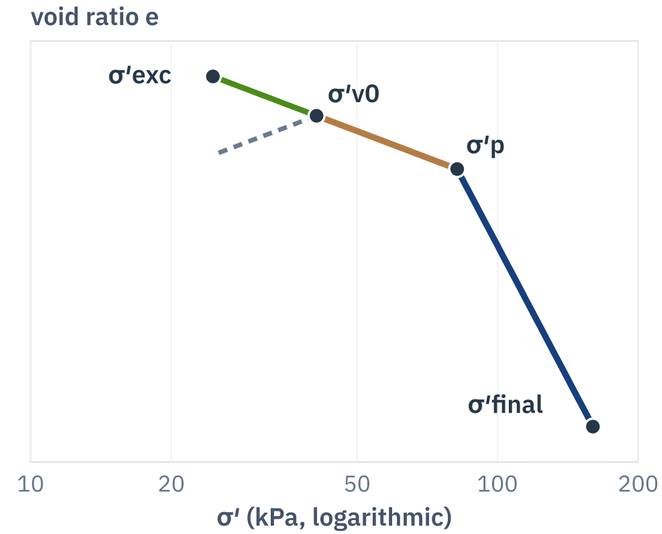
Geotechnical lecture presentation · Settlement

Consolidation settlement

Lecture: one-dimensional consolidation settlement of a clay layer; stress path, preconsolidation, excavation and heave, Boussinesq stresses, the mv method and time; worked examples on every topic.

Stress path | $C_c \cdot C_r \cdot C_s$ | Excavation and heave | Boussinesq | m_v method | Time-dependent settlement



What will we learn in this lecture?

- To follow the **stress path** of the clay: $\sigma'_{v0} \rightarrow \sigma'_{exc} \rightarrow \sigma'_{v0} \rightarrow \sigma'_{p} \rightarrow \sigma'_{final}$ (C_s, C_r, C_c)
- To determine the initial stresses (σ_{v0}, u_0, σ'_{v0}) and the **preconsolidation stress** ($\sigma'_p = OCR \cdot \sigma'_{v0}$)
- To distinguish the unloading and swelling effect of **the excavation** and **from which reference** the settlement is measured
- How to calculate the **Boussinesq** stress increase and the **influence depth**
- To apply the **four regimes** of slice compression (recompression, virgin compression); to compare with the $\mathbf{m_v}$ method
- **Time-dependent** consolidation (T_v, U, t_{50}, t_{90}) and plotting the settlement–time curve of the layered profile

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example soil: 6 m thick clay, GWT at ground surface (σ'_{v0} = 8.19 z), e_0 = 1.0, C_c = 0.30, C_r = 0.06; 4 × 6 m footing, q = 150 kPa. Settlements are in mm.

What does this module calculate?

THEORY

Every layer in the profile is clay; no other soil can be entered. The module computes the **primary consolidation** of the clay: when the load is applied, water is expelled slowly and the clay compresses along the stress path.

Immediate (elastic) settlement is in the Elastic Settlement modules; the sum of the two is in the Total Settlement (2D) module.

The solution is **analytical**: Boussinesq stress increase, stress path slice by slice and Terzaghi theory; hydrostatic pore water pressure; there is no finite element.

Out of scope: granular layers, secondary compression (C_α), SPT/CPT settlement of sands, rock settlement, 2D/3D consolidation, bearing capacity, liquefaction.

SIGN CONVENTION

Settlement is **positive**, heave is **negative**. Net deformation $S_{net} = S_c + S_{heave}$.

Not in a single step from initial to final stress

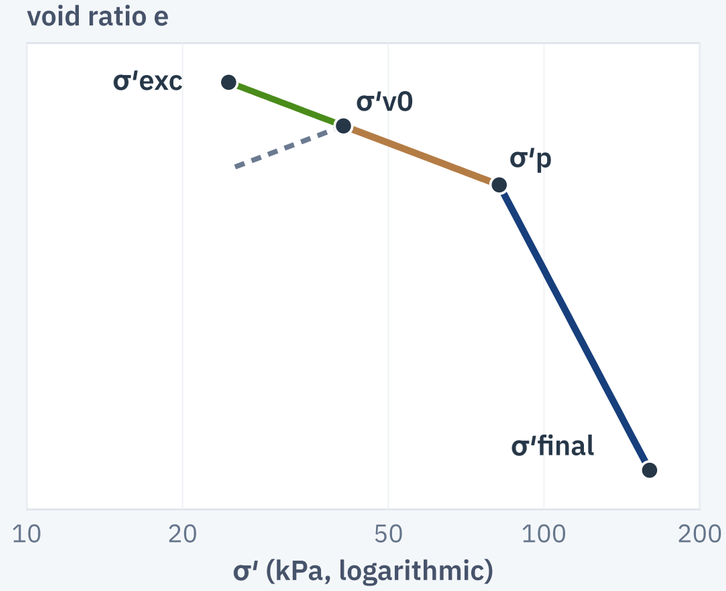
THEORY

The calculation never goes from the initial stress to the final stress in **a single step**. Excavation reduces the effective stress and the clay **swells** along the unloading line (C_s). When the structure reloads it, the clay first follows the same line back to σ'_{v0} (the swelling is reversed); then it recompresses along C_r up to σ'_p, and if the final stress exceeds σ'_p, it compresses along the virgin line (C_c):

$$\sigma'_{v0} \xrightarrow{C_s} \sigma'_{exc} \xrightarrow{C_s} \sigma'_{v0} \xrightarrow{C_r} \sigma'_p \xrightarrow{C_c} \sigma'_{final}$$

σ'_p is the maximum past stress; **it is not reduced by excavation**.

The loading–unloading cycle **closes** at σ'_{v0}: a load that merely restores the removed overburden does not produce settlement relative to the initial state, regardless of C_s and C_r.



Schematic: dashed line unloading, green line reloading (C_s), orange recompression (C_r), blue virgin compression (C_c).

