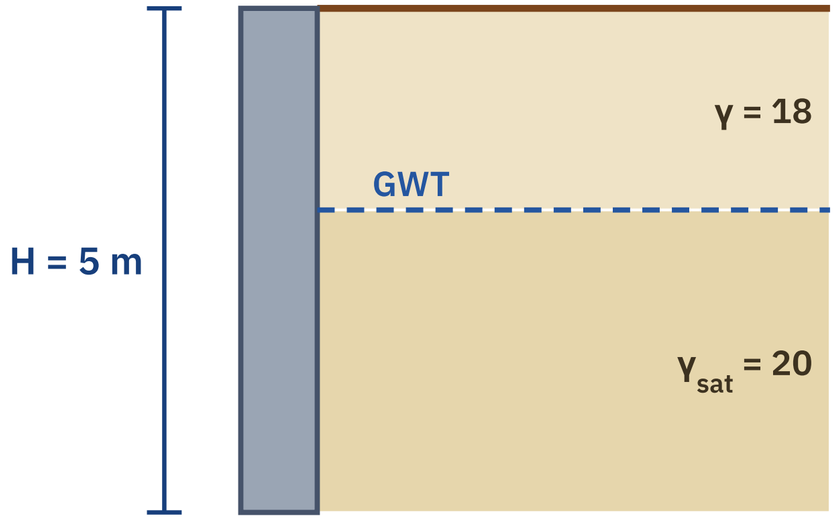
Geotechnical lecture presentation · Lateral earth pressure

At-rest earth pressure

Lecture: at-rest pressure behind a wall that cannot move (basement wall, arch abutment, culvert); relations for the K_0 coefficient, dry and submerged fill, layered soil; a worked example for each topic.

$K_0 = \sigma'_h / \sigma'_o$ · Jaky · Mayne–Kulhawy · Massarsch · Sherif · Layered fill · Groundwater



Introduction

What will we learn in this lecture?

- When the **at-rest pressure** develops and its relation to the active / passive state
- to calculate the **K_0 coefficient** in sand (Jaky, Sherif), in overconsolidated soil (Mayne–Kulhawy) and in clay (Massarsch)
- In dry fill, the force $P_0 = \frac{1}{2}K_0\gamma H^2$ and its line of action at height H/3
- How **groundwater** adds pressure; the jump of K_0 at a layer boundary in layered fill
- To compare the active – at-rest – passive states and the wall movement required

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Forces are per unit length of wall (kN/m); pressures in kPa, stresses are effective.

Non-yielding wall

THEORY

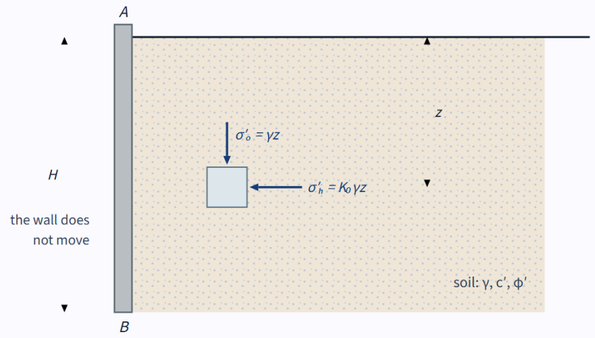
A wall restrained by basement floors, an arch springing on piles, or a culvert wall **cannot move**; the soil behind it retains the stress state it had in situ. The ratio of horizontal effective stress to vertical effective stress is the **coefficient of earth pressure at rest**:

$$K_0 = \frac{\sigma'_h}{\sigma'_o} \qquad \sigma'_o = \gamma z \qquad \sigma'_h = K_0 \gamma z$$

If the wall moves away from the soil the pressure **drops** to the active value; if it is pushed into the soil it **rises** to the passive value; $K_a < K_0 < K_p$.

The movements required are small in the active case and large in the passive case (Theory: Required movement).

K_0 is taken from testing (pressuremeter, dilatometer) or from a correlation; the module uses one for each layer.



