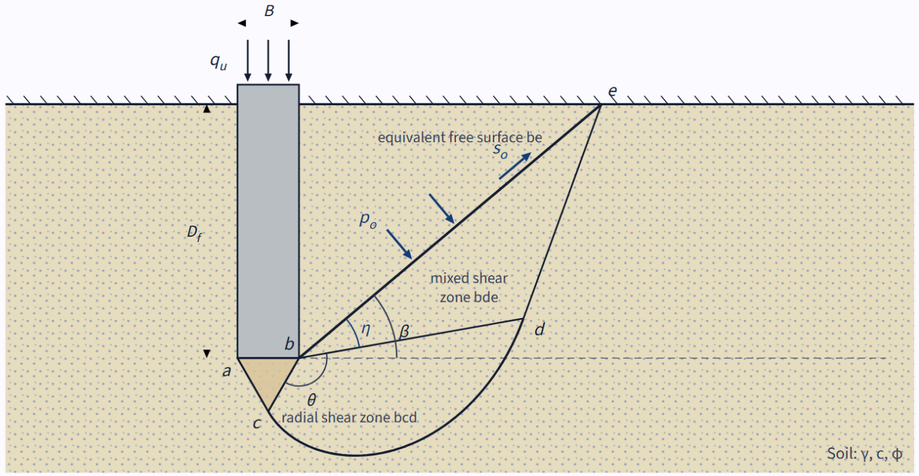
Slip line network

$N_c \cdot N_q \cdot N_\gamma$

Shape · depth · inclination

Groundwater

Worked examples



Based on the GeoTerva Theory page · geoterva.com

Introduction

What will we learn in this lecture?

- Recognising the three terms of the **general bearing capacity equation** and the shape, depth and inclination factors of each term
- To follow Meyerhof's **slip line net**, the equivalent free surface and how the N_c, N_q coefficients are derived
- How to calculate the **Reissner (N_q), Prandtl (N_c)** and Vesic (N_γ) coefficients for a surface footing
- To apply the **De Beer shape**, **Hansen depth** and **Meyerhof inclination** factors
- How to treat the undrained ($\varphi = 0$) case and the three **groundwater** cases
- To find and **check** the gross and net allowable pressure

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example foundation, the model opened in the module: a 1.2 m × 1.2 m square footing in sand, D_f = 1 m, groundwater at 0.5 m, load inclined 10° to the vertical.

Three terms, each with its own coefficient

THEORY

The module computes the **general bearing capacity equation**: the three terms of the Terzaghi equation (cohesion, surcharge, self-weight) and, for each term, a coefficient for **footing shape**, **depth**, and **load inclination**:

$$q_u = c\,N_c\,F_{cs}F_{cd}F_{ci} + q\,N_q\,F_{qs}F_{qd}F_{qi} + \tfrac{1}{2}\gamma B\,N_\gamma\,F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

N_c, N_q come from Meyerhof's (1951) theory; this page derives it **step by step**. For N_γ a trial method is described, while the module uses Vesic's closed form.

The footing base is **rough**; the soil beneath is homogeneous and very deep; strength $s = c + \sigma' \tan\varphi$. The load is central; inclined loads are handled with inclination factors.

Difference from Terzaghi: not only for shallow foundations; the soil above the base level is included in the mechanism via an **equivalent free surface**. In the equation this appears as **depth factors**: the module imposes no limit on D_f/B.

NOTATIONS

c: cohesion, φ′: angle of internal friction

$q = \gamma D_f$: surcharge at the base level

γ: soil unit weight below the base

B: short side of the footing, L: long side

$F_{.s}$ shape, $F_{.d}$ depth, $F_{.i}$ inclination

Wedge 45° + φ/2, radial zone and mixed zone

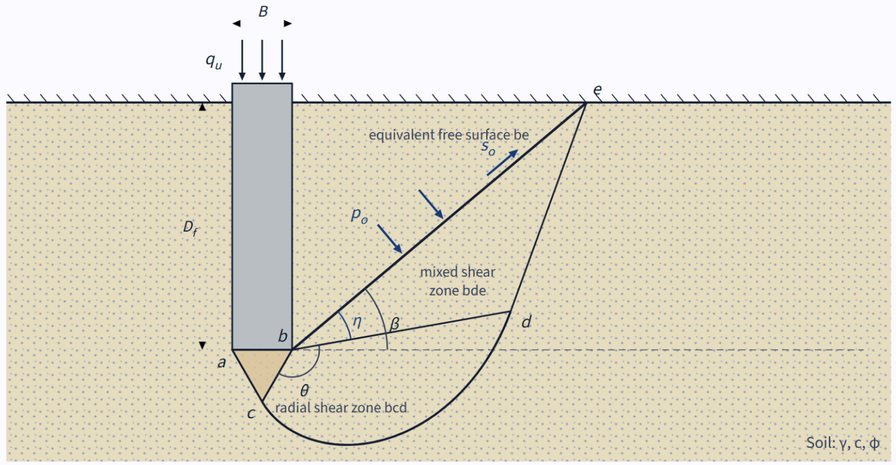
THEORY

Meyerhof (1951) adopted the failure surface shown in the Figure. Beneath the footing there is an **elastic wedge abc**, as experiments also show; its sides are inclined at **45° + φ/2** rather than φ (Terzaghi used φ). Adjacent to it is the **radial zone bcd**, bounded by the log spiral $r = r_0 e^{\theta \tan\varphi}$; beyond that, instead of Terzaghi's passive Rankine zone, there is a **mixed zone bde** in which the slip lies between radial and planar.

The soil above the bde zone is not neglected: its effect is represented by the **equivalent free surface** stresses p_o (normal) and s_o (shear) on the be plane. The plane is inclined at β to the horizontal; bd lies at η below it.

The bearing capacity is again built up by **superposition**: the contribution of c and p_o' (N_c, N_q) and the contribution of γ (N_γ):

$$q_u = c\,N_c + p_o\,N_q + \tfrac{1}{2}\gamma B\,N_\gamma$$



Partial mobilization m and Mohr circle

THEORY

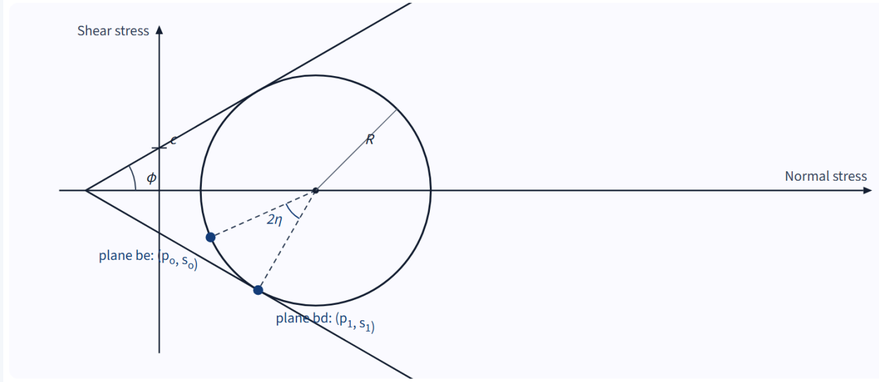
On the equivalent free surface the shear strength is mobilized only **partially**: Meyerhof wrote the shear stress as a fraction of the strength. In the linear bde zone, however, the soil is in plastic equilibrium: on the bd plane the full strength is mobilized:

$$0 \le m \le 1: \quad s_o = m\,(c + p_o \tan\varphi) \qquad s_1 = c + p_1 \tan\varphi$$

$$R = \frac{c + p_1 \tan\varphi}{\cos\varphi} \qquad \cos(2\eta + \varphi) = \frac{m\,(c + p_o \tan\varphi)\cos\varphi}{c + p_1 \tan\varphi}$$

