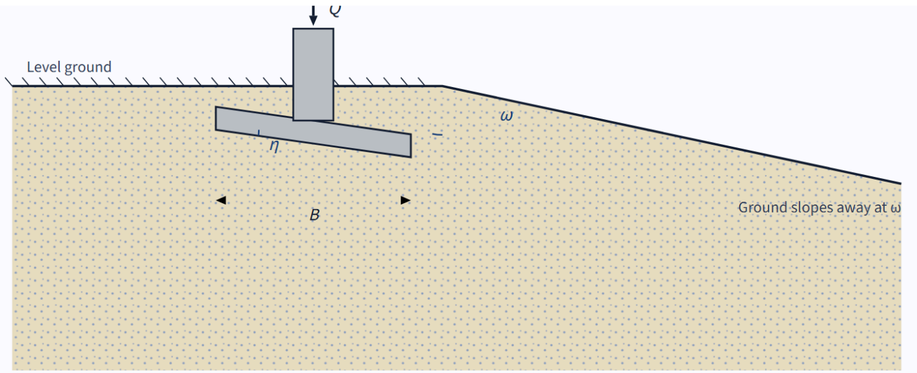
GeoTerva

presentation · Foundations

Hansen bearing capacity equation

Lecture: from Hansen's own N_γ coefficient to shape and depth corrections, from inclination coefficients depending on the actual load to base and ground inclination; a worked example in every topic.



Hansen N_γ

Figure · depth

Slope dependent on actual load

Base and ground inclination

Undrained additive form

Worked examples

Introduction

What will we learn in this lecture?

- Recognizing the three terms of the **Hansen equation** and the six correction coefficients of each term (shape, depth, inclination, base inclination, ground inclination)
- To compute Hansen's **own N_γ coefficient** and compare it with Vesic's
- Applying the **shape** and **depth** coefficients, including the transition at $D_f/B = 1$
- To compute the **slope coefficients** with the actual loads h and v and to check the shear limits
- Using the **base inclination η** and **ground inclination ω** coefficients
- Addressing the undrained ($\varphi = 0$) **total stress form** and groundwater conditions
- To find and **check** the gross and net allowable pressure

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example footing, the model opened by the module: 2 m × 2 m square footing on soil with c = 10 kPa, $\varphi' = 25°$, $D_f = 1$ m, $\gamma = 18$ kN/m³, no groundwater.

Three terms, each with six corrections

THEORY

Hansen (1970) derived the bearing capacity equation with its own failure mechanism, independently of Vesic. Each term carries **six correction** factors multiplied in:

$$
\begin{aligned}
q_u = {} & c\, N_c\, s_c d_c i_c b_c g_c \\
& + q\, N_q\, s_q d_q i_q b_q g_q \\
& + \tfrac{1}{2}\gamma B\, N_\gamma\, s_\gamma d_\gamma i_\gamma b_\gamma g_\gamma
\end{aligned}
$$

N_c and N_q are the same as in all the methods of this section (the closed-form Prandtl and Reissner expressions). **N_γ is specific to Hansen** and is smaller than Vesic's at the same φ′.

The **shape** (s), **depth** (d), **inclination** (i), **base inclination** (b) and **ground slope** (g) factors are Hansen's own set; inclined load, inclined base and sloping ground are treated in the same equation.

For a flat base, level ground and vertical load, **i = b = g = 1**: the equation reduces to the general equation with shape and depth factors.

NOTATIONS

c, φ′: cohesion, angle of internal friction

q = γ D_f: surcharge at base level

B, L: short and long side; B/L determines the shape

h, v: H/A and V/A pressures of the load case

η: inclination of the base from the horizontal

ω: inclination of the ground surface beside the foundation

Hansen's N_γ is smaller than Vesic's

THEORY

N_c and N_q are the same in every method:

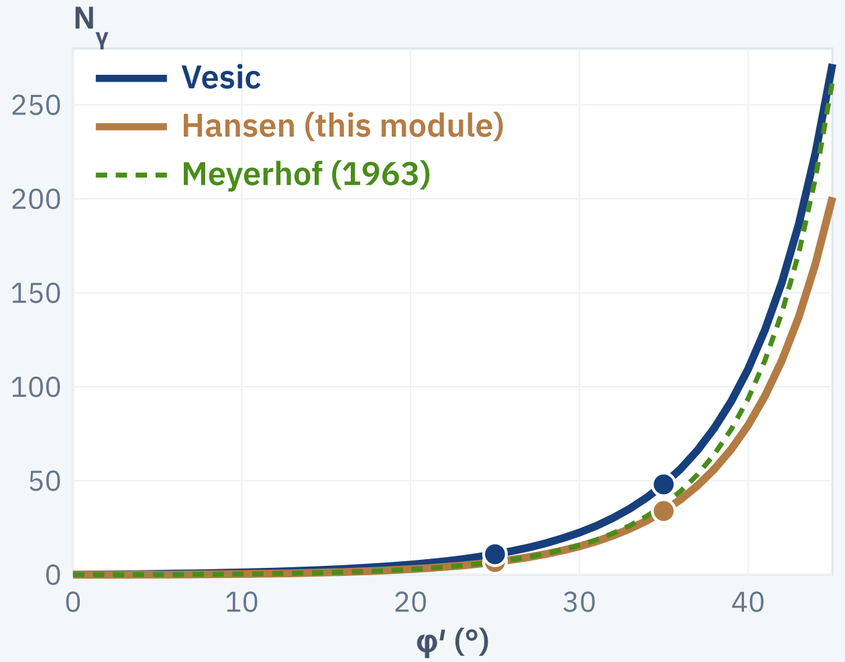
$$N_q = e^{\pi \tan\varphi} \tan^2\left(45^\circ + \tfrac{\varphi}{2}\right) \qquad N_c = (N_q - 1)\cot\varphi$$

There is no closed-form, universally accepted expression for N_γ. By idealizing the failure surface somewhat differently, Hansen obtained his own expression:

$$N_\gamma^{\text{Hansen}} = 1.5\,(N_q - 1)\tan\varphi \qquad N_\gamma^{\text{Vesic}} = 2\,(N_q + 1)\tan\varphi$$

Both are respected derivations; neither is absolutely "more correct". What matters is **to use the coefficients of the same method together**: Hansen's N_γ with Hansen's shape and inclination factors.

As φ′ increases the difference decreases proportionally but does not close; at φ′ = 25° Hansen is 62% of Vesic.



Problem statement

EXAMPLE

Given

Soil	$\varphi' = 25^\circ$ **(model in which the module is opened);** $\varphi' = 35^\circ$ **for comparison**
Methods	**Hansen:** $N_\gamma = 1.5(N_q - 1)\tan\varphi$ **· Vesic:** $N_\gamma = 2(N_q + 1)\tan\varphi$

REQUIRED

Find the **N_q, N_c** and **N_γ** values according to both methods; calculate by what percentage Hansen's value is smaller than Vesic's.

