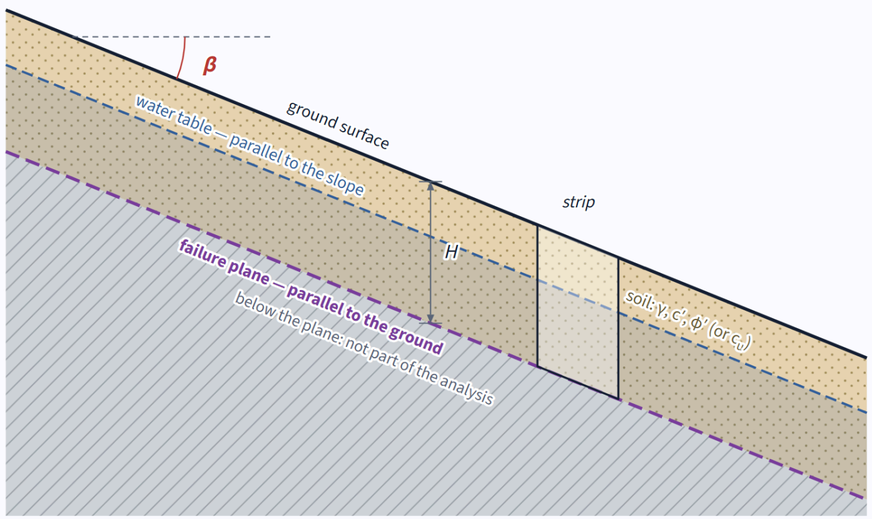
Geotechnical lecture presentation · Slope stability

Infinite slope stability

Lecture: infinite slope model (slip surface parallel to the surface); stresses, factor of safety, undrained clay, water, yield coefficient under earthquake, critical depth and required angle; a worked example on each topic.

Infinite slope | Shear strength and FS | Water · earthquake | Critical depth · required angle | Solvable by hand



What will we learn in this lecture?

- Why the **infinite slope** model is used for shallow slides: a thin soil layer sliding on a plane parallel to the surface
- The stresses on a plane from a single **strip** ($\sigma = \sigma_V \cos^2\beta$, $\tau = \sigma_V \sin\beta \cos\beta$) and the relation **$\tau/\sigma = \tan\beta$**
- **Mohr–Coulomb** factor of safety: $FS = c'/(\gamma H \cos^2\beta \tan\beta) + \tan\varphi'/\tan\beta$; critical depth H_{cr}, required depth H_{req}, required angle β_{req}
- **Undrained clay** (c_u, $\varphi = 0$) and the effect of **water**: groundwater table (seepage parallel to the slope), pore pressure ratio r_u
- The factor of safety under **pseudo-static earthquake** loading and the **yield coefficient k_y** (the horizontal acceleration at which the mass begins to slide)

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example slope: $\beta = 15°$, slip plane vertical at 6 m depth; soil $\gamma = 17.8$ kN/m³ (saturated 19.5), $c' = 10$ kPa, $\varphi' = 20°$; initially no pore pressure. The stresses are per unit area of the plane parallel to the surface.

A slice from the mass

THEORY

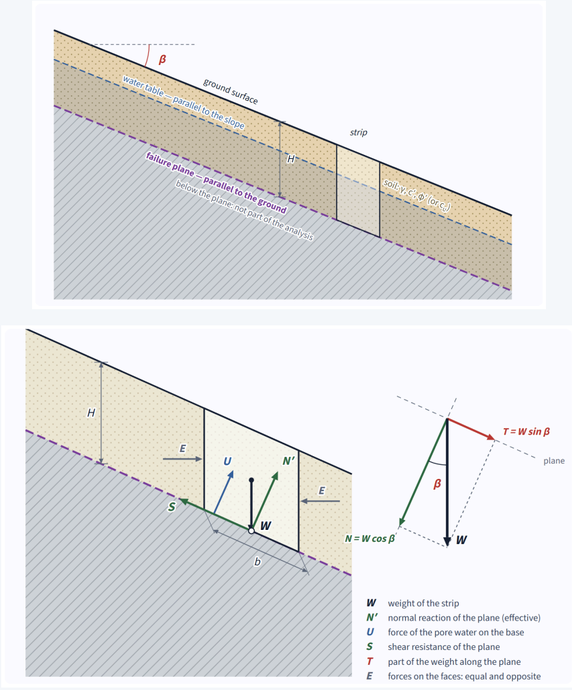
Many slope failures are shallow: a thin soil layer slides along a surface parallel to the surface at a depth of a few metres; the slope is so long that its ends hardly matter (a cut or fill slope after a wet season, a natural clay slope). The **infinite slope** is the model devised for this case: the slip plane is in the same condition everywhere, so one slice of the mass represents the whole, and its equilibrium takes only a few lines.

