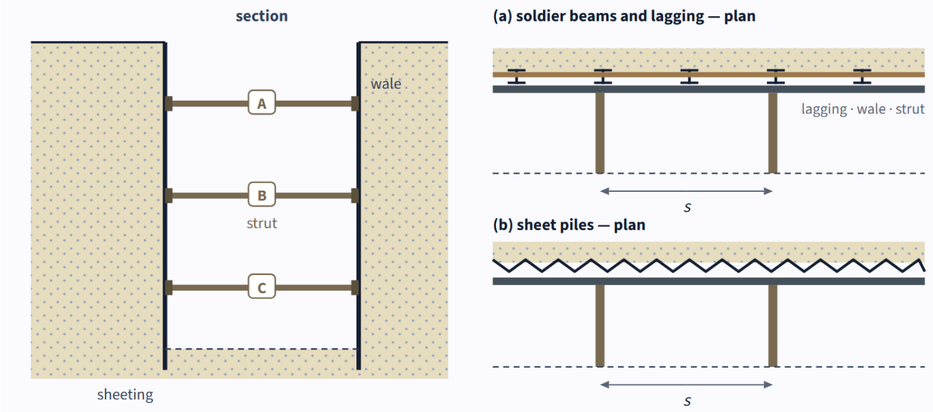
Geotechnical lecture presentation · Sheet piles and braced cuts

# Clay over sand: braced cut

Lecture: sand and clay in the same excavation (Peck equivalent cohesion); envelope, support loads, sheet pile and wale sizing, steel verification and base heave; a worked example in every topic.

Excavation support | Equivalent clay (sand + clay) | strut · wale · sheet pile | EN 1993 · AISC 360 | Base heave



Introduction

# What will we learn in this lecture?

- Why the pressure in a braced cut is **not Rankine** and how the envelopes are constructed; clay envelopes (with γH/c)
- When the excavation passes through sand and clay, Peck's **equivalent cohesion** $c_{av}$ and $\gamma_a$ are used; the envelope is selected by $\gamma_a H/c_{av}$
- How the National Plaza (Chicago) excavation first averages **multiple clays** by thickness and then combines them with the sand
- The addition of the surcharge next to the envelope; from the envelope, the **strut loads**, **sheet pile** and **wale** sizing
- Verifying the steel with **EN 1993-1-1 and AISC 360-22**; the **base heave** check in a layered profile

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example excavation: 3 m of sand at the top (γ = 17 kN/m³, φ′ = 30°) over clay (γ = 18 kN/m³, c = 35 kPa); H = 7.00 m, B = 6.00 m; struts at depths 1.00 · 3.50 · 6.00 m, horizontal spacing s = 3.00 m. Forces are per metre of wall (strut loads multiplied by s).

# Support system and its elements

THEORY

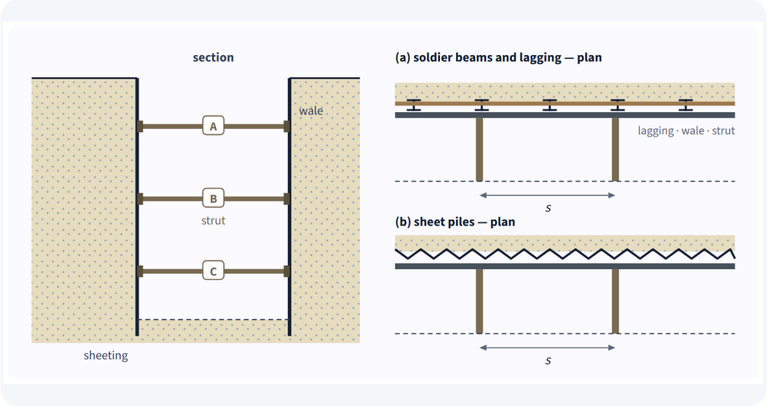
Deep excavations with vertical faces (urban basements, cut-and-cover tunnels) are kept open with a temporary **support system**. Two arrangements are common: **vertical piles** (steel or timber) driven in advance, with horizontal timber lagging placed between them as excavation proceeds; or interlocking **sheet piles** forming a continuous wall.

At each support level, horizontal **wales** are installed as soon as the excavation reaches that level; **struts** are horizontal steel compression members that cross the excavation from wale to wale.

Selecting the members requires the lateral pressure that the support system will take. The theory gives the total thrust, not its **distribution**: the distribution depends on the soil, the sequence and rate of the work, the crew and the equipment.

The design pressures are therefore **empirical envelopes**: drawn around the pressures back-calculated from support loads measured in actual excavations.

During construction, the strut spacing should be at least about 2.75 m vertically.



