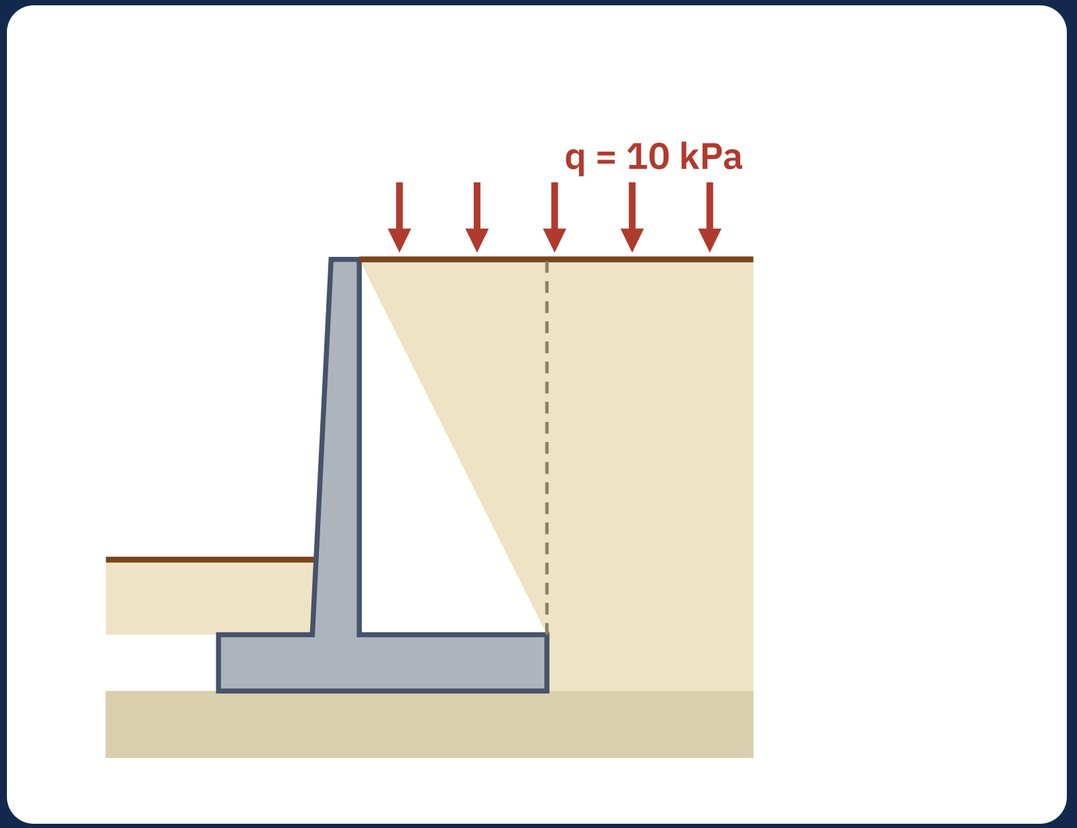
Geotechnical lecture slide · Retaining walls

Cantilever retaining wall

Lecture: external stability of a reinforced concrete cantilever retaining wall (sliding, overturning, bearing capacity) and the section forces in the stem, toe and heel slabs; heel block, earth pressure, water, earthquake, shear key and relief cantilever; worked examples on every topic.

- Stem + base slab
- Sliding · overturning · bearing capacity
- Section forces in stem, toe and heel
- Shear key · relief cantilever
- Water and earthquake



What will we learn in this lecture?

- Cantilever wall: **stem + base slab** (toe L_t, heel L_h), finding its weight and centroid from the component parts
- The **soil block** above the toe and the **vertical pressure plane** at the rear edge of the footing; active thrust, surcharge and front passive resistance
- Reading the **sliding, overturning, eccentricity and bearing capacity** checks and the earthquake condition (Mononobe–Okabe + inertia)
- **From the base pressure**, the shear and moment effects on the toe and heel plates, and on **the stem** from the thrust
- How the **shear key** and the **relieving cantilever** increase safety; the effect of water and uplift
- To see **how to revise** a failed section (sliding and eccentricity under earthquake)

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Everything is per 1.00 m length of wall. Example: H = 4.0 m stem, base slab 0.60 m, B = 3.5 m reinforced concrete wall (24 kN/m³), q = 10 kPa.

Stem, base slab and toe structure

THEORY

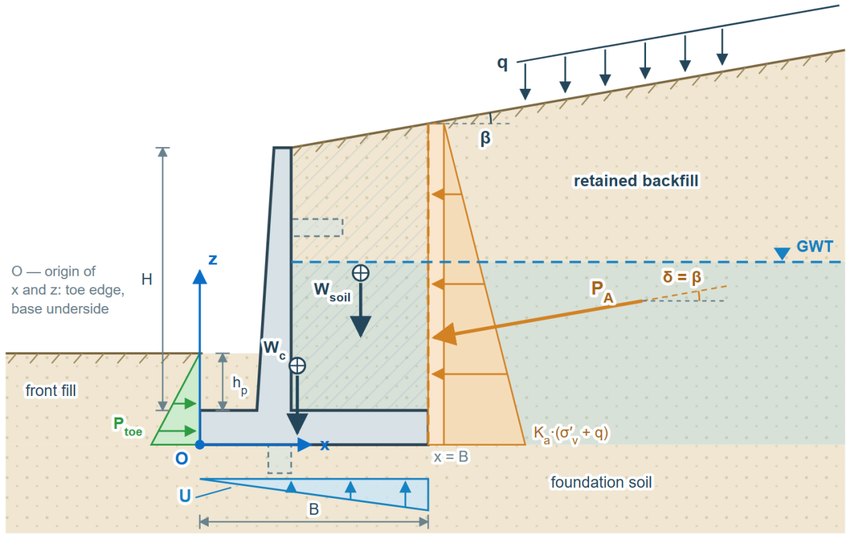
The cantilever wall also resists sliding and overturning **through the weight of the soil above the base slab**: not only the wall's own weight but also the soil block above the heel is stabilizing. Module:

$$B = L_t + t_b + L_h \qquad z_T = t_f + H$$

Stem: height H; thickness t_t at the top, t_b at the bottom (vertical back, inclined face); **base slab**: thickness t_f, toe L_t, heel L_h; concrete 24 kN/m³.

Soil: cohesionless fill (c′ = 0), level or sloping surface, surcharge q, groundwater, and a fill of height h_p on the base slab at the front.

Options: shear key, relief cantilever; **checks**: sliding, overturning, eccentricity, bearing capacity; **section forces**: stem, toe, heel, cantilever, key.



Base slab + stem rectangle + stem triangle

THEORY

The weight of the wall is divided into three simple shapes (known area, centroid); its moment about the toe is stabilizing:

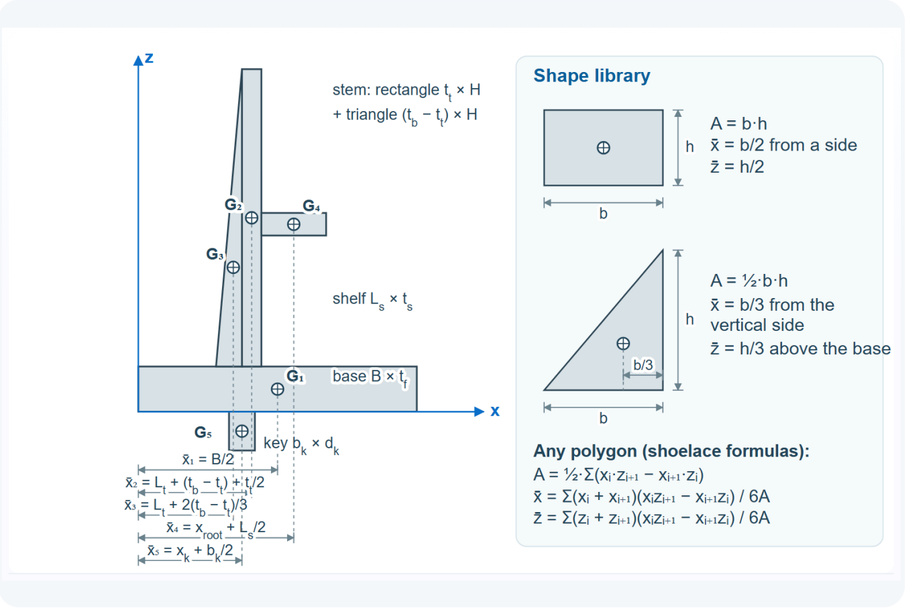
$$W_i = \gamma_c A_i \qquad W_c = \sum W_i \qquad \bar{x}_c = \frac{\sum W_i \bar{x}_i}{W_c} \qquad \bar{z}_c = \frac{\sum W_i \bar{z}_i}{W_c}$$

Base plate: A = B t_f, $\bar{x}$ = B/2, $\bar{z}$ = t_f/2.

Stem rectangle (adjacent to the back): A = t_tH, $\bar{x}$ = L_t + (t_b − t_t) + t_t/2, $\bar{z}$ = t_f + H/2.

Stem triangle (front batter): A = ½(t_b − t_t)H, $\bar{x}$ = L_t + 2(t_b − t_t)/3, $\bar{z}$ = t_f + H/3.

In the force table the wall is a single item: F_z = −W_c at $\bar{x}_c$; $\bar{z}_c$ is used only for horizontal inertia (earthquake).