Solution (1/2)

SOLUTION

1 N_q and N_c ($\varphi' = 25°$, $\tan 25° = 0.4663$)

$$N_q = e^{\pi \tan 25^\circ} \tan^2(57.5^\circ) = e^{1.4649}(2.4639) = \mathbf{10.66} \qquad N_c = \frac{10.66 - 1}{0.4663} = \mathbf{20.72}$$

2 N_γ: Hansen and Vesic

$$N_\gamma^{\text{Hansen}} = 1.5(10.66 - 1)(0.4663) = \mathbf{6.76}$$
$$N_\gamma^{\text{Vesic}} = 2(10.66 + 1)(0.4663) = \mathbf{10.88}$$

Hansen's N_γ is **62%** of Vesic's: on the same soil the self-weight term comes out about **38% smaller**.

Solution (2/2)

SOLUTION

φ′ (°)	N_q	N_c	N_γ Hansen	N_γ Vesic	Hansen / Vesic
10	2.47	8.34	0.39	1.22	%32
15	3.94	10.98	1.18	2.65	%45
20	6.40	14.83	2.95	5.39	%55
25	10.66	20.72	6.76	10.88	%62
30	18.40	30.14	15.07	22.40	%67
32	23.18	35.49	20.79	30.21	%69
35	33.30	46.12	33.92	48.03	%71
40	64.20	75.31	79.54	109.41	%73
45	134.87	133.87	200.81	271.75	%74

The same holds for φ′ = 35°: N_γ = 33.92 (Hansen) and 48.03 (Vesic), a ratio of 71%. The ratio rises from 32% to 74% as φ′ increases and is never 1: **do not mix methods**.

Check the solution in GeoTerva ↗

Three coefficients depending on the B/L ratio

THEORY

For a non-strip footing the failure is three-dimensional. Hansen's shape factors are similar to De Beer's; they include the **N_q/N_c ratio**:

$$s_c = 1 + \frac{B}{L}\frac{N_q}{N_c} \qquad s_q = 1 + \frac{B}{L}\sin\varphi \qquad s_\gamma = 1 - 0.4\,\frac{B}{L}$$

Under undrained loading ($\varphi = 0$) the ratio N_q/N_c behaves poorly; the cohesion coefficient takes its own simple form. This coefficient is **additive**, not a multiplier, so s'_c is written:

$$s'_c = 0.2\,\frac{B}{L} \qquad (\varphi = 0)$$

Strip footing (B/L → 0): $s_c = s_q = s_\gamma = 1$ and $s'_c = 0$. For square and circular footings, B/L = 1 is taken.

Comparison with De Beer: s_c and s_γ are the same; for s_q, De Beer uses tan φ, Hansen uses sin φ.

FOR φ′ = 25°

B/L	s_c	s_q	s_γ
0 (strip)	1,000	1,000	1,000
0.5	1.257	1.211	0.800
1 (square)	1.515	1.423	0.600

Problem statement

EXAMPLE

Given

Soil	$\varphi' = 25°$: $N_c = 20.72,\ N_q = 10.66,\ \sin 25° = 0.4226$ **(Example 1)**
Foundations	**(a) square 2 × 2 m · (b) rectangle 2 × 4 m (B/L = 0.5) · (c) strip**
Undrained	**φ = 0: square and 2 × 4 m**

REQUIRED

For each footing, find the values of **s_c, s_q, s_γ** (s'_c at φ = 0); for the square footing, compare s_q with that of De Beer.

Solution

SOLUTION

1 (a) Square footing (B/L = 1)

$$s_c = 1 + (1)\frac{10.66}{20.72} = \mathbf{1.515} \qquad s_q = 1 + \sin 25^\circ = \mathbf{1.423} \qquad s_\gamma = 1 - 0.4 = \mathbf{0.600}$$

2 (b) Rectangle 2 × 4 m (B/L = 0.5)

$$s_c = 1 + 0.5\frac{10.66}{20.72} = \mathbf{1.257} \qquad s_q = 1 + 0.5(0.4226) = \mathbf{1.211} \qquad s_\gamma = 1 - 0.2 = \mathbf{0.800}$$

3 (c) Strip and undrained case (φ = 0)

$$\text{strip}: \; s_c = s_q = s_\gamma = 1 \qquad \varphi = 0: \; s_c' = 0.2 \;(\text{square}),\; 0.1 \;(2 \times 4),\; 0 \;(\text{strip})$$

For a square footing the q term increases by **42%** and the self-weight term decreases by 40%. De Beer's $s_q = 1 + \tan\varphi = 1.466$, Hansen's 1.423: the difference is about 3%.

Check the solution in GeoTerva ↗

Linear form and arctangent form

THEORY

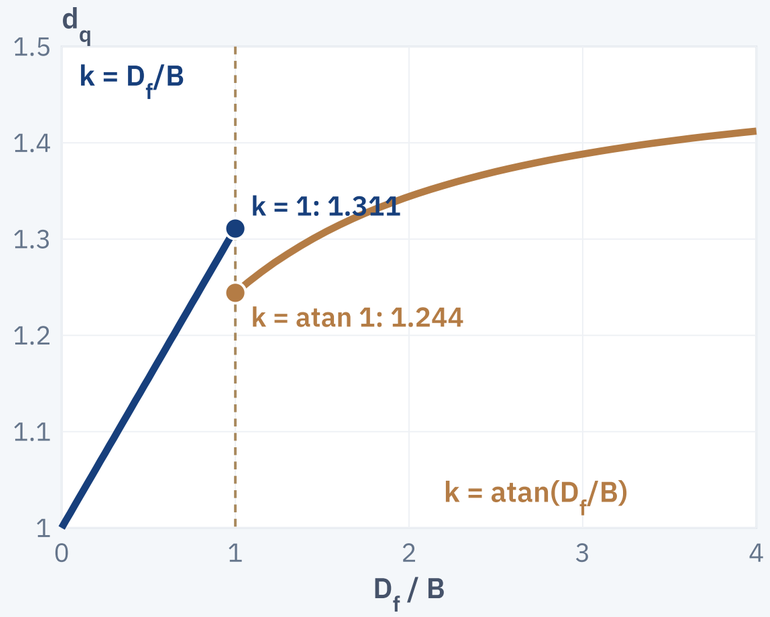
The soil above the base contributes to the bearing capacity. Hansen's depth factors depend on the ratio **k = D_f/B**; when D_f/B exceeds 1, k is taken as an arctangent (in radians):

$$d_c = 1 + 0.4\,k \qquad d_q = 1 + 2\tan\varphi\,(1-\sin\varphi)^2\,k \qquad d_\gamma = 1$$

$$k = \frac{D_f}{B}\ \left(\frac{D_f}{B} \le 1\right) \qquad k = \tan^{-1}\frac{D_f}{B}\ \left(\frac{D_f}{B} > 1\right)$$

At φ = 0, d_q = 1; in undrained additive form $d'_c = d_c - 1 = 0.4\,k$ is written.

The two forms meet at a step at D_f/B = 1: with k = 1, d_q = 1.311, just beyond it k = $\tan^{-1}1$ = 0.785 and d_q = 1.244. It is therefore advisable to compare adjacent D_f values.