The stress state in the region bde is **a single Mohr circle** (at the side): point bd lies on the envelope, and point be is ahead on the circle by 2η. This links the angle η with m.

Two limits: **m = 0** ⇒ cos(2η + φ) = 0, **η = 45° − φ/2**; **m = 1** ⇒ cos(2η + φ) = cosφ, **η = 0**.



How are N_c and N_q derived?

THEORY

On face bc the radial zone is also in failure: $s'_p = c + p'_p \tan\varphi$. From moments about pole b (only the normal stresses on the spiral and cohesion remain):

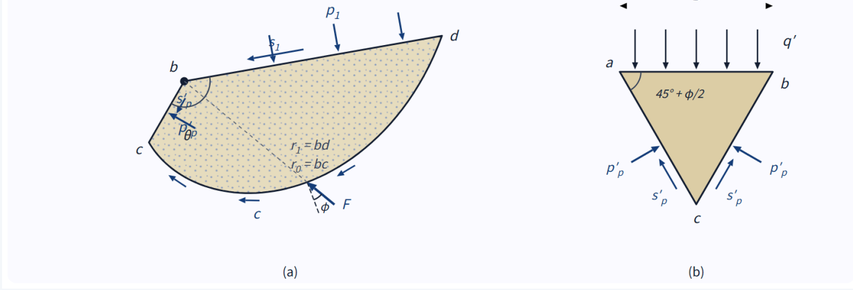
$$p'_p = p_1\, e^{2\theta \tan\varphi} + c \cot\varphi \left(e^{2\theta \tan\varphi} - 1\right)$$
$$s'_p = \left(c + p_1 \tan\varphi\right) e^{2\theta \tan\varphi}$$

Then the vertical equilibrium of wedge abc (each face of length B/2 / cos(45° + φ/2)):

$$q' = p'_p + s'_p \cot\left(45^\circ - \frac{\varphi}{2}\right)$$

Combining everything separates the c and p_o terms:

$$q' = c\, N_c + p_o\, N_q$$
$$N_q = \frac{\left(1 + \sin\varphi\right) e^{2\theta \tan\varphi}}{1 - \sin\varphi\, \sin(2\eta + \varphi)} \qquad N_c = \left(N_q - 1\right) \cot\varphi$$



How do the β and m coefficients increase it?

THEORY

The coefficients depend on φ′, on η through m, and through the inclination β of the equivalent free surface on the **angle θ** swept by the spiral (Figure 1 geometry):

$$\theta = 135^\circ + \beta - \eta - \frac{\varphi}{2}$$

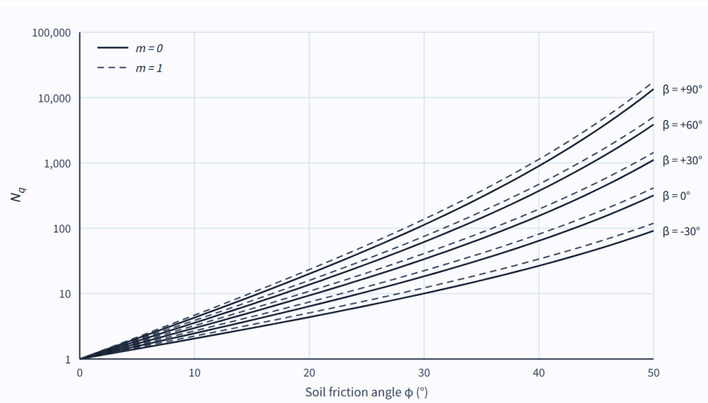
$$m = 0:\ \theta = 90^\circ + \beta \qquad m = 1:\ \theta = 135^\circ + \beta - \tfrac{\varphi}{2}$$

The equivalent free surface of a deep foundation is steeper, β is larger; the coefficients increase rapidly with it (at the side).

Surface footing (the condition in Terzaghi's derivation): β = 0, m = 0 ⇒ θ = π/2 and sin(2η + φ) = 1:

$$N_q = e^{\pi \tan\varphi} \tan^2\left(45^\circ + \tfrac{\varphi}{2}\right) \qquad N_c = (N_q - 1)\cot\varphi$$

These are **Reissner's (1924) N_q** and **Prandtl's (1921) N_c**; the module uses these. With $p_o = \gamma D_f = q$, the c and q terms become $cN_c + qN_q$; the soil above the base enters the calculation through **depth factors** instead of β and m. As φ → 0, $N_c \to \pi + 2 = 5.14$.



Problem statement

EXAMPLE

Given

Soil	$\varphi' = 30^\circ,\ c = 0$
Cases	**(a) surface footing: β = 0, m = 0 · (b) β = 15°, m = 0 · (c) β = 15°, m = 1**
Formulas	$\theta = 135^\circ + \beta - \eta - \varphi/2,\quad N_q = \frac{(1+\sin\varphi)e^{2\theta\tan\varphi}}{1-\sin\varphi\sin(2\eta+\varphi)}$

REQUIRED

For each case, find the values of **η, θ, N_q** and N_c. Does N_q at the ground surface agree with Reissner's closed form?