Sand, overconsolidated soil and clay

THEORY

$$K_0 = 1 - \sin\varphi' \qquad \text{(Jaky 1944, sand, NC)}$$

$$K_0 = (1 - \sin\varphi') + \left(\frac{\gamma_d}{\gamma_{d(min)}} - 1\right) 5.5 \qquad \text{(Sherif et al. 1984, compacted sand)}$$

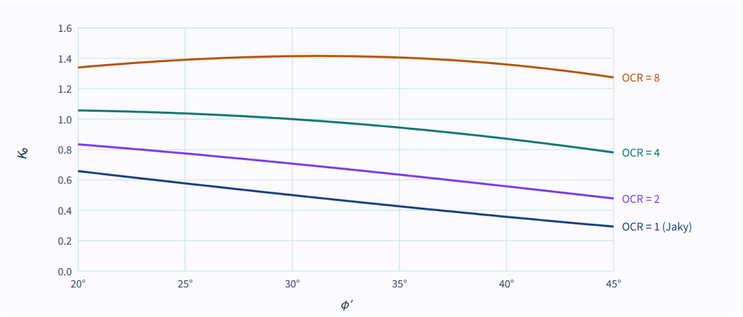
$$K_0 = (1 - \sin\varphi')\, OCR^{\sin\varphi'} \qquad \text{(Mayne–Kulhawy 1982)}$$

$$K_{0(NC)} = 0.44 + 0.42\frac{PI(\%)}{100} \qquad K_{0(OC)} = K_{0(NC)}\sqrt{OCR} \qquad \text{(Massarsch 1979, clay)}$$

Jaky **under**estimates the pressure in compacted sand: compaction makes the fill overconsolidated; Sherif corrects this with $\gamma_d/\gamma_{d,min}$.

OCR = ratio of the largest past effective stress to the present effective stress. For OCR = 4, φ′ = 30°, K_0 = 1: horizontal and vertical stress are equal.

K_0 – φ′ (OCR curves)



Problem statement

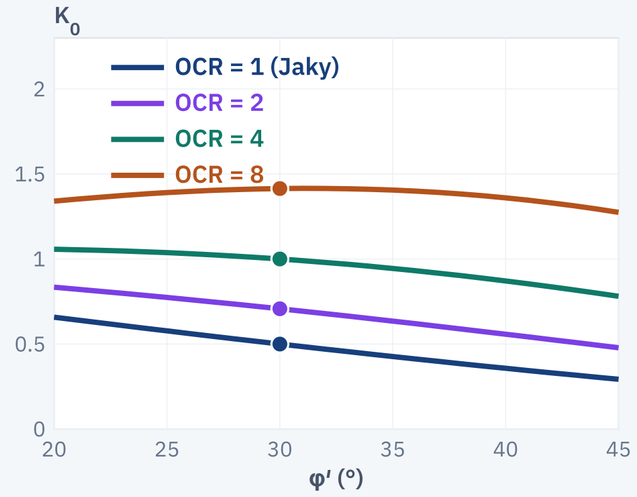
EXAMPLE

Given

Soil	**cohesionless sand, $\varphi' = 30^\circ$**
Cases	$OCR = 1,\ 2,\ 4,\ 8$ **(also $\varphi' = 25^\circ,\ 35^\circ,\ 40^\circ$ for $OCR = 1$)**

REQUIRED

K_0 for each case; OCR where $K_0 = 1$; comparison with Rankine K_a, K_p.



Solution

SOLUTION

1 NC sand (Jaky)

$$\varphi' = 30^\circ:\ K_0 = 1 - \sin 30^\circ = \mathbf{0.5000}$$

$$\varphi' = 25^\circ:\ 0.577 \qquad 35^\circ:\ 0.426 \qquad 40^\circ:\ 0.357$$

2 Overconsolidated (OCR exponent sin φ′ = 0.5)

OCR	1	2	4	8
$K_0 = 0.5 \cdot OCR^{0.5}$	0.500	0.707	1.000	1.414

K_0 increases as OCR increases: at OCR = 4, **$K_0 = 1$** (hydrostatic stress), at OCR = 8, 1.41. If Rankine K_a = 0.333 and K_p = 3.0, this lies outside all OCRs: $K_a < K_0 < K_p$ always holds.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Soil	**compacted sand fill: $\varphi' = 34^\circ$, dry unit weight $\gamma_d = 16.0\ \mathrm{kN/m^3}$; in the loosest state $\gamma_{d(min)} = 14.5\ \mathrm{kN/m^3}$**

REQUIRED

K_0 (Sherif) and comparison with Jaky.