Problem statement

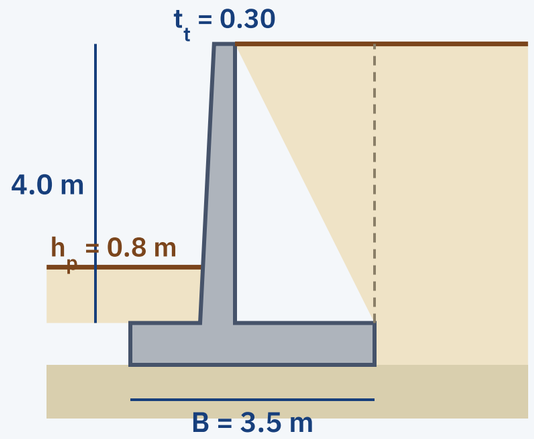
EXAMPLE

REQUIRED

Area, weight and location of the three parts; $\mathbf{W_c}$, $\mathbf{\bar{x}_c}$, $\mathbf{\bar{z}_c}$.

Given

Section	$H = 4.0$ **m;** $t_t = 0.30$**,** $t_b = 0.50$ **m;** $L_t = 1.00$**,** $L_h = 2.00$ **m;** $t_f = 0.60$ **m (B = 3.5 m)**
Material	**reinforced concrete:** $\gamma_c = 24\ \mathrm{kN/m^3}$



Solution

SOLUTION

1 Parts (e.g. triangle: A = ½(0.50 − 0.30)(4.0) = 0.40; $\bar{x}$ = 1.00 + 2(0.20)/3 = 1.133; $\bar{z}$ = 0.60 + 4.0/3 = 1.933)

Piece	A (m²)	$\bar{x}$ (m)	$\bar{z}$ (m)	W = 24A (kN/m)	W·$\bar{x}$	W·$\bar{z}$
Base plate	2.100	1.750	0.300	50.40	88.20	15.12
Shaft rectangle	1.200	1.350	2.600	28.80	38.88	74.88
Shaft triangle	0.400	1.133	1.933	9.60	10.88	18.56
Total				**88.80**	**137.96**	**108.56**

2 Centroid

$$\bar{x}_c = \frac{137.96}{88.80} = \mathbf{1.554}\ \text{m} \qquad \bar{z}_c = \frac{108.56}{88.80} = \mathbf{1.223}\ \text{m}$$

The wall weight **W_c = 88.80 kN/m**, $\bar{x}_C$ = 1.55 m from the toe, $\bar{z}_C$ = 1.22 m from the base; its moment M = −137.96 kN·m/m (restoring).

Check the solution in GeoTerva ↗

Soil and pressure plane moving together with the wall

THEORY

The soil above the heel of the base slab cannot slide relative to the slab: if the wall moves, it moves **together with it**. The failure planes start **at the rear edge of the heel**, not at the back of the stem. Free body = wall + heel block; its outer face is the **vertical pressure plane** at x = B (soil against soil: no wall friction, thrust δ = β):

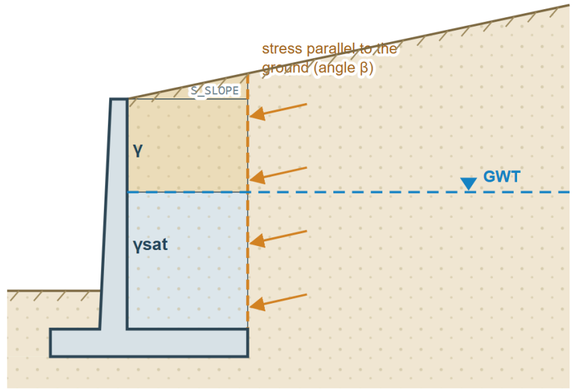
Weight of the heel block $W_s = \gamma L_h H$ (split into moist/saturated zones; since total stress is used, γ_{sat} is taken below water, with the water thrust kept as a separate item).

At the toe, the fill above the base plate (h_p) moves with the plate: vertical load W_{toe}.

Surcharge on the heel $\mathbf{Q_{heel} = qL_h}$ is favorable but, being a load that may also be absent, it **enters no check**; its lateral effect is always counted.

Height of the pressure plane $h = t_f + H$ (including the rise of the surface).

Vertical plane at the heel edge (all methods)



Problem statement

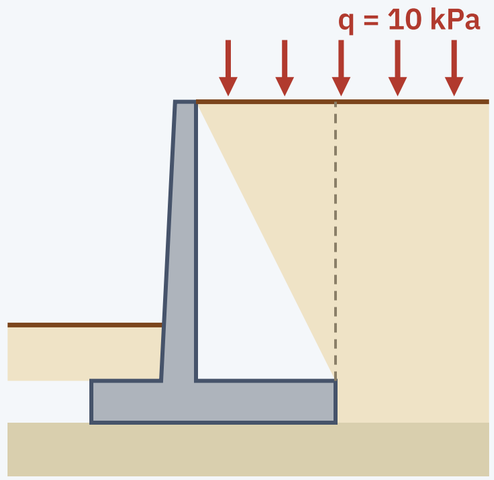
EXAMPLE

Given

Wall	**Section from Example 1; front fill height $h_p = 0.80$ m (above the base slab)**
Soil	**dry fill: $\gamma = 18\ \mathrm{kN/m^3}$, $\varphi' = 30^\circ$; horizontal surface**
Surcharge	$q = 10$ **kPa**

REQUIRED

The **weight and location** of the heel block; the **toe fill**; Q_{heel}; total vertical load N.



Solution

SOLUTION

1 Heel block (L_h × H rectangle, behind the stem)

$$A_s = L_h H = 2.00(4.0) = 8.0\ \text{m}^2 \qquad W_s = 18(8.0) = \mathbf{144.0}\ \text{kN/m}$$

$$\bar{x}_s = L_t + t_b + \frac{L_h}{2} = 2.50\ \text{m} \qquad \bar{z}_s = t_f + \frac{H}{2} = 2.60\ \text{m}$$

2 Toe fill (trapezoid: L_t = 1.00 at the base, $x_{toe}(h_p)$ = 1.04 at the top) and surcharge

$$A_{toe} = \tfrac{1}{2}(1.00 + 1.04)(0.80) = 0.816\ \text{m}^2 \quad W_{toe} = 18(0.816) = \mathbf{14.69}\ \text{kN/m}\ (\bar{x} = 0.510)$$

$$Q_{heel} = qL_h = 10(2.00) = \mathbf{20.0}\ \text{kN/m}\ (\text{not subject to any check})$$

N = W_c + W_s + W_{toe} = 88.80 + 144.00 + 14.69 = **247.49 kN/m**; Q_{heel} = 20.00 kN/m is the item left "unused" in the table.

Check the solution in GeoTerva ↗

Thrust in the vertical plane, surcharge and combined passive

THEORY

The pressure plane is at height h = t_f + H. Since the fill is horizontal and δ = β = 0, Rankine = Coulomb:

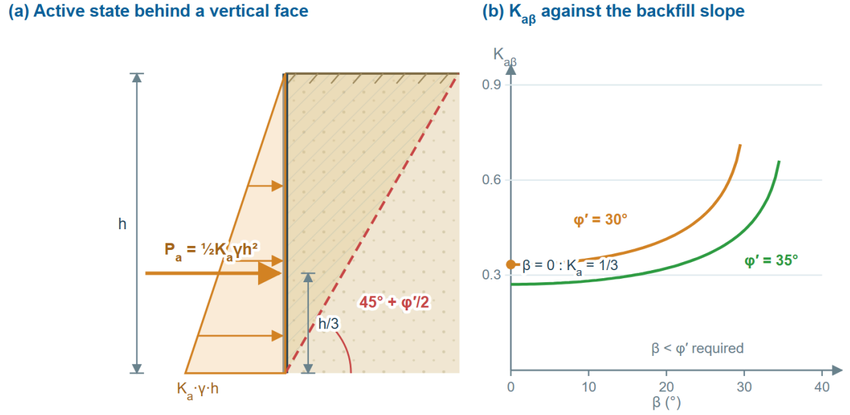
$$K_a = \frac{1-\sin\varphi'}{1+\sin\varphi'} \qquad P_A = \tfrac{1}{2}K_a\gamma h^2\ (h/3) \qquad P_Q = K_a q h\ (h/2)$$

In an inclined embankment the stress on the plane is parallel to the surface ($K_{a\beta}$), and the vertical component of the thrust is a resisting force. Front resistance (in sliding only):

$$K_{mob} = wK_p + (1-w)K_0 \qquad P_P = \tfrac{1}{2}K_{mob}\gamma D_p^2\ (D_p/3) \qquad D_p = t_f + h_p$$

w = 1/3 by default; if the front soil can be removed, the credit is reduced with h_p = 0 or h_{ignore}; **front resistance is not used in overturning and bearing capacity**.

Details (Rankine/Coulomb equivalence, K_{mob}, h_{ignore}) are in the gravity retaining wall presentation.



Problem statement

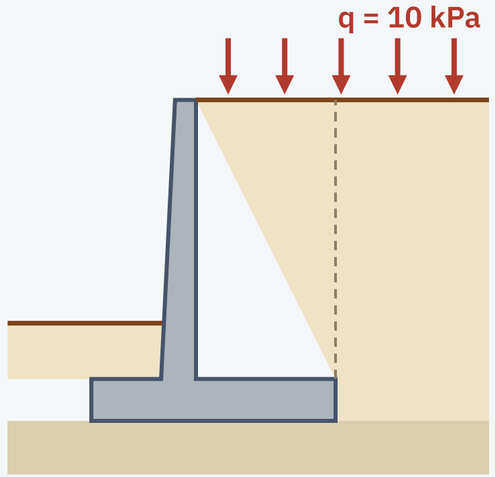
EXAMPLE

REQUIRED

P_A, P_Q, P_P and the overturning moment about the toe.

