Geotechnical lecture slide · Retaining walls

Gravity retaining wall

Lecture: external stability of a stone or concrete gravity retaining wall; the wall and the soil moving with it, active and passive earth pressure, water, uplift and earthquake; sliding, overturning and bearing capacity checks; a worked example on each topic.

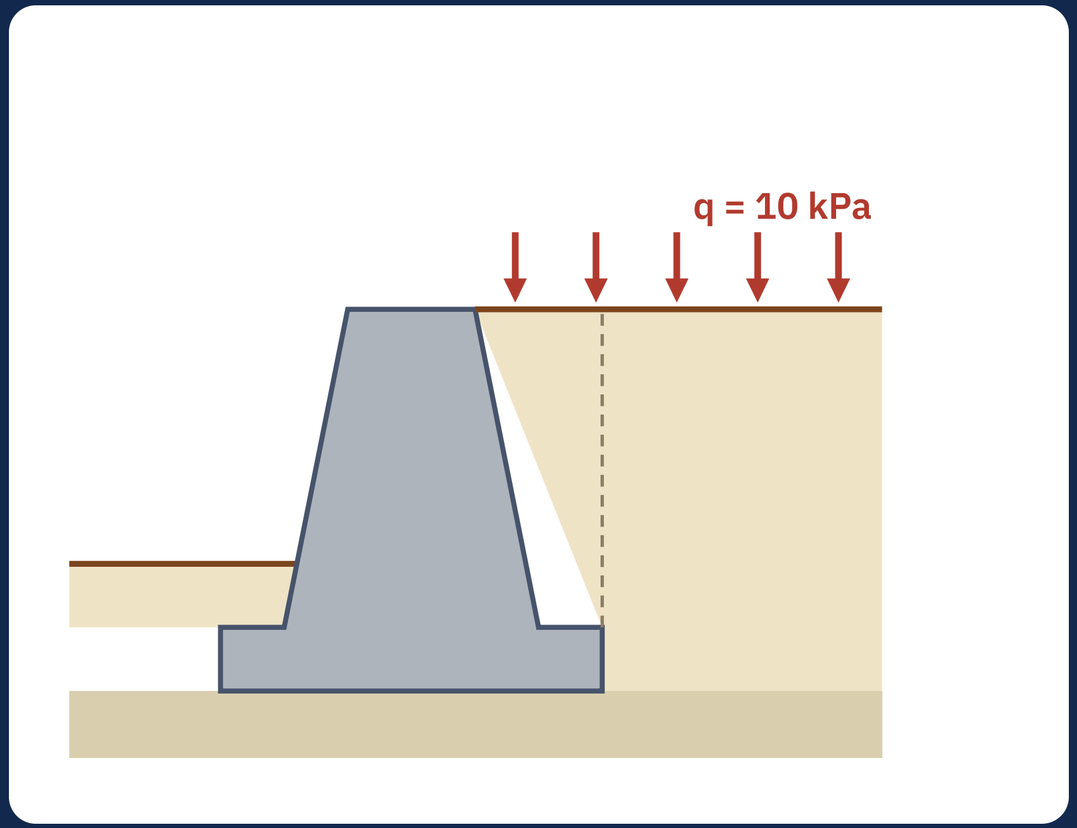
Sliding · overturning · bearing capacity

Rankine · Coulomb · Mononobe–Okabe

Water and uplift

Inertia in earthquakes

Force table



Introduction

What will we learn in this lecture?

- Computing the **geometry** and weight of a gravity wall by dividing it into segments (area, centroid, moment arm)
- The **soil block** moving together with the wall and the **vertical pressure plane** passing through the rear edge of the footing
- **Active** (Rankine = Coulomb, on a vertical plane), surcharge, **passive front resistance** (K_{mob}) and **water + uplift** forces
- **Sliding, overturning and bearing capacity** checks using the **force table** and a single sign convention ($M = x \cdot F_z - z \cdot F_x$)
- In an earthquake, the **Mononobe–Okabe** thrust, the seismic increment and the wall inertia
- How to read the result: factors of safety, eccentricity, base stresses

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Everything is per 1.00 m length of wall (kN/m, kN·m/m); pressures in kPa. Example wall: H = 3 m stone masonry wall (22 kN/m³).

What is solved, how is it drawn?

THEORY

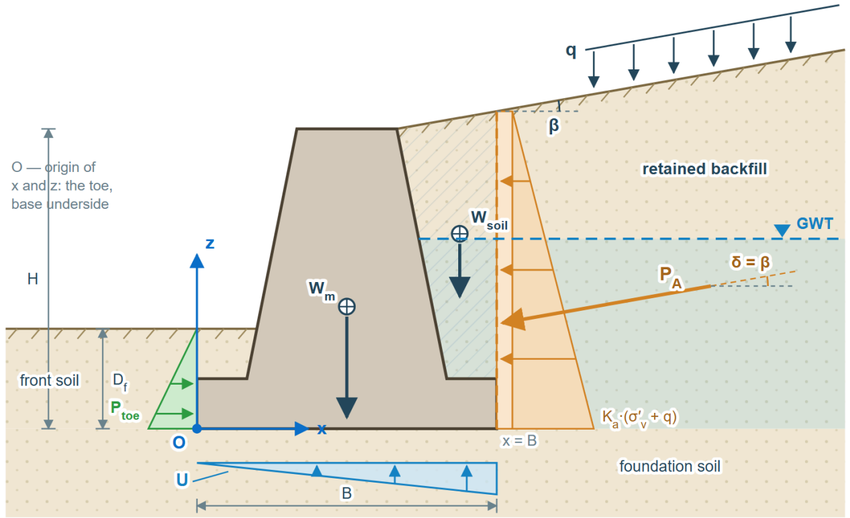
A gravity wall resists sliding and overturning **by its own weight**. The module examines a stone or mass-concrete wall with an inclined trapezoidal stem and stepped footings on both sides:

$$B = W_{bl} + W_{dl} + W_u + W_{dr} + W_{br}$$

Wall: H total height, W_u top width, W_{dl}, W_{dr} front and rear slope offsets, W_{bl}, W_{br} toe projections, H_{fl}, H_{fr} toe heights; unit weight 22 for stone, 24 kN/m³ for concrete.

Soil: cohesionless fill (c′ = 0), inclined surface β, surcharge q, groundwater, and soil and water of height D_f in front.

The **three theories**: Rankine, Coulomb (static), Mononobe–Okabe (earthquake). **Checks**: sliding, overturning, eccentricity, bearing capacity.



Free body, force table, single sign convention

THEORY

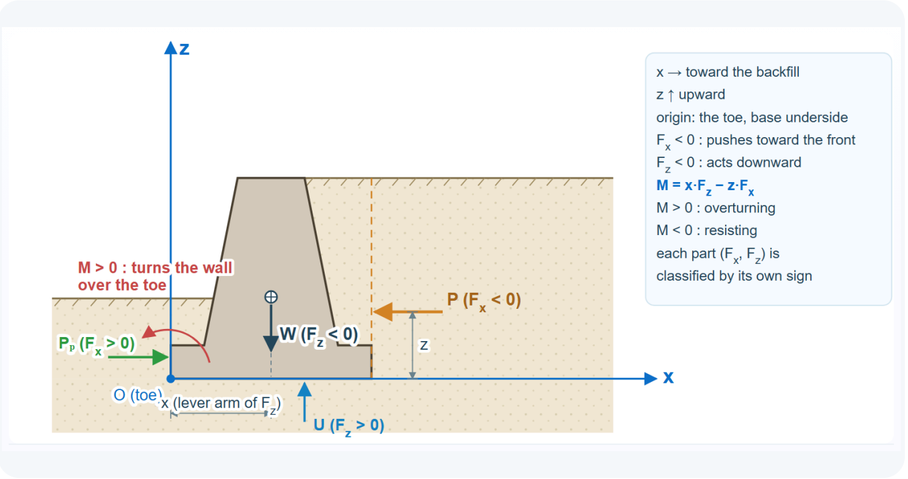
Stability is an **equilibrium** problem: first one decides what is in equilibrium and how the forces are to be signed. The origin O is at the **toe** (front bottom corner), x toward the fill, z upward. A force is written by its components (F_x, F_z) and its point of application; the moment about O:

$$M = x\,F_z - z\,F_x \qquad M > 0:\ \text{overturning}, \quad M < 0:\ \text{bond}$$

For weight ($F_z = -W$, $x > 0$), $M = -xW < 0$: it holds the wall. For a forward thrust ($F_x < 0$), $M = -zF_x > 0$: it overturns the wall. Uplift ($F_z > 0$) overturns it, front resistance ($F_x > 0$) holds it.