Solution

SOLUTION

1 Jaky (zero compaction)

$$K_0 = 1 - \sin 34^\circ = \mathbf{0.4408}$$

2 Sherif: compaction allowance

$$\frac{\gamma_d}{\gamma_{d(min)}} - 1 = \frac{16.0}{14.5} - 1 = 0.1034$$

$$K_0 = 0.4408 + (0.1034)(5.5) = \mathbf{1.010}$$

Compaction raised K_0 from 0.44 to **1.01** (2.3 times): a wall behind compacted fill must withstand a much larger pressure than the normally consolidated calculation gives.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Soil	**clay: plasticity index $PI = 25\%$**
Cases	**$OCR = 1$ and $OCR = 2$; also $PI =$ $10,\ 40,\ 60\%$**

REQUIRED

K_0 for normally and overconsolidated clay.

Solution

SOLUTION

1 PI = 25 %

$$K_{0(NC)} = 0.44 + 0.42\frac{25}{100} = \mathbf{0.5450}$$

$$K_{0(OC)} = K_{0(NC)}\sqrt{OCR} = 0.5450\sqrt{2} = \mathbf{0.7707}$$

2 Increase with plasticity

PI (%)	$K_{0(NC)}$	OCR = 2	OCR = 4
10	0.482	0.682	0.964
25	0.545	0.771	1.090
40	0.608	0.860	1.216
60	0.692	0.979	1.384

In clay, K_0 increases with plasticity; for OCR = 2 and PI = 25 %, it is **0.771**. In overconsolidated clay, K_0 is multiplied by √OCR (this multiplier is not part of the Massarsch relation but a common approximation).

Check the solution in GeoTerva ↗

Pressure triangle and force

THEORY

In a dry fill with unit weight γ, the horizontal effective stress increases linearly with depth; the force per unit length of wall is the area of the triangle and acts at H/3 above the base:

$$\sigma'_h = K_0 \gamma z \qquad P_0 = \tfrac{1}{2} K_0 \gamma H^2 \qquad \bar{z} = \tfrac{H}{3}$$

In layered fill σ'_o is summed layer by layer and each layer takes its own K_0 value: at every boundary where K_0 changes, **σ'_h jumps while σ'_o does not**.

The force grows with H^2; OCR and compression increase the force in direct proportion to K_0.

Problem statement

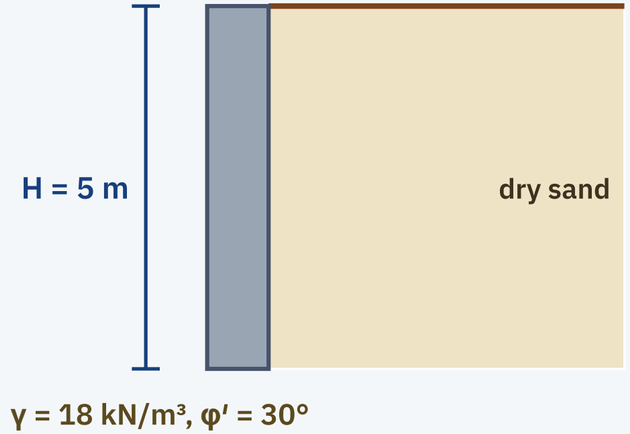
EXAMPLE

REQUIRED

For each case, **$\sigma'_h(H)$**, **P_0** and the height of the line of action.