Given

Wall and soil	**The wall in Examples 1–2: h = t_f + H = 4.60 m;** $\gamma = 18\ \mathrm{kN/m^3}$, $\varphi' = 30^\circ$**; horizontal surface;** $q = 10$ **kPa**
Front ground	$D_p = t_f + h_p = 1.40$ **m; w = 1/3**



Solution

SOLUTION

1 K_a = 0.3333; thrusts (h = 4.60 m)

$$P_A = \tfrac{1}{2} K_a \gamma h^2 = \tfrac{1}{2}(0.3333)(18)(21.16) = \mathbf{63.48}\ \mathrm{kN/m} \quad (z = 1.533\ \mathrm{m})$$

$$P_Q = K_a q h = 0.3333(10)(4.6) = \mathbf{15.33}\ \mathrm{kN/m} \quad (z = 2.30\ \mathrm{m})$$

2 Front resistance: K_{mob} = ⅓(3.00) + ⅔(0.50) = 1.333

$$P_P = \tfrac{1}{2}(1.333)(18)(1.40)^2 = \mathbf{23.52}\ \mathrm{kN/m} \quad (z = D_p/3 = 0.467\ \mathrm{m})$$

3 Horizontal thrust and overturning moment (about toe)

$$\Sigma H_{drive} = 63.48 + 15.33 = \mathbf{78.81}\ \mathrm{kN/m}$$

$$M_o = 63.48(1.533) + 15.33(2.30) = 97.34 + 35.27 = \mathbf{132.60}\ \mathrm{kN\cdot m/m}$$

Check the solution in GeoTerva ↗

Four checks from the same force table

THEORY

$$FS_s = \frac{\mu_b N + P_{toe}}{\Sigma H_{drive}} \qquad FS_o = \frac{\Sigma M_{res}}{\Sigma M_{ov}} \qquad x_R = -\frac{\Sigma M}{V} \qquad e = \frac{B}{2} - x_R$$

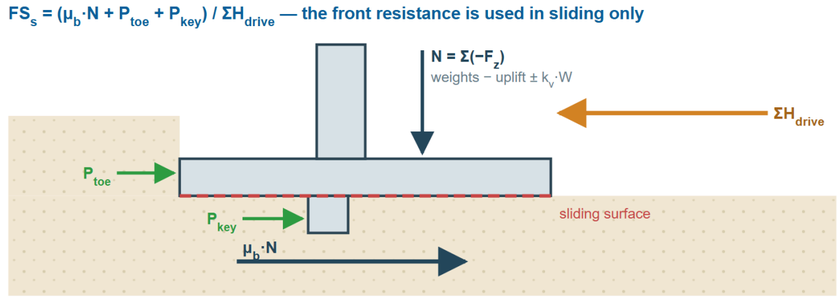
$$|e| \le \frac{B}{6}: \quad q_{max,min} = \frac{V}{B}\left(1 \pm \frac{6e}{B}\right) \qquad |e| > \frac{B}{6}: \quad q_{max} = \frac{2V}{3x_R}$$

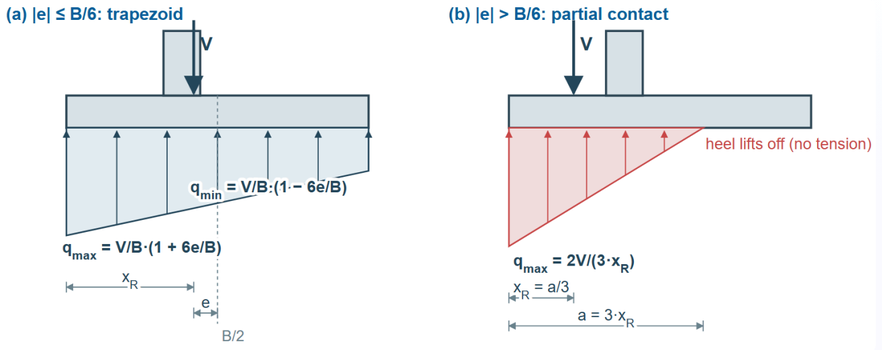
Sliding: N is the effective normal force (excluding Q_{heel}, uplift deducted); $\mu_b = \tan \delta_b$; no adhesion.

Overturning: about the toe (the front lower corner of the base slab); each item's $-zF_x$ and xF_z parts are counted with their own sign; front resistance does not enter.

Bearing capacity: if the resultant is within the middle third of the base ($|e| \le B/6$), the base is fully in contact; otherwise the soil cannot take tension and only a length $a = 3x_R$ is in contact.

Required: in static conditions $FS_s \ge 1.5$, $FS_o \ge 2.0$; under earthquake ≥ 1.1; $q_{max} \le q_{all}$.





Problem statement

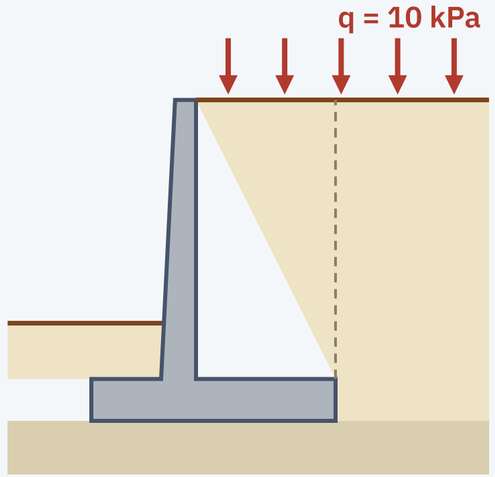
EXAMPLE

Given

Wall and loads	**Wall in Example 1–3:** $B = 3.5$ **m;** $W_c = 88.8,\ W_s = 144.0,\ W_{toe} = 14.69\ \mathrm{kN/m}$**;** $P_A = 63.48,\ P_Q = 15.33\ \mathrm{kN/m}$
Resistance	$\mu_b = 0.55$**;** $P_P = 23.52$ **kN/m;** $q_{all} = 200$ **kPa**
Required	$FS_s \geq 1.5,\ \ FS_o \geq 2.0$**; base in full contact**

REQUIRED

FS_s, FS_o, x_R, e, q_{max}, q_{min} and the results.



Solution (1/3)

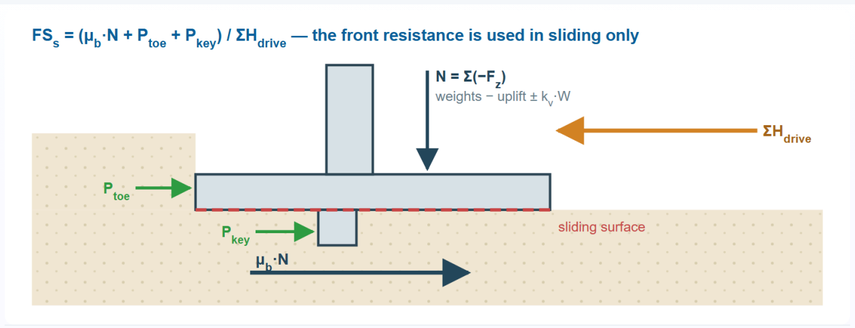
SOLUTION

1 Base friction ($N = W_c + W_s + W_{toe}$)

$$N = 88.8 + 144.0 + 14.69 = \mathbf{247.49}\ \mathrm{kN/m}$$

$$\mu_b N = 0.55(247.49) = 136.12\ \mathrm{kN/m}$$

$$FS_s = \frac{136.12 + 23.52}{78.81} = \mathbf{2.026}\ \geq 1.5\ \checkmark$$



Solution (2/3)

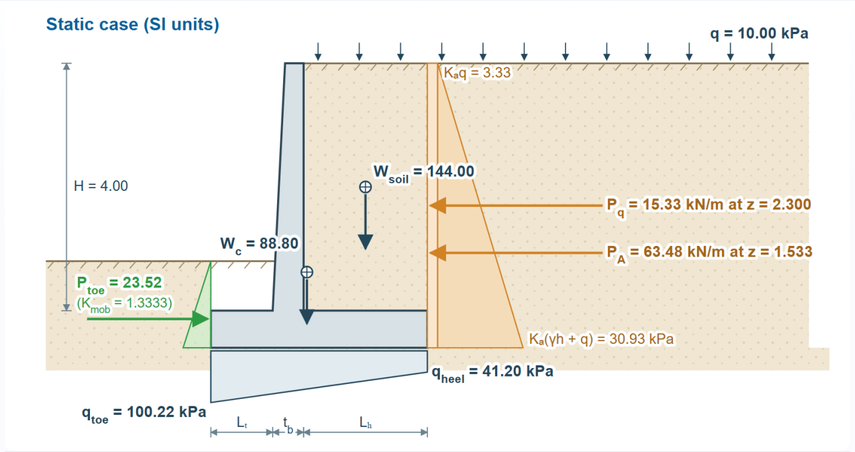
SOLUTION

2 Overturning moment (thrusts) and resisting moment (wall, heel block, front fill)

$$M_o = 97.34 + 35.27 = \mathbf{132.60}$$

$$M_r = 137.96 + 360.00 + 7.49 = \mathbf{505.45}\ \mathrm{kN\cdot m/m}$$

$$FS_o = \frac{505.45}{132.60} = \mathbf{3.812}\ \geq 2.0\ \checkmark$$



Solution (3/3)