No force is classified in advance as "resisting"; each component is counted with its own sign (the vertical component of the inclined thrust is correctly resisting).

Free body = wall + the soil moving with it; **in the force table** every item is read identically by all three checks: no force may be omitted in one and counted in another.



Breaking into parts, centers of gravity and arms

THEORY

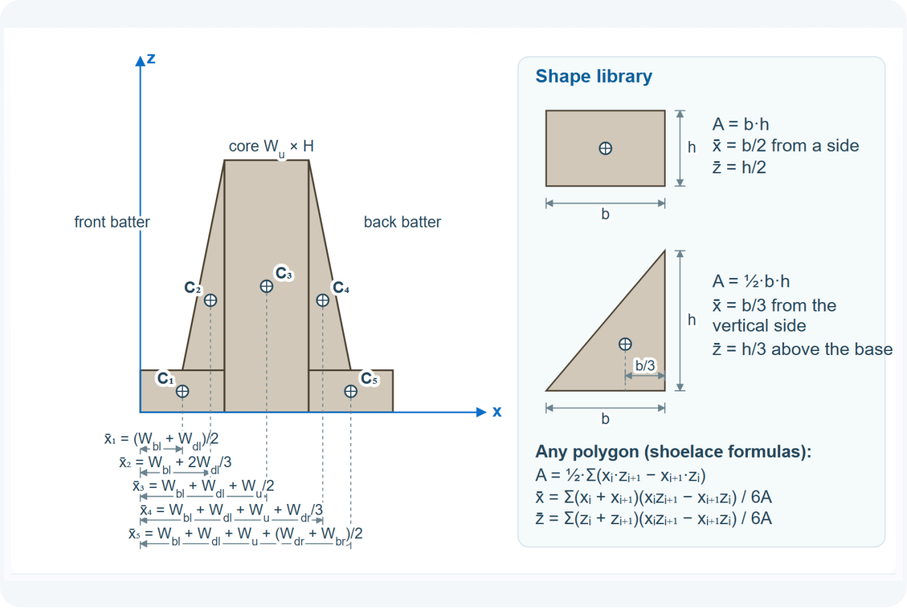
The weight of the wall and its moment about the toe are also the basis of both stability checks. The cross-section is divided into simple shapes with known area and centroid (toe, front slope, core, back slope, heel):

$$W_i = \gamma_m A_i \qquad W_m = \sum W_i \qquad \bar{x}_m = \frac{\sum W_i\,\bar{x}_i}{W_m} \qquad \bar{z}_m = \frac{\sum W_i\,\bar{z}_i}{W_m}$$

In the force table the wall is a single item: $F_z = -W_m$ at $\bar{x}_m$; its moment is $M = -W_m\bar{x}_m = -\Sigma M_i$.

The height $\bar{z}_m$ is required only for the horizontal inertia force $k_h W_m$ (earthquake).

The module also computes the entire section as a single polygon; the two agree within 10^{-9}.



Problem statement

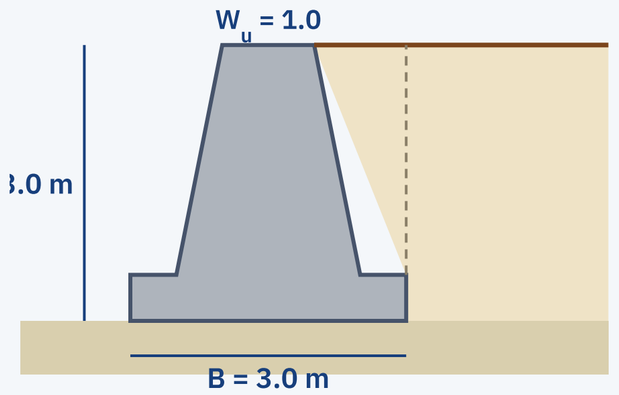
EXAMPLE

REQUIRED

Area and weight of the five parts; W_m, $\bar{x}_m$, $\bar{z}_m$.

Given

Section	$H = 3.0$ m; $W_u = 1.0$ m; $W_{dl} = W_{dr} = 0.5$ m; $W_{bl} = W_{br} = 0.5$ m; $H_{fl} = H_{fr} = 0.5$ m **(B = 3.0 m)**
Material	**stone wall:** $\gamma_m = 22\ \mathrm{kN/m^3}$



Solution

SOLUTION

1 Parts (e.g. front slope: $A = \tfrac{1}{2}(0.5)(3.0 - 0.5) = 0.625$; $\bar{x} = W_{bl} + 2W_{dl}/3 = 0.833$; $\bar{z} = H_{fl} + (H - H_{fl})/3 = 1.333$)

Piece	A (m²)	x̄ (m)	z̄ (m)	W = 22A (kN/m)	W·x̄	W·z̄
Toe	0.500	0.500	0.250	11.00	5.50	2.75
Front slope	0.625	0.833	1.333	13.75	11.46	18.33
Body (core)	3.000	1.500	1.500	66.00	99.00	99.00
Rear slope	0.625	2.167	1.333	13.75	29.79	18.33
Heel	0.500	2.500	0.250	11.00	27.50	2.75
Total	**5.250**			**115.50**	**173.25**	**141.17**

2 Center of gravity of the wall

$$\bar{x}_m = \frac{173.25}{115.50} = \mathbf{1.500\ m} \qquad \bar{z}_m = \frac{141.17}{115.50} = \mathbf{1.222\ m}$$

The weight of the wall is **W_m = 115.50 kN/m**, $\bar{x}_m$ = 1.50 m from the toe, $\bar{z}_m$ = 1.22 m from the base; its moment is M = −173.25 kN·m/m (restraining).

Check the solution in GeoTerva ↗

Vertical pressure plane and soil block

THEORY

Why does the earth pressure act on the **rear edge of the base** rather than on the rear slope of the wall? The soil above the heel cannot slide relative to the wall: if the wall moves, it moves with it. The failure planes in the fill start from the **rear edge of the heel**, not from the rear slope. Consistent free body: the wall plus this soil block; the **external pressure plane** on its outer face is vertical at x = B:

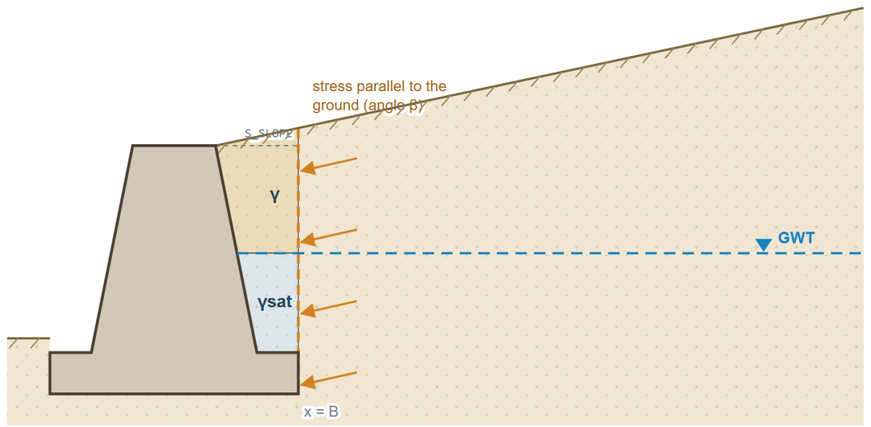
The plane runs from the surface above the back edge down to below the base; in an inclined fill it also includes the rise of the surface. Everything between the plane and the back of the wall is **weight**; the plane against the soil is soil against soil: wall friction is not counted, and the thrust acts with δ = β.