At the midpoint of each slice

THEORY

Each layer between the foundation level and the calculation depth z_{lim} is divided into **slices** of thickness at most Δz (truncated at the GWT); each stress is calculated at the midpoint of the slice:

$$\sigma_{v0} = \sum \gamma_i h_i \qquad u_0 = \gamma_w (z - z_w) \qquad \sigma'_{v0} = \sigma_{v0} - u_0$$

γ is used above the GWT, γ_{sat} below it; γ_w = 9.81 kN/m³.

The initial ground stresses are the **reference** for each subsequent stage.

The thinner the slice, the more accurate the result; in hand calculations a single slice gives an approximate result.

GWT AT THE GROUND SURFACE

For γ_{sat} = 18, γ′ = 18 − 9.81 = 8.19 kN/m³ and σ'_{v0} = 8.19 z. All the following examples use this soil.

Problem statement

EXAMPLE

Given

Soil	$\gamma = 17\ \mathrm{kN/m^3}$ **(above GWT),** $\gamma_{sat} = 18\ \mathrm{kN/m^3}$ **(below)**
GWT	**1.5 m from the ground surface**
Depths	**z = 1.5; 3.0; 4.5; 6.0 m**

REQUIRED

Find the **σ_{v0}, u_0, σ'_{v0}** values at each depth.

Solution

SOLUTION

1 z = 3.0 m (GWT 1.5 m)

$$\sigma_{v0} = 17(1.5) + 18(1.5) = 52.50\ \text{kPa}$$

$$u_0 = 9.81(3.0 - 1.5) = 14.715\ \text{kPa} \qquad \sigma'_{v0} = 52.50 - 14.715 = \mathbf{37.785}\ \text{kPa}$$

z (m)	σ_{v0} (kPa)	u_0 (kPa)	σ'_{v0} (kPa)
1.5	25.50	0.00	**25.50**
3.0	52.50	14.71	**37.78**
4.5	79.50	29.43	**50.07**
6.0	106.50	44.15	**62.35**

Below the GWT, σ'_{v0} increases more **slowly** (slope $\gamma' = 8.19$ kN/m³): at z = 6 m the total stress is 106.5, the effective stress only 62.4 kPa.

Check the solution in GeoTerva ↗

$\sigma'_p = OCR \cdot \sigma'_{v0}$ or the laboratory value

THEORY

A **single definition** is used for each model: either the overconsolidation ratio OCR or the laboratory σ'_p value:

$$\sigma'_p = OCR \cdot \sigma'_{v0} \qquad \text{or} \qquad \sigma'_p = \sigma'_{p,lab} \ (\geq \sigma'_{v0})$$

OCR = 1: normally consolidated clay; $\sigma'_p = \sigma'_{v0}$ and any loading beyond the initial stress is virgin compression.

If the laboratory σ'_p value is below σ'_{v0}, it is raised to σ'_{v0} with a warning: **lightly overconsolidated clay is out of scope**.

σ'_p represents the maximum past stress and does not decrease with excavation.

WHAT DOES OCR TELL US?

OCR = σ'_p/σ'_{v0}. A large OCR means a wide recompression region in which the clay will first compress under load with a **small C_r**.

Problem statement

EXAMPLE

Given

Slice	**midpoint $z = 3$ m: $\sigma'_{v0} = 24.57$ kPa (GWT at surface, $\gamma_{sat} = 18$)**
Cases	**(a) $OCR = 3$ · (b) laboratory $\sigma'_p = 60$ kPa · (c) laboratory $\sigma'_p = 20$ kPa**

REQUIRED

In each case **σ′p** and **OCR**; interpret the condition.

Solution

SOLUTION

1 (a) OCR given and (b) laboratory value

$$(\text{a}):\ \sigma'_p = 3(24.57) = \mathbf{73.71}\ \text{kPa} \qquad (\text{b}):\ OCR = \frac{60}{24.57} = \mathbf{2.44}$$

2 (c) laboratory value below σ′v0

$$\sigma'_{p,lab} = 20 < \sigma'_{v0} \ \Rightarrow\ \sigma'_p = \mathbf{24.57}\ \text{kPa},\ OCR = 1$$

(a) and (b) overconsolidated clay (σ′p > σ′v0); for (c) the module issues a warning and treats the clay as **normally consolidated** (σ′p = σ′v0 = 24.57 kPa): lightly overconsolidated clay is out of scope.

Check the solution in GeoTerva ↗

Unloading and heave

THEORY

The removed load is the **initial effective overburden stress** at the excavation base (GWT unchanged; below water the removed weight is computed with γ′). Unloading spreads with the Boussinesq influence of the excavation plan, I_{exc} (I_{exc} = 1 for a wide excavation):

$$q_{lift} = \sigma'_{v0}(D_{exc}) \qquad \Delta\sigma_{exc} = -q_{lift}\, I_{exc}(z') \qquad \sigma'_{exc} = \sigma'_{v0} + \Delta\sigma_{exc}$$

$$\Delta H_{heave} = -\frac{C_s\, \Delta H}{1 + e_0}\, \log_{10} \frac{\sigma'_{v0}}{\sigma'_{exc}}$$

Swelling is **negative** (settlement +, swelling –). If C_s is not given, $C_s = C_r$ is taken and stated in the report.

The net pressure $q_{net} = q_{gross} - q_{uplift}$ is reported only; the calculation is not done in a single step with the net pressure, the stress path is followed.

NEDEN KABARMA?

Excavation reduces the effective stress and the clay swells. When the structure reloads, the swelling is recovered: this is the **recompression of the swollen soil**.