# Rotates about the top of the wall

THEORY

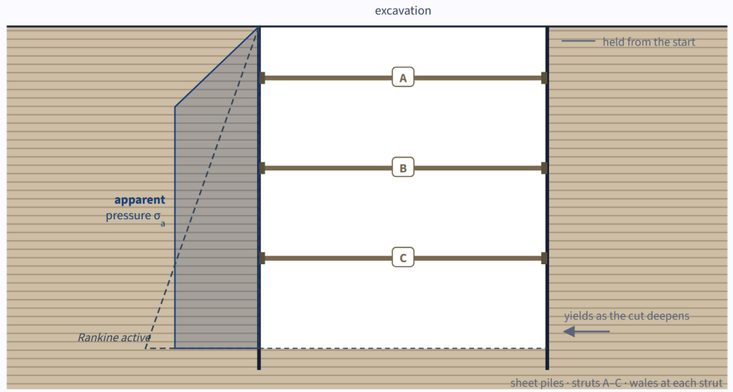
A cantilever retaining wall rotates about its base, and the Rankine active pressure (zero at the top, largest at the bottom) describes it well. **A braced wall moves in the opposite sense**: the first strut is placed while the excavation is shallow, so the upper part barely moves; each subsequent stage is excavated before the strut is installed, and the interval lengthens with depth, so the wall deflects progressively lower: when excavation is complete it has **rotated about its top**.

Near the top the pressure stays close to the **at-rest value** (the soil has moved very little).

At the bottom the pressure may drop below the Rankine active value (the soil has long exceeded plastic equilibrium).

The resultant thrust $P_a$ acts **above** the one-third point of the triangle: at height $n_aH$ from the base, $n_a > 1/3$.

The total thrust is found with the logarithmic spiral wedge ($r = r_0 e^{\theta \tan \varphi'}$); in granular soil it is 15–20% above Coulomb's value. But since its distribution and $n_a$ depend on the soil and the site, excavations are designed with **envelopes**.



# How were the envelopes constructed?

THEORY

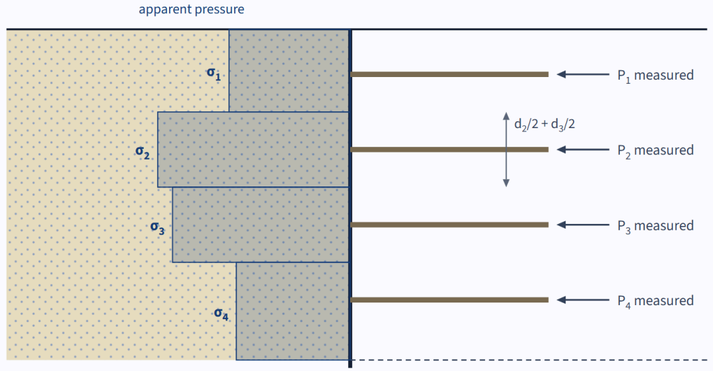
Pressure varies from point to point, and each support must be sized for the **maximum load** it may encounter. Envelopes are constructed for this purpose: in an instrumented excavation, the load on each support is measured, divided by the sheet-pile strip it supports (horizontal spacing s × half the distances to the supports above and below), and converted to an **apparent pressure** over this band:

$$\sigma_1 = \frac{P_1}{s\left(d_1 + \frac{d_2}{2}\right)} \qquad \sigma_i = \frac{P_i}{s\left(\frac{d_i}{2} + \frac{d_{i+1}}{2}\right)}$$

$d_1$ is the depth of the first strut, $d_i$ the spacings between subsequent struts. The envelope of all diagrams drawn one over the other is an **apparent pressure envelope**.

Peck's envelopes are based on sand loads measured in Berlin, Munich and New York subway excavations; he constructed them by the same method for soft–medium and stiff clays.

The envelope is **not** the pressure acting on the wall at any given instant: the load taken from it for a given support has been drawn so as to remain on the safe side of the measured loads.




each strut load spread over the band of wall it serves: s × (half the distance to each neighbour)

# Soft–medium or stiff: γH/c

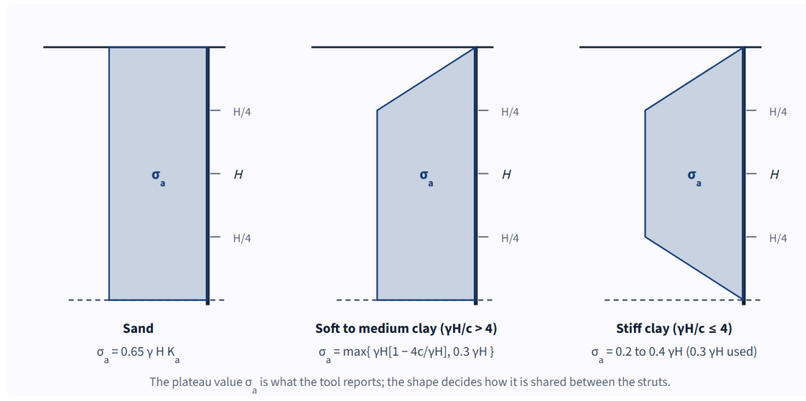
THEORY

In clay the shape of the envelope is governed by the **stability number γH/c** (c undrained cohesion, φ = 0). **Soft-to-medium clay** with γH/c > 4 is driven by the excavation far beyond its strength; **stiff clay** is closer to its elastic range:

$$\frac{\gamma H}{c} > 4: \quad \sigma_a = \max\left(\gamma H\left[1 - \frac{4c}{\gamma H}\right], 0.3\gamma H\right) \qquad \frac{\gamma H}{c} \le 4: \quad \sigma_a \approx 0.3\gamma H$$

The soft–medium clay envelope is zero at the surface, increases in the upper quarter, and forms a constant plateau down to the base; the stiff clay envelope increases in the upper quarter, is constant in the middle half, and returns to zero in the lower quarter. The larger of the two expressions governs.