At the front, the fill above the toe also moves with the wall: vertical load W_{toe}; entering D_f = 0 means the front soil may be taken into account.

The vertical part of the **surcharge** over the heel (Q_{heel}) is favorable but, since it is regarded as a load that may or may not be present, it **enters no check**; its lateral effect, however, is always counted.

The calculation is done with total stress: the submerged soil is weighed with γ_{sat}, and the water itself is added as a separate force (U_{rear}, uplift); γ′ and uplift are not taken together.

Vertical plane through the back edge of the footing (all methods)



Problem statement

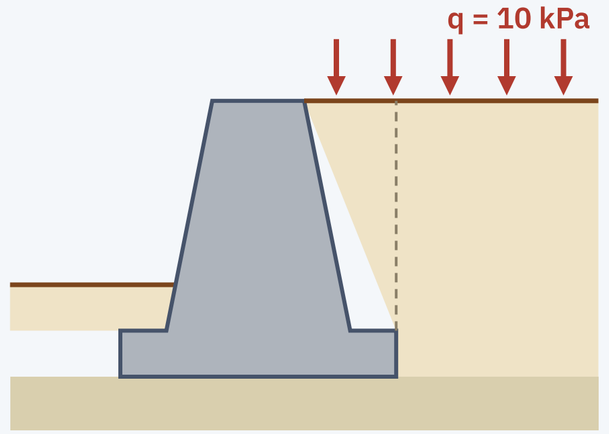
EXAMPLE

Given

Wall	**The cross-section in Example 1; front ground height $D_f = 1.0$ m**
Soil	**dry fill: $\gamma = 18\ \mathrm{kN/m^3}$, $\varphi' = 30^\circ$; horizontal surface**
Surcharge	$q = 10$ **kPa (at the surface)**

REQUIRED

Weight and location of the **soil block** moving with the wall; **front fill**; why does $\mathbf{Q_{heel}}$ not enter the checks?



Solution (1/2)

SOLUTION

1 Rectangular area above the heel + triangle beside the rear slope (up to the plane x = B = 3.0)

$$A_r = W_{br}(H - H_{fr}) = 0.5(2.5) = 1.250\ \text{m}^2\ (\bar{x} = 2.75,\ \bar{z} = 1.75)$$

$$A_t = \tfrac{1}{2} W_{dr}(H - H_{fr}) = 0.625\ \text{m}^2\ (\bar{x} = 2.333,\ \bar{z} = 2.167)$$

$$W_s = \gamma(A_r + A_t) = 18(1.875) = \mathbf{33.75}\ \text{kN/m}$$

2 Location

$$\bar{x}_s = \frac{1.250(2.75) + 0.625(2.333)}{1.875} = \mathbf{2.611}\ \text{m} \qquad \bar{z}_s = \mathbf{1.889}\ \text{m}$$

Solution (2/2)

SOLUTION

3 Front fill (above the toe, D_f = 1.0 m) and surcharge

$$A_{toe} = \tfrac{1}{2}(0.5 + 0.60)(0.5) = 0.275\ \mathrm{m}^2 \quad W_{toe} = 18(0.275) = \mathbf{4.95}\ \mathrm{kN/m}\ (\bar{x} = 0.276)$$

$$Q_{heel} = q(B - x_{BT}) = 10(3 - 2) = \mathbf{10.0}\ \mathrm{kN/m}\ (\text{not subject to any check})$$

Total vertical load N = W_m + W_s + W_{toe} = 115.50 + 33.75 + 4.95 = **154.20 kN/m**. Q_{heel} = 10.00 kN/m is the item marked "unused" in the table.

Check the solution in GeoTerva ↗

Rankine and Coulomb, on a vertical plane

THEORY

If the fill surface is inclined at β, in Rankine's semi-infinite slope solution the stress on the vertical plane is **parallel to the surface**: $\sigma'_a = K_{a\beta}\sigma'_v$:

$$K_{a\beta} = \cos\beta\,\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi'}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi'}} \qquad K_a = \frac{1-\sin\varphi'}{1+\sin\varphi'}\ (\beta = 0)$$

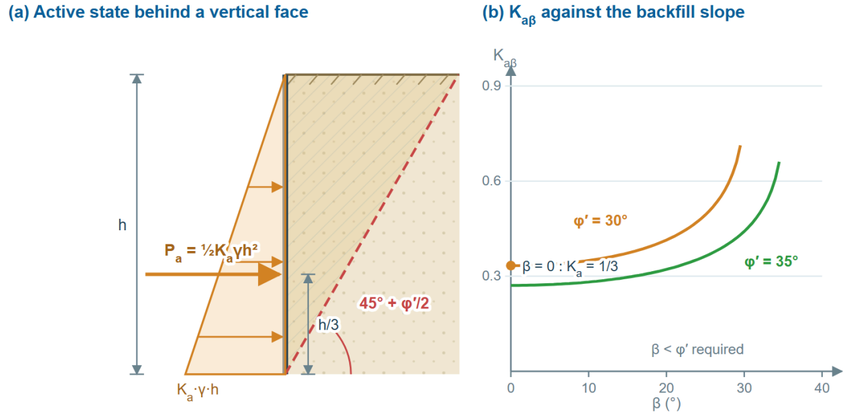
$$P_A = \tfrac{1}{2}K_a\gamma h^2\ \ (h/3) \qquad P_Q = K_a q h\ \ (h/2)$$

The square root requires β < φ′; if β ≥ φ′ the slope itself is unstable and the calculation is blocked.

Coulomb: for a vertical plane (α = 90°) and δ = β, K_a is **exactly equal** to the Rankine $K_{a\beta}$ **for every β**; the two methods give the same pressure on this plane. h = height of the plane (including the rise of the surface).

Horizontal component P cos β, vertical component P sin β (downward on a rising slope: resisting).

The surcharge is calculated twice (with and without q); P_Q is the difference.



Problem statement

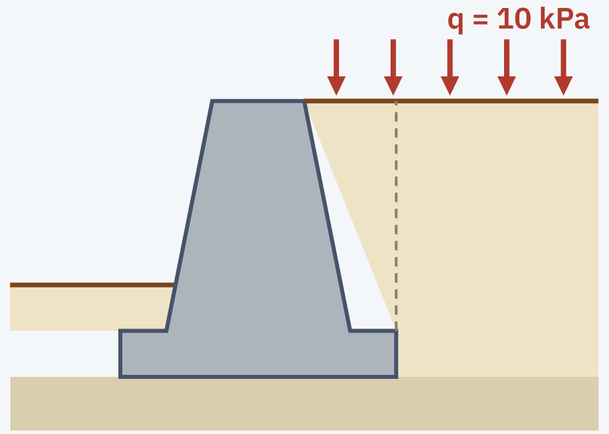
EXAMPLE

REQUIRED

With $\mathbf{K_a}$, the soil thrust $\mathbf{P_A}$ and surcharge thrust $\mathbf{P_Q}$ and their moments.

Given

Wall and soil	**Wall and fill in Example 2:** $h = H = 3.0$ **m;** $\gamma = 18\ \mathrm{kN/m^3}$, $\varphi' = 30^\circ$**; horizontal surface**
Surcharge	$q = 10$ **kPa**



Solution

SOLUTION

1 K_a = (1 − sin30°)/(1 + sin30°) = 0.3333; the thrusts (in the vertical plane, horizontal)

$$P_A = \tfrac{1}{2} K_a \gamma h^2 = \tfrac{1}{2}(0.3333)(18)(9) = \mathbf{27.00}\ \mathrm{kN/m} \quad (z = 1.00\ \mathrm{m})$$

$$P_Q = K_a q h = 0.3333(10)(3) = \mathbf{10.00}\ \mathrm{kN/m} \quad (z = 1.50\ \mathrm{m})$$

2 Total horizontal thrust and overturning moment (about the toe)

$$\Sigma H_{drive} = 27.0 + 10.0 = \mathbf{37.0}\ \mathrm{kN/m}$$

$$M_o = 27.0(1.00) + 10.0(1.50) = 27.0 + 15.0 = \mathbf{42.0}\ \mathrm{kN{\cdot}m/m}$$

Soil thrust 27.00, surcharge thrust 10.00 kN/m: total **37.00 kN/m**, about the toe **42.00 kN·m/m**.