Problem statement

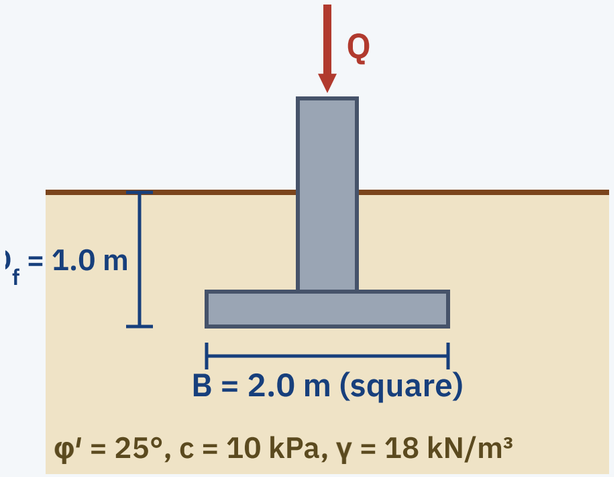
EXAMPLE

Given

Soil	$\varphi' = 25°$: $\tan\varphi = 0.4663$, $(1 - \sin 25°)^2 = 0.3334$
Foundation	**B = 2 m; D_f = 1 m (example), comparison D_f = 2 m and 3 m**
Undrained	**φ = 0: D_f = 1.5 m**

REQUIRED

For each case, find the values of d_c, d_q and d_γ; explain what happens when D_f increases from 2 m to 3 m.



Solution

SOLUTION

1 D_f = 1 m (D_f/B = 0.5 ≤ 1: k = 0.5)

$$d_q = 1 + 2(0.4663)(0.3334)(0.5) = \mathbf{1.155} \qquad d_c = 1 + 0.4(0.5) = \mathbf{1.200} \qquad d_\gamma = 1$$

2 D_f = 2 m (D_f/B = 1: linear, k = 1) and D_f = 3 m (D_f/B = 1.5: k = $\tan^{-1}1.5$ = 0.9828)

$$D_f = 2: \; d_q = 1 + 2(0.4663)(0.3334)(1) = \mathbf{1.311}, \; d_c = \mathbf{1.400}$$

$$D_f = 3: \; d_q = 1 + 2(0.4663)(0.3334)(0.9828) = \mathbf{1.306}, \; d_c = \mathbf{1.393}$$

3 Undrained (φ = 0): D_f/B = 1.5/2 = 0.75

$$d_c = 1 + 0.4(0.75) = \mathbf{1.300} \quad (d_c' = 0.3) \qquad d_q = 1$$

D_f = 2 m → 3 m: d_q **decreases** from 1.311 to 1.306, because k passes from 1 to $\tan^{-1}1.5$ = 0.98 (step). For D_f = 1 → 2 m, however, it increases from 1.155 to 1.311.

Check the solution in GeoTerva ↗

With the actual load, not the angle

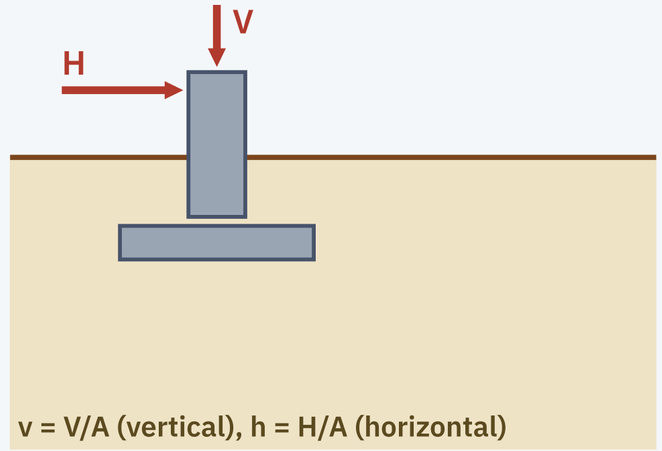
THEORY

Meyerhof's inclination coefficients depend only on the angle of the load to the vertical. Hansen's depend on the **actual horizontal and vertical stresses**; h = H/A and v = V/A are the stresses of the same load case (v = q_{app}):

$$i_q = \left(1 - \frac{0.5\,h}{v + c\cot\varphi}\right)^5 \qquad i_c = i_q - \frac{1 - i_q}{N_q - 1} \;\; (\geq 0)$$

$$i_\gamma = \left(1 - \frac{0.7\,h}{v + c\cot\varphi}\right)^5$$

$$\eta > 0: \;\; 0.7 \to 0.7 - \frac{\eta^\circ}{450^\circ}\;(i_\gamma) \qquad \varphi = 0: \;\; i'_c = 0.5\left(1 - \sqrt{1 - \frac{h}{c}}\right),\; i_q = i_\gamma = 1$$



If h > 0, $q_{applied}$ must be entered: the footing check with the inclination coefficients uses **the same load**.

As h/(v + c cot φ) grows, all three coefficients tend to zero; a footing approaching slip has no bearing capacity left.

The base reaches bearing capacity before sliding

THEORY

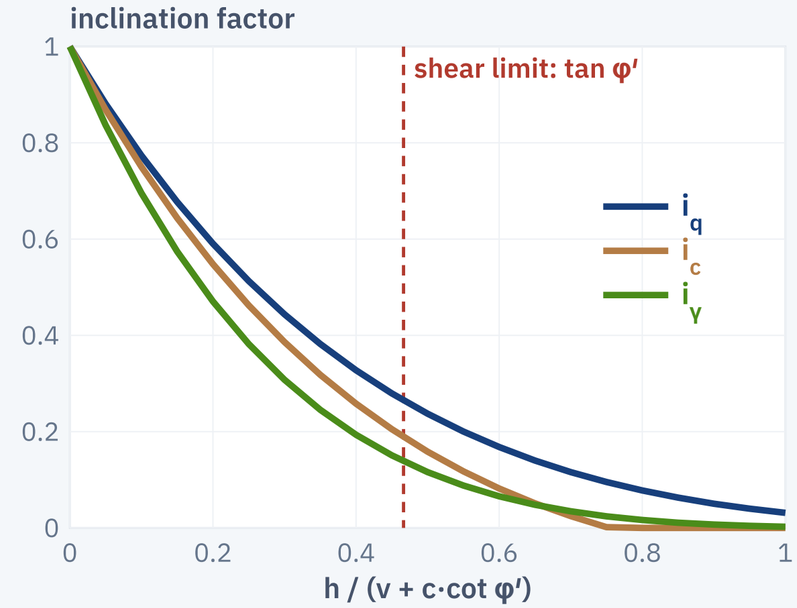
The foundation **slides** when the horizontal load exceeds the friction and adhesion resistance of the base. Upper bound (taking base friction φ and adhesion c):

$$h_{shear} = v \tan\varphi + c = \tan\varphi\,(v + c\cot\varphi)$$

h > v tan φ + c: the module warns that the sliding footing cannot reach its bearing capacity; check sliding separately for the actual base. On the chart this limit is the line **h/(v + c cot φ) = tan φ**.