Δσ below the center of the footing

THEORY

$$\Delta\sigma_{load} = q_{gross}\, I_z(z') \qquad z' = z - D_f$$

Rectangular B × L, under its centre: with the Newmark corner coefficient, $I_z = 4 \cdot I_{corner}$(m = B/2z′, n = L/2z′):

$$I_{köşe} = \frac{1}{4\pi}\left[\frac{2mn\sqrt{V}}{V + m^2n^2}\cdot\frac{V+1}{V} + \tan^{-1}\frac{2mn\sqrt{V}}{V - m^2n^2}\right] \qquad V = m^2 + n^2 + 1$$

(If $V < m^2n^2$, the arctangent term is increased by π). **Strip** (width B), below its centre line: $I_z = (\alpha + \sin\alpha)/\pi$, $\alpha = 2\tan^{-1}(B/2z')$. At $z' = 0$, $I_z = 1$.

$$\sigma'_{final} = \sigma'_{exc} + \Delta\sigma_{load}$$

NOTE

The stress increase is taken along the vertical below the centre of the footing (slices and time analysis); in the colour-band areas, for points outside the footing, it is divided into corner rectangles.

Problem statement

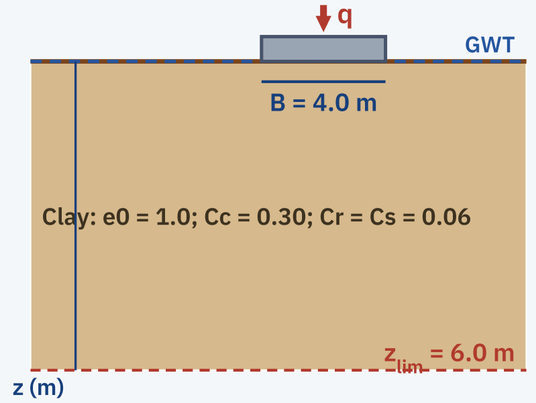
EXAMPLE

Given

Foundation	**4 × 6 m rectangle, $q = 150\ \mathrm{kPa}$, at ground surface ($D_f = 0$)**
Point	**slice midpoint z′ = 3 m, below the center**
Comparison	**B = 4 m wide strip**

REQUIRED

I_z and **Δσ**; the same values apply to the strip.



Solution (1/2)

SOLUTION

1 Rectangle: m = B/2z′ = 2/3 = 0.667; n = L/2z′ = 1.0

$$V = m^2 + n^2 + 1 = 2.4444, \quad m^2 n^2 = 0.4444, \quad \sqrt{V} = 1.5635$$

$$T_1 = \frac{2mn\sqrt{V}}{V + m^2 n^2} \cdot \frac{V+1}{V} = \frac{2.0846}{2.8889} \cdot \frac{3.4444}{2.4444} = 1.0168$$

$$T_2 = \tan^{-1} \frac{2.0846}{2.0000} = 0.8061 \qquad I_{köşe} = \frac{T_1 + T_2}{4\pi} = 0.1451$$

2 Rectangular and strip

$$\text{rectangular} : I;_z = 4(0.1451) = \mathbf{0.5803} \;\Rightarrow\; \Delta\sigma = 150(0.5803) = \mathbf{87.04}\ \text{kPa}$$

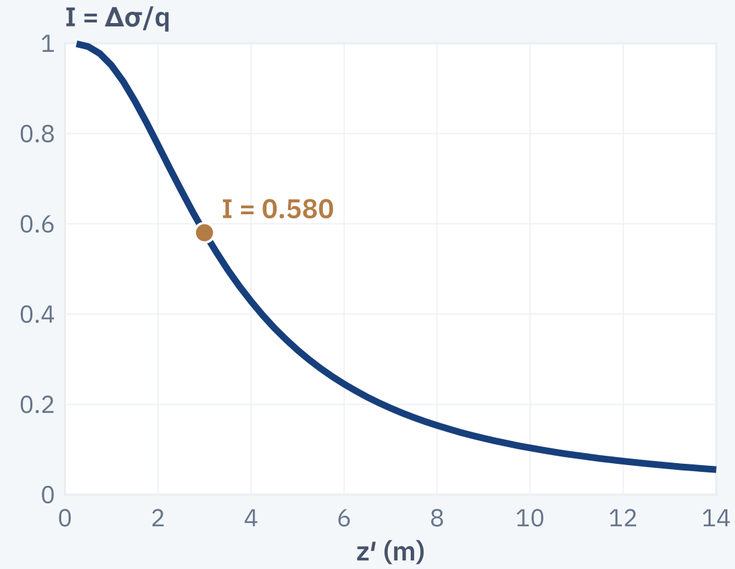
$$\text{strip} : \alpha = 2\tan^{-1}\tfrac{4}{6} = 1.1760 \;\Rightarrow\; I_z = \frac{1.1760 + 0.9231}{\pi} = 0.6682$$

$$\Delta\sigma = 150(0.6682) = \mathbf{100.22}\ \text{kPa}$$

Solution (2/2)

SOLUTION

z′ (m)	I_z	Δσ (kPa)
0.5	0.9927	148.9
1.0	0.9513	142.7
2.0	0.7746	116.2
3.0	0.5803	87.0
4.0	0.4283	64.2
6.0	0.2449	36.7
8.0	0.1532	23.0
10.0	0.1034	15.5



At the centre of the slice Δσ = **87.0 kPa** (58% of q); a narrow strip (same B) gives 100.2 kPa: **a long footing acts deeper**. The stress increase is still 10% even at z′ = 10 m.

Check the solution in GeoTerva ↗

Influence depth: I(z′) = p

THEORY

The stress increase decreases with depth but never becomes zero; the sum of slice compressions requires a lower limit: the **calculation depth z_{lim}**. In the influence-depth rule, clay compresses until the Boussinesq increase below the center falls to **p** times the applied pressure (commonly p = 10%):

$$I_z(z') = p \;\Rightarrow\; z_{lim} = D_f + z'$$

The same influence factor also governs the unloading of a pit with the same plan; therefore the depth is the same whether read in terms of q_{gross} or q_{net}.

If the profile ends above z_{lim}, the last layer continues with the same properties down to z_{lim}; if it ends below, the soil below z_{lim} is incompressible.