Given

Wall	$H = 5$ **m, cannot move**
Soil	**dry sand:** $\gamma = 18\ \mathrm{kN/m^3},\ \varphi' = 30^\circ$
Cases	$OCR = 1$ **and** $OCR = 2$



Solution

SOLUTION

1 OCR = 1 (Jaky: $K_0 = 0.5$)

$$\sigma'_h(5) = K_0 \gamma H = 0.5(18)(5) = \mathbf{45.00}\ \text{kPa}$$

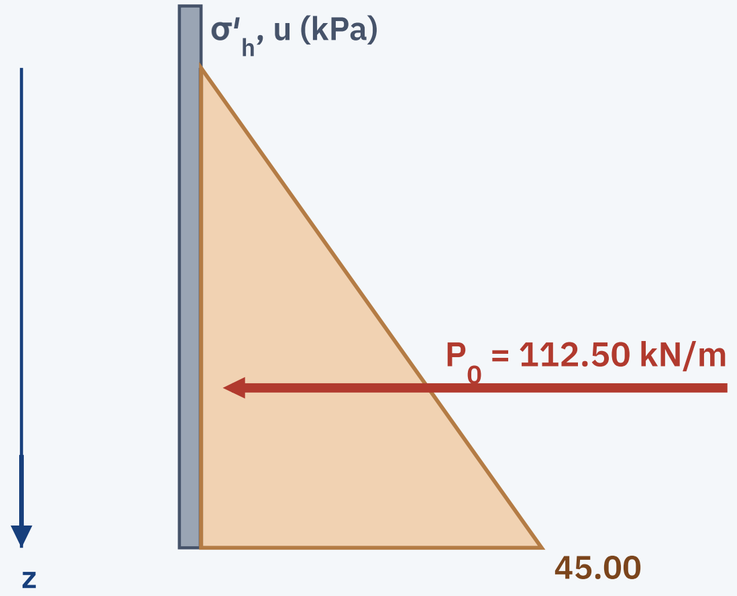
$$P_0 = \tfrac{1}{2}(45.00)(5) = \mathbf{112.50}\ \text{kN/m} \qquad \bar{z} = \tfrac{5}{3} = 1.67\ \text{m}$$

2 OCR = 2 (Mayne–Kulhawy: $K_0 = 0.7071$)

$$\sigma'_h(5) = 0.7071(18)(5) = \mathbf{63.64}\ \text{kPa}$$

$$P_0 = \tfrac{1}{2}(63.64)(5) = \mathbf{159.10}\ \text{kN/m}$$

P_0 = **112.50 kN/m** (OCR = 1) and **159.10 kN/m** (OCR = 2), both 1.67 m above the base; overconsolidation increased the force by 41%.



Check the solution in GeoTerva ↗

Effective stress + water pressure

THEORY

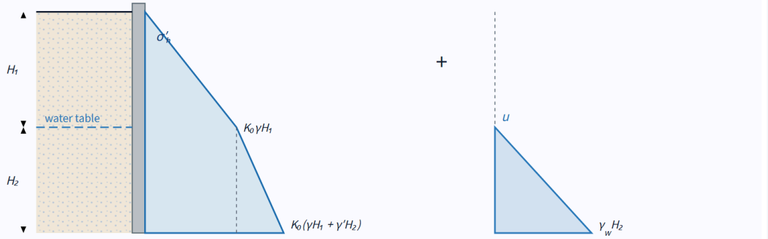
If the groundwater is at depth H_1 (no water in front of the wall), above it $\sigma'_h = K_0\gamma z$; below it the effective stress is found with $\gamma' = \gamma_{sat} - \gamma_w$, and the water pressure is added in full, **independent** of K_0:

$$\sigma'_h = K_0\left[\gamma H_1 + \gamma'(z - H_1)\right] \qquad u = \gamma_w(z - H_1) \qquad \sigma_h = \sigma'_h + u$$

$$P_0 = \tfrac{1}{2}K_0\gamma H_1^2 + K_0\gamma H_1 H_2 + \tfrac{1}{2}\left(K_0\gamma' + \gamma_w\right)H_2^2$$

The height of action is found from the moment of the areas about the base point.

$\gamma_w = 9.81$, $K_0\gamma' \approx 0.5\cdot 10 = 5$: **the water contribution is often larger than the soil contribution**.