Conditions: excavation depth ≳ 6 m; water below the base; clay undrained (pore water pressure is not taken into account).



# Peck's equivalent cohesion

THEORY

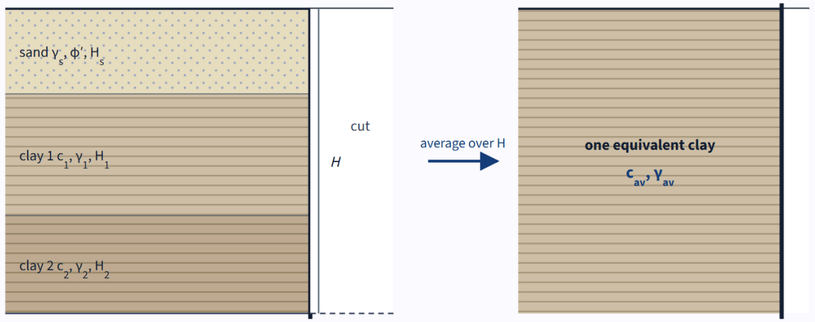
If the excavation contains both sand and clay, the Peck profile replaced **a single clay**: it averages the strength the soil can mobilize along the height. Sand contributes the friction of its own weight, clay its unconfined compressive strength (reduced for progressive failure):

$$c_{av} = \frac{1}{2H}\left[\gamma_s K_s H_s^2 \tan\varphi'_s + (H - H_s)\, n' q_u\right] \qquad \gamma_a = \frac{1}{H}\left[\gamma_s H_s + (H - H_s)\gamma_c\right]$$

$H_s$, $\gamma_s$, $\varphi'_s$: height, unit weight and effective friction angle of the sand; $K_s$ lateral pressure coefficient of the sand (≈ 1); $q_u = 2c$ unconfined compressive strength of the clay, $\gamma_c$ saturated unit weight; $n'$ progressive failure coefficient, 0.5–1.0 (average 0.75).

With $c_{av}$ and $\gamma_a$, the clay envelopes are applied over the full height; the envelope is selected with $\gamma_a H/c_{av}$.

If there are several clays beneath the sand, the source first averages them by thickness ($c_{av} = \Sigma c_i H_i / \Sigma H_i$, $\gamma_{av}$), then combines this equivalent clay with the sand (as in the National Plaza case).




sand: self-weight friction $\gamma K_s H_s^2 \tan\varphi'$ · clay: $n' q_u H$ · both summed over the height, then the clay rule applies

# Problem statement

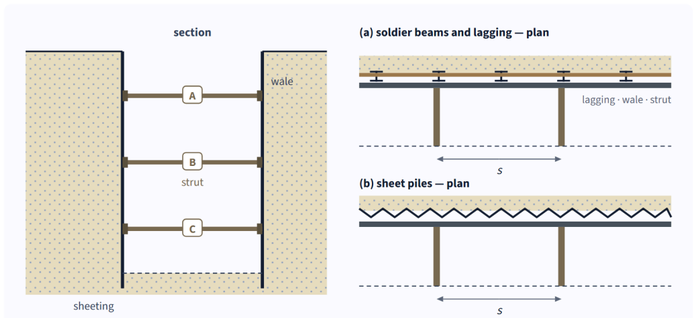
EXAMPLE

## Given

| | |
|---|---|
| Profile | **at top $H_s = 3$ m sand ($\gamma_s = 17$ kN/m³, $\varphi'_s = 30°$, $K_s = 1$), below it clay ($\gamma_c = 18$ kN/m³, $c = 35$ kPa)** |
| Excavation and coefficient | $H = 7.0$ **m;** $n' = 0.75$ **(mean); also** $n' = 0.5$ **and** $1.0$ |

## REQUIRED

$c_{av}$, $\gamma_a$, $\gamma_a H/c_{av}$, the type of envelope and $\sigma_a$; results from the variation of n′.



# Solution (1/2)

SOLUTION

**1** Equivalent cohesion and unit weight (n′ = 0.75)

$$c_{av} = \frac{1}{2(7)}\left[17(1)(3)^2\tan 30^\circ + (7-3)(0.75)(2\cdot 35)\right] = \frac{88.33 + 210.0}{14} = \mathbf{21.31}\ \text{kPa}$$

$$\gamma_a = \frac{17(3) + (7-3)18}{7} = \mathbf{17.57}\ \text{kN/m}^{\mathbf{3}} \qquad \frac{\gamma_a H}{c_{av}} = \frac{17.57(7)}{21.31} = \mathbf{5.77} > 4$$

Equivalent clay: $c_{av}$ = **21.31 kPa**, $\gamma_a$ = 17.57 kN/m³; $\gamma_a H/c_{av}$ = **5.77** > 4 → soft–medium clay envelope.