Solution (1/2)

SOLUTION

1. (a) Surface footing: m = 0 ⇒ η = 45° − φ/2 = 30°; θ = 135° + 0 − 30° − 15° = 90° (same as Reissner's $e^{\pi \tan\varphi}\tan^2(45° + \varphi/2)$ value)

$$N_q = \frac{1.5\, e^{2(\pi/2)(0.5774)}}{1 - 0.5 \sin 90^\circ} = \frac{1.5(6.134)}{0.5} = \mathbf{18.40}$$

2. (b) β = 15°, m = 0: η = 30°; θ = 135° + 15° − 30° − 15° = 105°; sin(2η + φ) = 1

$$N_q = \frac{1.5\, e^{2(1.8326)(0.5774)}}{0.5} = \mathbf{24.90} \qquad N_c = (24.90 - 1)\cot 30^\circ = \mathbf{41.39}$$

3. (c) β = 15°, m = 1: η = 0; θ = 135° + 15° − 0 − 15° = 135°; sin(2η + φ) = sin30° = 0.5

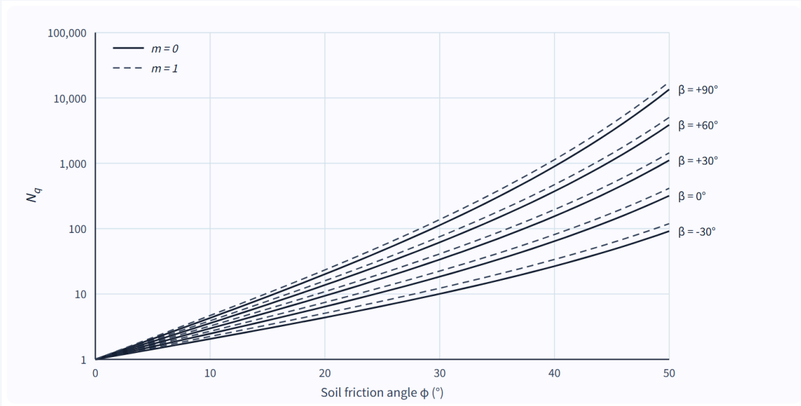
$$N_q = \frac{1.5\, e^{2(2.3562)(0.5774)}}{1 - 0.5(0.5)} = \frac{1.5(15.19)}{0.75} = \mathbf{30.38} \qquad N_c = \mathbf{50.89}$$

Solution (2/2)

SOLUTION

Condition	η (°)	θ (°)	N_q	N_c
(a) β = 0, m = 0 (Reissner/Prandtl)	30	90	18.40	30.14
(b) β = 15°, m = 0	30	105	24.90	41.39
(c) β = 15°, m = 1	0	135	30.38	50.89

The surface footing is **the same** as the closed form (18.40). As β increases and mobilization increases, the coefficients **increase rapidly**: (a) → (c) N_q by a factor of 1.65. The module accounts for this increase not through the mechanism but through **depth coefficients** (later sections).



Check the solution in GeoTerva ↗

N_γ: trial method, Meyerhof 1963 and Vesic

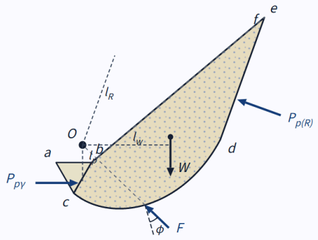
THEORY

As in Terzaghi, there is no closed form for N_γ; it is found by **trial**: φ, β and m are chosen; θ is found; a log spiral is drawn from the trial centre O; for the body bcdf the weight W and the Rankine passive force $P_{p(R)}$ on df; from the moments about O, the passive force on bc:

$$P_{p\gamma} = \frac{W\, l_w + P_{p(R)}\, l_R}{l_p}$$

By varying O the smallest $P_{p\gamma}$ is found; together with the wedge abc's own weight:

$$q'' = \frac{\gamma B}{2}\left[\frac{4P_{p\gamma}\sin(45^\circ+\varphi/2)}{\gamma B^2} - \tfrac{1}{2}\tan\left(45^\circ+\tfrac{\varphi}{2}\right)\right] = \tfrac{1}{2}\gamma B\, N_\gamma$$



$$N_\gamma = (N_q - 1)\tan(1.4\varphi) \text{ (Meyerhof 1963)}$$

$$N_\gamma = 2(N_q + 1)\tan\varphi \text{ (Vesic 1973, } \textbf{here}\text{)}$$

N_c, N_q, N_γ depend only on φ′

THEORY

The two sources are **the same** for N_c and N_q; they differ for N_γ: N_γ is the coefficient most dependent on the assumed wedge shape. At φ = 30°, Vesic gives 22.40, Meyerhof (1963) gives 15.67. With Meyerhof's N_γ, the self-weight term, and hence q_u of the frictional soil, would be lower.

$$N_q = e^{\pi \tan\varphi} \tan^2\left(45^\circ + \tfrac{\varphi}{2}\right) \qquad N_c = (N_q - 1)\cot\varphi \qquad N_\gamma = 2(N_q + 1)\tan\varphi$$

COEFFICIENTS

φ (°)	N_c	N_q	N_γ	N_γ (M63)
0	5.14	1.00	0.00	0.00
10	8.34	2.47	1.22	0.37
20	14.83	6.40	5.39	2.87
26	22.25	11.85	12.54	8.00
30	30.14	18.40	22.40	15.67
32	35.49	23.18	30.21	22.02
35	46.12	33.30	48.03	37.15
40	75.31	64.20	109.41	93.69

Problem statement

EXAMPLE

Given

Soil	$\varphi' = 32^\circ$ **(tan 32° = 0.6249)**
Method	**Reissner, Prandtl, Vesic; for comparison, Meyerhof (1963)**

REQUIRED

Find the **N_q, N_c, N_γ** values using closed-form expressions; compare Vesic's N_γ with Meyerhof (1963).

Solution

SOLUTION

1 Reissner N_q

$$N_q = e^{\pi \tan 32^\circ} \tan^2(61^\circ) = e^{1.9631}(3.2546) = \mathbf{23.18}$$

2 Prandtl N_c

$$N_c = (N_q - 1)\cot\varphi = \frac{23.18 - 1}{0.6249} = \mathbf{35.49}$$

3 Vesic N_γ (used by the module) and Meyerhof (1963)

$$N_\gamma = 2(N_q + 1)\tan\varphi = 2(23.18 + 1)(0.6249) = \mathbf{30.21}$$

$$N_\gamma^{M63} = (N_q - 1)\tan(1.4\varphi) = (23.18 - 1)\tan 44.8^\circ = 22.02$$

Vesic's N_γ is **37%** larger than Meyerhof's (φ′ = 32°). Consistent with the table values: N_c = 35.49, N_q = 23.18, N_γ = 30.21.

Check the solution in GeoTerva ↗

De Beer: non-strip footing

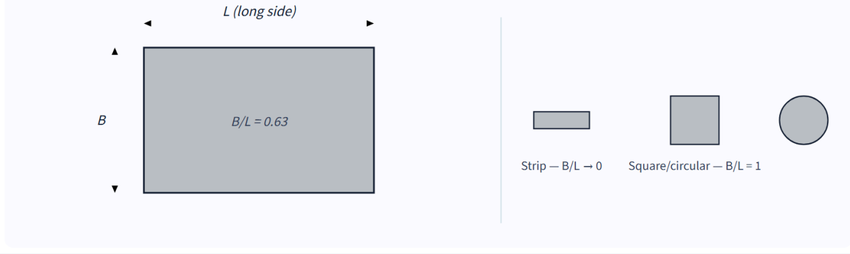
THEORY

The derivation is for a continuous (strip) footing (plane strain). For a rectangular footing the failure is three-dimensional: the soil can also move from the ends of the wedge and from the sides. **De Beer** (1970) corrects each term with the ratio of the short side to the long side, B/L:

$$F_{cs} = 1 + \frac{B}{L}\frac{N_q}{N_c} \qquad F_{qs} = 1 + \frac{B}{L}\tan\varphi \qquad F_{\gamma s} = 1 - 0.4\,\frac{B}{L}$$

For a strip footing (B/L → 0) all three coefficients equal 1; the equation reduces to the plane strain form.