SOLUTION

3 Location of the resultant and eccentricity

$$\Sigma M = M_o - M_r = -372.85$$

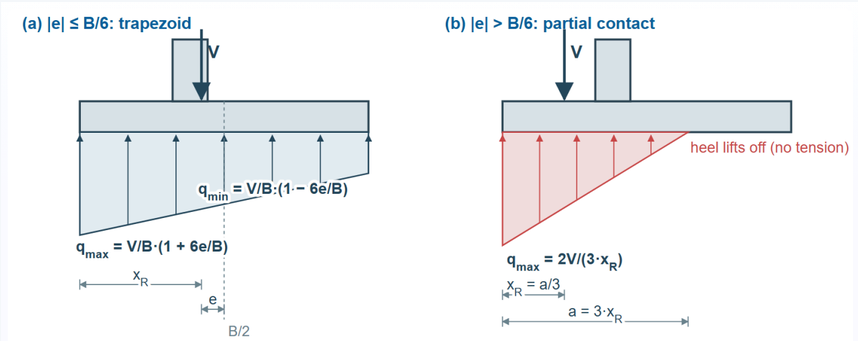
$$x_R = -\frac{\Sigma M}{V} = \frac{372.85}{247.49} = \mathbf{1.5065}\ \text{m}$$

$$e = \frac{B}{2} - x_R = \mathbf{0.2435}\ \text{m} \le \frac{B}{6} = 0.583\ \checkmark$$

4 Base stresses (full contact)

$$q_{max} = \frac{247.49}{3.5}\left(1 + \frac{6(0.2435)}{3.5}\right) = \mathbf{100.22}\ \text{kPa}$$

$$q_{min} = \mathbf{41.20}\ \text{kPa}$$



FS_s = **2.03**, FS_o = **3.81**, e = 0.243 m, q_{max} = **100.2 kPa** ≤ 200, q_{min} = 41.2 kPa > 0: **all static checks are satisfied.**

Check the solution in GeoTerva ↗

Mononobe–Okabe thrust and addition of inertia

THEORY

In the earthquake case, the **Mononobe–Okabe** thrust P_{AE} is taken instead of the static thrust P_A + P_Q; the inertia of the wall, the heel block and the front fill $k_h W$ also pushes forward at their centers of gravity (if $k_v > 0$, both directions are analyzed). In the vertical plane, for horizontal fill:

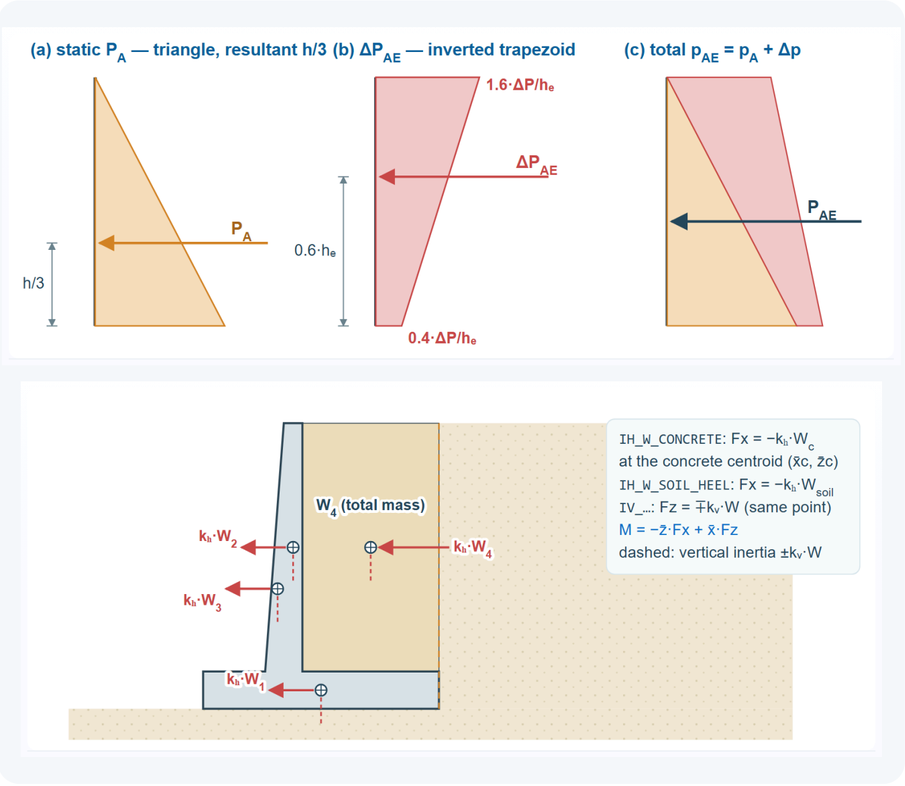
$$\theta = \tan^{-1}\frac{k_h}{1-k_v} \qquad K_{AE} = \frac{\cos^2(\varphi'-\theta)}{\cos^2\theta\left[1+\sqrt{\dfrac{\sin\varphi'\sin(\varphi'-\theta)}{\cos\theta}}\right]^2}$$

$$P_{AE} = \left(\tfrac{1}{2}\gamma h^2 + qh\right)K_{AE} \qquad \Delta P_{AE} = P_{AE} - (P_A + P_Q)\ \ (0.6h)$$

ΔP_{AE} is distributed as an inverted trapezoid (1.6ΔP/h at the top, 0.4ΔP/h at the bottom); the static part retains its shape. If water is present, the hydrodynamic pressure $(7/12)k_h\gamma_w h_w^2$ is added.

The front passive credit is multiplied by $\eta_{p,E}$; **default 0** (sliding safety comes from base friction alone).

If the resultant falls outside the middle third of the base, **partial contact** occurs ($q_{max} = 2V/(3x_R)$).



Problem statement

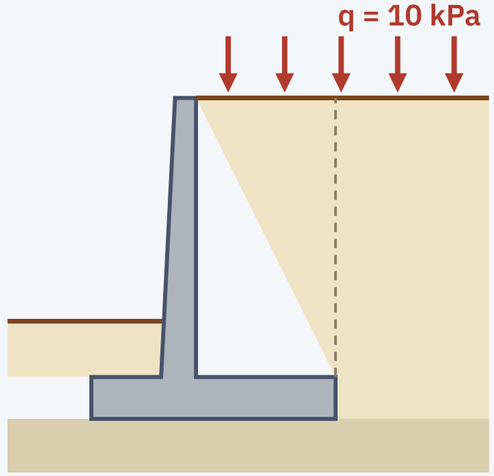
EXAMPLE

Given

Wall and soil	**The wall in Examples 1–4 (static FS_s = 2.03; FS_o = 3.81)**
Earthquake	$k_h = 0.15$, $k_v = 0$; $\eta_{p,E} = 0$; **required** $FS_s,\ FS_o \geq 1.1$; **the base must be fully in contact (|e| ≤ B/6)**

REQUIRED

K_{AE}, P_{AE}, $\bar{z}$, inertial forces; FS_s, FS_o, e, q_{max} and the result of each check.



Solution (1/4)

SOLUTION

1 Seismic angle and coefficient

$$\theta = \tan^{-1} 0.15 = 8.53^\circ$$

$$\sqrt{\sin 30^\circ \sin(30^\circ - 8.53^\circ)/\cos 8.53^\circ} = 0.4302$$

$$K_{AE} = \frac{\cos^2(21.47^\circ)}{\cos^2 8.53^\circ (1 + 0.4302)^2} = \mathbf{0.4329}$$

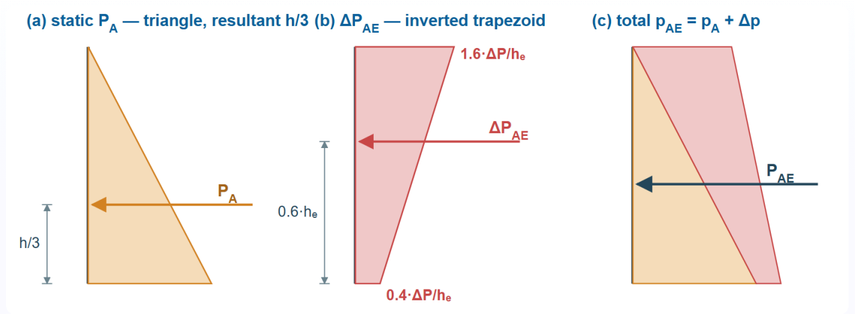
2 Dynamic thrust (static P_A + P_Q = 78.81 kN/m) and its location

$$P_{AE} = (190.44 + 46.0)(0.4329) = \mathbf{102.36}\ \text{kN/m}$$

$$\Delta P_{AE} = 102.36 - 78.81 = 23.55\ \text{kN/m}\ \ (0.6h)$$

$$\bar{z}_{AE} = [63.48(1.533) + 15.33(2.30) + 23.55(2.76)]/102.36$$

$$= \mathbf{1.930}\ \text{m}$$



Solution (2/4)