Check the solution in GeoTerva ↗

Problem statement

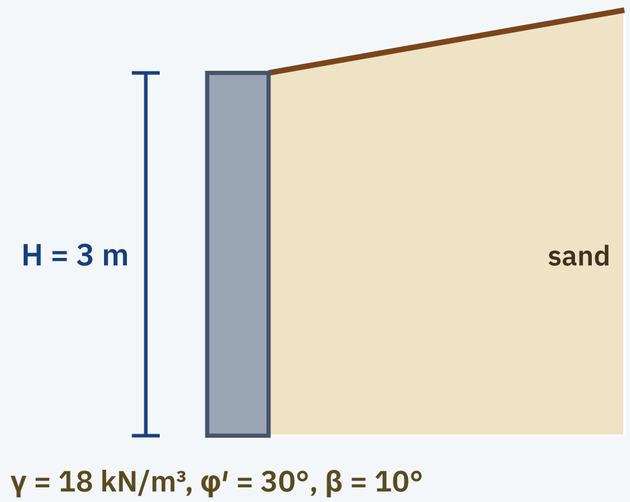
EXAMPLE

Given

Wall and soil	**The wall of Example 2; the same backfill but with the surface rising $\beta = 10^\circ$ (away from the wall)**
Surcharge	$q = 10$ **kPa**

REQUIRED

$\mathbf{K_{a\beta}}$, thrust components; variation of the result relative to the horizontal fill (sliding, overturning, eccentricity).



Solution (1/2)

SOLUTION

1 Coefficient $K_{a\beta}$ ($\cos\beta = 0.9848$, $\sqrt{(\cos^2\beta - \cos^2\varphi')} = 0.4689$)

$$K_{a\beta} = \cos\beta\,\frac{\cos\beta - r}{\cos\beta + r} = 0.9848\,\frac{0.5159}{1.4537} = \mathbf{0.3495}\ \ (\text{horizontally } 0.3333)$$

2 Driving forces (h = height of the plane = H + rise of the surface; engine)

$$P_A:\ F_x = 31.25,\ F_z = 5.51\ \text{kN/m}\ \ (\text{horizontal and vertical})$$

$$P_Q:\ F_x = 10.93,\ F_z = 1.93\ \text{kN/m}$$

Solution (2/2)

SOLUTION

3 Checks (engine)

$$\Sigma F_z = -N = -163.23\ \text{kN/m}\ \ (\text{soil block } 35.34)$$

$$FS_s = 2.412 \qquad FS_o = 5.734 \qquad e = 0.037\ \text{m} \qquad q_{max} = 58.4\ \text{kPa}$$

Raising the slope increases the thrust ($K_{a\beta}$ 0.333 → 0.350), but the **vertical component of the thrust** and the heavier soil block are stabilizing: slip FS 2.62 → 2.41, overturning FS 6.26 → 5.73; eccentricity 0.068 → 0.037 m.

Check the solution in GeoTerva ↗

Combined coefficient between at-rest and passive

THEORY

The soil in front of the wall (c′ = 0) is defined by a **blend** of the Rankine passive and Jaky at-rest coefficients; full passive resistance would require a displacement magnitude that a functional wall would not allow:

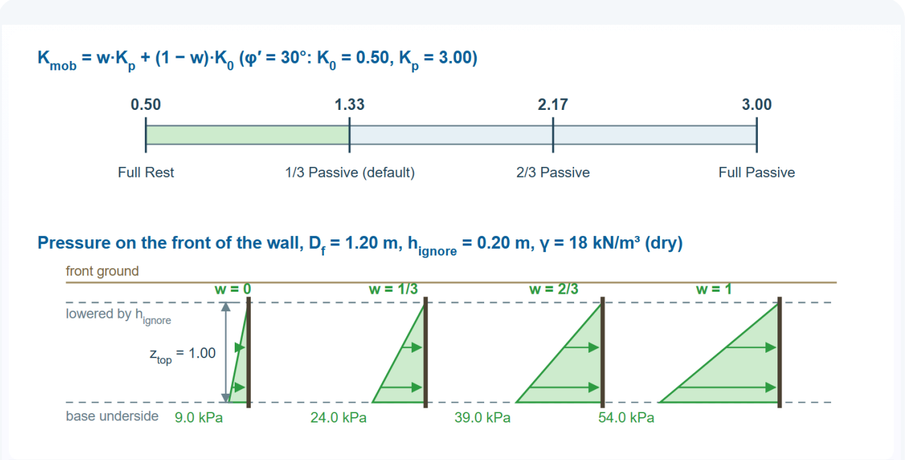
$$K_{mob} = w\,K_p + (1-w)\,K_0 \qquad K_p = \tan^2(45^\circ + \varphi'/2) \qquad K_0 = 1 - \sin\varphi'$$

$$p(z) = K_{mob}\,\sigma'_v(z) \qquad P_P = \tfrac{1}{2}K_{mob}\gamma D_f^2 \;\; (D_f/3)$$

w = 0 full at-rest, 1/3 (default), 2/3, 1 full passive. The top **h_{ignore}** thickness of the front soil (excavatable, may soften) is not taken into account.

Front resistance is credited **only in sliding** (full in static, with $\eta_{p,E}$ in earthquakes); it is **never** used in **overturning or bearing capacity**.

The thrust of the front water (h_{wf}) is directed towards the back and reduces the net thrust.



Problem statement

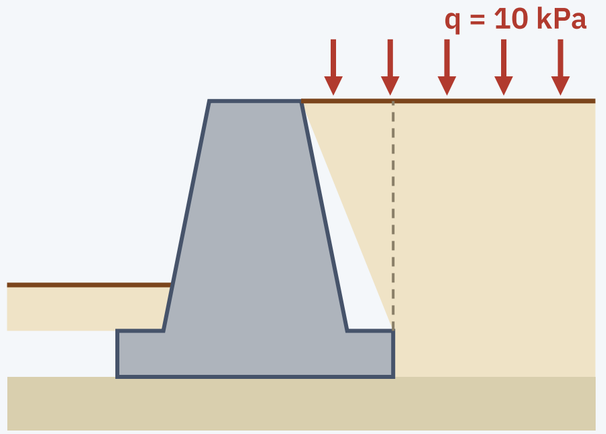
EXAMPLE

REQUIRED

K_{mob}, P_P and the sliding factor of safety FS_S (for the wall in Example 2).

Given

Front ground	$D_f = 1.0$ **m;** $\gamma = 18\ \mathrm{kN/m^3}$, $\varphi' = 30°$**;** $h_{ignore} = 0$
Cases	**full at-rest, 1/3, 2/3, full passive; and if the frontal resistance is not credited at all**



Solution

SOLUTION

1 K_{mob} and thrust (default w = 1/3)

$$K_{mob} = \tfrac{1}{3}(3.000) + \tfrac{2}{3}(0.500) = \mathbf{1.333}$$

$$P_P = \tfrac{1}{2} K_{mob} \gamma D_f^2 = \tfrac{1}{2}(1.333)(18)(1)^2 = \mathbf{12.0}\ \mathrm{kN/m} \quad (z = D_f/3 = 0.333\ \mathrm{m})$$

2 Four cases ($FS_s = (\mu_b N + P_P)/\Sigma H_{drive} = (84.81 + P_P)/37.0$)

Condition	w	K_{mob}	P_P (kN/m)	FS_s
Full at-rest	0	0.500	4.50	2.414
1/3 passive + 2/3 at-rest (default)	1/3	1.333	12.00	2.616
2/3 passive + 1/3 at-rest	2/3	2.167	19.50	2.819
Full passive	1	3.000	27.00	3.022
End resistance is not credited	–	–	0	2.292

Credit for the front resistance changes FS_s between 2.29 and 3.02 (default **2.62**); if the front soil can be excavated, the credit is reset to zero: D_f and h_{ignore} must be chosen deliberately.