Problem statement

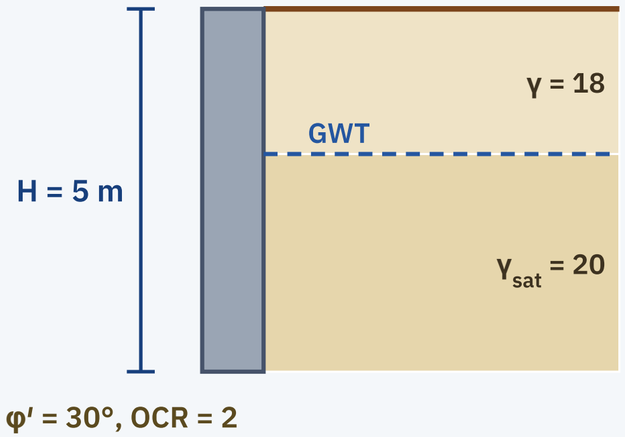
EXAMPLE

Given

Wall	$H = 5$ **m, cannot move**
Soil	**sand:** $c' = 0,\ \varphi' = 30^\circ,\ OCR = 2$**;** $\gamma = 18\ \mathrm{kN/m^3}$ **(upper),** $\gamma_{sat} = 20\ \mathrm{kN/m^3}$ **(lower)**
Groundwater	**at 2 m depth; no water in front of the wall**

REQUIRED

Pressure diagram, total P_0 (soil + water) and line of action height. The module opens with this wall.



Solution (1/2)

SOLUTION

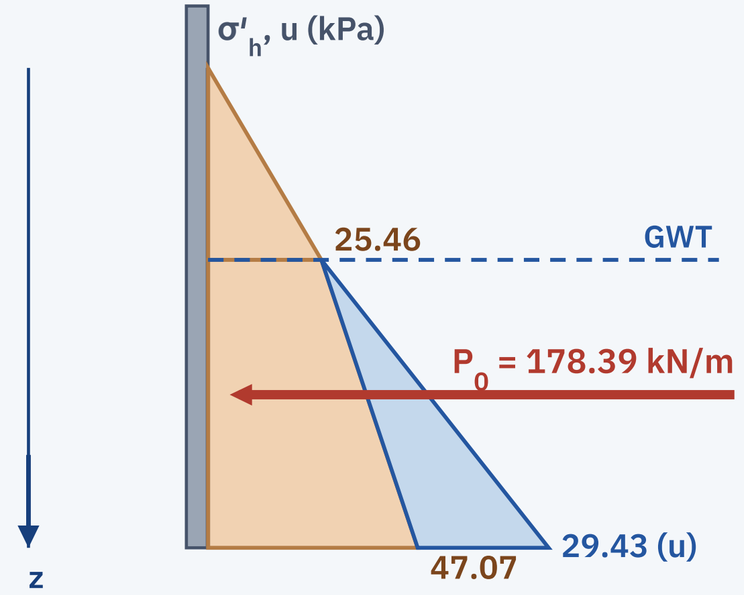
1 K_0 and stresses

$$K_0 = (1 - \sin 30^\circ)\, 2^{\sin 30^\circ} = \mathbf{0.7071}$$

$$z = 2:\ \sigma'_o = 36 \ \rightarrow\ \sigma'_h = 0.7071(36) = 25.46\ \text{kPa}$$

$$z = 5:\ \sigma'_o = 36 + 3(20 - 9.81) = 66.57 \ \rightarrow\ \sigma'_h = \mathbf{47.07}\ \text{kPa}$$

$$u = 9.81(3) = \mathbf{29.43}\ \text{kPa}$$



Solution (2/2)

SOLUTION

2 Areas: upper triangle · lower rectangle · lower triangle · water triangle

$$P_0 = \tfrac{1}{2}(2)(25.46) + 3(25.46) + \tfrac{1}{2}(3)(21.62) + \tfrac{1}{2}(3)(29.43)$$
$$= 25.46 + 76.37 + 32.42 + 44.14 = \mathbf{178.39}\ \mathrm{kN/m}$$

3 Height of action (moment arms 3.67; 1.50; 1.00; 1.00 m)

$$\bar{z} = \frac{25.46(3.67) + 76.37(1.50) + 32.42(1.00) + 44.14(1.00)}{178.39} = \mathbf{1.59}\ \mathrm{m}$$

P_0 = **178.39 kN/m**, **1.59 m** above the base; soil 134.25, water 44.14 kN/m (water share 25%).