SOLUTION

3 Inertial forces $k_h W$ and sliding (no credit for passive resistance)

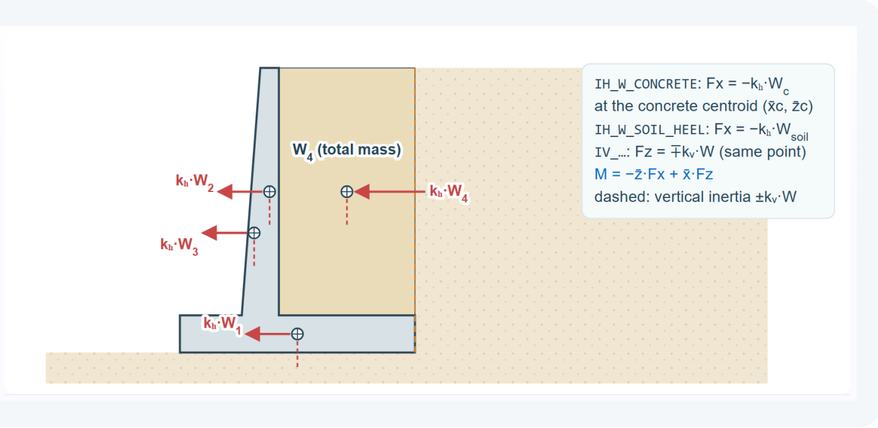
$$IH_c = 0.15(88.8) = 13.32 \;\; (z = 1.223\ \text{m})$$

$$IH_s = 0.15(144.0) = 21.60 \;\; (z = 2.60\ \text{m})$$

$$IH_{toe} = 0.15(14.69) = 2.20\ \text{kN/m}$$

$$\Sigma H_{drive} = 102.36 + 13.32 + 21.60 + 2.20 = 139.49$$

$$FS_{s,E} = \frac{\mu_b N}{\Sigma H_{drive}} = \frac{136.12}{139.49} = \mathbf{0.976} \;<\; 1.1 \;\; \text{✗}$$



Solution (3/4)

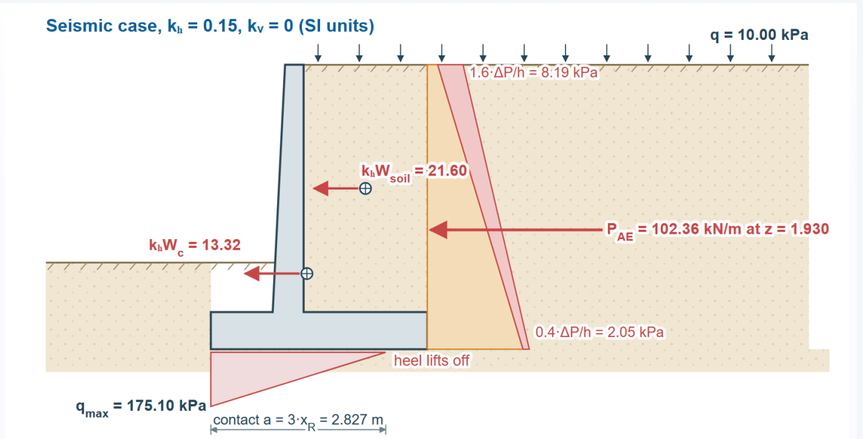
SOLUTION

4 Overturning (moment arms: $\bar{z}_{AE}$ = 1.930; z_c = 1.223; 2.60; 1.003 m)

$$M_{o,E} = 197.60 + 16.28 + 56.16 + 2.21 = 272.26$$

$$FS_{o,E} = \frac{505.45}{272.26} = \mathbf{1.857} \;\geq 1.1 \;\checkmark$$

$$x_R = \frac{505.45 - 272.26}{247.5} = 0.942 \text{ m}$$



Solution (4/4)

SOLUTION

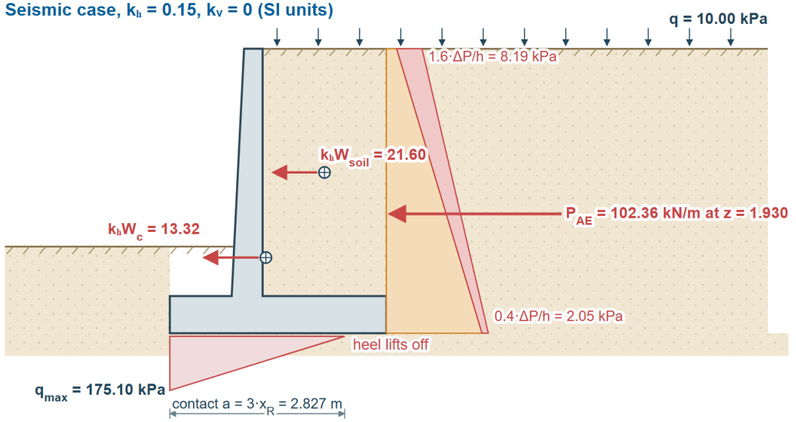
5 Eccentricity and partial contact

$$e = \frac{B}{2} - x_R = 1.750 - 0.942 = \mathbf{0.808}\ \text{m} > \frac{B}{6} = 0.583 \quad ✗$$

$$a = 3x_R = 2.827\ \text{m} \quad \text{(only this length in contact)}$$

$$q_{max} = \frac{2V}{a} = \frac{2(247.5)}{2.827} = \mathbf{175.1}\ \text{kPa} \quad q_{min} = 0$$

Under earthquake, **sliding (FS_s = 0.98 < 1.1)** and **eccentricity (e = 0.81 m > B/6)** are not satisfied; overturning (FS_o = 1.86) and q_{max} = 175.1 kPa are adequate. The section **must be revised** (Example 6).



Check the solution in GeoTerva ↗

Problem statement

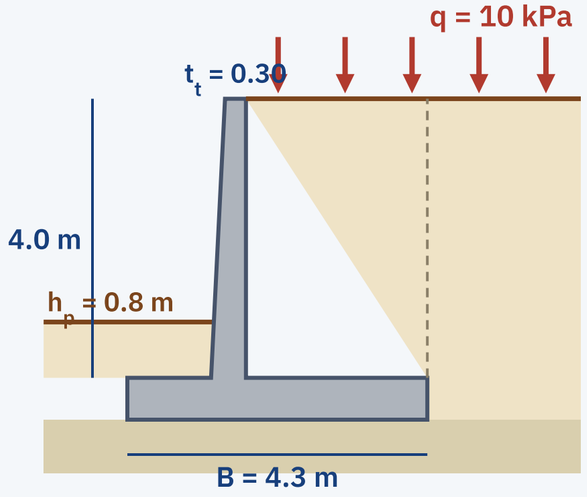
EXAMPLE

Given

Problem	**In Example 5, under earthquake FS_s = 0.98 < 1.1 and e = 0.81 m > B/6 = 0.58 m**
Proposed modification	**toe and heel extended: $L_t = 1.20$ m, $L_h = 2.60$ m (B = 4.30 m); other dimensions and soil unchanged**

REQUIRED

With the new section, static and earthquake **FS_s**, **FS_o**, **e**, **q_{max}**; is the section adequate?



Solution

SOLUTION

1 Same loading, new geometry (module; B = 4.30 m, W_c = 100.3 kN/m, N = 305.1 kN/m)

	Current (B = 3.5 m)	Revised (B = 4.3 m)	Required
Static FS_s	2.026	2.427	≥ 1.5
Static FS_o	3.812	5.753	≥ 2.0
Static e (m) / q_{max} (kPa)	0.243 / 100.2	0.084 / 79.3	|e| ≤ B/6
Earthquake FS_s	0.976 ✗	1.133 ✓	≥ 1.1
Earthquake FS_o	1.857	2.630	≥ 1.1
Earthquake e (m) / contact	0.808 / partial ✗	0.600 / full ✓ (B/6 = 0.717)	|e| ≤ B/6
Earthquake q_{max} (kPa)	175.1	130.4	≤ 200

Extending the toe and heel increased N from 247 to 305 kN/m and raised the friction: under earthquake **FS_s = 1.13 ≥ 1.1**, the resultant fell within the middle third (e = 0.60 ≤ B/6 = 0.717): **all checks are satisfied**. Alternatives: shear key + $\eta_{p,E}$ (Example 9), a wider heel.

Check the solution in GeoTerva ↗

Stem, toe and heel: critical sections

THEORY

Once external stability is satisfied, the **section forces** of the elements are determined. The **stem** is a cantilever loaded by the back pressure and fixed at its base (above the base slab):

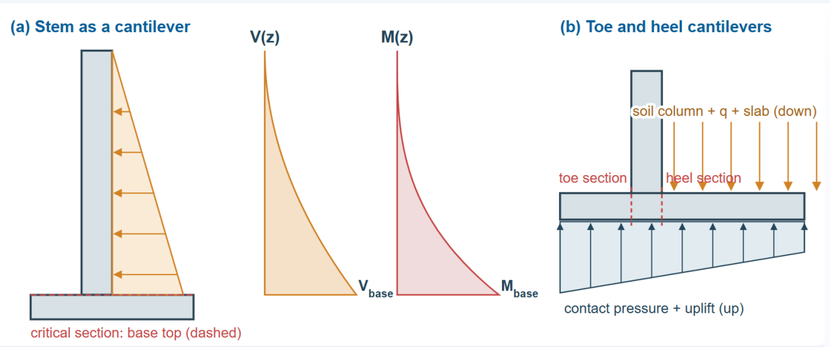
$$V_{st} = K_a\left(\tfrac{1}{2}\gamma H^2 + qH\right) \qquad M_{st} = K_a\left(\frac{\gamma H^3}{6} + \frac{qH^2}{2}\right)$$