If **h ≥ v + c cot φ** (h ≥ c when φ = 0) the module stops: the slope coefficients lose their meaning.

Once i_c, i_q fall below $1/N_q$ they are held at zero; the cohesion term never becomes negative.



Problem statement

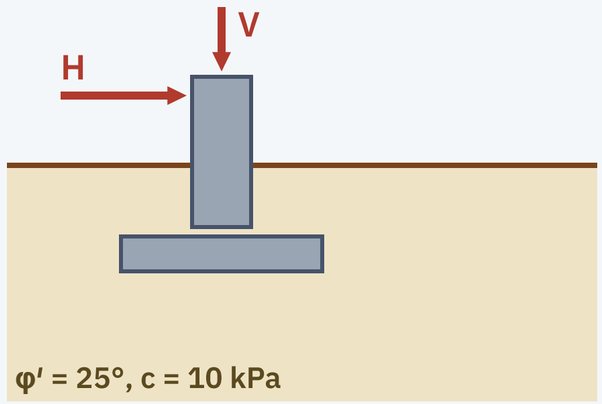
EXAMPLE

Given

Foundation, soil	**module model:** $c = 10\ \mathrm{kPa},\ \varphi' = 25^\circ,\ N_q = 10.66$
Load case	$v = q_{apply} = 150\ \mathrm{kPa},\ h = 30\ \mathrm{kPa}$ **(ek: 60, 90, 120)**
Base	**straight (η = 0)**

REQUIRED

Find the i_c, i_q, i_γ coefficients and check the slip limit.



Solution (1/2)

SOLUTION

1 Ratio: v + c cot φ = 150 + 10 (2.1445) = 171.45 kPa

$$x = \frac{h}{v + c\cot\varphi} = \frac{30}{171.45} = \mathbf{0.1750}$$

2 Coefficients

$$i_q = (1 - 0.5 \cdot 0.175)^5 = \mathbf{0.633} \qquad i_\gamma = (1 - 0.7 \cdot 0.175)^5 = \mathbf{0.520}$$

$$i_c = i_q - \frac{1 - i_q}{N_q - 1} = 0.633 - \frac{0.367}{9.66} = \mathbf{0.595}$$

3 Slip boundary

$$v\tan\varphi + c = 150(0.4663) + 10 = \mathbf{79.9}\ \text{kPa} \geq h = 30 \ \Rightarrow\ \text{no slip}$$

$$v + c\cot\varphi = 171.4\ \text{kPa} > h$$

Solution (2/2)

SOLUTION

h (kPa)	h/(v + c cot φ)	i_c	i_q	i_γ	sliding check
0	0.000	1.000	1.000	1.000	passes
30	0.175	0.595	0.633	0.520	passes
60	0.350	0.318	0.382	0.245	passes
90	0.525	0.137	0.218	0.101	**warning: h > 79.9**
120	0.700	0.025	0.116	0.035	**warning: h > 79.9**

h = 30 kPa (20% of v) reduces the q term by **37%**, the c term by **41%** and the self-weight term by **48%**. If the base is inclined at 10°, $i_\gamma = (1 - (0.7 - 10/450)\cdot 0.175)^5 = 0.532$. At h = 90 and 120 kPa, which exceed the limit, the footing slides: the coefficients are still computed but sliding must also be checked.

Check the solution in GeoTerva ↗

Non-uniform base and sloping ground

THEORY

Real footings are not always flat-based and on level ground: the base may be deliberately inclined or the footing may be at the crest of a slope. Hansen's coefficients cover both (η and ω in degrees; in exponential expressions, radians):

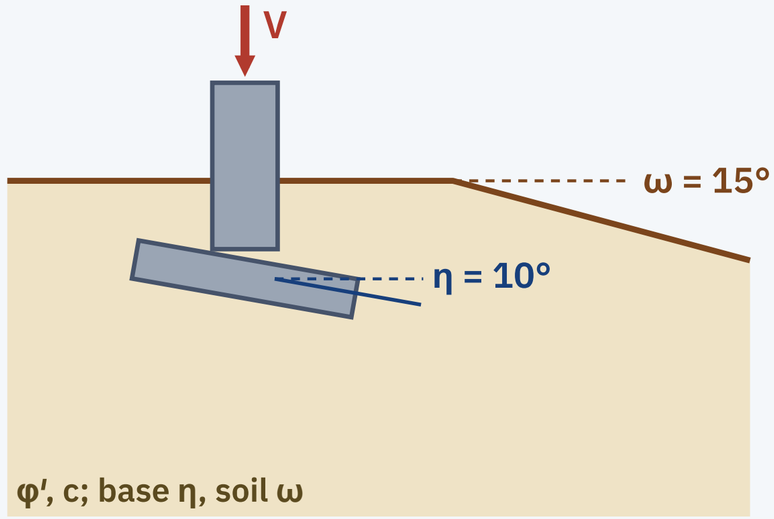
$$b_c = 1 - \frac{\eta^\circ}{147^\circ} \qquad b_q = e^{-2\eta \tan\varphi} \qquad b_\gamma = e^{-2.7\,\eta \tan\varphi}$$

$$g_c = 1 - \frac{\omega^\circ}{147^\circ} \qquad g_q = g_\gamma = (1 - 0.5\tan\omega)^5$$

The exponent of b_γ (2.7) is larger than that of b_q (2): an inclined base harms the self-weight term more than the surcharge term.

η and ω are entered at most 45°; if ω > φ the slope is not in equilibrium on its own and the module stops.

(1 – tan ω)² is **Vesic's** form, not Hansen's, and is slightly larger: at ω = 5° it gives 0.833; Hansen's gives 0.800. It should not be confused with Hansen's equation.



Problem statement

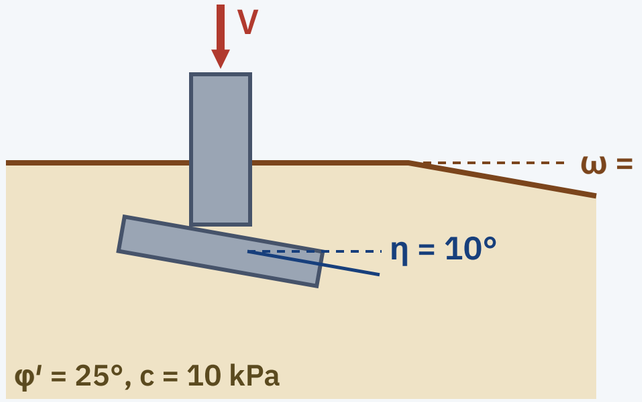
EXAMPLE

Given

Soil	$\varphi' = 25^\circ,\ \tan\varphi = 0.4663$
Base slope	**η = 10°**
Ground slope	**ω = 10° and ω = 15°**

REQUIRED

Find the values of $\mathbf{b_c}$, $\mathbf{b_q}$, $\mathbf{b_\gamma}$ and $\mathbf{g_c}$, $\mathbf{g_q}$, $\mathbf{g_\gamma}$; compare g_q with Vesic's $(1 - \tan\omega)^2$ form.