The alternative rule is the **base of the profile**: a rigid base is assumed. (The $\Delta\sigma = 0.2\ \sigma'_{v0}$ criterion goes deeper for a small footing at depth; the module's rule depends on geometry only.)

z′/B (p = %10 | %20)

L/B	p = %10	p = %20
1.0	2.09 B	1.40 B
1.5	2.55 B	1.70 B
2.0	2.92 B	1.94 B
5.0	4.33 B	2.68 B
strip	6.34 B	3.13 B

Problem statement

EXAMPLE

Given

Foundations	**(a) 4 × 6 m, p = 10% and 20% · (b) 20 × 30 m raft, p = 10% and 20%**
Rule	**influence depth; $D_f = 0$**

REQUIRED

In each case **z′** and **z_{lim}**; check of I(z′) for 4 × 6 m.

Solution

SOLUTION

1 Check: 4 × 6 m, p = 10% ⇒ z′ = 10.19 m

$$m = \frac{2}{10.19} = 0.1963,\ n = \frac{3}{10.19} = 0.2945 \ \Rightarrow\ I_z = 4(0.0250) = \mathbf{0.1000}$$

Foundation	p = %10: z′ (m)	p = %20: z′ (m)
4 × 6 m (L/B = 1.5)	10.19 (2.55 B)	6.82 (1.70 B)
20 × 30 m raft	50.9 (2.55 B)	34.1 (1.70 B)

z′ is **proportional to the size of the footing**: for a 4 × 6 m footing z_{lim} = 10.19 m, for a 20 × 30 m raft **50.9 m**. The raft accounts for the clay compressing to a depth of about 51 m.

Check the solution in GeoTerva ↗

Slice pinching on reloading

THEORY

$f = \Delta H/(1 + e_0)$ for the slice; with $\sigma'_{exc} \le \sigma'_{v0} \le \sigma'_p$, the compression of the slice according to the position of the final stress:

Condition	Slice pinching on reloading
$\sigma'_{exc} \le \sigma'_{son} \le \sigma'_{v0}$	$S_b = C_s\, f \log(\sigma'_{final}/\sigma'_{exc})$: part of the heave is recovered
$\sigma'_{v0} < \sigma'_{final} \le \sigma'_p$	$S_b = C_s\, f \log(\sigma'_{v0}/\sigma'_{exc}) + S_r = C_r\, f \log(\sigma'_{final}/\sigma'_{v0})$
$\sigma'_{v0} < \sigma'_p < \sigma'_{final}$	$S_b + S_r = C_r\, f \log(\sigma'_p/\sigma'_{v0}) + S_v = C_c\, f \log(\sigma'_{final}/\sigma'_p)$
$\sigma'_p = \sigma'_{v0} < \sigma'_{final}$ (NC)	$S_b + S_v = C_c\, f \log(\sigma'_{final}/\sigma'_{v0})$
$\sigma'_{final} < \sigma'_{exc}$	Additional swelling with C_s (negative)

IF THERE IS NO EXCAVATION

$\sigma'_{exc} = \sigma'_{v0}$ and $S_b = 0$: classical recompression and virgin compression, from the initial stress.

$$S_c = \sum (S_b + S_r + S_v) \qquad S_{net} = S_c + S_{heave}$$

Problem statement

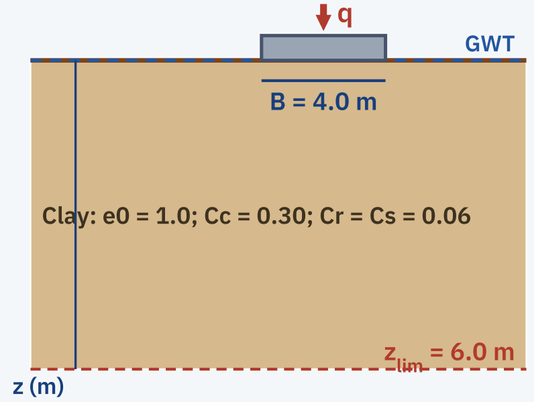
EXAMPLE

Given

Soil	**single clay layer, 6 m thickness;** $\gamma_{sat} = 18\ \mathrm{kN/m^3}$**; GWT at ground surface;** $e_0 = 1.0,\ C_c = 0.30,\ C_r = C_s = 0.06$
Foundation	**4 × 6 m rectangle,** $q = 150\ \mathrm{kPa}$**, at the ground surface (no excavation)**
Slice	**single slice, H = 6 m, midpoint** $z = 3\ \mathrm{m}$**:** $\sigma'_{v0} = 8.19\,z = 24.57\ \mathrm{kPa}$
Overconsolidation	**(a) OCR = 1 (NC) · (b) OCR = 6 · (c) OCR = 3**
Stress increase	$\Delta\sigma = 150\,I_z(3) = 87.04\ \mathrm{kPa}$ **(Example 3)** ⇒ $\sigma'_{final} = 111.61\ \mathrm{kPa}$

REQUIRED

The **regime** and consolidation settlement $\mathbf{S_c}$ for each clay.



Solution (1/2)

SOLUTION

1 Common: f = H/(1 + e_0) = 6/2 = 3 m; log($\sigma'_{final}/\sigma'_{v0}$) = log(111.61/24.57) = 0.6573

$$\text{(a) NC}: \quad S_v = C_c f \log \frac{\sigma'_{final}}{\sigma'_{v0}} = 0.30(3)(0.6573) = \mathbf{591.6\ mm}$$

2 (b) OCR = 6: σ'_p = 6(24.57) = 147.4 kPa > σ'_{final} ⇒ recompression only

$$S_r = C_r f \log \frac{\sigma'_{final}}{\sigma'_{v0}} = 0.06(3)(0.6573) = \mathbf{118.3\ mm}$$

Same clay, same load: the normally consolidated clay settles **591.6 mm**, the one with OCR = 6 settles **118.3 mm**; the difference comes from the ratio C_c/C_r = 5.