Slice: length b along the slip plane, 1 m width along the slope; the plane is at vertical depth H below the surface; the side faces are vertical and **cancel each other** (the slope is infinitely long, the soil is the same everywhere).

$$W = \gamma H b \cos\beta \qquad N = W\cos\beta \quad T = W\sin\beta$$

$$\sigma = \frac{N}{b} = \sigma_v \cos^2\beta \qquad \tau = \frac{T}{b} = \sigma_v \sin\beta\cos\beta \qquad \frac{\tau}{\sigma} = \tan\beta$$

In a slice with side faces removed, the stresses on the plane are inclined at the slope angle: τ/σ = tan β. If there are two unit weights above the plane (water table), then $\sigma_v = \gamma(H - h_w) + \gamma_{sat} h_w$; the equations remain the same.



Problem statement

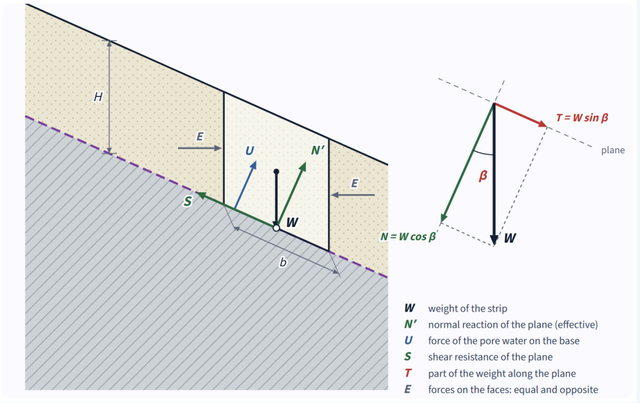
EXAMPLE

Given

Slope	$\beta = 15^\circ$; slip plane vertical at depth $H = 6$ m
Soil	$\gamma = 17.8$ kN/m³; no pore pressure

REQUIRED

Vertical stress σ_V, normal stress σ and shear stress τ on the plane.



Solution

SOLUTION

1 Vertical stress

$$\sigma_v = \gamma H = 17.8 \times 6 = \mathbf{106.80}\ \mathrm{kPa}$$

2 Stress normal and tangential to the plane

$$\sigma = \sigma_v \cos^2\beta = 106.80 \times 0.93301 = \mathbf{99.65}\ \mathrm{kPa}$$

$$\tau = \sigma_v \sin\beta \cos\beta = 106.80 \times 0.25000 = \mathbf{26.70}\ \mathrm{kPa} \qquad \frac{\tau}{\sigma} = 0.2679 = \tan 15^\circ \ \checkmark$$

On a unit area of the plane, a normal stress **σ = 99.65 kPa** and a shear stress **τ = 26.70 kPa** act; the ratio τ/σ = 0.2679 = tan 15°, i.e. the stress is inclined as much as the slope.

Check the solution in GeoTerva ↗

Mohr–Coulomb, closed-form solution, critical depth

THEORY

The soil on the plane carries the shear strength τ_f: in drained analysis, Mohr–Coulomb strength depending on the effective normal stress ($\sigma' = \sigma - u$). The factor of safety is the ratio of strength to stress: cohesion and friction are reduced by the same factor until the strip is just held:

$$\tau_f = c' + \sigma' \tan\varphi' \qquad FS = \frac{\tau_f}{\tau} \qquad \sigma' = \sigma - u$$

When there is no pore pressure $\sigma' = \sigma$ and a **closed-form solution** follows from the stresses:

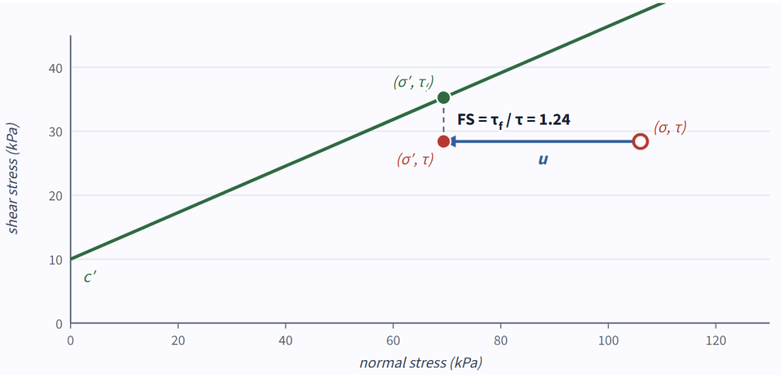
$$FS = \frac{c'}{\gamma H \cos^2\beta \, \tan\beta} + \frac{\tan\varphi'}{\tan\beta}$$

Cohesionless soil ($c' = 0$): FS = tan φ′/tan β; if the slope is flatter than φ′, it is safe at any depth.

Cohesive soil: cohesion holds the shallow plane (FS large); as it deepens FS decreases and reduces to friction alone.