Solution (1/2)

SOLUTION

1 Base inclination η = 10° (0.1745 rad)

$$b_c = 1 - \frac{10}{147} = \mathbf{0.932} \quad b_q = e^{-2(0.1745)(0.4663)} = \mathbf{0.850} \quad b_\gamma = e^{-2.7(0.1745)(0.4663)} = \mathbf{0.803}$$

2 Ground slope ω = 10° (tan 10° = 0.1763)

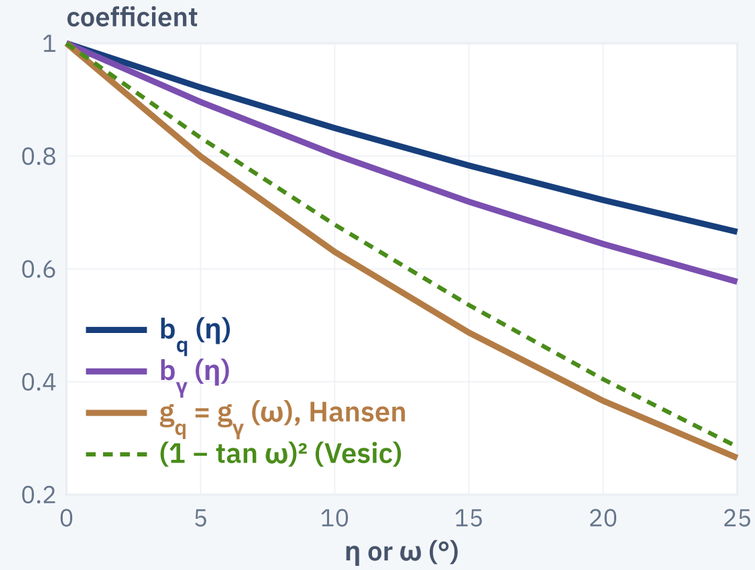
$$g_c = 1 - \frac{10}{147} = \mathbf{0.932} \quad g_q = g_\gamma = (1 - 0.5 \cdot 0.1763)^5 = (0.9118)^5 = \mathbf{0.630}$$

Solution (2/2)

SOLUTION

ω (°)	g_c	$g_q = g_\gamma$ Hansen	g_q Vesic $(1 - \tan \omega)^2$
10	0.932	0.630	0.678
15	0.898	0.487	0.536

Hansen's g_q is smaller than Vesic's (ω = 10°: 0.630 and 0.678): it is more **conservative**. A base inclination of η = 10° reduces the self-weight term by 20% and the surcharge term by 15%.



Check the solution in GeoTerva ↗

Combining the six coefficients

THEORY

Although the six coefficients look daunting, **in the ordinary case they are all 1**; the full equation comes into play only when one assumption is violated. q_u is the vertical pressure at failure; the module keeps gross and net pressure separate:

$$q_{u,net} = q_u - q \qquad q_{all,g} = \frac{q_u}{FS} \qquad q_{all,n} = \frac{q_{u,net}}{FS}$$

The applied pressure $q_{applied}$ is the **vertical gross pressure at the base** for the same loading case (V/A). The footing is acceptable under the following conditions:

$$q_{apply} \le q_{all,g} \qquad FS_{\text{actual}} = \frac{q_u}{q_{apply}} \qquad q_{apply} - q \le q_{all,n}$$

Allowable vertical load $Q_{all} = q_{all,g} \cdot A$. If $h > 0$ and $q_{applied}$ is not entered, the module cannot compute the inclination factors either, not only the capacities.

WHEN IS THE COEFFICIENT 1?

Coefficient	case where it is 1
s	strip footing (B/L = 0)
d	$D_f = 0$ (base at the surface)
i	h = 0 (vertical load)
b	η = 0 (smooth base)
g	ω = 0 (level ground)

Coefficients unchanged; q and γ change

THEORY

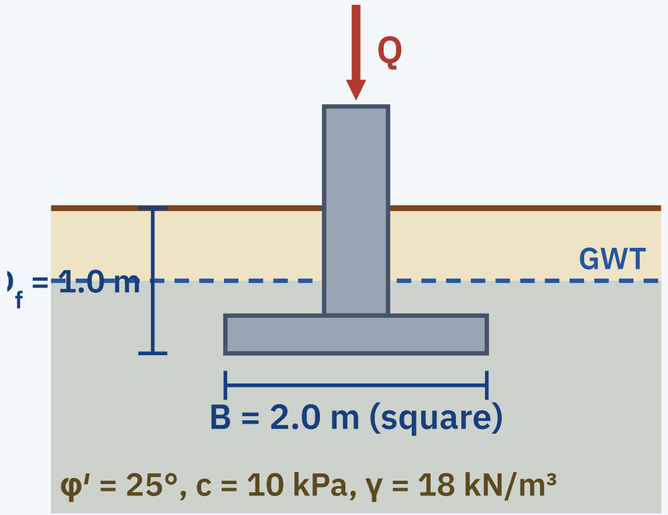
The Hansen module treats the groundwater with **the same three cases** as the other bearing modules. Below the GWT the effective unit weight is $\gamma' = \gamma_{sat} - \gamma_w$ (γ_w = 9.81). D_W is the depth of the groundwater from the surface:

$$\text{I } (D_w \le D_f):\ q = \gamma D_w + \gamma'(D_f - D_w),\ \ \gamma \to \gamma' \text{ (third term)}$$

$$\text{III } (D_f < D_w \le D_f + B):\ q = \gamma D_f,\ \ \bar{\gamma} = \frac{1}{B}\left[\gamma D + \gamma'(B - D)\right],\ D = D_w - D_f$$

For $D_W = D_f$, $q = \gamma D_f$ and γ' is used in the third term.

Water deeper than one B below the base is **ineffective**. Water enters only through **q and γ**; s, d, i, b, g remain the same.



Problem statement

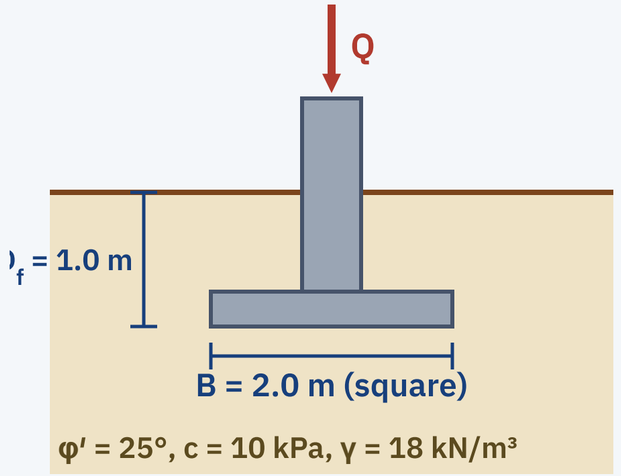
EXAMPLE

Given

Foundation	**square, B = L = 2.0 m, base depth D_f = 1.0 m**
Soil	$c = 10\ \mathrm{kPa},\ \varphi' = 25^\circ$
Unit weight	$\gamma = 18\ \mathrm{kN/m^3}$**; no groundwater**
Load	**vertical, central**
Safety	**FS = 3**

REQUIRED

q_u, gross and net q_{all}, and the allowable load Q_{all}; find the contribution of each term.