Check the solution in GeoTerva ↗

Problem statement

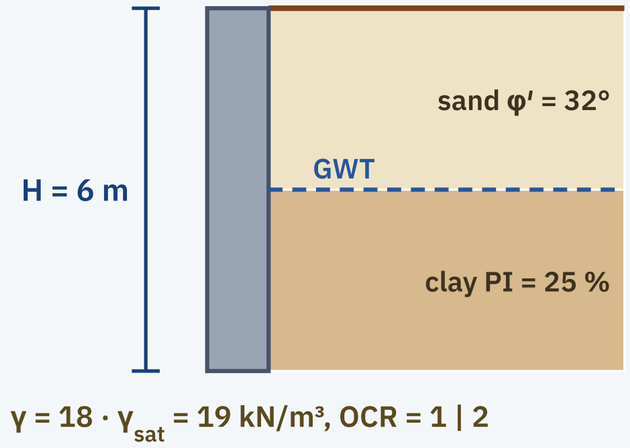
EXAMPLE

Given

Wall	$H = 6$ **m, cannot move**
Upper layer (0–3 m)	**sand:** $\gamma = 18\ \mathrm{kN/m^3},\ \varphi' = 32°,\ OCR = 1$
Lower layer (3–6 m)	**clay:** $\gamma_{sat} = 19\ \mathrm{kN/m^3},\ PI = 25\%,\ OCR = 2$
Groundwater	**above the clay (3 m)**

REQUIRED

Pressure jump at the boundary, $\mathbf{P_0}$ and $\bar{\mathbf{z}}$.



Solution (1/2)

SOLUTION

1 K_0 values: sand Jaky, clay Massarsch

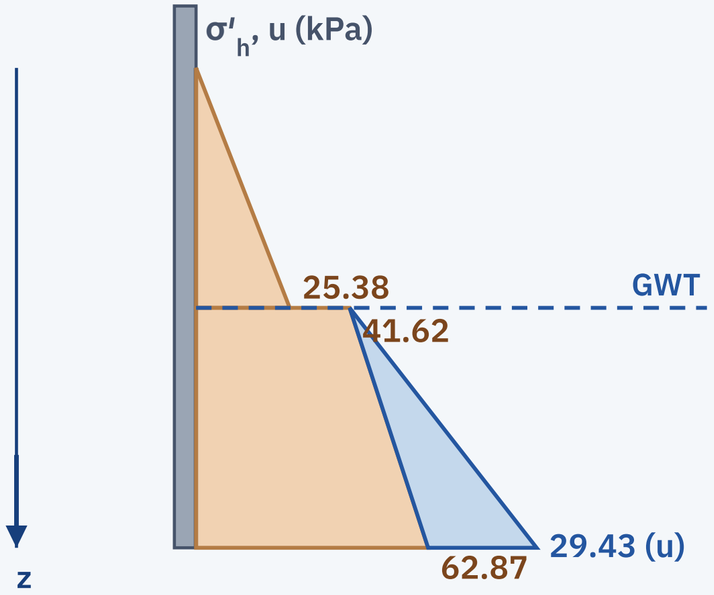
$$K_{0(kum)} = 1 - \sin 32^\circ = \mathbf{0.4701}$$

$$K_{0(kil)} = \left[0.44 + 0.42\frac{25}{100}\right]\sqrt{2} = 0.5450\sqrt{2} = \mathbf{0.7707}$$

2 Stresses ($\sigma'_o(3) = 54$; $\sigma'_o(6) = 54 + 3(19 - 9.81) = 81.57$)

$$z = 3:\ \sigma'_h = 0.4701(54) = 25.38\ (\text{sand});\ \ 0.7707(54) = 41.62\ (\text{clay})$$

$$z = 6:\ \sigma'_h = 0.7707(81.57) = \mathbf{62.87}\ \text{kPa};\ \ u = \mathbf{29.43}\ \text{kPa}$$