If β > φ′, there is a **critical depth** where FS = 1 and a **required depth** where FS = FS_{req}; for cohesionless soil, β_{req}: tan β_{req} = tan φ′/FS_{req}:

$$H_{cr} = \frac{c'}{\gamma} \frac{1}{\cos^2\beta(\tan\beta - \tan\varphi')} \qquad H_{req} = \frac{c'}{\gamma \cos^2\beta \, (FS_{req} \tan\beta - \tan\varphi')}$$



Problem statement

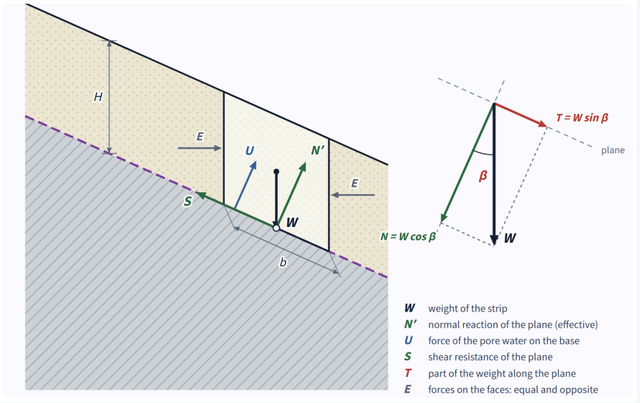
EXAMPLE

Given

Slope and soil	**Example 1:** $\beta = 15^\circ$, $H = 6$ **m,** $\gamma = 17.8$ **kN/m³;** $c' = 10$ **kPa,** $\varphi' = 20^\circ$**; no pore pressure**

REQUIRED

The plane's τ_f and **FS**; verification by closed-form solution.



Solution

SOLUTION

1 Strength and factor of safety (with the stresses in Example 1)

$$\tau_f = c' + \sigma \tan \varphi' = 10 + 99.65 \times 0.36397 = \mathbf{46.27}\ \mathrm{kPa}$$

$$FS = \frac{\tau_f}{\tau} = \frac{46.27}{26.70} = \mathbf{1.733}$$

2 Closed-form solution (check)

$$FS = \frac{10}{17.8(6)(0.93301)(0.26795)} + \frac{0.36397}{0.26795} = 0.3745 + 1.3584 = \mathbf{1.7329}$$

Factor of safety **1.733**. The cohesion term is 0.3745 and the friction term is tan 20°/tan 15° = 1.3584: the total gives the same number. Since the slope (15°) is flatter than the friction angle (20°), friction alone holds it; cohesion adds margin.

Check the solution in GeoTerva ↗

Problem statement

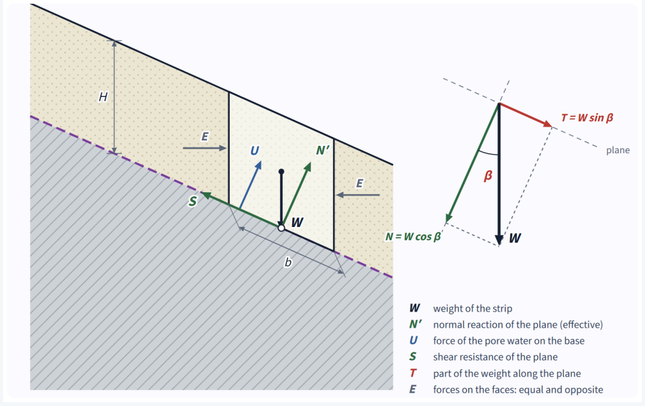
EXAMPLE

Given

Soil	**Soil in Example 2 ($c' = 10$ kPa, $\varphi' = 20°$, $\gamma = 17.8$ kN/m³)**
Required	**required factor of safety $FS_{req} = 2.0$; also the critical depth for a steeper slope $\beta = 25°$**

REQUIRED

H_{req} (β = 15°); **β_{req}** (H = 6 m); **H_{cr}** for β = 25°.



Solution

SOLUTION

1 Required depth (β = 15°; FS_{req} = 2)

$$H_{req} = \frac{c'}{\gamma \cos^2\beta(FS_{req}\tan\beta - \tan\varphi')} = \frac{10}{17.8(0.93301)(2(0.26795) - 0.36397)} = \mathbf{3.50}\ \mathrm{m}$$

2 Required angle (H = 6 m): the root of the equation FS(β) = 2; critical depth (β = 25°)

$$FS(\beta) = \frac{10}{106.8\cos^2\beta\tan\beta} + \frac{0.36397}{\tan\beta} = 2 \Rightarrow \beta_{req} = \mathbf{13.02}^\circ$$

$$H_{cr}(25^\circ) = \frac{10}{17.8(0.8214)(0.4663 - 0.3640)} = \mathbf{6.68}\ \mathrm{m}$$

For FS = 2, the plane can be at most **3.50 m** deep; at 6 m, the slope must not be steeper than **13.0°** for the same safety. Since β = 25° > φ′, there is a critical depth at which FS drops to 1: **6.68 m**. For the existing slope (β < φ′), there is no critical depth.