Solution (1/2)

SOLUTION

1 Bearing capacity factors (φ′ = 25°)

$$N_c = \mathbf{20.72}, \quad N_q = \mathbf{10.66}, \quad N_\gamma = 1.5(10.66 - 1)(0.4663) = \mathbf{6.758}$$

2 Correction coefficients (B/L = 1; D_f/B = 0.5; h = η = ω = 0)

$$s_c = 1.515,\ s_q = 1.423,\ s_\gamma = 0.6 \qquad d_c = 1.200,\ d_q = 1.155,\ d_\gamma = 1 \qquad i = b = g = 1$$

Solution (2/2)

SOLUTION

3 Bearing capacity ($q = \gamma D_f = 18$ kPa)

$$q_u = 10(20.72)(1.515)(1.200) + 18(10.66)(1.423)(1.155)$$
$$+ \tfrac{1}{2}(18)(2)(6.758)(0.6) = 376.6 + 315.5 + 73.0 = \mathbf{765.05}\ \text{kPa}$$

4 Allowable pressure and load ($A = 2^2 = 4$ m²)

$$q_{all,g} = \frac{765.05}{3} = \mathbf{255.02}\ \text{kPa} \qquad q_{all,n} = \frac{765.05 - 18}{3} = 249.02\ \text{kPa}$$

$$Q_{all} = 255.02(4) = \mathbf{1{,}020}\ \text{kN}$$

49% of the bearing capacity comes from cohesion, **41%** from surcharge, and only **10%** from self-weight: the small N_γ and $s_\gamma = 0.6$ weaken the self-weight term.

Check the solution in GeoTerva ↗

Problem statement

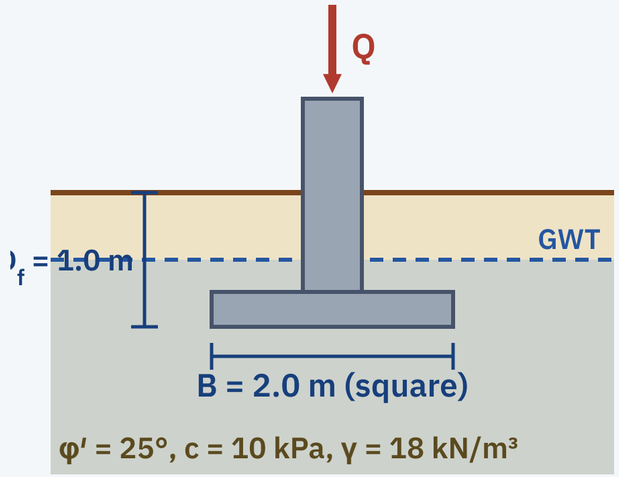
EXAMPLE

Given

Foundation	**square, B = L = 2.0 m, D_f = 1.0 m**
Soil	$c = 10\ \mathrm{kPa},\ \varphi' = 25^\circ$
Unit weight	$\gamma = 18$ **(above),** $\gamma_{sat} = 19\ \mathrm{kN/m^3}$ **(below the GWT)**
Groundwater	**D_w = 0.5; 1.0; 2.0 and 4.0 m**

REQUIRED

For each case, find **q**, the unit weight in the third term, and the **q_u** values. (Factors as in Example 6.)



Solution (1/2)

SOLUTION

1 Case I ($D_W \le D_f$): $\gamma' = 19 - 9.81 = 9.19$ kN/m³

$$D_w = 0.5: \;\; q = 18(0.5) + 9.19(0.5) = \mathbf{13.595}\ \mathrm{kPa}$$

$$D_w = 1.0: \;\; q = 18(1.0) = \mathbf{18.0}\ \mathrm{kPa}$$

2 Case III ($D_W = 2.0 \Rightarrow D = 1.0 \le B$) and water without effect ($D_W = 4.0 \Rightarrow D = 3.0 > B$)

$$D_w = 2.0: \;\; \bar{\gamma} = \frac{1}{2}\left[18(1) + 9.19(1)\right] = \mathbf{13.595}\ \mathrm{kN/m^3},\ q = 18\ \mathrm{kPa}$$

$$D_w = 4.0: \;\; q = 18\ \mathrm{kPa},\ \gamma = 18\ \mathrm{kN/m^3}$$

Solution (2/2)

SOLUTION

D_w (m)	Condition	q (kPa)	γ third term	q_u (kPa)
0.5	I	13.59	9.19	**652.1**
1.0	I	18.00	9.19	**729.3**
2.0	III	18.00	13.59	**747.2**
4.0	negligible	18.00	18.00	**765.1**

With water at 0.5 m, q_u = 652, and when deep, 765 kPa (15% difference). The source of the difference is **only q and γ**: the cohesion term (376.6 kPa) does not change at all.

Check the solution in GeoTerva ↗

Problem statement

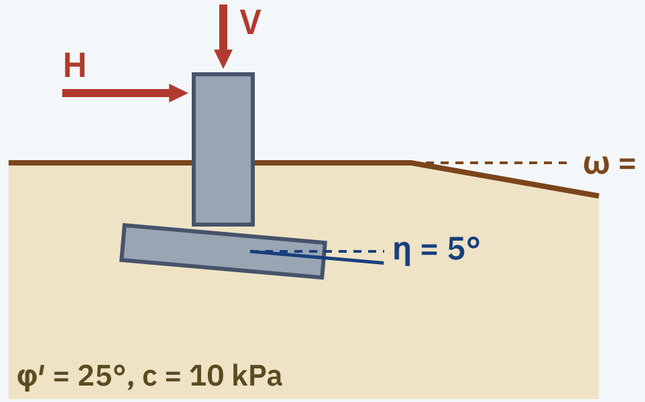
EXAMPLE

Given

Foundation	**square, B = L = 2.0 m, base depth D_f = 1.0 m**
Soil	$c = 10\ \mathrm{kPa},\ \varphi' = 25^\circ$
Unit weight	$\gamma = 18\ \mathrm{kN/m^3}$**; no groundwater**
Load case	$v = q_{apply} = 150\ \mathrm{kPa},\ h = 30\ \mathrm{kPa}$
Base, soil	**η = 5°, ω = 10°**
Safety	**FS = 3**

REQUIRED

Find the values of $\mathbf{q_u}$ and $\mathbf{q_{all}}$; check whether the footing is adequate for $q_{applied}$ = 150 kPa; check sliding.