Solution (2/2)

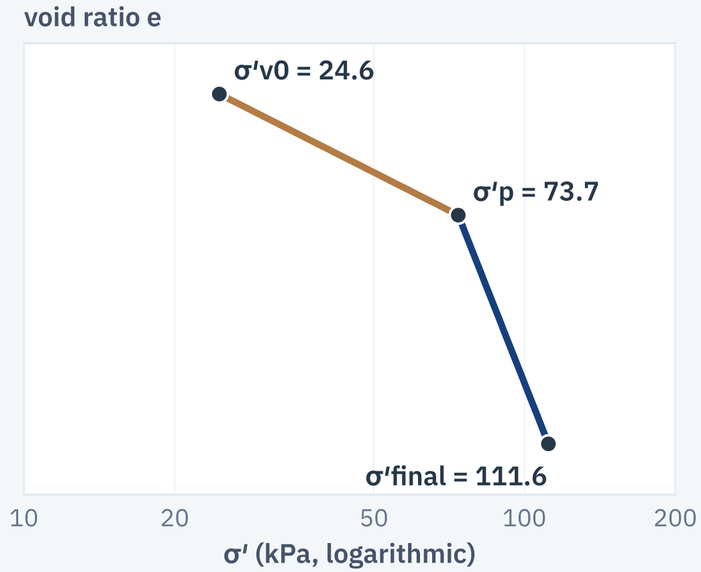
SOLUTION

3 (c) OCR = 3: $\sigma'_p = 3(24.57) = 73.71$ kPa < $\sigma'_{final} = 111.61$ ⇒ recompression + virgin compression

$$S_r = 0.06(3)\log\frac{73.71}{24.57} = 85.9\ \text{mm}$$

$$S_v = 0.30(3)\log\frac{111.61}{73.71} = 162.2\ \text{mm} \qquad S_c = \mathbf{248.0\ mm}$$

OCR	σ'_p (kPa)	regime	S_r (mm)	S_v (mm)	S_c (mm)
1	24.6	intact	0	591.6	**591.6**
6	147.4	recompression	118.3	0	**118.3**
3	73.7	recompression + virgin	85.9	162.2	**248.0**



Check the solution in GeoTerva ↗

From the original ground or from the heaved base?

THEORY

After the excavation comes the foundation: settlement depends on **which reference** the displacements are measured from (the *post-excavation displacements* option at the excavation stage):

Preserved — from the original ground (default): heave is also shown separately; the foundation settlement is the net deformation at the end of loading: $\mathbf{S = S_c + S_{heave}}$ = total compression beyond σ'_{v0}. A load not greater than the removed overburden gives S = 0; if it is smaller, the soil remains above the original level (reported, settlement 0).

Set to zero before loading: the displacements of the excavation stage are zeroed; settlement is measured from the heaved excavation base: $\mathbf{S = S_c = S_b + S_r + S_v}$. Recompression of the heave counts as settlement: a fully compensated foundation settles as much as the base heaved. If the excavation remained open until the structure was built and heave developed, this is the correct answer.

Both measures are always printed; the choice determines which is reported as the footing settlement, the time curve and the result.

SUMMARY

Preserved: $S = S_c + S_{heave}$ (small). Set to zero: $S = S_c$ (large).

Problem statement

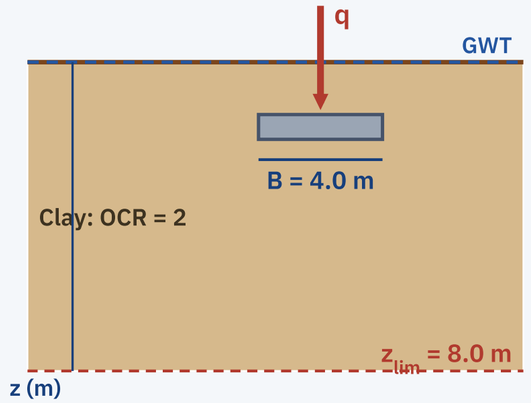
EXAMPLE

Given

Soil	**clay 0–8 m, $\gamma_{sat} = 18$, GWT at surface;** $e_0 = 1.0,\ C_c = 0.30,\ C_r = C_s = 0.06$; $OCR = 2$
Excavation	**wide excavation (I_{exc} = 1), depth 2 m; at the foundation excavation base**
Foundation	**4 × 6 m, $q_{gross} = 100$ kPa**
Slice	**single slice 2–8 m, $H = 6$ m, $z = 5$ m, $f = 3$ m: $\sigma'_{v0} = 8.19(5) = 40.95$ kPa**

REQUIRED

σ'_{exc}, heave, S_b, S_r, S_v, S_c; settlement of the foundation (from the original soil and after resetting).



Solution (1/3)

SOLUTION

1 Excavation: removed load, unloading and heave

$$q_{lift} = \sigma'_{v0}(2) = 8.19(2) = 16.38 \text{ kPa} \qquad \sigma'_{exc} = 40.95 - 16.38 = \mathbf{24.57} \text{ kPa}$$

$$\Delta H_{heave} = -0.06(3)\log\frac{40.95}{24.57} = -0.06(3)(0.2218) = \mathbf{-39.9 \text{ mm}}$$

2 Loading: $I_z(z' = 3) = 0.5803$ (Example 3); $\sigma'_p = 2(40.95) = 81.9$ kPa

$$\Delta\sigma = 100(0.5803) = 58.03 \text{ kPa} \quad \sigma'_{final} = 24.57 + 58.03 = \mathbf{82.60} \text{ kPa}$$

Solution (2/3)

SOLUTION

3 Regime: $\sigma'_{v0} < \sigma'_p = 81.9 < \sigma'_{final} = 82.60$

$$S_b = 0.06(3)\log\frac{40.95}{24.57} = 39.9\ \text{mm} \qquad S_r = 0.06(3)\log\frac{81.9}{40.95} = 54.2\ \text{mm}$$

$$S_v = 0.30(3)\log\frac{82.60}{81.9} = 3.3\ \text{mm} \qquad S_c = \mathbf{97.4\ mm}$$

On reloading, the swell is first recovered (S_b = 39.9 mm = the entire swell), then 54.2 mm of recompression with C_r and 3.3 mm of virgin compression slightly exceeding σ'_p.