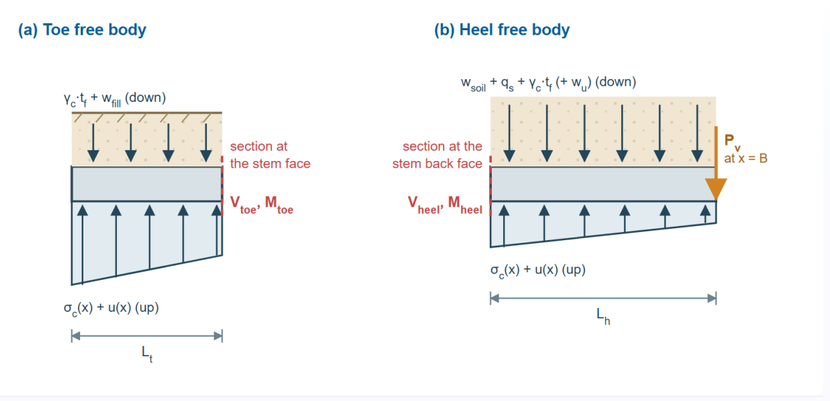
Under earthquake: $V_E = V + \Delta P_{AE,st} + k_h W_{body}$, $M_E = M + \Delta P_{AE,st}(0.6H) + k_h W_{body}\bar{z}$ ($\Delta P_{AE,st} = K_{AE}(\tfrac{1}{2}\gamma H^2 + qH) - V_{st}$). The **toe and heel** are fixed plates on the body faces; load = base stress − the weight of the plate and the soil above it:

$$V_{toe} = L_t\frac{q_0 + q_L}{2} - w_{toe}L_t \qquad M_{toe} = \frac{L_t^2(2q_0 + q_L)}{6} - \frac{w_{toe}L_t^2}{2}$$

$$V_{heel} = w_h L_h - L_h\frac{q_a + q_b}{2} \qquad M_{heel} = \frac{w_h L_h^2}{2} - \frac{L_h^2(q_a + 2q_b)}{6}$$

$w_{toe} = \gamma_c t_f + \gamma h_p$; $w_h = \gamma H + q + \gamma_c t_f$; q_0, q_L, q_a, q_b are the base stresses at the critical sections (linear distribution).





Problem statement

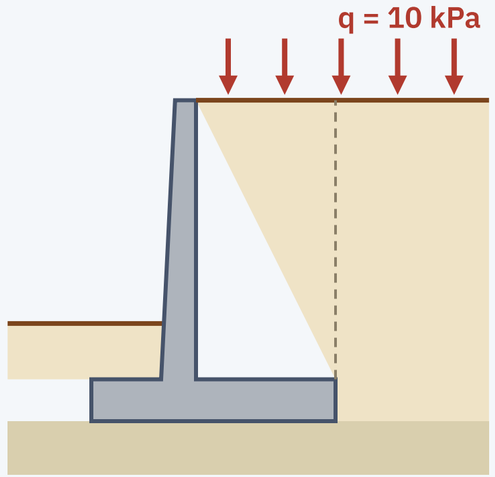
EXAMPLE

REQUIRED

The **V** and **M** at the base of the stem: in the static and earthquake cases.

Given

Wall and soil	**Wall in Examples 1–5:** $H = 4.0$ **m;** $\gamma = 18\ \mathrm{kN/m^3}$, $\varphi' = 30^\circ$**;** $q = 10$ **kPa; stem:** $t_t = 0.30$, $t_b = 0.50$ **m**
Earthquake	$k_h = 0.15$ $(K_{AE} = 0.4329)$



Solution (1/2)

SOLUTION

1 Static (at wall base, H = 4.0 m)

$$V_{st} = K_a\left(\tfrac{1}{2}\gamma H^2 + qH\right) = 0.3333(144 + 40) = \mathbf{61.33}\ \mathrm{kN/m}$$

$$M_{st} = K_a\left(\frac{\gamma H^3}{6} + \frac{qH^2}{2}\right) = 0.3333(192 + 80) = \mathbf{90.67}\ \mathrm{kN\cdot m/m}$$

Solution (2/2)

SOLUTION

2 Earthquake: seismic increment and inertia of the wall

$$\Delta P_{AE,st} = K_{AE}(184) - V_{st} = 79.66 - 61.33 = 18.33\ \text{kN/m} \quad (0.6H = 2.40\ \text{m})$$

$$k_h W_{st} = 0.15(24)(1.2 + 0.4) = 5.76\ \text{kN/m}; \qquad M_i = 10.56\ \text{kN·m/m}$$

3 Total earthquake effects

$$V_E = 61.33 + 18.33 + 5.76 = \mathbf{85.42}\ \text{kN/m}$$

$$M_E = 90.67 + 18.33(2.40) + 10.56 = \mathbf{145.21}\ \text{kN·m/m}$$

At the base of the stem, static **V = 61.33 kN/m, M = 90.67 kN·m/m**; earthquake **V_E = 85.42, M_E = 145.21** (a 60% increase in moment). The reinforced-concrete design is carried out with these effects.

Check the solution in GeoTerva ↗

Problem statement

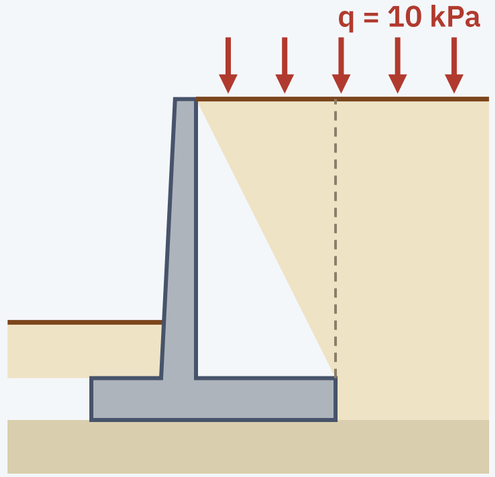
EXAMPLE

Given

Base stresses (Example 4)	$q_{max} = 100.22$ **(toe),** $q_{min} = 41.20$ **kPa (heel tip);** $B = 3.5$ **m**
Slab loads	$L_t = 1.00$, $L_h = 2.00$ **m,** $t_b = 0.50$ **m,** $t_f = 0.60$ **m;** $\gamma_c = 24$**; front fill** $h_p = 0.8$ **m**

REQUIRED

The **V** and **M** values of the toe and heel slabs at the stem face (static); earthquake values.



Solution (1/2)

SOLUTION

1 Base stresses at critical sections (linear): q_0 at the toe, q_L and q_a at the stem face, q_b at the heel end

$$q_0 = 100.22 \qquad q_L = q_0 + (q_{min} - q_0)\frac{L_t}{B} = 83.36$$

$$q_a = q_0 + (q_{min} - q_0)\frac{L_t + t_b}{B} = 74.93 \qquad q_b = 41.20\ \text{kPa}$$

2 Toe (w_{toe} = 24(0.60) + 18(0.80) = 28.8 kPa)

$$V_{toe} = 1.00\frac{100.22 + 83.36}{2} - 28.8(1.00) = \mathbf{62.99}\ \text{kN/m}$$

$$M_{toe} = \frac{1^2(2\cdot 100.22 + 83.36)}{6} - \frac{28.8(1)^2}{2} = \mathbf{32.90}\ \text{kN·m/m}$$

Solution (2/2)

SOLUTION

3 Toe (w_h = 18(4.0) + 10 + 24(0.60) = 96.4 kPa; load downward, base stress upward)

$$V_{heel} = 96.4(2.00) - 2.00\frac{74.93+41.20}{2} = \mathbf{76.67}\ \mathrm{kN/m}$$

$$M_{heel} = \frac{96.4(2)^2}{2} - \frac{2^2(74.93+2\cdot 41.20)}{6} = \mathbf{87.92}\ \mathrm{kN\cdot m/m}$$

Toe: **V = 62.99, M = 32.90**; heel: **V = 76.67, M = 87.92** (toe sloping upward, heel sloping downward). In the earthquake (partial contact, module): toe V = 115.33, M = 62.83; heel V = 138.28, M = 168.69: **the heel moment increases 1.9 times**.

Check the solution in GeoTerva ↗

Simplified shear key method

THEORY

If base friction is insufficient, a **key** is cast below the base into the undisturbed soil; the front face of the key adds the passive resistance of the soil in front of it to the slip surface. The method is deliberately simple: the key face takes a **mobilized** pressure $K_{mob}\sigma'_v$ and is fully credited in sliding; the key **internal forces** are obtained from the **gross full passive** diagram:

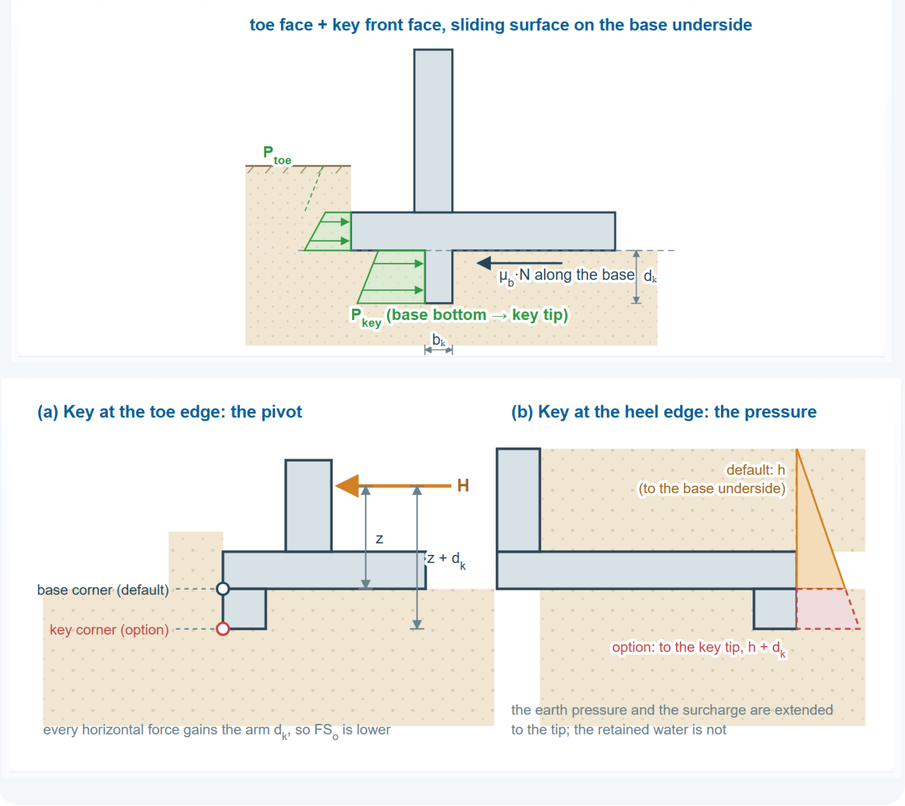
$$P_{key} = \tfrac{1}{2}\left(p_{top} + p_{bot}\right) d_k \quad (p = K_{mob}\sigma'_v) \qquad V_{key} = \tfrac{1}{2}\left(p^g_{top} + p^g_{bot}\right) d_k \quad (p^g = K_p\sigma'_v)$$

$$M_{key} = p^g_{top}\frac{d_k^2}{2} + \left(p^g_{bot} - p^g_{top}\right)\frac{d_k^2}{3}$$

The heel weight (to N_{eff}) is included in both the static and the earthquake case. Passive resistances are used **only in sliding**; in the earthquake both are multiplied by $\eta_{p,E}$ (default 0, i.e. no credit).