Check the solution in GeoTerva ↗

Total stress: c_u and $\varphi = 0$

THEORY

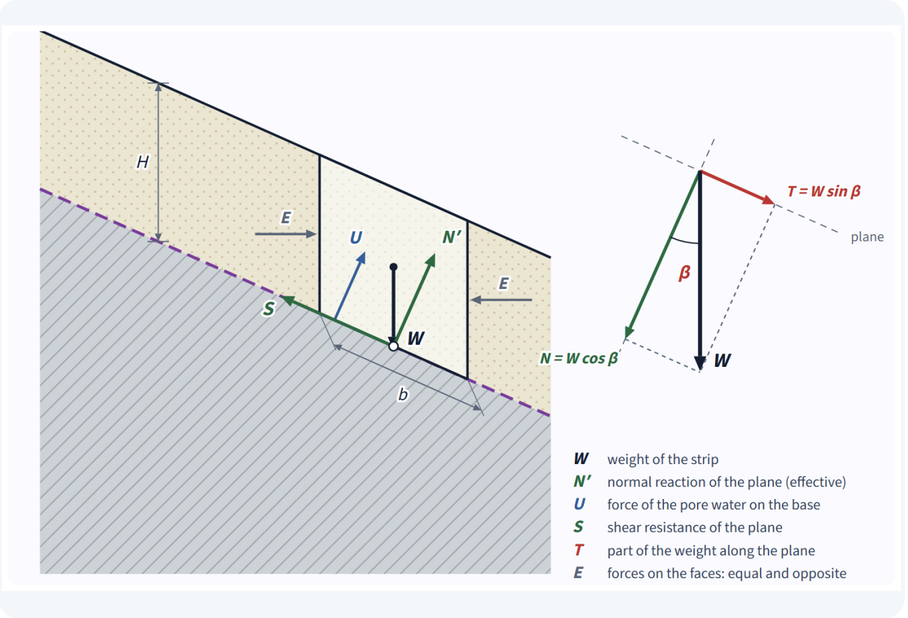
Saturated clay loaded or cut too fast for drainage (the short-term condition of a cut in soft clay) is solved in **total stress** with c_u and $\varphi = 0$. Strength does not depend on normal stress; pore pressure is not entered:

$$FS = \frac{c_u}{\tau} = \frac{c_u}{\gamma H \sin\beta \cos\beta} = \frac{2c_u}{\gamma H \sin 2\beta}$$

$\tau = (\gamma H/2)\sin 2\beta$ is greatest for a 45° slope; FS is **inversely proportional** to depth: a plane twice as deep has half the FS.

A clay that is stable undrained in the short term may not be stable once drained: the long term is a **drained analysis** with groundwater pore pressures and drained c', φ'.

$H_{cr} = c_u/(\gamma \sin\beta \cos\beta)$; $H_{req} = c_u/(FS_{req}\, \gamma \sin\beta \cos\beta)$; $\sin 2\beta_{req} = 2c_u/(FS_{req}\, \gamma\, H)$.



Problem statement

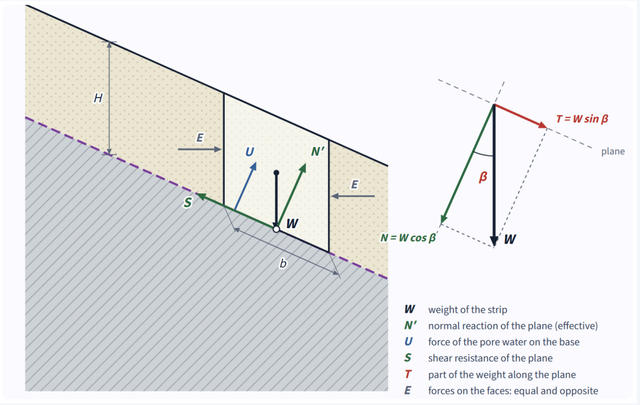
EXAMPLE

Given

Slope	$\beta = 15^\circ$, $H = 6$ **m**, $\gamma = 17.8$ **kN/m³;** **undrained clay** $c_u = 40$ **kPa** ($\varphi = 0$)
Required	$FS_{req} = 2.0$

REQUIRED

FS, H_{cr}, H_{req}, β_{req}.



Solution

SOLUTION

1 Factor of safety and critical depth

$$FS = \frac{c_u}{\gamma H \sin\beta \cos\beta} = \frac{40}{17.8(6)(0.2500)} = \frac{40}{26.70} = \mathbf{1.498}$$

$$H_{cr} = \frac{c_u}{\gamma \sin\beta \cos\beta} = \frac{40}{17.8(0.2500)} = \mathbf{8.99}\ \text{m}$$

2 Required depth and angle (FS_{req} = 2)

$$H_{req} = \frac{c_u}{FS_{req}\,\gamma \sin\beta \cos\beta} = \frac{8.99}{2} = \mathbf{4.49}\ \text{m}$$

$$\sin 2\beta_{req} = \frac{2c_u}{FS_{req}\gamma H} = \frac{80}{2(106.8)} = 0.3745 \;\Rightarrow\; \beta_{req} = \mathbf{11.00}^\circ$$

With c_u = 40 kPa, **FS = 1.50** (1.73 in the drained case). For FS = 2 the slope is at most **11.0°**; H_{cr} = 8.99 m.