For a square footing B/L = 1: the N_γ term decreases by 40% ($F_{\gamma S}$ = 0.6), while the q and c terms increase.



Problem statement

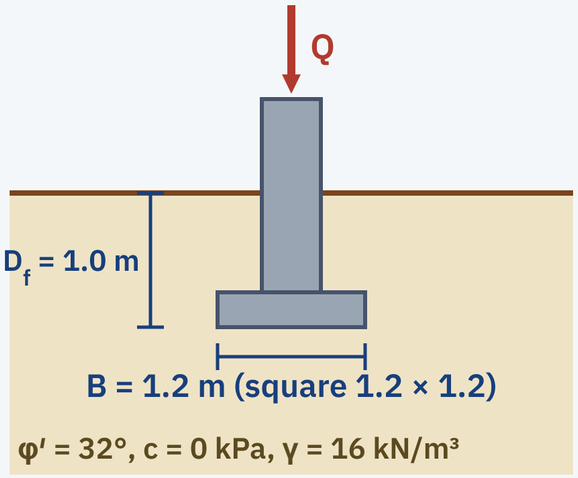
EXAMPLE

Given

Soil	$\varphi' = 32°$: $N_c = 35.49,\ N_q = 23.18$ **(Example 2)**
Foundation	**square B = L = 1.2 m; comparison: B/L = 0.5 and strip**

REQUIRED

Find the F_{cs}, F_{qs}, $F_{\gamma s}$ values for a square footing; also find the same factors for B/L = 0.5.



Solution

SOLUTION

1 Square footing (B/L = 1)

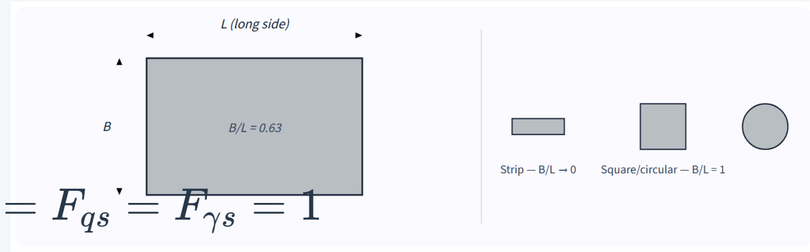
$$F_{cs} = 1 + (1)\frac{23.18}{35.49} = \mathbf{1.653}$$

$$F_{qs} = 1 + (1)\tan 32^\circ = \mathbf{1.625}$$

$$F_{\gamma s} = 1 - 0.4(1) = \mathbf{0.600}$$

2 B/L = 0.5 and strip

$$B/L = 0.5: \; F_{cs} = 1.327,\; F_{qs} = 1.312,\; F_{\gamma s} = 0.800 \qquad \text{strip}: \; F_{cs} = F_{qs} = F_{\gamma s} = 1$$



For a square footing the q term increases by **62%**, the self-weight term decreases by 40%. As B/L decreases, the coefficients tend to the strip value (1).

Check the solution in GeoTerva ↗

Hansen: contribution of the soil above the base

THEORY

Terzaghi considers only the soil above the base as **dead load**. In Meyerhof's theory, however, it **also offers resistance** through an equivalent free surface; in the general equation this is accounted for by the **Hansen (1970) depth factors**, which increase with D_f/B:

$$D_f/B \le 1: \; F_{qd} = 1 + 2\tan\varphi(1-\sin\varphi)^2\frac{D_f}{B} \qquad D_f/B > 1: \; F_{qd} = 1 + 2\tan\varphi(1-\sin\varphi)^2\tan^{-1}\frac{D_f}{B}$$

$$F_{cd} = F_{qd} - \frac{1-F_{qd}}{N_c\tan\varphi}\;(\varphi>0) \qquad F_{cd} = 1 + 0.4\,k\;(\varphi=0) \qquad F_{\gamma d} = 1$$

k = D_f/B (≤ 1) or $\tan^{-1}(D_f/B)$ (> 1, radians). Beyond $D_f/B = 1$ the arctangent continues to increase the coefficients slowly: **unlike Terzaghi's, this equation requires no depth limit**.

Problem statement

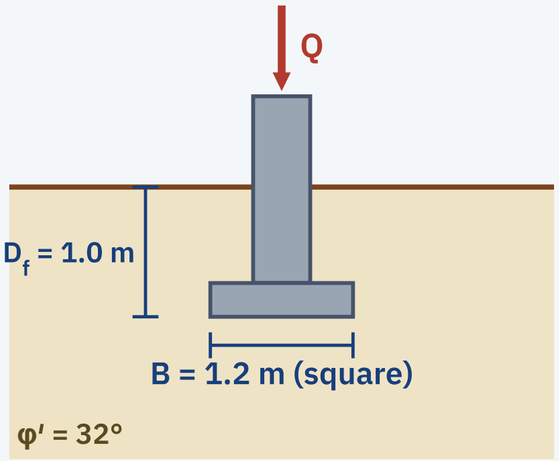
EXAMPLE

Given

Soil	$\varphi' = 32^\circ$: $N_c = 35.49$, $\tan\varphi' = 0.6249$, $(1 - \sin 32^\circ)^2 = 0.2210$
Foundation	**B = 1.2 m; D_f = 1.0 m (example) and comparison D_f = 2.4 m**
Undrained	**φ = 0: B = 2 m, D_f = 1.5 m**

REQUIRED

Find the values of **F_{qd}, F_{cd}** and $F_{\gamma d}$ for each case.



Solution

SOLUTION

1 $D_f/B = 1.0/1.2 = 0.833 \le 1$ (linear form)

$$F_{qd} = 1 + 2(0.6249)(0.2210)(0.833) = \mathbf{1.230}$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\varphi} = 1.230 + \frac{0.230}{(35.49)(0.6249)} = \mathbf{1.241}$$

2 $D_f/B = 2.4/1.2 = 2 > 1$ (arctangent: $\tan^{-1} 2 = 1.1071$ rad)

$$F_{qd} = 1 + 2(0.6249)(0.2210)(1.1071) = \mathbf{1.306} \qquad F_{cd} = 1.306 + \frac{0.306}{22.18} = \mathbf{1.320}$$

3 Undrained ($\varphi = 0$): $F_{qd} = 1$; $D_f/B = 1.5/2 = 0.75$

$$F_{cd} = 1 + 0.4(0.75) = \mathbf{1.300} \qquad F_{qd} = 1$$

D_f/B 0.83 → 2: F_{qd} 1.230 → 1.306; the depth contribution increases but **progressively slows**.