Check the solution in GeoTerva ↗

Three checks from the same force table

THEORY

Sliding: the base is held by friction; required shear $\Sigma H_{drive} - \Sigma P_{resist}$, available shear $\mu_b N$ (no adhesion; N is the effective normal force, including uplift). Resistance is summed in the denominator:

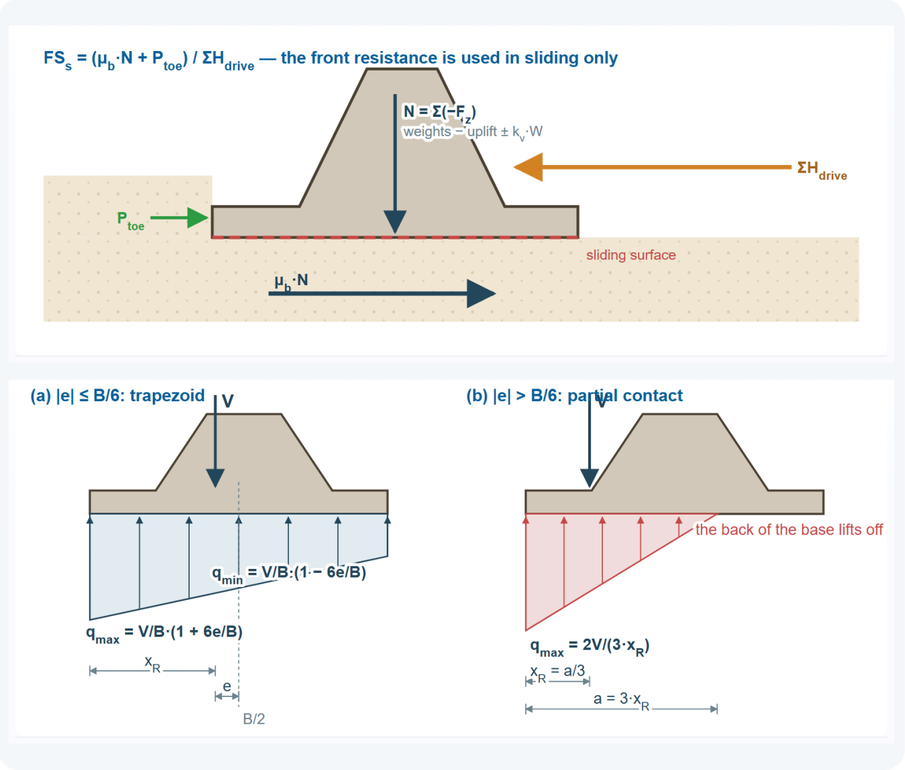
$$FS_s = \frac{\mu_b N + P_{toe}}{\Sigma H_{drive}} \qquad \mu_b = \tan \delta_b$$

Overturning is about the toe: the $-zF_x$ and xF_z components of each element enter $\Sigma M_{overturning}$ or $\Sigma M_{resisting}$ according to their sign; the front resistance is not included:

$$FS_o = \frac{\Sigma M_{res}}{\Sigma M_{ov}} \qquad x_R = -\frac{\Sigma M}{V} \qquad e = \frac{B}{2} - x_R$$

Bearing capacity: if $|e| \le B/6$, the base is fully in contact; otherwise the soil cannot take tension, and only a portion is in contact:

$$q_{max,min} = \frac{V}{B}\left(1 \pm \frac{6e}{B}\right) \qquad |e| > \frac{B}{6}: \quad q_{max} = \frac{2V}{3x_R}$$



Problem statement

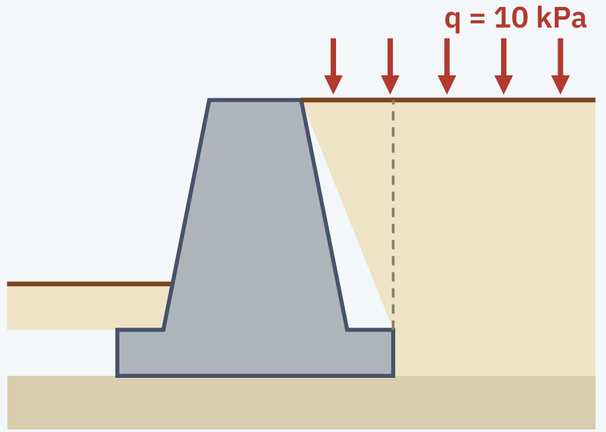
EXAMPLE

Given

Wall and loads	**Wall in Example 1–5:** $B = 3.0$ **m;** $W_m = 115.5,\ W_s = 33.75,\ W_{toe} = 4.95\ \mathrm{kN/m}$**;** $P_A = 27.0,\ P_Q = 10.0\ \mathrm{kN/m}$
Resistance	$\mu_b = 0.55$**; tip resistance** $P_P = 12.0$ **kN/m (w = 1/3);** $q_{all} = 200$ **kPa**
Required	$FS_s \geq 1.5,\ \ FS_o \geq 2.0$**; the base must be fully in contact**

REQUIRED

FS_s, FS_o, x_R, e, q_{max}, q_{min} and the result of each check.



Solution (1/3)

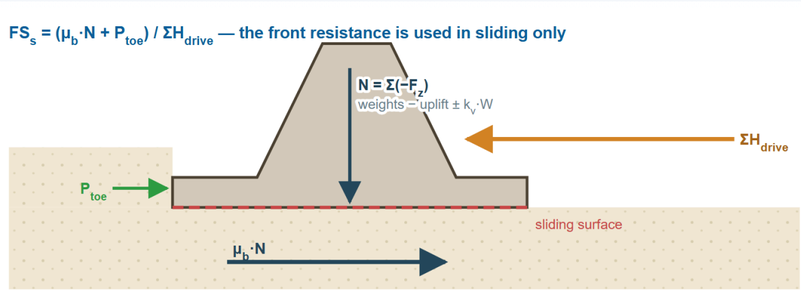
SOLUTION

1 Base friction: N = W_m + W_s + W_{toe} (excluding Q_{heel})

$$N = 115.50 + 33.75 + 4.95 = \mathbf{154.20}\ \text{kN/m}$$

$$\mu_b N = 0.55(154.20) = 84.81\ \text{kN/m}$$

$$FS_s = \frac{84.81 + 12.0}{37.0} = \mathbf{2.616}\ \geq 1.5\ \checkmark$$



Solution (2/3)

SOLUTION

2 Overturning moment (about the toe; active forces only)

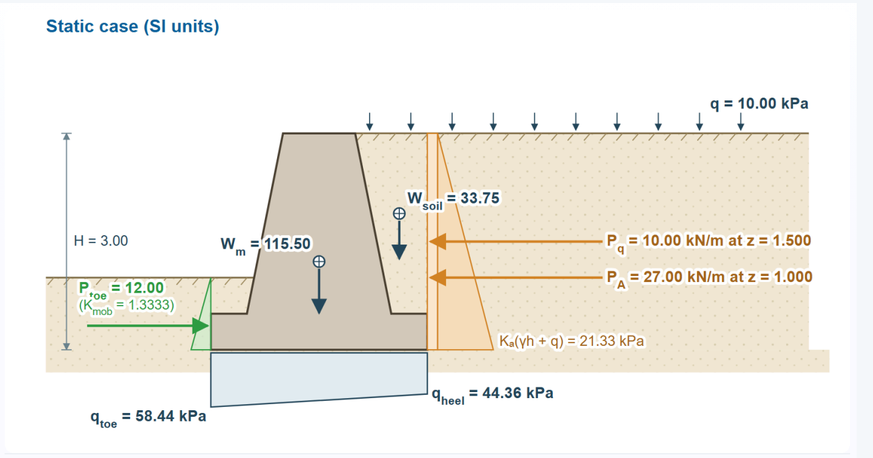
$$M_o = P_A(1.00) + P_Q(1.50) = 27.0 + 15.0 = \mathbf{42.0}\ \text{kN·m/m}$$