Check the solution in GeoTerva ↗

Seepage parallel to the slope; pore pressure ratio r_u

THEORY

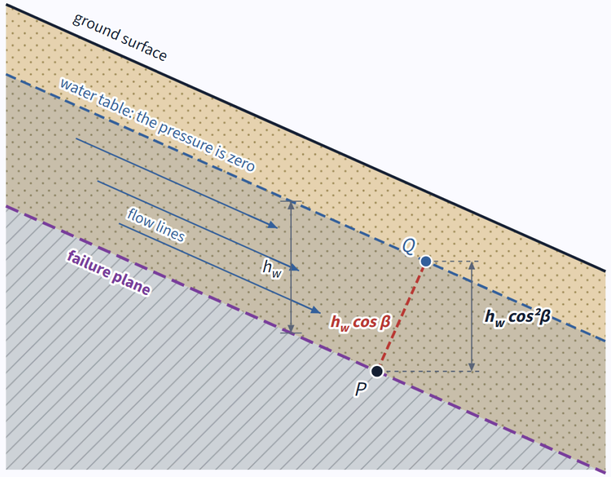
Water reduces the effective stress through pore pressure u, thereby removing strength. If the water table is at depth z_w below the surface (vertical), the soil above it has unit weight γ and below it γ_{sat}; the height of water above the plane is $h_w = H - z_w$. If the water flows parallel to the surface, the equipotential line passing through point P is perpendicular to the slope; the pressure head is $h_w \cos^2\beta$:

$$u = \gamma_w h_w \cos^2\beta \qquad \sigma' = \sigma - u \qquad FS = \frac{c' + \sigma' \tan\varphi'}{\tau}$$

water at surface: $FS = \dfrac{c'}{\gamma_{sat} H \cos^2\beta \tan\beta} + \dfrac{\gamma'}{\gamma_{sat}} \dfrac{\tan\varphi'}{\tan\beta}$ $\qquad r_u = \dfrac{u}{\gamma H}$: $FS = \dfrac{c'}{\gamma H \cos^2\beta \tan\beta} + \left(1 - \dfrac{r_u}{\cos^2\beta}\right) \dfrac{\tan\varphi'}{\tan\beta}$

For cohesionless soil with the water table at the surface, FS = (γ′/γ_{sat}) tan φ′/tan β: since γ′/γ_{sat} ≈ ½, a sand slope that in the dry state stands at close to φ′ requires, under seepage, a slope whose tangent is halved.

If the pore pressure comes from a piezometer or a flow net, the ratio of the soil weight is given as r_u. If σ′ ≤ 0 results (water lifts the soil off the plane), friction is set to zero, only cohesion remains, and this is stated.



On the equipotential line PQ the head is the same at P and at Q.

At Q the pressure is zero, so the pressure head at P is the rise from P to Q:

$u / \gamma_w = h_w \cos^2\beta$

Problem statement

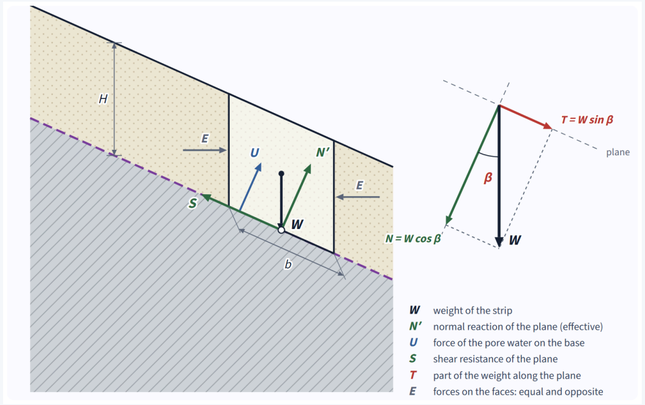
EXAMPLE

Given

Slope and soil	**The slope of Example 2; $\gamma = 17.8$ (dry), $\gamma_{sat} = 19.5$ kN/m³**
Water conditions	**(a) water table at $z_w = 2$ m, seepage parallel to the slope; (b) water at the surface ($z_w = 0$); (c) $r_u = 0.25$**

REQUIRED

In every case **u**, **σ′**, **τ_f** and **FS**; comparison with the dry FS.



Solution (1/2)

SOLUTION

1 (a) Water table at 2 m: $h_w = 6 - 2 = 4$ m

$$\sigma_v = 17.8(2) + 19.5(4) = 113.6 \text{ kPa}$$

$$\sigma = \sigma_v \cos^2\beta = 105.99 \text{ kPa} \quad \tau = \sigma_v \sin\beta \cos\beta = 28.40 \text{ kPa}$$

$$u = \gamma_w h_w \cos^2\beta = 9.81(4)(0.9330) = 36.61 \text{ kPa} \quad \sigma' = \sigma - u = 69.38 \text{ kPa}$$

$$FS = \frac{10 + 69.38(0.36397)}{28.40} = \frac{35.25}{28.40} = \mathbf{1.241}$$

With the water table at 2 m, FS drops from the dry case's 1.73 to **1.24**: water increases both the soil weight (σ_V) and the pore pressure (36.6 kPa).