Check the solution in GeoTerva ↗

Meyerhof: inclined load

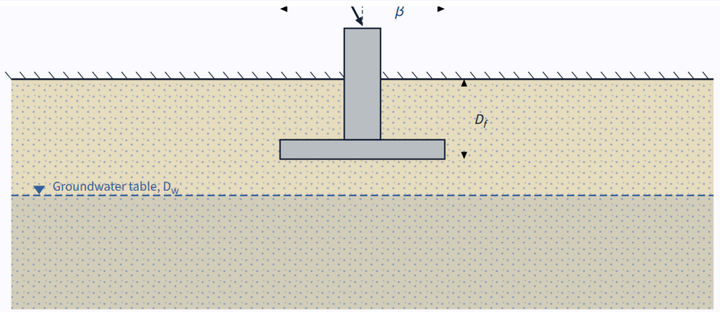
THEORY

A load inclined at angle β to the vertical mobilizes less vertical resistance than a vertical load: part of the load pushes the footing sideways. The equation gives the bearing capacity only for the **vertical component** of the load. The **Meyerhof** (1963) inclination factors reduce every term:

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2 \qquad F_{\gamma i} = \left(1 - \frac{\beta}{\varphi}\right)^2 \ (\beta < \varphi) \ ; \ F_{\gamma i} = 0 \ (\beta \geq \varphi)$$

At β = 0 all three equal 1. $F_{\gamma i}$ reaches zero at β = φ and **stays** there: the module does not allow $(1 - \beta/\varphi)^2$ to grow again. β is accepted **up to 89°**, below the point at which $(1 - \beta/90°)^2$ would start to grow again.

Inclined load reduces the bearing capacity, particularly through the self-weight term.



The equation returns the ultimate capacity for the VERTICAL component of an inclined load only.

Problem statement

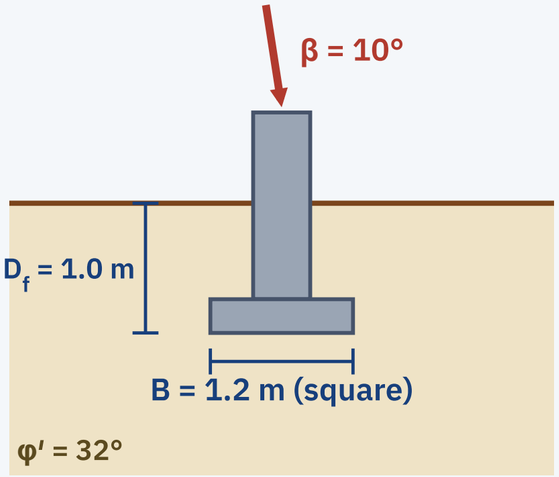
EXAMPLE

Given

Foundation	**square, B = L = 1.2 m, base depth D_f = 1.0 m**
Soil	**sand:** $\varphi' = 32^\circ,\ c = 0$
Unit weight	$\gamma = 16\ \mathrm{kN/m^3}$ **(above),** $\gamma_{sat} = 19.5\ \mathrm{kN/m^3}$ **(below the GWT)**
Load	**β = 10° from vertical (supplement: 20°, 30°, 32°)**

REQUIRED

Find the values of F_{ci}, F_{qi}, $F_{\gamma i}$ for β = 10°; what happens as β increases?



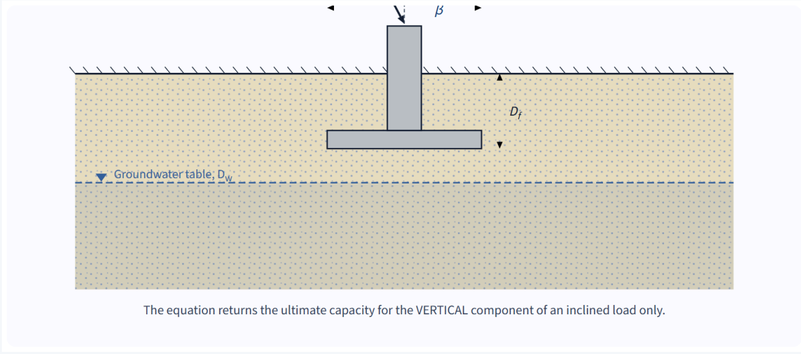
Solution

SOLUTION

1 β = 10°, φ′ = 32° (β < φ)

$$F_{ci} = F_{qi} = \left(1 - \frac{10}{90}\right)^2 = \mathbf{0.790}\ ; \quad F_{\gamma i} = \left(1 - \frac{10}{32}\right)^2 = \mathbf{0.473}$$

β (°)	$F_{ci} = F_{qi}$	$F_{\gamma i}$
0	1.000	1.000
10	0.790	0.473
20	0.605	0.141
30	0.444	0.004



The equation returns the ultimate capacity for the VERTICAL component of an inclined load only.

A 10° load reduces the q and c terms by **21%** and the self-weight term by **53%**; at β = φ′ = 32° the self-weight term vanishes entirely.

Check the solution in GeoTerva ↗

General equation and undrained case

THEORY

The general equation is completed with the coefficients in §5–10. Under vertical load, on a **strip footing** at the base level all correction coefficients are 1 and the equation takes the form of Terzaghi's: only, instead of Terzaghi's coefficients, Reissner N_q, Prandtl N_c and Vesic N_γ appear.

Undrained loading ($\varphi = 0$, $c = c_u$): $N_c = 5.14$, $N_q = 1$, $N_\gamma = 0$:

$$q_u = 5.14\, c_u\, F_{cs} F_{cd} F_{ci} + q\, F_{qs} F_{qd} F_{qi}$$

> Groundwater enters the equation as in Terzaghi (§12): through q and the unit weight of the third term, **never through the coefficients**.

Problem statement

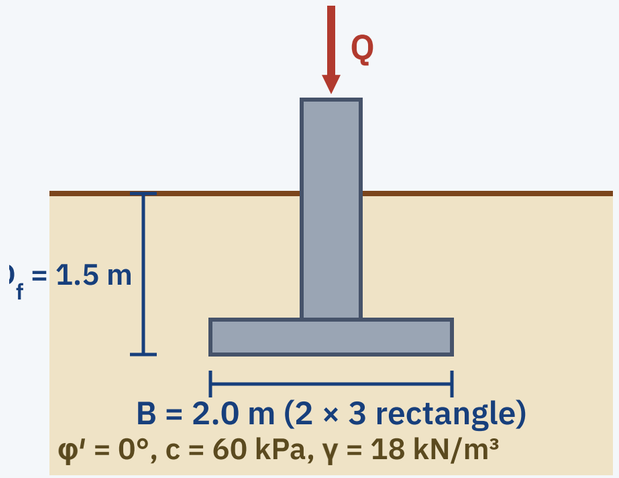
EXAMPLE

Given

Foundation	**rectangular 2 m × 3 m, D_f = 1.5 m, central vertical load**
Soil	**clay,** $\varphi = 0,\ c_u = 60\ \mathrm{kPa},\ \gamma = 18\ \mathrm{kN/m^3}$ **; no groundwater**
Safety	**FS = 3**

REQUIRED

Find q_u, gross q_{all} and net bearing capacity.