Solution (3/3)

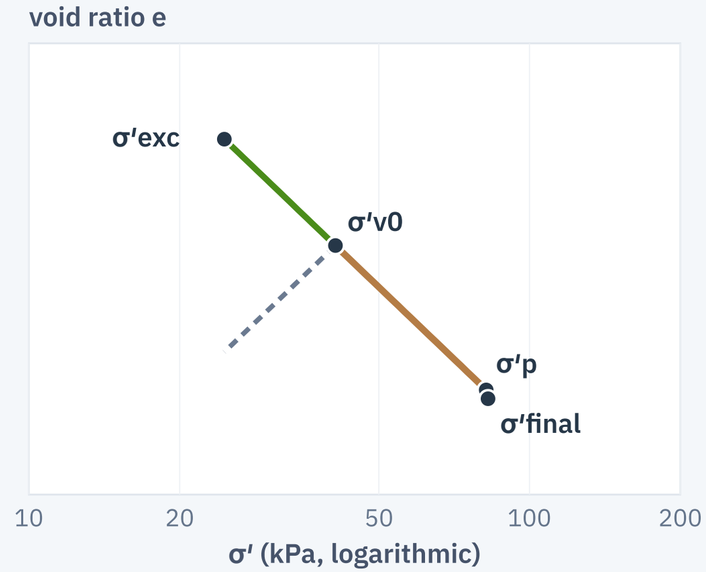
SOLUTION

4 Settlement of the footing

$$\text{is preserved}: \quad S = S_c + S_{heave} = 97.4 - 39.9 = \mathbf{57.5\ mm}$$

$$\text{becomes zero}: \quad S = S_c = \mathbf{97.4\ mm}$$

Settlement measured from the original ground surface is **57.5 mm**; measured from the heave surface it is **97.4 mm**. The difference is equal to the heave (39.9 mm): the choice of reference changes the result by 69%.



Check the solution in GeoTerva ↗

Linear path: $\Delta H = m_v\, \Delta\sigma'\, H$

THEORY

It is a separate and **linear** calculation route: the same m_v is used for both unloading and reloading; C_c, C_r, C_s and e_0 do not enter:

$$\Delta H = m_v\, \Delta\sigma'\, H_{slice}$$

$$\text{heave} = -m_v\,(\sigma'_{v0} - \sigma'_{exc})\,H \qquad S_c = m_v\,\left(\sigma'_{final} - \sigma'_{exc}\right)H$$

The part of S_c up to σ'_{v0} is the recovered swell (S_b).

Since it is linear, the loop closes by itself.

The unit of m_v is 1/kPa (coefficient of volume compressibility); it must be selected according to the stress range.

DIFFERENCE FROM e – log σ′

The m_v method ignores the stress level (no σ'_p): every stress increment produces the same compression.

Problem statement

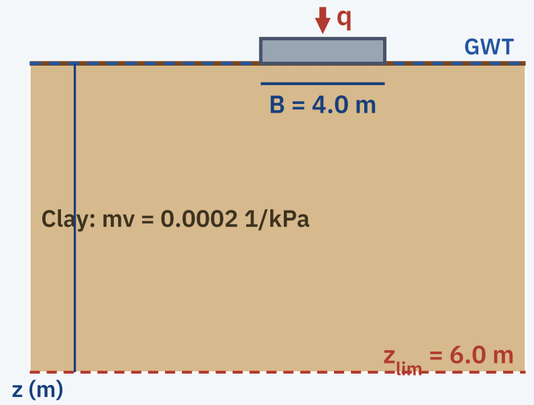
EXAMPLE

Given

Soil	**6 m clay, $m_v = 0.0002\ 1/\mathrm{kPa}$, GWT at surface**
Case (a)	**unbraced: 4×6 m, $q = 150$ kPa, $\Delta\sigma = 87.04$ kPa (Example 3)**
Case (b)	**The cut and foundation in Example 6 (2 m cut, $q = 100$ kPa, 2–8 m slice)**

REQUIRED

S_c in each case; for (b), heave, S_b and net settlement.



Solution

SOLUTION

1 (a) No excavation

$$S_c = m_v\,\Delta\sigma'\,H = 0.0002(87.04)(6) = \mathbf{104.4\ mm}$$

2 (b) Excavated: σ'_{v0} = 40.95; σ'_{exc} = 24.57; $\Delta\sigma_{load}$ = 58.03; σ'_{final} = 82.60 kPa

$$\text{heave} = -0.0002(40.95 - 24.57)(6) = -19.7\ \text{mm} = -S_b$$

$$S_r = 0.0002(82.60 - 40.95)(6) = 50.0\ \text{mm} \qquad S_c = S_b + S_r = \mathbf{69.6\ mm}$$

Net settlement (from the original ground) S = S_c + heave = **50.0 mm**; only the 41.65 kPa increment exceeding σ'_{v0} produces settlement. In the linear route, the e – log σ′ method has no σ'_p effect.

Check the solution in GeoTerva ↗

Terzaghi: T_v, U and t

THEORY

$$T_v = \frac{C_v\, t}{H_{dr}^2} \qquad U(T_v) = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} e^{-M^2 T_v} \qquad M = \frac{\pi(2m+1)}{2}$$