# Solution (2/2)

SOLUTION

**2** Soft–medium clay envelope and n′ sensitivity

$$\sigma_a = \gamma_a H \left[1 - \frac{4c_{av}}{\gamma_a H}\right] = 17.57(7) - 4(21.31) = \mathbf{37.75}\ \text{kPa}$$

| n′ | $c_{av}$ (kPa) | $\gamma_a H/c_{av}$ | $\sigma_a$ (kPa) | $P_{max}$ (kN) |
|---|---|---|---|---|
| 0.50 | 16.31 | 7.54 | 57.75 | 424.5 |
| **0.75** | 21.31 | 5.77 | 37.75 | 277.5 |
| 1.00 | 26.31 | 4.67 | 36.90 | 271.2 |

Equivalent clay: $c_{av}$ = **21.31 kPa**, $\gamma_a H/c_{av}$ = 5.77 > 4: soft-to-medium clay envelope, **$\sigma_a$ = 37.75 kPa**. If n′ = 0.5 is taken, $c_{av}$ drops to 16.31 and $\sigma_a$ rises to 57.8 kPa: the progressive failure coefficient strongly affects the result.

Check the solution in GeoTerva ↗

# Problem statement

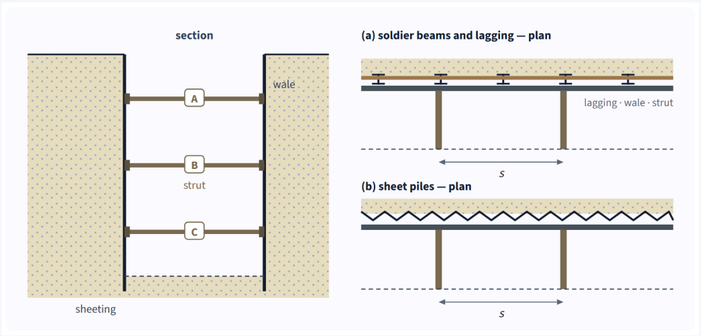
EXAMPLE

## Given

| | |
|---|---|
| Excavation | $H = 70$ ft $= 21.34$ m deep, six supports below; 13 ft sand below the fill, four clays (table below, SI) |
| Coefficients | $K_s = 1$, $\varphi'_s = 30°$, $n' = 0.75$ |

## REQUIRED

Thickness-averaged clay $c_{av,clay}$; $c_{av}$, $\gamma_{av}$, $\gamma H/c$ and $\sigma_a$.



# Solution (1/2)

SOLUTION

**1** Profile (SI) and thickness-averaged clay

| Layer | Thickness (m) | γ (kN/m³) | c (kPa) | Contribution term (kN/m) |
|---|---|---|---|---|
| Sand fill | 3.96 | 17.28 | — | 156.6 |
| Clay 1 | 10.06 | 19.95 | 19.2 | 289.0 |
| Clay 2 | 3.35 | 20.42 | 33.5 | 168.6 |
| Clay 3 | 1.83 | 21.21 | 95.8 | 262.7 |
| Clay 4 | 2.13 | 21.21 | 191.5 | 612.9 |

$$c_{av,clay} = \frac{\Sigma c_i H_i}{\Sigma H_i} = \mathbf{51.16}\ \mathrm{kPa}$$

The thickness-weighted average of the four clays is **51.16 kPa**; the sand term $\gamma_s K_s H_s^2 \tan \varphi'_s$ = 156.6, the clay term $n' q_u \Sigma H_i$ = 1,333.2 kN/m.

# Solution (2/2)

SOLUTION

2 Equivalent cohesion, unit weight and envelope

$$c_{av} = \frac{\gamma_s K_s H_s^2 \tan\varphi'_s + n' q_u \Sigma H_i}{2H} = \frac{156.6 + 1{,}333.2}{2(21.34)} = \mathbf{34.91}\ \text{kPa}$$

$$\gamma_{av} = \frac{\Sigma \gamma_i H_i}{H} = \mathbf{19.76}\ \text{kN/m}^{\mathbf{3}} \qquad \frac{\gamma_{av} H}{c_{av}} = \frac{19.76(21.34)}{34.91} = \mathbf{12.08}\ > 4$$

$$\sigma_a = \gamma_{av} H \left[1 - \frac{4c_{av}}{\gamma_{av} H}\right] = \mathbf{282.0}\ \text{kPa}$$

The soft-to-medium clay envelope is applied: **$\sigma_a$ = 282.0 kPa** ($c_{av}$ = 34.91 kPa, $\gamma_{av}$ = 19.76 kN/m³, $\gamma_{av}H/c_{av}$ = 12.08). A comparison of this envelope with observed strut loads is given in the section 'Envelopes in real excavations' (Peck envelope ≈ 1.8 times).

Check the solution in GeoTerva ↗

# Added next to the envelope

THEORY

**Surcharge q** is not included in the envelope (the empirical coefficients would then also apply to it); it is carried as its own uniform pressure over the full height, and in undrained clay $K_a = 1$: $\Delta p_q = q$.

**Water**: clay envelopes are undrained, total-stress constructs; in clay the water is already in the unit weight and in the undrained strength. Water above the base is drawn and noted, not added as a separate load; it changes only the soil weight: saturated unit weight below water, and in a layer cut by water, the thickness-weighted average of the two values.