Solution

SOLUTION

1 Surcharge and coefficients (φ = 0: $N_c = 5.14$, $N_q = 1$, $N_\gamma = 0$)

$$q = \gamma D_f = 18(1.5) = 27\ \text{kPa} \qquad F_{cs} = 1 + \frac{2}{3}\cdot\frac{1}{5.14} = 1.130 \qquad F_{cd} = 1 + 0.4\frac{1.5}{2} = 1.300$$

2 Bearing capacity ($F_{ci} = F_{qi} = 1$; $F_{qs} = F_{qd} = 1$)

$$q_u = 5.14(60)(1.130)(1.300) + 27(1) = 452.9 + 27 = \mathbf{479.9}\ \text{kPa}$$

3 Gross and net allowable pressure

$$q_{all,g} = \frac{479.9}{3} = \mathbf{160.0}\ \text{kPa} \qquad q_{all,n} = \frac{q_u - q}{3} = \frac{452.9}{3} = 151.0\ \text{kPa}$$

In undrained clay the bearing capacity comes almost entirely from the **cohesion term** (453 / 480 kPa): the shape (1.13) and depth (1.30) coefficients increase it by about 47%; since $N_\gamma = 0$ at φ = 0, there is no self-weight term.

Check the solution in GeoTerva ↗

Coefficients unchanged; q and γ change

THEORY

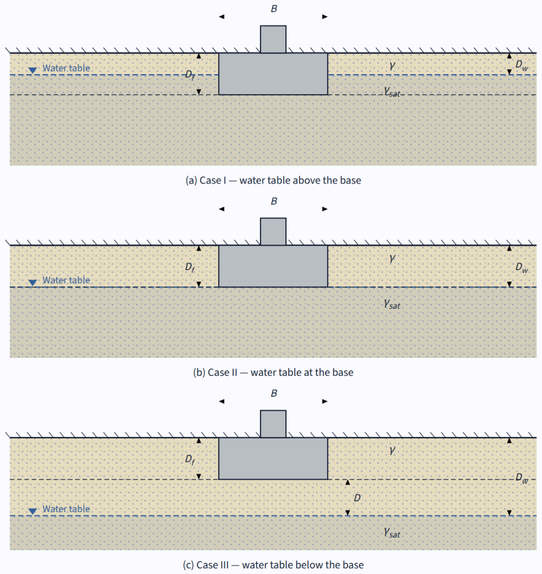
Below the GWT the effective unit weight $\gamma' = \gamma_{sat} - \gamma_w$ ($\gamma_w = 9.81$) acts. The module distinguishes three cases; D_W is the depth of the groundwater level from the surface:

$$\text{I } (D_w \le D_f):\ q = \gamma D_w + \gamma'(D_f - D_w),\ \ \gamma \to \gamma' \text{ (third term)}$$

$$\text{III } (D_f < D_w \le D_f + B):\ q = \gamma D_f,\ \ \bar{\gamma} = \frac{1}{B}\left[\gamma D + \gamma'(B - D)\right],\ D = D_w - D_f$$

Case II ($D_W = D_f$) is a special case of I: $q = \gamma D_f$, with γ' in the third term.

Water **has no effect** deeper than one B below the base.



Problem statement

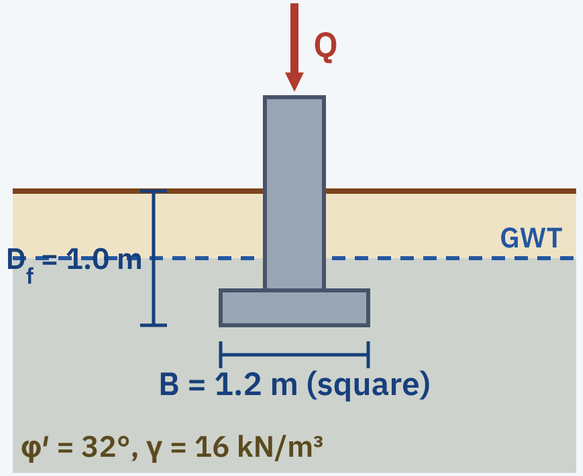
EXAMPLE

Given

Foundation	**square, B = L = 1.2 m, base depth D_f = 1.0 m**
Soil	**sand:** $\varphi' = 32^\circ,\ c = 0$
Unit weight	$\gamma = 16\ \mathrm{kN/m^3}$ **(above),** $\gamma_{sat} = 19.5\ \mathrm{kN/m^3}$ **(below the GWT)**
Load	**vertical, central; no ε**

REQUIRED

Find **q**, the unit weight in the third term, and $\mathbf{q_u}$ values for GWT D_W = 0.5; 1.0; 1.5 and 2.6 m. (F_{qs} = 1.625; F_{qd} = 1.230; $F_{\gamma s}$ = 0.6; N_q = 23.18; N_γ = 30.21; no inclination)



Solution (1/2)

SOLUTION

1 Case I (D_w = 0.5 < D_f) and D_w = D_f

$$D_w = 0.5: \;\; q = 16(0.5) + 9.69(0.5) = 12.845\ \text{kPa},\ \gamma' = 9.69$$

$$D_w = 1.0: \;\; q = 16(1.0) = 16.0\ \text{kPa},\ \gamma' = 9.69$$

2 Case III (D_w = 1.5 ⇒ D = 0.5 m ≤ B) and water has no effect (D_w = 2.6 ⇒ D = 1.6 > B)

$$\bar{\gamma} = \frac{1}{1.2}\left[16(0.5) + 9.69(0.7)\right] = 12.319\ \text{kN/m}^3\ ; \;\; q = 16\ \text{kPa}$$

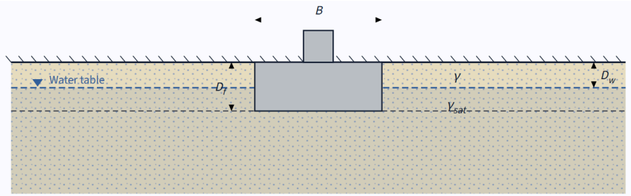
$$D_w = 2.6: \;\; q = 16,\ \gamma = 16$$

Solution (2/2)