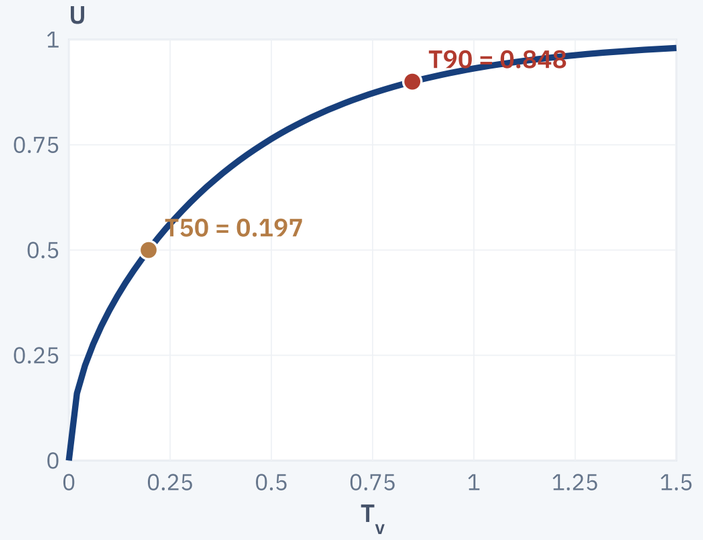
$$S_i(t) = U_i(t)\, S_{c,i} \qquad S(t) = \sum S_i(t) \qquad t = \frac{T_v\, H_{dr}^2}{C_v}$$

H_{dr} = H (one-way drainage) or H/2 (two-way); H is the layer thickness below the foundation level (up to z_{lim} for the last layer extending beyond the profile).

$T_{v,50}$ = 0.197; $T_{v,90}$ = 0.848. For T_v < 0.05 the early-time solution $U = 2\sqrt{(T_v/\pi)}$ is used.

Layers with different C_v and H_{dr} values consolidate at different rates: the curve of the profile is the **sum of the layer curves**; its own t_{50} and t_{90} values are read from this sum.

The curve belongs to the load settlement S_c; the heave is assumed to be completed before loading.



Problem statement

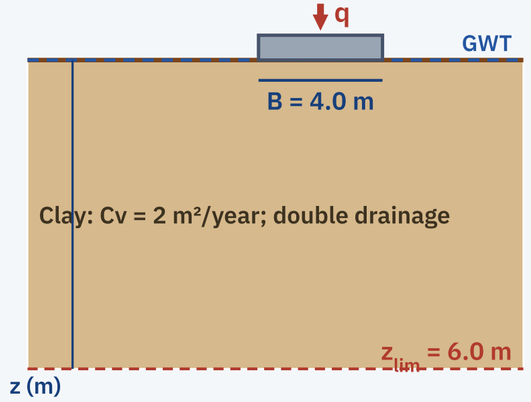
EXAMPLE

Given

Clay	**6 m, normally consolidated (Example 5a): $S_c = 591.6$ mm**
Consolidation	**$C_v = 2\ \mathrm{m}^2/\mathrm{year}$; drainage at top and bottom (double-sided)**
Times	**t = 0.25; 0.5; 1; 2; 4; 8 years**

REQUIRED

H_{dr}, t_{50}, t_{90} and S(t); t_{90} with single drainage.



Solution (1/2)

SOLUTION

1 Drainage path and characteristic times

$$H_{dr} = \frac{H}{2} = 3\ \text{m} \qquad t_{50} = \frac{0.197(3^2)}{2} = \mathbf{0.89}\ \text{year} \qquad t_{90} = \frac{0.848(3^2)}{2} = \mathbf{3.82}\ \text{year}$$

2 One-way drainage (H_{dr} = 6 m)

$$t_{90} = \frac{0.848(6^2)}{2} = \mathbf{15.3}\ \text{year}$$

With double drainage the drainage path halves and the time drops to **one quarter**: t_{90} = 3.82 years, versus 15.3 years for single drainage.

Solution (2/2)

SOLUTION

t (year)	$T_v = C_v t/H_{dr}^2$	U	$S(t) = U\,S_c$ (mm)
0.25	0.056	0.266	**157**
0.50	0.111	0.376	**222**
1.00	0.222	0.531	**314**
2.00	0.444	0.729	**431**
4.00	0.889	0.910	**538**
8.00	1.778	0.990	**586**

In 1 year only 53% of the settlement is completed, in 4 years 91% (S_c = 592 mm); in 8 years U = 0.990. For small T_v values (< 0.05), $U = 2\sqrt{(T_v/\pi)}$ is used.

Check the solution in GeoTerva ↗

Problem statement

EXAMPLE

Given

Foundation	**4 × 6 m, q = 150 kPa, at ground surface; GWT at surface (γ_{sat} = 18)**
Layer A (0–4 m)	**soft, NC: $e_0 = 1.10,\ C_c = 0.35,\ C_r = 0.07$; $C_v = 1$; double drainage**
Layer B (4–8 m)	**stiff, $OCR = 2$: $e_0 = 0.80,\ C_c = 0.20,\ C_r = 0.04$; $C_v = 3$; single-sided drainage**
Calculation	**base of the profile (8 m); each layer as a single slice (A: z = 2 m, B: z = 6 m)**

REQUIRED

The **S_c** value, t_{50} and t_{90} of each layer; the S(t) curve of the system.

Solution (1/3)

SOLUTION

1 A: z = 2 m, σ'_{v0} = 16.38; $I_z(2)$ = 0.7746 ⇒ Δσ = 116.19; σ'_{final} = 132.57; f = 4/2.10 = 1.9048 m; NC

$$S_{c,A} = C_c f \log \frac{\sigma'_{final}}{\sigma'_{v0}} = 0.35(1.9048) \log \frac{132.57}{16.38} = \mathbf{605.4\ mm}$$

2 B: z = 6 m, σ'_{v0} = 49.14; σ'_p = 98.28; $I_z(6)$ = 0.2449 ⇒ Δσ = 36.74; σ'_{final} = 85.88 < σ'_p; f = 4/1.80 = 2.2222 m

$$S_{c,B} = C_r f \log \frac{\sigma'_{final}}{\sigma'_{v0}} = 0.04(2.2222) \log \frac{85.88}{49.14} = \mathbf{21.6\ mm}$$

Total consolidation settlement **627.0 mm**: the soft NC upper layer carries 97% of the total; the stiff, overconsolidated lower layer only 21.6 mm.