Center of rotation: for a heel at the toe edge, the overturning pivot can be chosen as the front bottom corner of the base slab (default) or the front bottom corner of the heel; for a heel at the heel edge, the active pressure can be extended to the heel tip.

$\Delta FS = P_{key}/\Sigma H_{drive}$ is reported in every case.



Problem statement

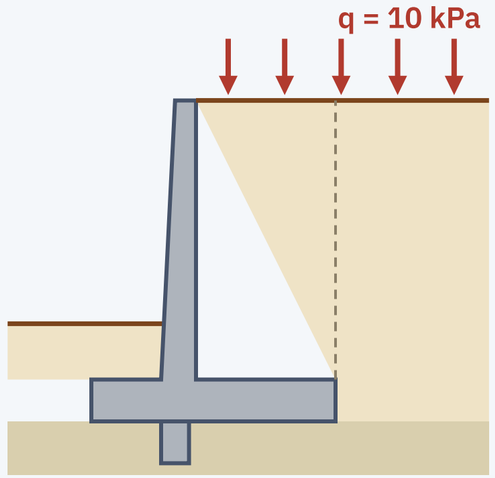
EXAMPLE

Given

Wall	**The wall in Examples 1–5; heel near the toe edge: $x_k = 1.0$ m, $b_k = 0.40$ m, $d_k = 0.60$ m (below the base)**
Soil	$\gamma = 18\ \mathrm{kN/m^3}$, $\varphi' = 30°$**; front soil** $D_f = 1.4$ **m (2.0 m below the base);** $w = 1/3$ **(** $K_{mob} = 1.333$**)**
Earthquake	$\eta_{p,E} = 0$ **(default) and** $\eta_{p,E} = 0.5$

REQUIRED

Shear key resistance **P_{key}**, static **FS_s**; internal forces in the key **V_{key}**, **M_{key}**; effect of $\eta_{p,E}$ under earthquake.



Solution (1/2)

SOLUTION

1 Combined pressure on the face of the shear key ($\sigma'_v = 18\,z$; $z = 1.4 \rightarrow 2.0$ m)

$$p_{top} = K_{mob}(25.2) = 33.6 \qquad p_{bot} = K_{mob}(36.0) = 48.0\ \text{kPa}$$

$$P_{key} = \tfrac{1}{2}(33.6 + 48.0)(0.60) = \mathbf{24.48}\ \text{kN/m}$$

2 Static sliding (tooth weight 5.76 kN/m enters N; toe passive is credited with 23.52)

$$N_{eff} = 253.25 \qquad \mu_b N = 139.29\ \text{kN/m}$$

$$FS_s = \frac{139.29 + 23.52 + 24.48}{78.81} = \mathbf{2.376}\ \ (\text{unnotched } 2.066)$$

Solution (2/2)

SOLUTION

3 Tooth section forces: gross full passive (K_p = 3.00): p^g_{top} = 75.6; p^g_{bot} = 108.0 kPa

$$V_{key} = \tfrac{1}{2}(75.6 + 108.0)(0.60) = \mathbf{55.08}\ \mathrm{kN/m}$$

$$M_{key} = 45.36(0.30) + 9.72(0.40) = \mathbf{17.50}\ \mathrm{kN\cdot m/m}$$

The tooth raised the static sliding factor of safety from 2.07 to **2.38** (ΔFS = 0.31). In an earthquake, with $\eta_{p,E}$ = 0 the tooth **provides no credit**; with $\eta_{p,E}$ = 0.5, $FS_{s,E}$ goes from 0.98 to **1.16** (≥ 1.1 is satisfied), but the eccentricity (0.80 m > B/6) persists: this is why the geometry change in Example 6 is more effective.

Check the solution in GeoTerva ↗

Hydrostatic water on both sides, linear uplift at the base

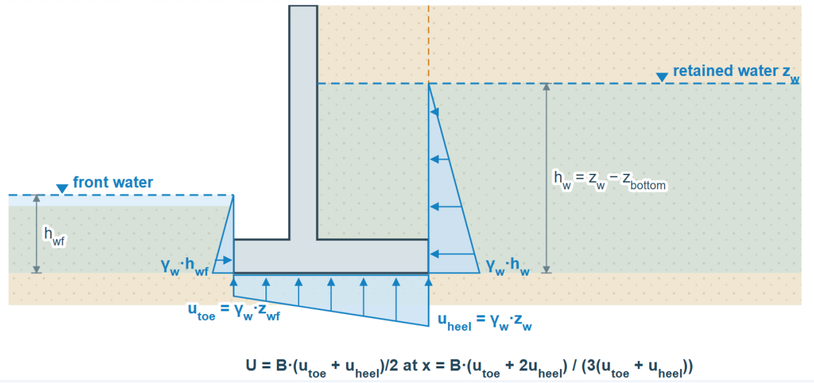
THEORY

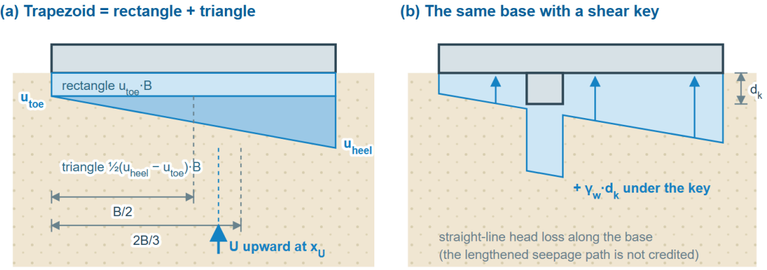
Water pressure is isotropic and acts normal to every surface. The free body is surrounded by water on three boundaries: the back pressure plane, the front face of the wall and the base underside. Each is a separate item in the force table; the soil is weighed with total stress (γ_{sat}):

$$U_{rear} = \tfrac{1}{2}\gamma_w h_w^2 \quad (h_w/3) \qquad U = \frac{B(u_{toe} + u_{heel})}{2} \qquad x_U = \frac{B(u_{toe} + 2u_{heel})}{3(u_{toe} + u_{heel})}$$

Uplift acts upward: it reduces N′ (and hence friction) and adds an overturning moment. The pressure under the base varies linearly between the water loads at the toe and at the heel.

In the earthquake case, the hydrodynamic pressure of the free pore water $(7/12)k_h\gamma_w h_w^2$ $(0.4h_w)$ is additionally added.





Problem statement

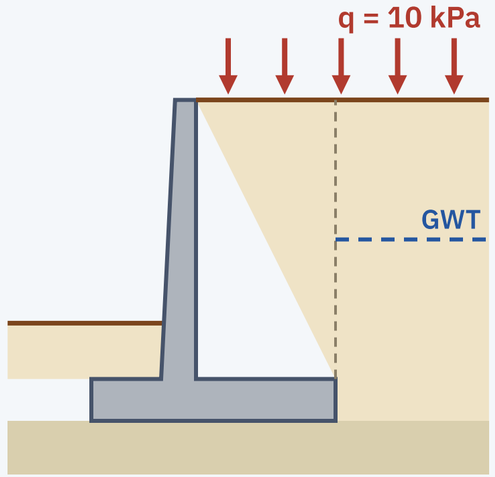
EXAMPLE

REQUIRED

Force table; **FS_s**, **FS_o**, **e**, **q_{max}**; effect of water.

Given

Wall and soil	**Wall and fill in Examples 1–4 ($\gamma = 18,\ \gamma_{sat} = 20\ \mathrm{kN/m^3}$); static condition**
Water	**in the fill the groundwater level is 2.0 m below the wall top; linear uplift below the base**



Solution (1/2)

SOLUTION

1 Force table (module; $M = xF_Z - zF_X$, overturning +)

Item	F_X (kN/m)	F_Z (kN/m)	x (m)	z (m)	M (kN·m/m)
Wall	0.00	−88.80	1.55	1.22	−137.96
Heel block (with γ_{sat})	0.00	−152.00	2.50	2.55	−380.00
Q_{heel} (not used)	0.00	−20.00	2.50	4.60	−50.00
Front fill	0.00	−14.69	0.51	1.00	−7.49
Soil thrust P_A	−54.68	0.00	3.50	1.64	89.71
Surcharge thrust P_Q	−15.33	0.00	3.50	2.30	35.27
Back water thrust	−33.16	0.00	3.50	0.87	28.74
Uplift	0.00	44.64	2.33	0.00	104.15
Front resistance	23.52	0.00	0.00	0.47	−10.98

Solution (2/2)

SOLUTION

2 Checks

$$N = 210.85\ \text{kN/m} \quad FS_s = \frac{116.0 + 23.5}{103.2} = \mathbf{1.352} \quad FS_o = \frac{525.5}{257.9} = \mathbf{2.038}$$

$$e = 0.481\ \text{m}\ (\leq B/6 = 0.583) \qquad q_{max} = 109.9\ \text{kPa} \qquad q_{min} = 10.6\ \text{kPa}$$

It changed the sliding safety from 2.03 → **1.35** (< 1.5 ✗), the overturning safety from 3.81 → **2.04**, and e from 0.243 → 0.481 m; uplift reduced N by 44.6 kN/m. **Static sliding is not satisfied**: drainage, a shear key, or a wider base is required.