Solution (2/2)

SOLUTION

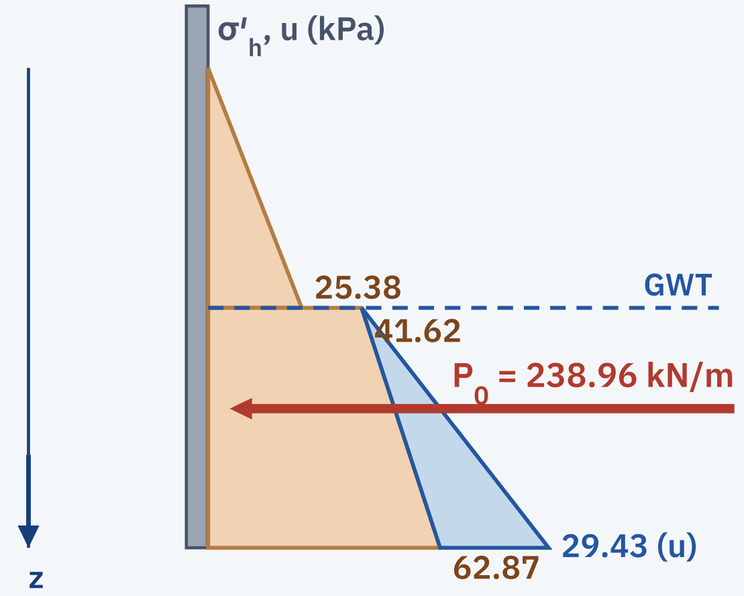
3 Forces

$$P_0 = \tfrac{1}{2}(3)(25.38) + 3(41.62) + \tfrac{1}{2}(3)(21.25) + \tfrac{1}{2}(3)(29.43)$$
$$= 38.08 + 124.86 + 31.87 + 44.14 = \mathbf{238.96}\ \mathrm{kN/m}$$

4 Height of action (moment arms 4.00; 1.50; 1.00; 1.00 m)

$$\bar{z} = \frac{38.08(4) + 124.86(1.5) + 31.87(1) + 44.14(1)}{238.96} = \mathbf{1.74}\ \mathrm{m}$$

P_0 = **238.96 kN/m**, $\bar{z}$ = **1.74 m**. σ'_h jumps from 25.38 → 41.62 kPa at the top of the clay layer (K_0 changes); σ'_o = 54 kPa is the same on both sides.



Check the solution in GeoTerva ↗

Movements reaching active and passive states

THEORY

If the wall cannot move, at-rest pressure develops. If the wall moves away from the soil, the pressure drops from K_0 to K_a; if it moves toward the soil, it rises to K_p; the required movement relative to the height (active very small, passive ten times more):

Soil	Active $\Delta L_a/H$	Passive $\Delta L_p/H$
Loose sand	0.001–0.002	0.01
Dense sand	0.0005–0.001	0.005
Soft clay	0.02	0.04
Stiff clay	0.01	0.02

WHY DOES IT MATTER?

A basement wall, an arch springing tied to a pile cap or a culvert wall cannot move: the design pressure is the **at-rest** value, not the active one, and depends only on the K_0 relation.

Problem statement

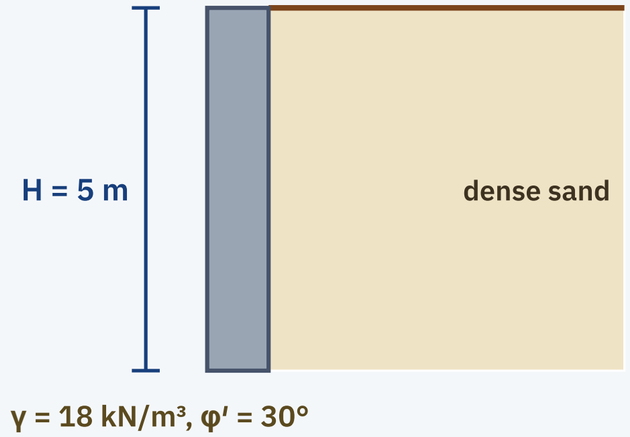
EXAMPLE

Given

Wall	$H = 5$ **m, dense sand fill**
Soil	**dry:** $\gamma = 18\ \mathrm{kN/m^3}$, $\varphi' = 30°$**, NC (Jaky)**

REQUIRED

Active, at-rest and passive **force**; the wall movement required for each (dense sand).