3 Resisting moment: wall + soil block + front fill

$$M_r = W_m\bar{x}_m + W_s\bar{x}_s + W_{toe}\bar{x}_{toe}$$

$$= 173.25 + 88.13 + 1.36 = \mathbf{262.74}$$

$$FS_o = \frac{262.74}{42.0} = \mathbf{6.256}\ \geq 2.0\ \checkmark$$



Solution (3/3)

SOLUTION

4 Location of the resultant and eccentricity

$$\Sigma M = M_o - M_r = 42.00 - 262.74 = -220.74$$

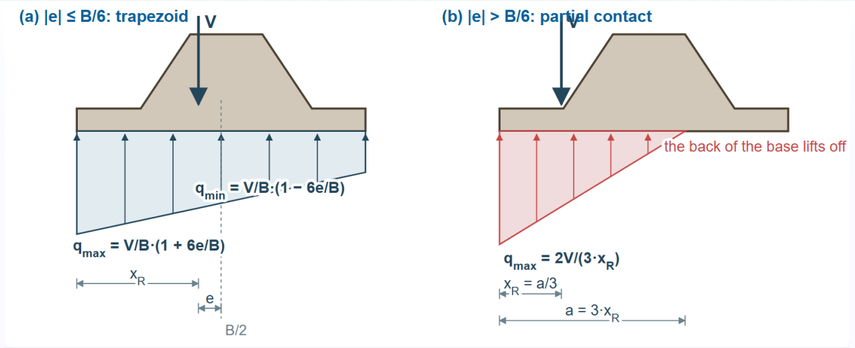
$$x_R = -\frac{\Sigma M}{V} = \frac{220.74}{154.2} = \mathbf{1.4315}\ \text{m}$$

$$e = \frac{B}{2} - x_R = \mathbf{0.0685}\ \text{m} \ \leq\ \frac{B}{6} = 0.500\ \checkmark$$

5 Base stresses (full contact)

$$q_{max} = \frac{154.2}{3}\left(1 + \frac{6(0.0685)}{3}\right) = \mathbf{58.44}\ \text{kPa}$$

$$q_{min} = \mathbf{44.36}\ \text{kPa}$$



FS_s = **2.62**, FS_o = **6.26**, e = 0.068 m, q_{max} = **58.4 kPa** ≤ 200, q_{min} = 44.4 kPa > 0: **all checks are satisfied in the static case**.

Check the solution in GeoTerva ↗

Hydrostatic water on both sides, linear uplift at the base

THEORY

Water pressure is isotropic and acts perpendicular to every surface it touches. The free body is surrounded by water on three boundaries: the back pressure plane, the front face of the wall, and the bottom face of the base. Each is a separate item in the force table:

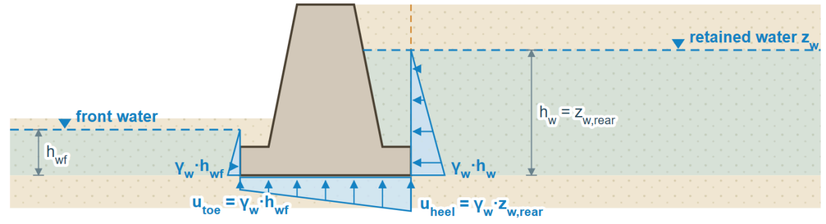
$$U_{rear} = \tfrac{1}{2}\gamma_w h_w^2 \;\; (h_w/3) \qquad U_{front} = \tfrac{1}{2}\gamma_w h_{wf}^2 \;\; (h_{wf}/3,\ \text{back})$$

The pore water pressure beneath the base varies **linearly** from the load at the toe to the load at the heel (straight-line load loss):

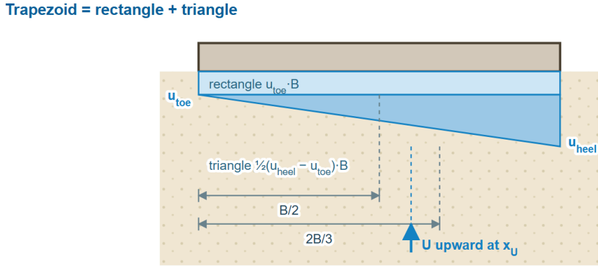
$$U = \frac{B\,(u_{toe} + u_{heel})}{2} \qquad x_U = \frac{B\,(u_{toe} + 2u_{heel})}{3\,(u_{toe} + u_{heel})}$$

Uplift acts upward (F_z = +U): it reduces N′ and hence the base friction, and adds an overturning moment $U \cdot x_U$.

$h_W = H - z_W$ (z_W: depth of the groundwater from the top of the wall); the front water height h_{wf} acts towards the back and reduces the net thrust. If the water levels differ, hydraulic stability (seepage) must also be considered separately.



U = B·(u_toe + u_heel)/2 at x = B·(u_toe + 2u_heel) / (3(u_toe + u_heel))



Problem statement

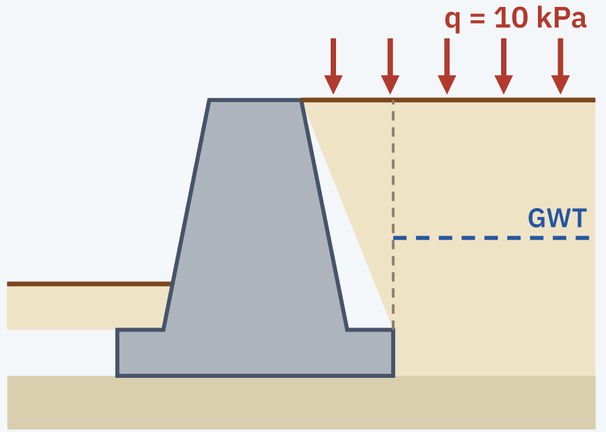
EXAMPLE

Given

Wall and soil	**Wall and fill in Examples 1–5 ($\gamma = 18,\ \gamma_{sat} = 20\ \mathrm{kN/m^3},\ \varphi' = 30^\circ,\ q = 10$ kPa); static condition**
Water	**in the fill, the groundwater level is 1.5 m below the wall top ($h_w = 1.5$ m); no water in front; linear uplift below the base**

REQUIRED

Change in earth thrust and water; **uplift** and **N**; FS_s, FS_o, e, q_{max}.



Solution (1/2)

SOLUTION

1 Earth thrust (γ on top, γ′ = 20 − 9.81 = 10.19 below) and water thrust

$$P_A = \tfrac{1}{2}K_a\gamma d_1^2 + K_a\gamma d_1 d_2 + \tfrac{1}{2}K_a\gamma' d_2^2$$

$$= 6.75 + 13.50 + 3.82 = \mathbf{24.07}\ \text{kN/m}\ \ (z = 1.061\ \text{m})$$

$$U_{rear} = \tfrac{1}{2}(9.81)(1.5)^2 = \mathbf{11.04}\ \text{kN/m}\ \ (z = 0.50\ \text{m}) \qquad P_Q = 10.0$$

2 Uplift (u = 0 at the toe, u = 9.81·1.5 = 14.72 kPa at the heel; triangular)

$$U = \tfrac{1}{2}(14.72)(3.0) = \mathbf{22.07}\ \text{kN/m (upward)}\ \ \text{at}\ x_U = \tfrac{2B}{3} = 2.00\ \text{m}$$

Solution (2/2)

SOLUTION

3 Effective normal force and friction (with soil block γ_{sat}: W_s = 34.95)

$$N = W_m + W_s + W_{toe} - U = 115.50 + 34.95 + 4.95 - 22.07 = \mathbf{133.33}\ \text{kN/m}$$

$$FS_s = \frac{0.55(133.33) + 12.0}{24.07 + 10.0 + 11.04} = \frac{85.33}{45.11} = \mathbf{1.892}$$

4 Overturning and base

$$M_o = 25.54 + 15.00 + 5.52 + 44.14 = 90.20\ \ (P_A,\ P_Q,\ U_{rear},\ \text{uplift}) \qquad M_r = 265.98$$

$$FS_o = \frac{265.98}{90.20} = \mathbf{2.949} \qquad x_R = \frac{175.78}{133.33} = 1.318\ \text{m} \quad e = 0.182\ \text{m}$$

$$q_{max} = \mathbf{60.58} \qquad q_{min} = \mathbf{28.30}\ \text{kPa}$$

Water reduced FS_s from 2.62 to **1.89**, FS_o from 6.26 to **2.95**, and q_{min} from 44.4 to 28.3 kPa (uplift reduced N by 22.1 kN/m and added 44.1 kN·m/m of overturning); the checks are still satisfied.