Solution (2/2)

SOLUTION

2 Summary of the three water conditions (same equations)

Condition	σ_v (kPa)	σ (kPa)	τ (kPa)	u (kPa)	σ' (kPa)	τ_f (kPa)	FS
Dry	106.8	99.65	26.70	0.00	99.65	46.27	**1.733**
(a) z_w = 2 m	113.6	105.99	28.40	36.61	69.38	35.25	**1.241**
(b) water at surface	117.0	109.16	29.25	54.92	54.25	29.74	**1.017**
(c) r_u = 0.25	106.8	99.65	26.70	26.70	72.95	36.55	**1.369**

With water at the surface, FS = **1.02**, almost at the limit value; for r_u = 0.25 it is **1.37**. A slope with ample safety in the dry state has no margin at all when wet: the module shows the dry and water-at-surface FS together alongside the result.

Check the solution in GeoTerva ↗

Horizontal and vertical force; yield coefficient k_y

THEORY

The shaking is replaced by a constant acceleration (a fraction of g) applied to the mass above the plane. In addition to its weight W, the slice carries a horizontal force **$k_h W$** down the slope and a vertical force **$k_v W$** upward/downward; k_h and k_v are the engineer's coefficients (the module does not take them from a code). Per unit area W/A = $\sigma_v \cos\beta$:

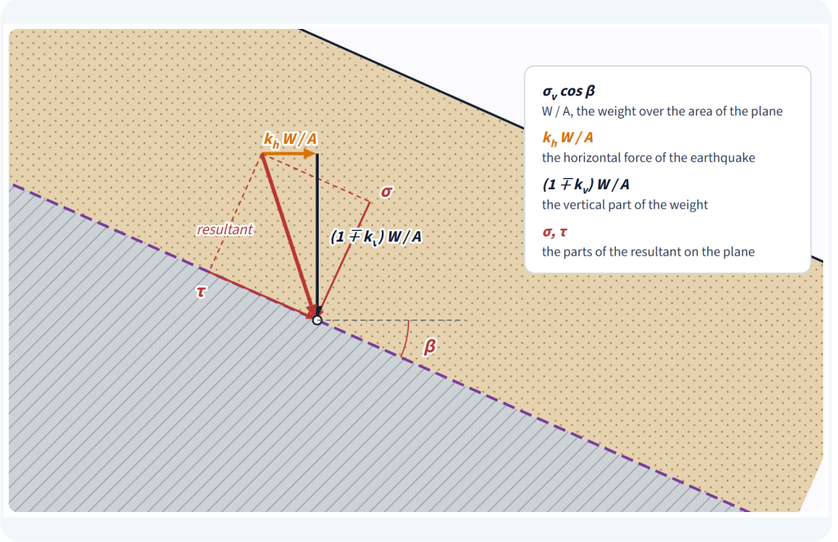
$$\sigma = \sigma_v \cos\beta\,[(1 \mp k_v)\cos\beta - k_h \sin\beta] \qquad \tau = \sigma_v \cos\beta\,[(1 \mp k_v)\sin\beta + k_h \cos\beta]$$

Pore pressure is the static value (no shaking added): $\sigma' = \sigma - u$; $FS_{eq} = \tau_f/\tau$ with the new σ and τ. The horizontal force reduces the normal stress and increases the shear stress.

Both directions of the vertical force are tried, and the direction giving the **lower FS** is adopted: in frictional soil it is usually the upward one (it lightens the mass and reduces the normal stress); in clay under total stress, since c_u does not depend on normal stress, it is the downward direction, which increases the shear stress.

Yield coefficient k_y: the horizontal coefficient that makes $FS_{eq} = 1$. Dry, cohesionless, $k_v = 0$: $k_y = \tan(\varphi' - \beta)$; in other cases k_h is increased from zero until found. A shaking not exceeding $k_y g$ does not start the mass; one that exceeds it does (how far it moves is the sliding-block analysis, not included in the module).

$$k_y = \frac{\tan\varphi' - \tan\beta}{1 + \tan\varphi' \tan\beta} = \tan(\varphi' - \beta)$$