Since the thickness of the sand layer determines the equivalent clay, the **layer sequence** of the profile directly affects the result: as the sand thickens, $c_{av}$ changes (example below).

$$\Delta p_q = K_a q = q \;\; (\varphi = 0 \Rightarrow K_a = 1)$$

$$\gamma = \frac{\gamma z_w + \gamma_{sat}(H - z_w)}{H}$$

# Problem statement

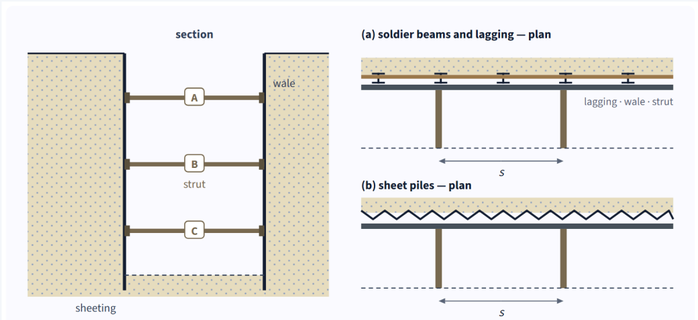
EXAMPLE

## Given

| | |
|---|---|
| Excavation | **The cut in Example 1 ($H = 7$ m, $n' = 0.75$): sand $3$ m (excavation)** |
| Changes | **(a) sand $5$ m thick (clay $2$ m); (b) surcharge $q = 10$ kPa** |

## REQUIRED

In every case $c_{av}$, $\gamma_a H/c_{av}$, $\sigma_a$, total load R and maximum strut load.



# Solution (1/2)

SOLUTION

**1** (a) Sand 5 m: equivalent clay reconsolidated

$$c_{av} = \frac{1}{14}\left[17(5)^2 \tan 30^\circ + (2)(0.75)(70)\right] = 25.03 \text{ kPa}$$

$$\gamma_a = \frac{17(5) + 2(18)}{7} = 17.29 \text{ kN/m}^3$$

$$\sigma_a = \max\left(\gamma_a H - 4c_{av},\, 0.3\gamma_a H\right) = \max(20.91, 36.31) = \mathbf{36.31} \text{ kPa}$$

When the sand is 5 m the equivalent cohesion becomes 21.3 → **25.0 kPa** and the envelope is governed by **0.3γ$_a$H = 36.3 kPa**.

# Solution (2/2)

SOLUTION

**2** (b) Surcharge: separately $\Delta p_q = q = 10$ kPa; summary of the three cases

| Condition | $c_{av}$ (kPa) | $\gamma_a$ (kN/m³) | $\gamma_a H/c_{av}$ | $\sigma_a$ (kPa) | R (kN/m) | $P_{max}$ (kN) |
|---|---|---|---|---|---|---|
| Excavation (sand 3 m) | 21.31 | 17.57 | 5.77 | 37.75 | 231.2 | 277.5 |
| (a) sand 5 m | 25.03 | 17.29 | 4.84 | 36.31 | 222.4 | 266.9 |
| (b) q = 10 kPa | 21.31 | 17.57 | 5.77 | 37.75 | 301.2 | 351.0 |

As the sand thickness increases, $c_{av}$ goes from 21.3 to 25.0 kPa and the envelope drops from 37.8 to **36.3 kPa** (frictional sand adds more strength than clay; the remaining $0.3\gamma_a H$ term governs). Surcharge does not affect the envelope: it only adds $\Delta p_q = 10$ kPa (R from 231.2 to 301.2 kN/m).

Check the solution in GeoTerva ↗

# From envelope to support loads

THEORY

Struts are horizontal members: they carry the pressure along their axes and bend under their own weight; capacity depends on slenderness, and wide excavations may require additional support. The source gives a simple and conservative method for the load:

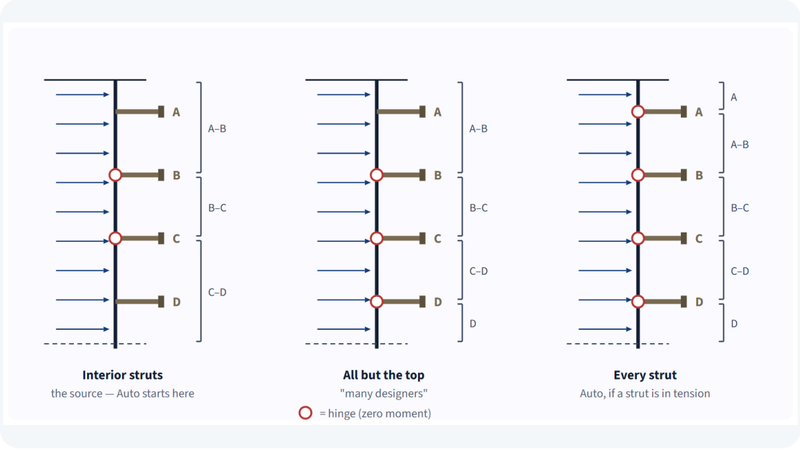
> Draw the pressure envelope and the support levels. Take the sheet pile **hinged at every support level** except the topmost and the lowest (many designers do so at all except the topmost).