Check the solution in GeoTerva ↗

Mononobe–Okabe thrust and inertia of the wall

THEORY

In the pseudo-static method each mass is subjected to a horizontal force $k_h$mg and a vertical force ±$k_v$mg; the critical direction is that in which inertia pushes forward (in the same direction as the thrust). Gravity and inertia together act like a gravity field rotated by θ; the Coulomb wedge is solved with the inertial forces (Mononobe–Okabe). In the vertical plane, for a horizontal fill (δ = β = 0):

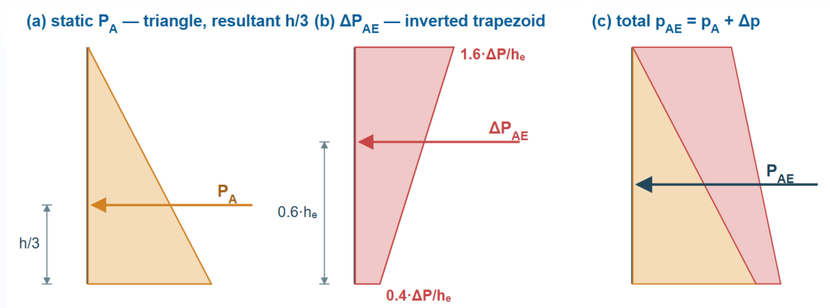
$$\theta = \tan^{-1}\frac{k_h}{1-k_v} \qquad K_{AE} = \frac{\cos^2(\varphi' - \theta)}{\cos^2\theta\left[1+\sqrt{\dfrac{\sin\varphi'\sin(\varphi'-\theta)}{\cos\theta}}\right]^2} \qquad P_{AE} = \left(\tfrac{1}{2}\gamma h^2 + qh\right)K_{AE}$$

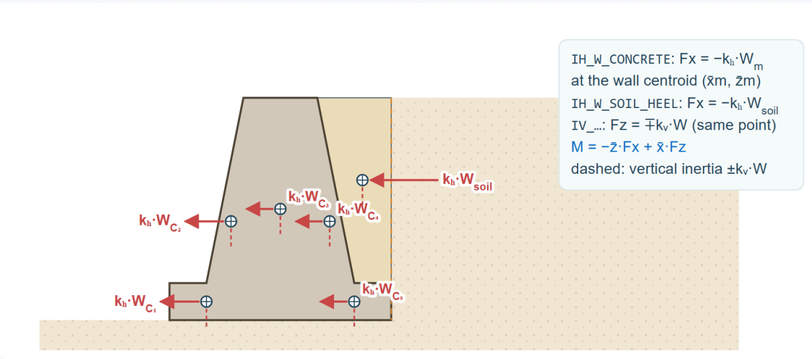
If φ′ − θ < 0, no wedge can remain in equilibrium: "no solution", the checks are deemed not satisfied.

Distribution: P_{AE} gives the magnitude, not the location. $\Delta P_{AE} = P_{AE} - P_{A,static}$ as an inverted trapezoid (1.6ΔP/h at the top, 0.4ΔP/h at the bottom; resultant at **0.6h**); the static part keeps its static shape.

Inertia: the wall, the soil block and the front fill push forward with force k_hW at their center of gravity; if $k_v > 0$, both cases are analyzed with $(1 \pm k_v)$ and the smallest FS governs. The hydrodynamic water pressure $(7/12)k_h\gamma_w h_w^2$ (at $0.4h_w$) is added.

The front passive resistance is multiplied by $\eta_{p,E}$ in an earthquake; default **0** (no credit).





Problem statement

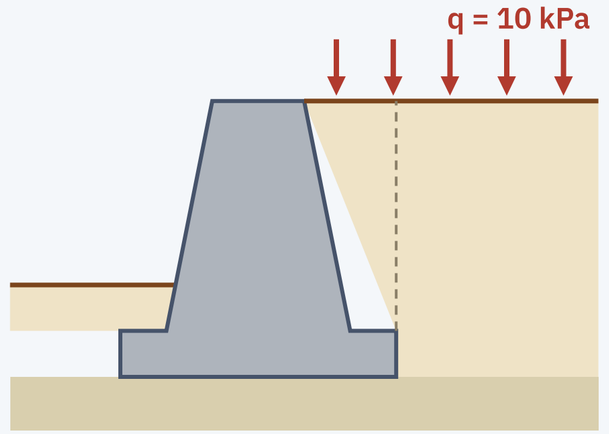
EXAMPLE

Given

Wall and soil	**The wall and fill in Examples 1–6 (static FS_s = 2.62; FS_o = 6.26)**
Earthquake	$k_h = 0.15$, $k_v = 0$; **front resistance credit** $\eta_{p,E} = 0$; **required** $FS_s,\ FS_o \geq 1.1$

REQUIRED

θ, **K_{AE}**, **P_{AE}**, **ΔP_{AE}**, **$\bar{z}$**; inertial forces; FS_s, FS_o, e, q_{max}.



Solution (1/3)

SOLUTION

1 Seismic angle and coefficient

$$\theta = \tan^{-1} 0.15 = 8.53^\circ$$

$$\sqrt{\sin 30^\circ \sin(30^\circ - 8.53^\circ)/\cos 8.53^\circ} = 0.4302$$

$$K_{AE} = \frac{\cos^2(21.47^\circ)}{\cos^2 8.53^\circ\,(1+0.4302)^2} = \mathbf{0.4329}$$

2 Dynamic thrust and seismic increment (static P_A + P_Q = 37.0 kN/m)

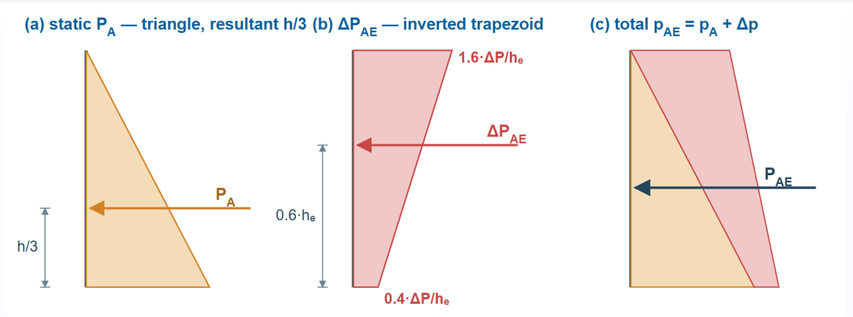
$$P_{AE} = (81+30)(0.4329) = \mathbf{48.06}\ \text{kN/m}$$

$$\Delta P_{AE} = 48.06 - 37.0 = \mathbf{11.06}\ \text{kN/m}\ \ (0.6h)$$

3 Height of load application

$$\bar{z}_{AE} = [27.0(1.00) + 10.0(1.50) + 11.06(1.80)]/48.06$$

$$= 61.90/48.06 = \mathbf{1.288}\ \text{m}$$



Solution (2/3)