Problem statement

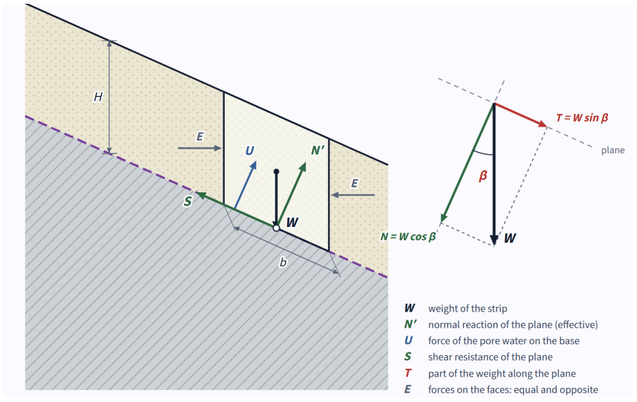
EXAMPLE

Given

Slope and soil	**Dry slope from Example 2 ($c' = 10$ kPa, $\varphi' = 20°$, $\beta = 15°$, $H = 6$ m)**
Earthquake	$k_h = 0.15$, $k_v = 0.05$ **(two directions are tried); also for dry sand** $\varphi' = 32°$, $\beta = 25°$

REQUIRED

σ, **τ**, **FS_{eq}** (critical direction); the FS(k_h) table and **k_y**; k_y in dry sand.



Solution (1/2)

SOLUTION

1 Critical direction: vertical force upward (k_v = +0.05 relieves)

$$\sigma = \sigma_v \cos\beta\,[0.95\cos\beta - 0.15\sin\beta] = 103.16\,[0.8788] = 90.66\ \text{kPa}$$

$$\tau = \sigma_v \cos\beta\,[0.95\sin\beta + 0.15\cos\beta] = 103.16\,[0.3908] = 40.31\ \text{kPa}$$

$$FS_{eq} = \frac{10 + 90.66(0.36397)}{40.31} = \frac{43.00}{40.31} = \mathbf{1.067} \qquad (k_v \text{ below: } 1.085)$$

The upward vertical force governs: FS_{eq} = **1.067** (1.085 downward).

Solution (2/2)

SOLUTION

2 FS(k_h) and yield coefficient (where FS = 1)

k_h	σ (kPa)	τ (kPa)	τ_f (kPa)	FS_{eq}
0.00	94.66	25.36	44.45	**1.753**
0.05	93.33	30.35	43.97	**1.449**
0.10	91.99	35.33	43.48	**1.231**
0.15	90.66	40.31	43.00	**1.067**
0.1746	90.00	42.76	42.76	**1.000**
0.20	89.32	45.29	42.51	**0.939**

The earthquake reduces FS from 1.73 to **1.07** (the upward vertical force is decisive). **k_y = 0.175**: if k_h exceeds this, the mass begins to slide. In dry sand ($\varphi' = 32°$, $\beta = 25°$), $k_y = \tan 7° = 0.1228$.

Check the solution in GeoTerva ↗

FS; H_{req}, β_{req}, H_{cr}

THEORY

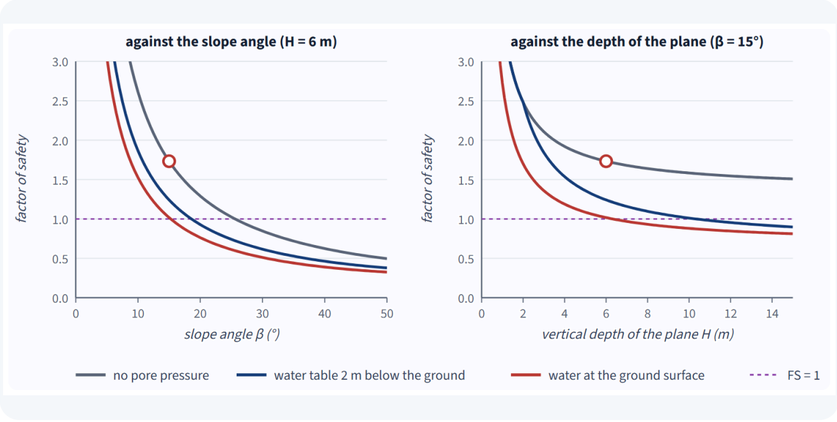
FS against slope: FS decreases steadily with β; the spacing of the curves shows how much of the margin the water takes away; the angle at which the curve crosses FS_{req} is β_{req}.

FS versus depth: the curve for cohesive soil falls from a large value at the surface to the level governed by friction; where it crosses FS = 1 is the critical depth. The water table leaves a mark at the depth where water first reaches the plane; above it the dry curve is followed.

FS against water: it increases as the water table deepens and decreases as r_u grows; it shows how strongly a drained slope depends on the difficult-to-know pore pressure.

In the seismic case each graph carries a second (orange) line; the fifth graph of FS versus k_h gives k_y at the point where it intersects FS = 1.

Alongside FS and the comparison with the required value, the Results tab gives: **H_{req}** (the first depth at which FS drops to FS_{req}; "no limit" if it does not), **β_{req}** (the steepest angle, rising from the flat angle, at which FS_{req} is still satisfied), **H_{cr}**, the two water extremes (no pore pressure / water at the surface) and, for earthquakes, FS_{eq}, k_y.