> Divide the wall at the hinges into **simple cantilevers at the ends and simple beams in between**; find the reactions of each part by statics (for a four-support cut, A, $B_1$, $B_2$, $C_1$, $C_2$, D).

> The load of a support is the sum of the reactions it receives × the horizontal spacing s.

$$P_A = A\,s \quad P_B = (B_1 + B_2)\,s \quad P_C = (C_1 + C_2)\,s \quad P_D = D\,s$$

Each segment is statically determinate: it carries two supports; moment and horizontal force equilibrium about one of the supports gives both of them. Since the hinge support belongs to both sides, it collects **two reactions**. The reactions are per unit length of the wall (kN/m); s converts them to a support load.



# Problem statement

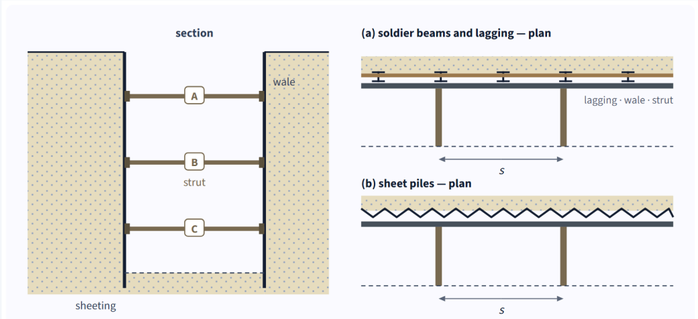
EXAMPLE

## Given

| | |
|---|---|
| Excavation | $H = 7.00$ m; $B = 6$ m; struts $z = 1.00, 3.50, 6.00$ m (A, B, C); horizontal spacing $s = 3.00$ m |
| Pressure | Example 1: equivalent clay, soft–medium envelope $\sigma_a = 37.75$ kPa (increases in the upper quarter, constant down to the base); sheet pile hinged at support B (except top and bottom) |

## REQUIRED

The **resultant W** of the parts and its arm **z̄**; the reactions **A, $B_1$, $B_2$, C**; the support loads **$P_A$, $P_B$, $P_C$**.



# Solution (1/2)

SOLUTION

**1** Divide the envelope into parts at the hinges; the resultant and moment arm of each part (depth from the top surface)

| Piece | Range (m) | W (kN/m) | z̄ (m) | Supports |
|---|---|---|---|---|
| 1 | 0.00 – 3.50 | 99.09 | 2.139 | A (z = 1.00), B (z = 3.50) |
| 2 | 3.50 – 7.00 | 132.12 | 5.250 | B (z = 3.50), C (z = 6.00) |

**2** First part: moment equilibrium about the first support

$$R_{B_2} = \frac{W(\bar{z} - z_A)}{z_B - z_A} = \frac{99.09(2.139 - 1.00)}{3.50 - 1.00} = \mathbf{45.14} \qquad R_A = W - R_{B_2} = \mathbf{53.95}\ \mathrm{kN/m}$$

The first span carries the two struts with 53.95 and 45.14 kN/m; the reaction of the uppermost support (A) is larger because it also includes the load of the cantilever portion.

# Solution (2/2)

SOLUTION

**3** Second part

$$R_C = \frac{W(\bar{z} - z_B)}{z_C - z_B} = \frac{132.12(5.250 - 3.50)}{6.00 - 3.50} = \mathbf{92.49} \qquad R_{B_1} = W - R_C = \mathbf{39.64}\ \mathrm{kN/m}$$

**4** Each support sums the reactions it receives; load = total × s (s = 3.00 m)

| Level | z (m) | Reactions received (kN/m) | Total (kN/m) | P = R·s (kN) |
|---|---|---|---|---|
| A | 1.00 | 53.95 | 53.95 | **161.85** |
| B | 3.50 | 45.14 + 39.64 | 84.78 | **254.34** |
| C | 6.00 | 92.49 | 92.49 | **277.47** |

Maximum support load **$P_C$ = 277.47 kN** (level C): this load governs the strut sizing. Since support B belongs to two parts, it sums two reactions.

Check the solution in GeoTerva ↗

# Maximum moment and required section modulus

THEORY

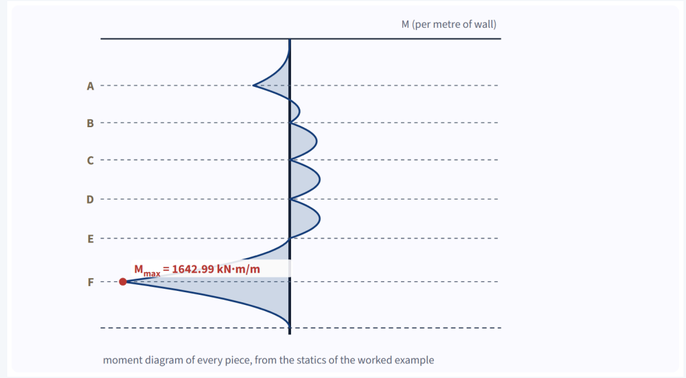
The **sheet pile** is designed from the same parts: the maximum bending moment is found in each part (where the shear changes sign: within the span or at the support where the cantilever meets the strut); the largest is the design moment for unit length of wall. The required section modulus follows from the allowable bending stress, and a section with at least that value is selected from the table:

$$M_{max} = \max_{parts} |M| \qquad S = \frac{M_{max}}{\sigma_{allow}}$$

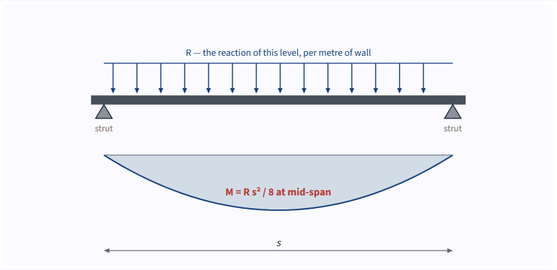
**Struts** may be considered continuous if properly connected; conservatively, they are taken as **hinged** at the supports: a simply supported beam of span s, with the level reaction R (per unit length of wall) as a distributed load:

$$M_{max} = \frac{R\,s^2}{8} \qquad S = \frac{M_{max}}{\sigma_{allow}}$$

The level with the largest reaction is governing. Struts are sometimes connected to the sheet pile at points that will provide the lateral support they require.




moment diagram of every piece, from the statics of the worked example



# Problem statement

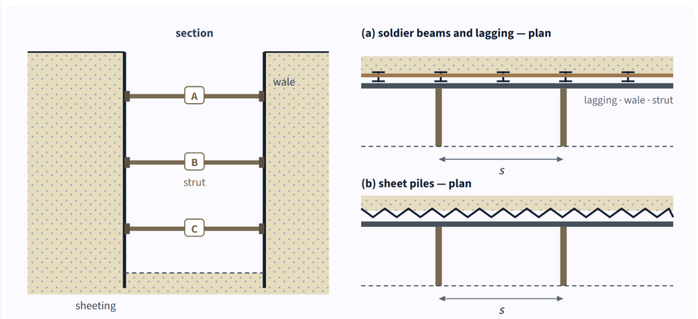
EXAMPLE

## Given

| | |
|---|---|
| Excavation | **The excavation in Example 1 ($H = 7$ m, equivalent clay, $\sigma_a = 37.75$ kPa); struts at 1.0 · 3.5 · 6.0 m; $s = 3.0$ m; hinge at B** |
| Steel | **allowable bending stress $\sigma_{allow} = 170\,000$ kPa** |

## REQUIRED

The sheet pile's **$M_{max}$** and required **S** (cm³/m); at the level of the largest reaction, the wale's **$M_{max}$** and **S** (cm³).



# Solution (1/2)

SOLUTION

**1** Sheet pile: maximum moment in each part

| Piece | Range (m) | $M_{max}$ (kN·m/m) | Location z (m) |
|---|---|---|---|
| 1 | 0.00 – 3.50 | 26.99 | 2.30 |
| 2 | 3.50 – 7.00 | 20.81 | 4.55 |

**2** Required section modulus

$$S = \frac{M_{max}}{\sigma_{allow}} = \frac{26.99}{170{,}000} = 0.0001588\ \mathrm{m^3/m} = \mathbf{158.8}\ \mathrm{cm^3/m}$$

In the sheet pile **$M_{max}$ = 26.99 kN·m/m** (segment 1, z = 2.30 m): **S ≥ 158.8 cm³/m**.

# Solution (2/2)

SOLUTION

**3** Wale: simple beam at each level (s = 3.000 m)

| Level | R (kN/m) | M = R s²/8 (kN·m) | S = M/$\sigma_{allow}$ (cm³) |
|---|---|---|---|
| A | 53.95 | 60.69 | 357 |
| B | 84.78 | 95.38 | 561 |
| C | 92.49 | 104.05 | 612 |

The largest reaction is **at level C** (R = 92.49 kN/m): the wale is sized with **$M_{max}$ = 104.05 kN·m, S ≥ 612 cm³**; the wales at the other levels may be smaller.

Check the solution in GeoTerva ↗

# Design effects, class, bending and buckling

THEORY

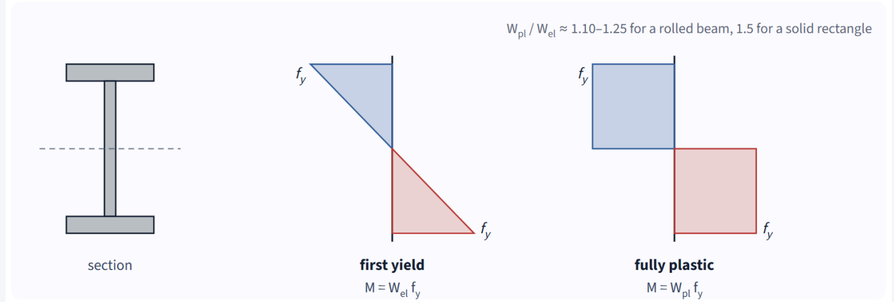
The loads from the envelope are **working values** (the Peck envelope is an upper bound on the measured loads); they are factored to the design value once: $E_d = \gamma_F E_k$. Each element is verified with the yield strength of its own steel grade ($f_y$); the section is given by the element's **section modulus** (sheet pile: $W_{el}$ per metre; wale: W; strut: A and I about the weak axis).