Solution

SOLUTION

1 $P = \frac{1}{2}K\gamma H^2 = \frac{1}{2}K(18)(25) = 225\ K$

Condition	K	P (kN/m)	Movement (dense sand, H = 5 m)
Active	0.3333	75.0	0.0005–0.001·5 = 2.5–5 mm
At-rest	0.50	112.5	0 (wall does not move)
Passive	3.00	675.0	0.005·5 = 25 mm

Active **75.0** < at-rest **112.5** < passive **675.0 kN/m**; the at-rest force is 1.5 times the active. If the wall can move 2.5–5 mm, active pressure is taken as the basis; if it cannot move at all, at-rest pressure; passive resistance requires at least 25 mm of movement.

Check the solution in GeoTerva ↗

Scope of the solution

THEORY

The **wall is assumed frictionless and vertical**; the at-rest pressure acts horizontally ($\delta' = 0$).

Surface horizontal; for inclined surface and inclined wall see the Rankine (inclined fill) and Coulomb modules.

K_0 is separate and constant for each layer: Jaky, Sherif, Mayne–Kulhawy, Massarsch or a value entered from the test.

Water is hydrostatic; there is no water in front of the wall; water pressure does not depend on K_0.

Surcharge, compaction stress and earthquake are not included in this module; the compaction effect is reflected in the K_0 value (Sherif) or in the OCR.

Summary

Key points

At-rest pressure: in a wall that cannot move $\sigma'_h = K_0\sigma'_o$; $K_a < K_0 < K_p$

K_0: Jaky $1 - \sin\varphi'$; Mayne–Kulhawy $\times\ OCR^{\sin\varphi'}$; Massarsch $0.44 + 0.42\ PI/100$ ($\times\ \sqrt{OCR}$ for overconsolidated); Sherif for compacted sand $+\ (\gamma_d/\gamma_{d,min} - 1)\cdot 5.5$

Dry fill: $P_0 = \frac{1}{2}K_0\gamma H^2$, at $H/3$ from the base

Water: effective stress with γ', $u = \gamma_w(z - H_1)$ is fully added; the water contribution is often large

In a **layered** fill σ'_o is continuous, while σ'_h jumps at the boundary where K_0 changes; $\bar{z}$ is found from the moments of the areas

Movement: $\Delta L/H \sim 0.001$ for active; ~ 0.01 for passive (half in dense sand); at-rest conditions govern if the wall cannot move

References

Das, B.M. (2018). Principles of Geotechnical Engineering, 9th ed. Cengage Learning.

Jaky, J. (1944). The coefficient of earth pressure at rest. Journal of the Society of Hungarian Architects and Engineers, 7, 355–358.

Mayne, P.W. and Kulhawy, F.H. (1982). K_o–OCR relationships in soil. J. Geotech. Eng. Div., ASCE, 108(GT6), 851–872.

Massarsch, K.R. (1979). Lateral earth pressure in normally consolidated clay. Proc. 7th European Conf. SMFE, Brighton, 2, 245–250.

Sherif, M.A., Fang, Y.S. and Sherif, R.I. (1984). KA and K0 behind rotating and non-yielding walls. J. Geotech. Eng., ASCE, 110(GT1), 41–56.

NOTE

All numerical examples were produced with the module's own calculation engine; the module gives the same values. The module opens with the wall in Example 5 (single layer; for two layers see Example 6).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter your own wall's layers, K_0 method, OCR value and groundwater in GeoTerva; the module gives the pressure diagram, the force and the height of action together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com