Problem statement

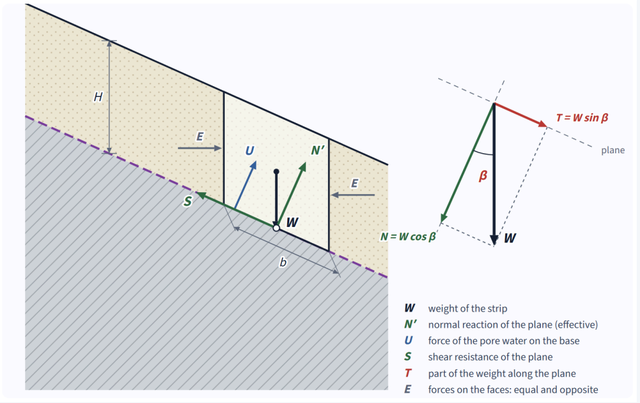
EXAMPLE

Given

Soil	**The soil in Example 2 ($c' = 10$ kPa, $\varphi' = 20^\circ$, $\gamma = 17.8$, $\gamma_{sat} = 19.5$ kN/m³)**
Table	**Values of FS for $\beta = 10,\ 15,\ 20,\ 25,\ 30^\circ$ and $H = 2,\ 4,\ 6,\ 8$ m with dry and with water table at $z_w = 2$ m (by closed-form solution)**

REQUIRED

$FS_{dry}(\beta, H)$ table; for GWT at H = 6 m; beyond which angle is FS < 1?



Solution

SOLUTION

1 FS = c′/(γH cos²β tan β) + tan φ′/tan β (dry); in the water table case, from the stresses (bold: FS < 1)

β (°)	H = 2 m	H = 4 m	H = 6 m	H = 8 m	H = 6, water z_w = 2 m
10	3.707	2.885	2.612	2.475	1.866
15	2.482	1.920	1.733	1.639	1.241
20	1.874	1.437	1.291	1.219	**0.928**
25	1.514	1.147	1.025	**0.964**	**0.741**
30	1.279	**0.955**	**0.847**	**0.793**	**0.616**

FS decreases regularly with angle and decreases with depth (the cohesion contribution). From β = 20° and H = 8 m even a dry slope approaches the limit; a groundwater table at z_w = 2 m, at H = 6 m, drops FS **below 1** from 20° onward (0.62 at 30°). Therefore β_{req} and H_{req} are sought.

Check the solution in GeoTerva ↗

What the model assumes

THEORY

Model: infinite slope: ground plane, slip surface parallel to it at vertical depth H; the ends of the mass play no role; conditions are the same everywhere; two-dimensional, 1 m along the slope.

Soil: a single soil above the plane (given strength and unit weight); layers, soil varying along the slope or with depth, and strength increasing with depth are not included in this model; for a weak layer interface, the properties of that layer may be used, or the Slope Stability module.

Water: a water table flowing parallel to the slope ($u = \gamma_w h_w \cos^2\beta$) or r_u; pore pressure is not computed from rainfall or permeability. **Strength**: Mohr–Coulomb in effective stress or c_u; if $\sigma' \leq 0$ the friction is zero. No residual strength, progressive failure or root effect.

Earthquake: pseudo-static only; shaking does not add pore pressure; amplification, duration, residual pore pressure of loose saturated soil and permanent displacement are not analyzed; the coefficients are the engineer's choice.

None: surcharge, tension crack, reinforcement, toe wall. FS is the value for a single plane at a single depth; no weaker plane that is not parallel to the surface is sought. If the slope is short relative to depth, or the soil/water varies along the slope, the ends matter: the Slope Stability module is used.

Summary

Key points

Strip: $\sigma = \sigma_v \cos^2\beta$, $\tau = \sigma_v \sin\beta \cos\beta$, $\tau/\sigma = \tan\beta$ (in the example $\sigma = 99.65$, $\tau = 26.70$ kPa)

FS $= (c' + \sigma' \tan\varphi')/\tau = c'/(\gamma H \cos^2\beta \tan\beta) + \tan\varphi'/\tan\beta =$ **1.733**; $H_{req} = 3.50$ m ($FS_{req} = 2$), $\beta_{req} = 13.0°$; if $\beta > \varphi'$ then H_{cr}

Undrained clay: $FS = c_u/(\gamma H \sin\beta \cos\beta) = 1.50$; inversely proportional to depth

Water: $u = \gamma_w h_w \cos^2\beta$; FS 1.73 → 1.24 ($z_w = 2$ m) → 1.02 (at the surface); $r_u = 0.25$ → 1.37

Earthquake: horizontal $k_h W$ and vertical $k_v W$; the critical direction is taken (upward); FS 1.73 → 1.07; **$k_y = 0.175$**; $\tan(\varphi' - \beta)$ for dry cohesionless soil

Limit: single plane, infinite slope; if ends or layers matter, use Slope Stability (slices)

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NOTE

All numerical examples were generated with the module's own calculation engine; the module yields the same values. The worked example on the module page is also this slope (β = 15°, H = 6 m, γ = 17.8, c′ = 10 kPa, φ′ = 20°).

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