Solution (1/3)

SOLUTION

1 Slope coefficients (v + c cot φ = 171.45 kPa; base inclined: 0.7 → 0.7 − 5/450 = 0.6889)

$$x = \frac{30}{171.45} = 0.175 \qquad i_q = (1 - 0.5x)^5 = \mathbf{0.633} \qquad i_c = \mathbf{0.595}$$

$$i_\gamma = (1 - 0.6889x)^5 = \mathbf{0.526}$$

2 Base and ground slope factors (η = 5°, ω = 10°)

$$b_c = 0.966,\ b_q = 0.922,\ b_\gamma = 0.896$$

$$g_c = 0.932,\ g_q = g_\gamma = (1 - 0.5\tan 10^\circ)^5 = 0.630$$

Solution (2/3)

SOLUTION

3 Three terms (shape and depth factors as in Example 6)

$$c\text{ term} = 10(20.72)(1.515)(1.20)(0.595)(0.966)(0.932) = 201.6$$

$$q\text{ term} = 18(10.66)(1.423)(1.155)(0.633)(0.922)(0.630) = 116.0$$

$$\gamma\text{ term} = \tfrac{1}{2}(18)(2)(6.758)(0.6)(1)(0.526)(0.896)(0.630) = 21.7$$

4 Bearing capacity and allowable pressure

$$q_u = 201.6 + 116.0 + 21.7 = \mathbf{339.3}\ \text{kPa} \qquad q_{all,g} = \frac{339.3}{3} = \mathbf{113.1}\ \text{kPa}$$

Solution (3/3)

SOLUTION

5 Check (q_{app} = 150 kPa)

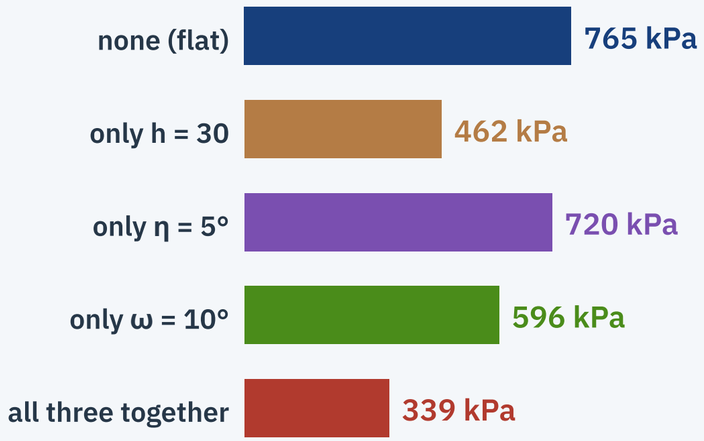
$$q_{apply} = 150 > q_{all,g} = 113.1 \;\Rightarrow\; \textbf{does not pass}$$

$$FS_{\text{actual}} = \frac{339.3}{150} = \mathbf{2.26} < 3$$

6 Slip boundary

$$h = 30 \le v\tan\varphi + c = 79.9\ \text{kPa} \;\Rightarrow\; \text{no slip warning}$$

q_u, which is 765 kPa for the level model, drops to **339 kPa** (56%) under this load case. Individually: horizontal load **−40%**, ground slope −22%, base slope −6%. The footing is not acceptable (FS = 2.26 < 3); remedies: increase B or D_f, reduce h, level the base.



Check the solution in GeoTerva ↗

φ = 0: the cohesion term is written additively

THEORY

At φ = 0 the multiplicative cohesion term is not used. Hansen writes it as **additive**: corrections are added to or subtracted from 1, so that the cohesion term does not vanish when h, η and ω are zero. $N_c = 5.14$, $N_q = 1$, $N_\gamma = 0$:

$$q_u = c\,N_c\,(1 + s'_c + d'_c - i'_c - b'_c - g'_c) + q\,g_q$$

$$s'_c = 0.2\frac{B}{L} \quad d'_c = 0.4k \quad i'_c = 0.5\left(1 - \sqrt{1 - \frac{h}{c}}\right) \quad b'_c = \frac{\eta^\circ}{147^\circ} \quad g'_c = \frac{\omega^\circ}{147^\circ}$$

The surcharge term reduces to q g_q: on level ground, only q. There is no self-weight term ($N_\gamma = 0$).

h < c must hold; if h ≥ c the base slides and the module stops. The bracket must also remain positive (if h, η and ω are all very large, the cohesion term would become negative).

EXAMPLE: c = 40 kPa, 2 m square, D_f = 1 m

Coefficient	Value
$s'_c = 0.2(B/L)$	0.2
$d'_c = 0.4\ k$ (k = 0.5)	0.2
i'_c, b'_c, g'_c	0
parenthesis	1.4

$q_u = 40 \times 5.14 \times 1.4 + 18 =$ **305.84 kPa**

Problem statement

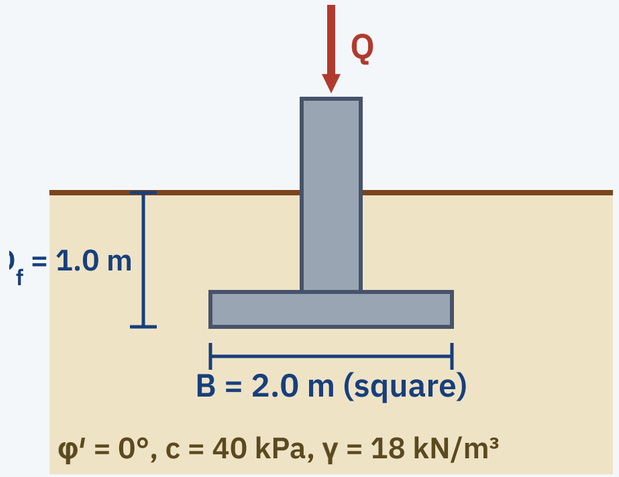
EXAMPLE

Given

Foundation	**square B = L = 2 m, D_f = 1 m**
Soil	**clay:** $\varphi = 0,\ c_u = 40\ \mathrm{kPa},\ \gamma = 18\ \mathrm{kN/m^3}$**; no groundwater**
Cases	**(a) vertical load · (b)** $v = 100,\ h = 10\ \mathrm{kPa}$ **· (c)** $h = 10$ **and** $\eta = \omega = 10^\circ$

REQUIRED

Find the q_u value for each case.



Solution (1/2)

SOLUTION

1 (a) Vertical load: bracket $1 + s'_c + d'_c = 1 + 0.2 + 0.2$

$$q_u = 40(5.14)(1.4) + 18 = 287.84 + 18 = \mathbf{305.84}\ \text{kPa}$$

2 (b) h = 10 kPa (h/c = 0.25 < 1)

$$i'_c = 0.5\left(1 - \sqrt{0.75}\right) = 0.0670 \ \Rightarrow\ q_u = 40(5.14)(1.4 - 0.0670) + 18 = \mathbf{292.07}\ \text{kPa}$$

Solution (2/2)

SOLUTION

3 (c) h = 10 kPa, η = ω = 10°: $b'_c = g'_c = 10/147 = 0.068$; $g_q = (1 - 0.5 \tan 10°)^5$

$$1.4 - 0.067 - 0.068 - 0.068 = 1.197$$

$$q_u = 40(5.14)(1.197) + 18(0.630) = 246.1 + 11.3 = \mathbf{257.4}\ \mathrm{kPa}$$

Condition	parenthesis	c N_c(...)	q g_q	q_u (kPa)
(a) vertical	1,400	287.8	18.0	**305.8**
(b) h = 10	1.333	274.1	18.0	**292.1**
(c) h, η, ω	1.197	246.1	11.3	**257.4**

The surcharge h = 10 kPa alone reduces the bearing capacity by 5%, and together with h the base and ground inclination reduce it by 16%. Thanks to the additive form, the cohesion term does not vanish in the flat, vertical case: 305.84 kPa.