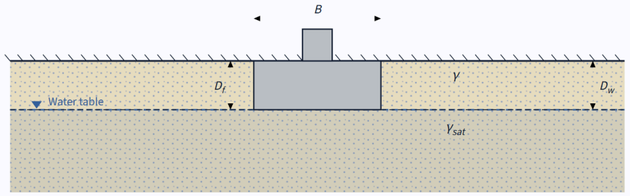
SOLUTION

D_w (m)	Condition	q (kPa)	γ third term	q_u (kPa)
0.5	I	12.84	9.69	**520.0**
1.0	I	16.00	9.69	**635.5**
1.5	III	16.00	12.32	**649.0**
2.6	negligible	16.00	16.00	**667.9**

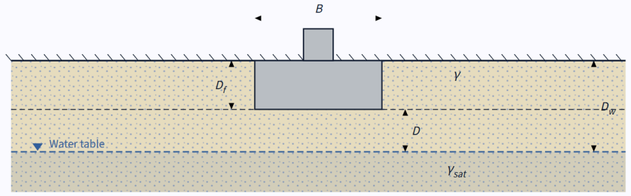
Groundwater reduces bearing capacity in two ways: the surcharge q decreases and the self-weight term is calculated with γ′. With water at 0.5 m, q_u = 520; with water deep, 668 kPa (22% difference). The coefficients never change: **water enters only through q and γ**.



(a) Case I — water table above the base



(b) Case II — water table at the base



(c) Case III — water table below the base

Check the solution in GeoTerva ↗

Problem statement

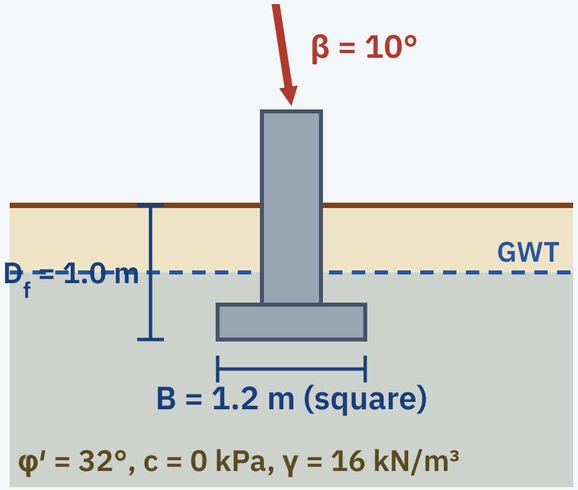
EXAMPLE

Given

Foundation	**square, B = L = 1.2 m, base depth D_f = 1.0 m**
Soil	**sand:** $\varphi' = 32^\circ,\ c = 0$
Unit weight	$\gamma = 16\ \mathrm{kN/m^3}$ **(above),** $\gamma_{sat} = 19.5\ \mathrm{kN/m^3}$ **(below the GWT)**
Groundwater	**D_w = 0.5 m (case I)**
Load	**inclined at β = 10° to the vertical**
Safety	**FS = 3**

REQUIRED

q_u, gross **q_{all}**, allowable vertical load and **allowable resultant** load; then q_u for c = 5 kPa.



Solution (1/2)

SOLUTION

1 Coefficients (φ′ = 32°; B/L = 1; D_f/B = 0.833; β = 10° < φ′)

$$N_c = 35.49,\ N_q = 23.18,\ N_\gamma = 30.21$$

$$F_{qs} = 1.625,\ F_{\gamma s} = 0.600,\ F_{qd} = 1.230,\ F_{qi} = 0.790,\ F_{\gamma i} = 0.473$$

2 Case I: q = 16(0.5) + (19.5 – 9.81)(0.5); γ′ = 9.69 kN/m³ (in the third term)

$$q = \mathbf{12.845}\ \mathrm{kPa} \qquad \gamma' = \mathbf{9.69}\ \mathrm{kN/m^3}$$

Solution (2/2)

SOLUTION

3 Bearing capacity (c = 0: no cohesion term)

$$q_u = (12.845)(23.18)(1.625)(1.230)(0.790) + \tfrac{1}{2}(9.69)(1.2)(30.21)(0.600)(1)(0.473) = 470.17 + 49.82 = \mathbf{519.99}\ \text{kPa}$$

4 Allowable pressure and loads (A = 1.2² = 1.44 m²)

$$q_{all,g} = \frac{519.99}{3} = 173.33\ \text{kPa}$$

$$Q_{all} = 173.33(1.44) = \mathbf{249.59}\ \text{kN} \qquad \frac{Q_{all}}{\cos 10^\circ} = \mathbf{253.44}\ \text{kN}$$

5 c = 5 kPa: cohesion term (F_{cd} = 1.241, F_{cs} = 1.653, F_{ci} = 0.790)

$$c\,N_c F_{cs} F_{cd} F_{ci} = 5(35.49)(1.653)(1.241)(0.790) = 287.51 \ \Rightarrow\ q_u = \mathbf{807.50}\ \text{kPa}$$

Check the solution in GeoTerva ↗

Gross and net, vertical and resultant

THEORY

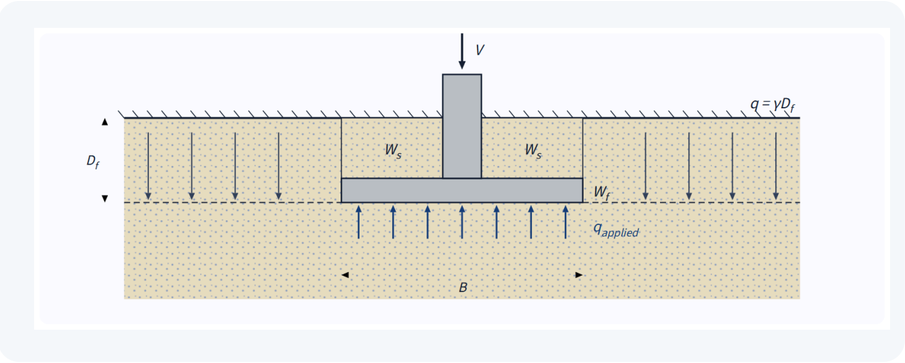
q_u is the vertical pressure at the moment of failure. The module treats **gross** and **net** pressure separately:

$$q_{u,net} = q_u - q \qquad q_{all,g} = \frac{q_u}{FS} \qquad q_{all,n} = \frac{q_{u,net}}{FS}$$

The applied pressure q_{app} is the **vertical gross pressure** at the base under service loads, V/A; under eccentric load it is obtained by dividing the vertical component, not the resultant, by the area. The footing is acceptable under the following conditions:

$$q_{apply} \le q_{all,g} \qquad FS_{actual} = \frac{q_u}{q_{apply}} \qquad q_{apply} - q \le q_{all,n}$$

Allowable vertical load $Q_{all} = q_{all,g}\,A$; the allowable resultant at inclination β is $Q_{all}/\cos\beta$. If q_{app} is not entered, the module gives only the capacities.