SOLUTION

4 Inertial forces $k_h W$ (at the centers of gravity, forward)

$$IH_m = 0.15(115.50) = 17.325 \quad (z = 1.222\ \text{m})$$

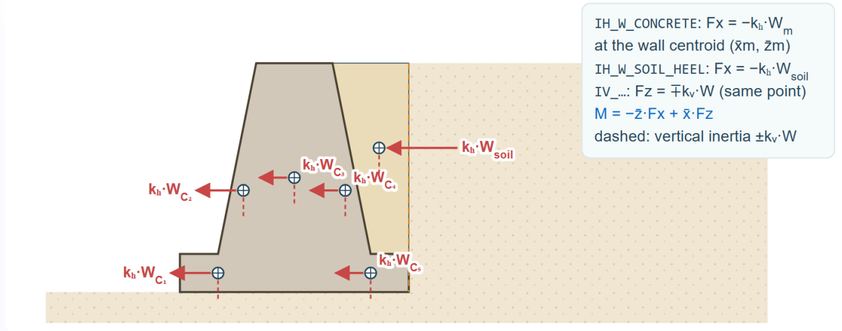
$$IH_s = 0.15(33.75) = 5.062 \quad (z = 1.889\ \text{m})$$

$$IH_{toe} = 0.15(4.95) = 0.743 \quad (z = 0.758\ \text{m})$$

5 Sliding (no credit for front resistance)

$$\Sigma H_{drive} = 48.06 + 17.32 + 5.06 + 0.74 = 71.19$$

$$FS_{s,E} = \frac{\mu_b N}{\Sigma H_{drive}} = \frac{84.81}{71.19} = \mathbf{1.191} \ \geq 1.1 \ \checkmark$$



Solution (3/3)

SOLUTION

6 Overturning and location of the resultant

$$M_{o,E} = P_{AE}\bar{z} + \Sigma IH\, z = 61.90 + 21.18 + 9.56 + 0.56 = 93.20$$

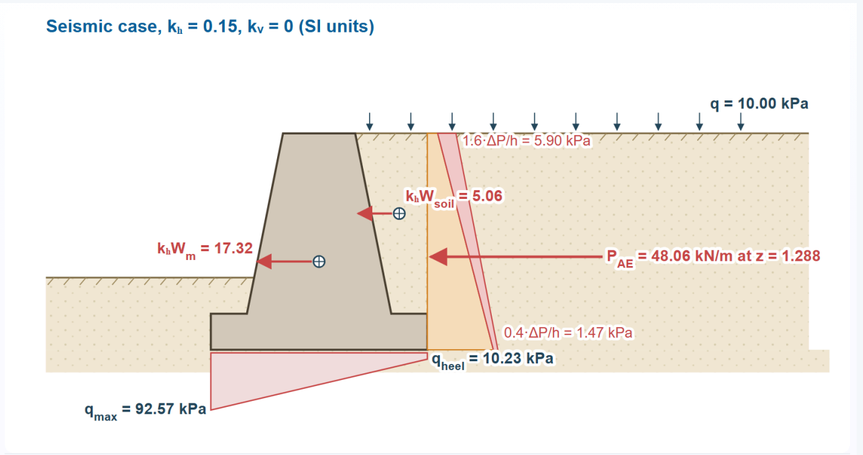
$$FS_{o,E} = \frac{262.74}{93.20} = \mathbf{2.819} \;\geq 1.1$$

$$x_R = \frac{262.74 - 93.20}{154.2} = 1.099 \text{ m} \qquad e = \mathbf{0.401} \text{ m}$$

7 Base stresses (|e| = 0.401 m ≤ B/6 = 0.50 m: full contact)

$$q_{max} = \frac{154.2}{3}\left(1 + \frac{6(0.401)}{3}\right) = \mathbf{92.57} \text{ kPa}$$

$$q_{min} = \mathbf{10.23} \text{ kPa}$$



Under earthquake FS_s = **1.19**, FS_o = **2.82**, e = **0.401 m**, q_{max} = **92.6 kPa**; the earthquake changed FS_s from 2.62 to 1.19 and e from 0.068 to 0.401 m: **sliding** and **eccentricity** are the governing checks.

Check the solution in GeoTerva ↗

Where the checks end

THEORY

These checks treat the wall on its foundation as **a single rigid block**: the **external stability** of the wall. A stone or mass-concrete gravity wall can also fail **within** itself (especially in a tall or slender section, in dry-stone masonry, with weak mortar, and under earthquake); these checks are made from the effects produced by this module.

Cohesionless fill ($c' = 0$); two-soil or layered fill is not included in this module. The fill surface is either level or has a single slope.

Base friction $\mu_b = \tan \delta_b$ (generally $2/3\varphi'–\varphi'$ for masonry/concrete); no adhesion is used. The full front passive resistance is added to the friction allowance and is not used in overturning or bearing capacity.

The **analysis** is per 1.00 m length of the wall; strip and point surcharge lateral pressures are added by Boussinesq (rigid wall).

Overturning effects (hydraulic stability, seepage, global stability, settlement) are outside the scope of this module.

Summary

Key points

Free body = wall + the soil moving with it; the pressure acts **on the vertical plane at the rear edge of the base** (δ = β); single sign convention: $M = x \cdot F_z - z \cdot F_x$, overturning +

Weight: $W_i = \gamma_m A_i$, $\bar{x} = \Sigma W_i \bar{x}_i / \Sigma W_i$; the soil block and front fill also move with the wall; the surcharge on the rear heel (Q_{heel}) enters no check

Active: $P_A = \frac{1}{2} K_a \gamma h^2$ (h/3), $P_Q = K_a q h$ (h/2); for an inclined fill $K_{a\beta}$ (Rankine on a vertical plane = Coulomb); **passive**: $K_{mob} = wK_p + (1 - w)K_0$, for sliding only

Sliding $FS_s = (\mu_b N + P_P)/\Sigma H_{drive}$; **overturning** $FS_o = \Sigma M_{res}/\Sigma M_{ov}$ (toe); **bearing**: $x_R = -\Sigma M/V$, $e = B/2 - x_R$, $q = V/B(1 \pm 6e/B)$, if $|e| > B/6$ then $q_{max} = 2V/(3x_R)$

Water: weight with γ_{sat} + U_{rear} as a separate item; uplift u_{toe} at the toe – u_{heel} at the heel, linear: reduces N′ and adds overturning

Earthquake: $\theta = \tan^{-1} k_h/(1 - k_v)$; $P_{AE} = (\frac{1}{2}\gamma h^2 + qh)K_{AE}$; ΔP_{AE} at 0.6h; $k_h W$ inertia; front passive credit $\eta_{p,E}$ (default 0)

References

The three theories of the tool and the source whose presentation of their closed forms (Ka, Kp, KAE) the manual follows. Everything not derived from them is an assumption of this module and is marked so in the text.

Coulomb C.A. (1776) Essai sur une application des règles de maximis et minimis à quelques problèmes de statique relatifs à l'architecture. Mémoires de l'Académie Royale des Sciences 7, Paris.

Das B.M. (2016) Principles of Foundation Engineering, 8th ed. Cengage Learning, Boston.

Mononobe N., Matsuo H. (1929) On the determination of earth pressures during earthquakes. Proceedings of the World Engineering Congress, Tokyo, 9:177–185.

Okabe S. (1926) General theory of earth pressure. Journal of the Japanese Society of Civil Engineers 12(1).

Rankine W.J.M. (1857) On the stability of loose earth. Philosophical Transactions of the Royal Society of London 147:9–27.

NOTE

All numerical examples have been produced with the module's own computation engine; the module gives the same values. The step-by-step example on the module page is also this wall (H = 3 m stone wall, q = 10 kPa, k_h = 0.15).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own wall's section, fill, surcharge, water and earthquake coefficients in GeoTerva; the module provides the force table, the three checks and the base stresses together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com