Solution (2/3)

SOLUTION

3 Drainage path and durations ($t = 0.197\ H_{dr}^2/C_v$; $0.848\ H_{dr}^2/C_v$)

$$A:\ H_{dr} = 2\ \mathrm{m}:\ t_{50} = \frac{0.197(4)}{1} = 0.788,\ t_{90} = \frac{0.848(4)}{1} = 3.39\ \mathrm{year}$$

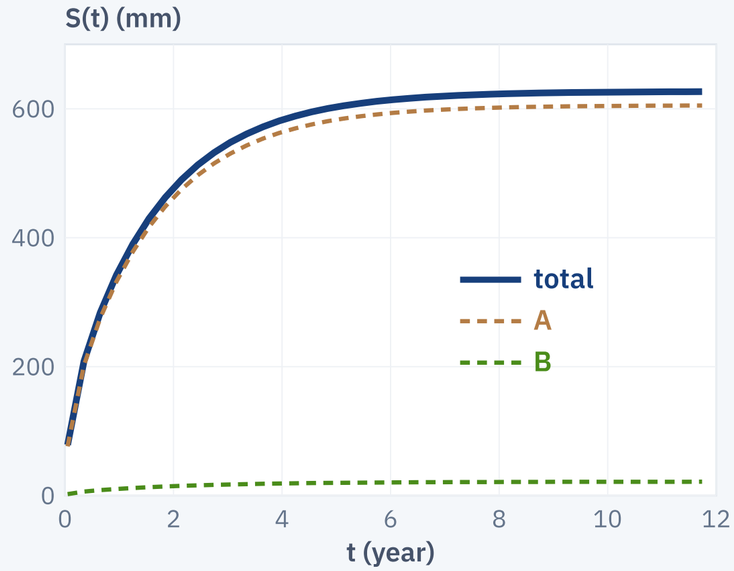
$$B:\ H_{dr} = 4\ \mathrm{m}:\ t_{50} = \frac{0.197(16)}{3} = 1.051,\ t_{90} = \frac{0.848(16)}{3} = 4.52\ \mathrm{year}$$

The curve of the system is the sum $S(t) = U_A(t)\ S_A + U_B(t)\ S_B$; its own times are read from this sum: **t_{50} = 0.794 yr, t_{90} = 3.43 yr** (close to the dominant layer A).

Solution (3/3)

SOLUTION

t (year)	S_A (mm)	S_B (mm)	S(t) (mm)	U
0.5	242	7.4	**249**	0.397
1.0	340	10.5	**351**	0.560
2.0	463	14.6	**477**	0.761
5.0	583	19.8	**603**	0.961
10.0	604	21.4	**626**	0.998
20.0	605	21.6	**627**	1.000



56% of the settlement is completed in 1 year and 96% in 5 years; it finishes in about 10 years (627 mm). The curve is governed by the **soft layer A**.

Check the solution in GeoTerva ↗

Points to keep in mind when reading the results

THEORY

Footing level = excavation base (ground surface if no excavation); **hydrostatic** pore water pressure; the GWT is not affected by the excavation.

The stress increase is taken **on the vertical below the footing center** (slices and time analysis); the color zones follow the same stress path at every point.

e_0 is the initial void ratio at each stage; **there is no secondary compression (C_α)**.

Settlement is positive, heave is negative; the toggle in the header reverses this and switches to upward-positive display (the engine stays classical).

Color bands: zones that decrease rapidly with depth, such as settlement, swell and deformation, are painted with the classes $1 \cdot 1.5 \cdot 2 \cdot 3 \cdot 5 \cdot 7 \times 10^n$; equal bands would compress the lower zone into a single color.

Summary

Key points

Stress path: $\sigma'_{v0} \rightarrow \sigma'_{exc} \rightarrow \sigma'_{v0}$ (C_s) $\rightarrow \sigma'_p$ (C_r) $\rightarrow \sigma'_{final}$ (C_c); $\sigma'_p = OCR \cdot \sigma'_{v0}$, does not decrease with excavation

Slice compression $f = \Delta H/(1 + e_0)$: NC: C_c f $\log(\sigma'_{final}/\sigma'_{v0})$; OC and $\sigma'_{final} \leq \sigma'_p$: C_r f log(...); $\sigma'_{final} > \sigma'_p$: $C_r + C_c$ parts

Excavation: $q_{unload} = \sigma'_{v0}(D_{exc})$, heave = $-C_s$ f $\log(\sigma'_{v0}/\sigma'_{exc})$; $S = S_c + S_{heave}$ (original ground) or S_c (re zeroed)

Boussinesq: $\Delta\sigma = q\ I_z(z')$; rectangle centre 4 I_{corner}, strip $(\alpha + \sin\alpha)/\pi$; $\mathbf{z_{lim}}$: $I(z') = p$ (square 2.09 B, L = 1.5 B 2.55 B, strip 6.34 B)

m_v method: $\Delta H = m_v\ \Delta\sigma'\ H$, linear; heave = $-m_v(\sigma'_{v0} - \sigma'_{exc})\ H$

Time: $T_v = C_v\ t/H_{dr}^2$; $T_{v,50} = 0.197$, $T_{v,90} = 0.848$; H_{dr} = H (single) or H/2 (double); $S(t) = \Sigma\ U_i\ S_{c,i}$

References

Terzaghi, K. (1943). Theoretical Soil Mechanics. Wiley.

Newmark, N.M. (1935). Simplified computation of vertical pressures in elastic foundations. Univ. of Illinois Bulletin 429.

Das, B.M. (2016). Principles of Geotechnical Engineering, 9th ed. Cengage.

Holtz, R.D., Kovacs, W.D., Sheahan, T.C. (2011). An Introduction to Geotechnical Engineering, 2nd ed. Pearson.

Craig, R.F., Knappett, J.A. (2012). Craig’s Soil Mechanics, 8th ed. Spon.

NOTE

All numerical examples have been produced with the module's own calculation engine; the module gives the same values (example slices: dz = layer thickness).

GeoTerva

Online geotechnical analysis

From theory to practice

Calculate the examples in this presentation step by step in GeoTerva; the number of layers and the excavation are entered freely in the module.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com