Check the solution in GeoTerva ↗

Problem statement

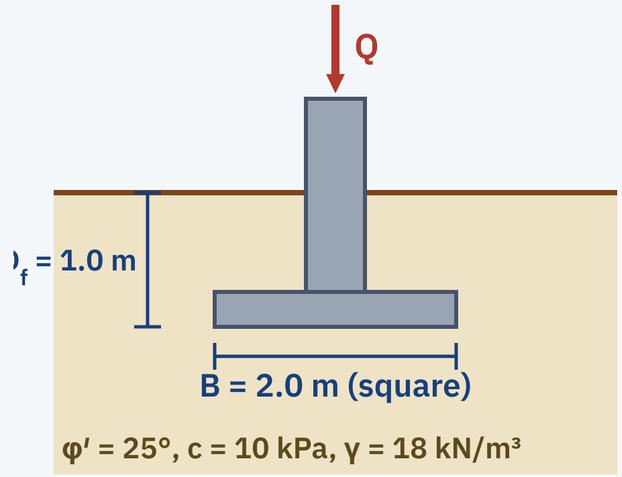
EXAMPLE

Given

Foundation	**square, B = L = 2.0 m, base depth D_f = 1.0 m**
Soil	$c = 10\ \mathrm{kPa},\ \varphi' = 25^\circ$
Unit weight	$\gamma = 18\ \mathrm{kN/m^3}$**; no groundwater**
Comparison	**Vesic equation: De Beer shape, Vesic N_γ, $d_c = d_q - (1 - d_q)/(N_c \tan \varphi)$**
Load case (supplementary)	**Example 8: q_{app} = 150, h = 30 kPa, η = 5°, ω = 10°**

REQUIRED

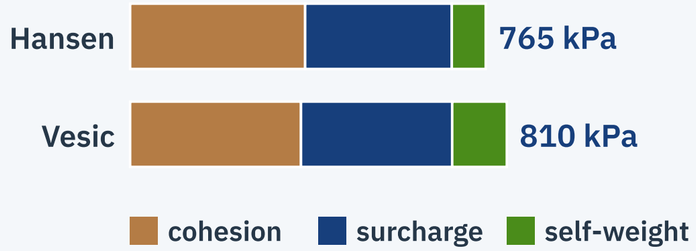
Compare the q_u values of the two equations term by term; then also for the load case of Example 8.



Solution (1/2)

SOLUTION

	Hansen	Vesic
N_γ	6.76	10.88
s_q	1.423	1.466
d_c	1.200	1.172
c term	376.6	367.7
q term	315.5	325.2
γ term	73.0	117.5
q_u (kPa)	**765.1**	**810.3**



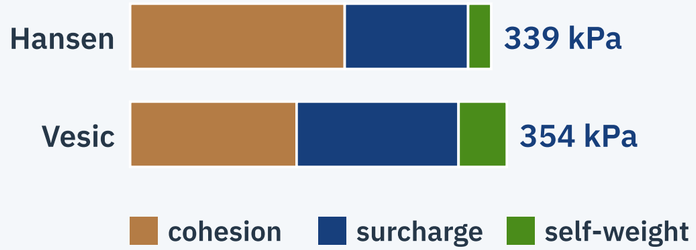
Hansen is **5.6% smaller** (765 and 810 kPa). The difference lies almost entirely in the self-weight term (73 and 117 kPa): N_γ is small.

Solution (2/2)

SOLUTION

Coefficient	Hansen	Vesic
i_c, i_q, i_γ	0.595 · 0.633 · 0.526	0.723 · 0.749 · 0.618
b_c, b_q, b_γ	0.966 · 0.922 · 0.896	0.912 · 0.920 · 0.920
g_c, g_q, g_γ	0.932 · 0.630 · 0.630	0.645 · 0.678 · 0.678
q_u (kPa)	**339.3**	**354.0**

For the same load case the difference is 4.3%: Vesic's inclination exponents m = 1.5 (square) and the forms b = $(1 - \eta \tan \varphi)^2$, g = $(1 - \tan \omega)^2$ give slightly **less penalty** than Hansen's. Do not mix the two equations; each method's coefficients are consistent with its own N_γ.



Check the solution in GeoTerva ↗

Summary

Key points

Hansen equation: $q_u = cN_c(s\ d\ i\ b\ g) + qN_q(s\ d\ i\ b\ g) + \tfrac{1}{2}\gamma BN_\gamma(s\ d\ i\ b\ g)$; in the ordinary case $i = b = g = 1$

$N_\gamma = 1.5(N_q - 1)\tan\varphi$: smaller than Vesic's ($\varphi' = 25°$: 6.76 and 10.88); the coefficients of the method are used together

Shape: $s_c = 1 + (B/L)(N_q/N_c)$, $s_q = 1 + (B/L)\sin\varphi$, $s_\gamma = 1 - 0.4\ B/L$ · **Depth**: $k = D_f/B$ or $\tan^{-1}(D_f/B)$; step at $D_f/B = 1$

Inclination: with true h and v; $i_q = (1 - 0.5h/(v + c\cot\varphi))^5$, $i_\gamma = (1 - 0.7h/(\ldots))^5$; warning for sliding when $h > v\tan\varphi + c$, stoppage when $h \geq v + c\cot\varphi$

Base inclination $b_q = e^{-2\eta\tan\varphi}$, $b_\gamma = e^{-2.7\eta\tan\varphi}$; **ground inclination** $g_q = g_\gamma = (1 - 0.5\tan\omega)^5$ (Vesic's $(1 - \tan\omega)^2$ differs)

Undrained ($\varphi = 0$): $cN_c(1 + s' + d' - i' - b' - g') + qg_q$; $N_c = 5.14$, $N_\gamma = 0$; $h < c$

Groundwater changes only q and γ; **check**: $q_{applied} \leq q_{all,g}$, FS_{actual} and net

References

Hansen's complete bearing-capacity equation — his own Nγ, shape factors, inclination factors, and base-tilt and ground-slope factors — follows Hansen, J.B., A Revised and Extended Formula for Bearing Capacity, Danish Geotechnical Institute Bulletin 28, 1970, as restated consistently across standard geotechnical references. The depth factors used across this whole section are Hansen's, as set out in Das, B.M., Principles of Geotechnical Engineering, 9th ed., Cengage Learning, 2018 (Chapter 16).

NOTE

Some of the figures in this presentation are taken from the GeoTerva Theory page; all numerical examples were produced with the module's own calculation library, and the module gives the same values.

GeoTerva

Online geotechnical analysis

From theory to practice

Compute the example foundation in this lecture step by step in GeoTerva: the module opens with this model.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com