Problem statement

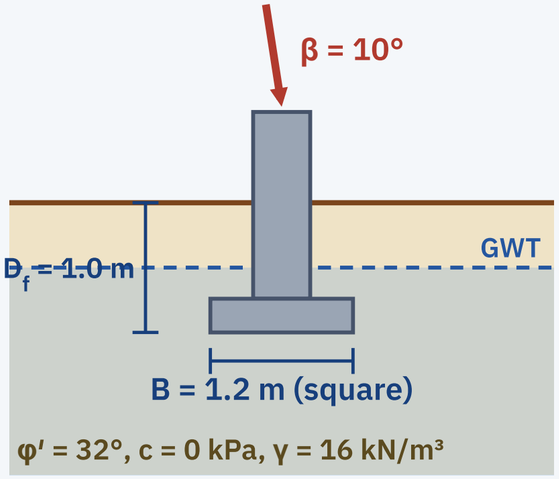
EXAMPLE

Given

Foundation	**square, B = L = 1.2 m, base depth D_f = 1.0 m**
Soil	**sand:** $\varphi' = 32^\circ,\ c = 0$
Unit weight	$\gamma = 16\ \mathrm{kN/m^3}$ **(above),** $\gamma_{sat} = 19.5\ \mathrm{kN/m^3}$ **(below the GWT)**
Groundwater, slope	**D_w = 0.5 m, β = 10° (Example 8)**
Applied	**$q_{applied}$ = 150 kPa (vertical, gross)**
Result (Example 8)	**q_u = 519.99 kPa, q = 12.845 kPa, FS = 3**

REQUIRED

Check the three conditions (gross, FS_{actual}, net).



Solution

SOLUTION

1 Gross allowable pressure

$$q_{apply} = 150\ \text{kPa} \le q_{all,g} = 173.33\ \text{kPa} \ \Rightarrow\ \text{valid}$$

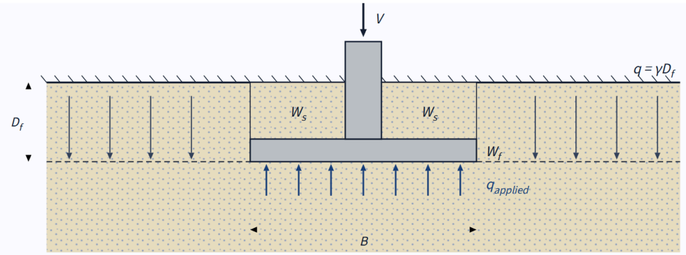
2 Actual factor of safety

$$FS_{actual} = \frac{q_u}{q_{apply}} = \frac{519.99}{150} = \mathbf{3.47} \ \ge 3$$

3 Net

$$q_{apply} - q = 150 - 12.845 = 137.2\ \text{kPa}$$

$$q_{all,n} = \frac{519.99 - 12.845}{3} = 169.0\ \text{kPa} \ \Rightarrow\ \text{valid}$$



The footing is acceptable under all three conditions. Vertical load V = 150 × 1.44 = 216 kN; resultant 216/cos10° = 219.3 kN ≤ allowable resultant 253.4 kN.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Foundation	**square, B = L = 1.2 m, base depth D_f = 1.0 m**
Soil	**sand:** $\varphi' = 32^\circ,\ c = 0$
Unit weight	$\gamma = 16\ \mathrm{kN/m^3}$ **(above),** $\gamma_{sat} = 19.5\ \mathrm{kN/m^3}$ **(below the GWT)**
Groundwater, slope	**D_w = 0.5 m, β = 10°**
Comparison	**Vesic N_γ = 30.21 and Meyerhof (1963) N_γ = 22.02**

REQUIRED

What would q_u and gross q_{all} in Example 8 become with Meyerhof's N_γ?

Solution

SOLUTION

1 Only the third term changes

$$\tfrac{1}{2}(9.69)(1.2)(22.02)(0.600)(0.473) = \mathbf{36.31}\ \text{kPa} \quad (\text{Vesic: } 49.82)$$

2 q_u and q_{all}

$$q_u = 470.17 + 36.31 = \mathbf{506.48}\ \text{kPa} \qquad q_{all,g} = 168.83\ \text{kPa}$$

For this footing the difference is only **2.6%** (520 → 506 kPa): the self-weight term is about 10% of q_u. **For a wide footing** and low surcharge, the self-weight term becomes dominant and the choice of N_γ changes the result markedly.

Check the solution in GeoTerva ↗

Summary

Key points

General equation: $q_u = cN_cF_c. + qN_qF_q. + ½\gamma BN_\gamma F_\gamma.$; with shape (s), depth (d) and inclination (i) factors

Meyerhof theory: wedge at 45° + φ/2, radial zone, mixed zone; represented by an equivalent free surface above the base (p_o, s_o, m, η, β)

Shallow foundation: $N_q = e^{\pi \tan\varphi}\tan^2(45° + \varphi/2)$ (Reissner), $N_c = (N_q - 1)\cot\varphi$ (Prandtl); $N_\gamma = 2(N_q + 1)\tan\varphi$ (Vesic)

De Beer shape, **Hansen** depth, **Meyerhof** inclination factors; at φ = 0, $N_c = 5.14$, $N_q = 1$, $N_\gamma = 0$

Groundwater changes only q and γ (3 cases), not the coefficients

Check: $q_{applied} \leq q_{all,g}$, FS_{actual} and net; under inclined load the safe resultant $Q_{all}/\cos\beta$

References

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Meyerhof, G.G. (1963). Some recent research on the bearing capacity of foundations. Canadian Geotechnical Journal, 1(1), 16–26.

Das, B.M. (2017). Shallow Foundations: Bearing Capacity and Settlement, 3rd ed. CRC Press — Sections 2.4–2.5; this page follows the order of its derivation; “the source” in the text of this page.

Das, B.M. (2016). Principles of Foundation Engineering, 8th ed. Cengage — the general bearing-capacity equation and its factors; “the second source” in the text of this page.

Reissner, H. (1924). Zum Erddruckproblem. Proc. 1st Int. Congress of Applied Mechanics, Delft, 295–311.

Prandtl, L. (1921). Über die Eindringungsfestigkeit (Härte) plastischer Baustoffe und die Festigkeit von Schneiden. ZAMM, 1(1), 15–20.

Vesic, A.S. (1973). Analysis of ultimate loads of shallow foundations. JSMFD, ASCE, 99(SM1), 45–73.

De Beer, E.E. (1970). Experimental determination of the shape factors and the bearing capacity factors of sand. Géotechnique, 20(4), 387–411.

Hansen, J.B. (1970). A revised and extended formula for bearing capacity. Danish Geotechnical Institute Bulletin 28.

The figures are drawn from the mechanism itself; the chart of Nq is computed from the equations of this page.

NOTE

The figures in this presentation are taken from the GeoTerva Theory page; all numerical examples were produced with the module's own calculation library, and the module gives the same values.

GeoTerva

Online geotechnical analysis

From theory to practice

Compute the example foundation in this lecture step by step in GeoTerva: the module opens with this model.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com