Check the solution in GeoTerva ↗

Cantilever slab lightening the stem

THEORY

The relieving console is a reinforced-concrete slab extending from the back of the stem into the fill; it is used for tall walls (the stem moment grows with H³) or to strengthen an inadequate wall. It has three effects: (1) it carries the soil above it; (2) it **relieves** the stem beneath it: the self-weight pressure starts again under the console; (3) it adds weight to the wall.

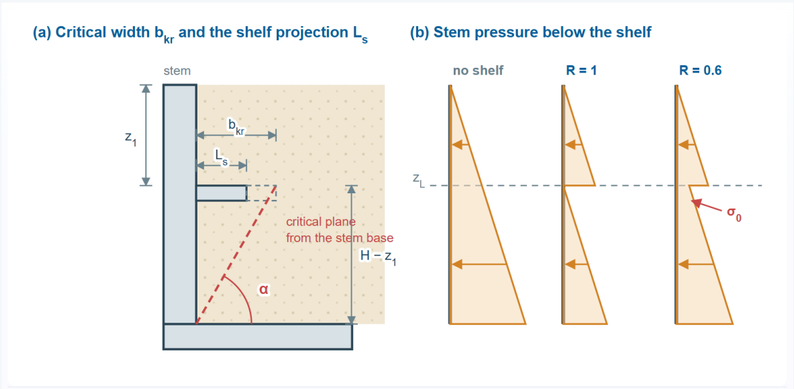
The cantilever is fully effective only if its tip reaches the **critical slip plane** rising from beneath the heel of the stem (Rankine, horizontal fill: α = 45° + φ′/2). The required critical width of the cantilever and the relief ratio:

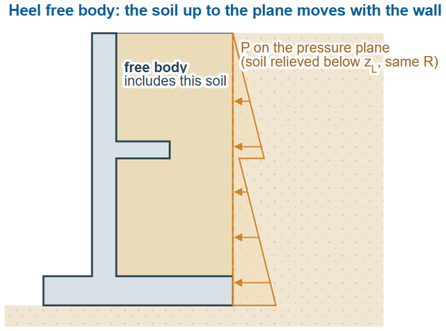
$$b_{kr} = (H - z_1)\,(\cot\alpha - \tan\beta_w) \qquad R = \min\left(\frac{L_s}{b_{kr}}, 1\right)$$

$$p(z) = K\,[\sigma'_v(z) - R\,\sigma'_v(z_L)] \qquad z > z_L \;\; \text{(below cantilever)}$$

R = 1: two separate triangles (full relief); R < 1: partial relief, a linear interpolation between an unpropped wall and a wall with full relief. Only the static **soil weight** pressure is relieved; surcharge, water and earthquake increment are not relieved.

The moment of the cantilever's load about the stem is not credited to the stem (conservative); the cantilever is a separate cantilever under its own load (M_{root}).





Problem statement

EXAMPLE

Given

Wall and soil	**The wall in Example 1–4 (H = 4.0 m; static); horizontal fill, $\varphi' = 30^\circ$**
Cantilever	**depth below the cantilever $z_1 = 2.15$ m (from the wall top), projection $L_s = 1.00$ m, thickness $t_s = 0.30$ m**

REQUIRED

The critical angle α, the critical width $\mathbf{b_{kr}}$, **R**; the variation of V and M at the base of the stem; external stability.

Solution (1/2)

SOLUTION

1 Critical plane and width (vertical back: $\beta_w = 0$)

$$\alpha = 45^\circ + \frac{\varphi'}{2} = 60.0^\circ \qquad b_{kr} = (4.0 - 2.15)\cot 60^\circ = 1.068\ \text{m}$$

$$R = \min(1.00/1.068,\ 1) = \mathbf{0.9362}\ \ (\text{partial relief})$$

2 Effects at the base of the stem (cantilever → with cantilever)

$$V:\ 61.33 \rightarrow 38.99\ \text{kN/m}\ \ (-\%36.4)$$

$$M:\ 90.67 \rightarrow 70.00\ \text{kN·m/m}\ \ (-\%22.8)$$

Solution (2/2)

SOLUTION

3 Cantilever load and external stability (same relief in the pressure plane)

$$W_{soil} = 43.3\ \mathrm{kN/m}, \quad W_{self} = 7.2\ \mathrm{kN/m}, \quad M_{root} = 25.25\ \mathrm{kN\cdot m/m}$$

$$FS_s:\ 2.03 \rightarrow 3.26 \qquad FS_o:\ 3.81 \rightarrow 5.28 \qquad e:\ 0.243 \rightarrow 0.094\ \mathrm{m}$$

The counterfort (L_s = 1.00 m < b_{cr} = 1.07 m: R = 0.94) reduces the shear at the base of the stem by 36% and the moment by 23%; the load and weight of the counterfort are added to the wall: sliding FS 2.03 → 3.26, eccentricity 0.243 → 0.094 m. The counterfort is also designed for M_{root} = 25.3 kN·m/m at its own root.

Check the solution in GeoTerva ↗

Scope of the solution

THEORY

External stability treats the wall as a single rigid block; **section forces** (stem, toe, heel, cantilever, shear key) are obtained from the effects given by this module; reinforced-concrete design (reinforcement, shear) is outside the scope of this presentation.

Cohesionless fill ($c' = 0$); single layer, horizontal or single-sloping surface; water level horizontal and hydrostatic; surcharge uniformly distributed (strip and point lateral pressures are added via Boussinesq).

Base friction $\mu_b = \tan \delta_b$; no adhesion. Front passive resistance is used only in sliding; it is not used in overturning or bearing capacity; under earthquake it is applied with $\eta_{p,E}$ (default 0).

Pseudo-static earthquake: Mononobe–Okabe + inertia + hydrodynamic water; if $k_v > 0$, both directions are also analyzed.

The relief console and the shear key are modelled with simplified methods (console: partial/full relief, key: combined passive); overall stability and settlement are outside the scope.

Summary

Key points

Wall: base slab + stem; $W_c = \Sigma \gamma_c A_i$; the **heel block** and the front fill move with the wall; the pressure acts on the vertical plane at the rear edge of the foundation ($h = t_f + H$); Q_{heel} enters no check

Checks from the same force table: $FS_s = (\mu_b N + P_P)/\Sigma H$, $FS_o = \Sigma M_{res}/\Sigma M_{ov}$, x_R, e, $q = V/B(1 \pm 6e/B)$; if $|e| > B/6$ then $q_{max} = 2V/(3x_R)$

Earthquake: P_{AE} (Mononobe–Okabe) + ΔP_{AE} at $0.6h$ + $k_h W$ inertia forces; the example wall **fails in sliding and eccentricity** under earthquake, and is satisfied when revised with a wider toe/heel

Section forces: stem $V = K_a(\frac{1}{2}\gamma H^2 + qH)$, $M = K_a(\gamma H^3/6 + qH^2/2)$; toe and heel are fixed at the stem face: base stress – slab and soil weight

Shear key: P_{key} from combined passive (in sliding), V, M from gross K_p; in earthquake with $\eta_{p,E}$

Relief shelf: $b_{kr} = (H - z_1)(\cot \alpha - \tan \beta_w)$, $R = \min(L_s/b_{kr}, 1)$; below the shelf the weight pressure resumes; water and surcharge are not relieved

Water: weight with γ_{sat} + separate rear water thrust; uplift reduces N' and adds overturning (in the example static sliding is not satisfied)

References

The three theories of the tool and the source whose presentation of their closed forms (Ka, Kp, KAE) the manual follows. Everything not derived from them is an assumption of this module and is marked so in the text.

Coulomb C.A. (1776) Essai sur une application des règles de maximis et minimis à quelques problèmes de statique relatifs à l'architecture. Mémoires de l'Académie Royale des Sciences 7, Paris.

Das B.M. (2016) Principles of Foundation Engineering, 8th ed. Cengage Learning, Boston.

Mononobe N., Matsuo H. (1929) On the determination of earth pressures during earthquakes. Proceedings of the World Engineering Congress, Tokyo, 9:177–185.

Okabe S. (1926) General theory of earth pressure. Journal of the Japanese Society of Civil Engineers 12(1).

Rankine W.J.M. (1857) On the stability of loose earth. Philosophical Transactions of the Royal Society of London 147:9–27.

NOTE

All numerical examples are produced with the module's own calculation engine; the module gives the same values. The step-by-step example on the module page is also this wall (H = 4 m, q = 10 kPa, k_h = 0.15).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter the stem of your own wall, base slab, backfill, water, seismic coefficients, shear key and cantilever into GeoTerva; the module provides the force table, checks and sectional forces together.

Open the module →

Theory page →

Open on your phone
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com