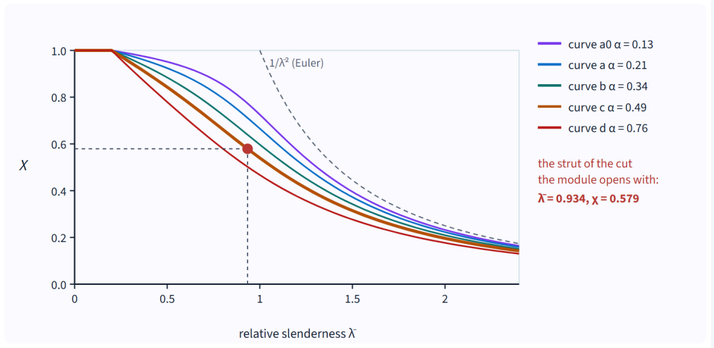
$$M_{c,Rd} = \frac{W f_y}{\gamma_{M0}} \qquad \eta = \frac{M_{Ed}}{M_{c,Rd}} \le 1 \qquad W = W_{pl}\ (\text{class 1–2}),\ W_{el}\ (\text{class 3})$$

$$N_{cr} = \frac{\pi^2 EI}{L_{cr}^2} \qquad \bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}} \qquad \Phi = 0.5\left[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2\right] \qquad \chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \le 1 \qquad N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}}$$

The strut buckles about its weak axis: I is the small moment of inertia, $L_{cr} = K L_b$ is the effective length (excavation width, if no king pile or cross bracing passes in between). α comes from the buckling curve (curve c: 0.49).

For sheet piles the same resistance applies per unit width (EN 1993-5; interlock slip $\beta_B$ for U-profiles, 1.0 for Z-profiles). Since the compression flange of the wale bears against the wall, lateral torsional buckling is not governing.



# Problem statement

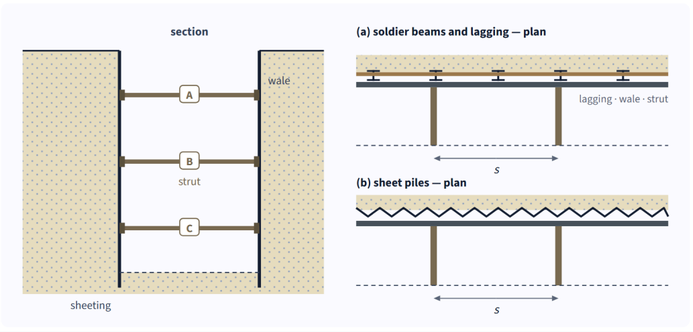
EXAMPLE

## Given

| | |
|---|---|
| Excavation and loads | **The excavation in Example 1 and the loads in Example 4: $P_{max} = 277.47$ kN** |
| Elements | **S275 ($f_y = 275$ MPa), $\gamma_F = 1.35$, $\gamma_{M0} = \gamma_{M1} = 1.00$: sheet pile $W_{el} = 600$ cm³/m; wale $W_{el,y} = 1\,000$ cm³; strut $A = 65.7$ cm², $I = 3\,598$ cm⁴, curve c, $L_{cr} = 6.00$ m** |

## REQUIRED

**η** for the sheet pile, the most loaded wale and the most loaded strut (≤ 1).



# Solution (1/2)

SOLUTION

1 Sheet pile and wale (bending): $\eta = \gamma_F M/(W f_y)$

$$\eta_{sp} = \frac{1.35(26.99)}{600 \cdot 10^{-6}(275\,000)} = \frac{36.44}{165.0} = \mathbf{0.221}$$

$$\eta_{belt} = \frac{1.35(104.05)}{1\,000 \cdot 10^{-6}(275\,000)} = \frac{140.47}{275.0} = \mathbf{0.511}$$

Sheet pile $\eta$ = **0.221**, wale $\eta$ = **0.511**: both are sufficient ($\eta \leq 1$).

# Solution (2/2)

SOLUTION

2 Strut (buckling): $N_{Ed} = 1.35\ P_{max} = 1.35(277.47) = 374.58$ kN

$$N_{cr} = \frac{\pi^2(210\,000\,000)(3\,598 \cdot 10^{-8})}{6.00^2} = 2{,}071.5\ \text{kN}$$

$$\bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}} = \sqrt{\frac{1\,806.8}{2{,}071.5}} = 0.9339$$

$$\Phi = 0.5\left[1 + 0.49(0.9339 - 0.2) + 0.9339^2\right] = 1.1159$$

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} = 0.5791 \quad N_{b,Rd} = \chi A f_y = 1{,}046.4\ \text{kN}$$

$\eta_{strut}$ = 374.6 / 1,046.4 = **0.358**. All three members are adequate ($\eta \le 1$); **buckling** governs the strut ($N_{c,Rd}$ = 1,806.8 kN, $N_{b,Rd}$ = 1,046.4 kN).

Check the solution in GeoTerva ↗

# LRFD or ASD; classes, bending, shear, column curve

THEORY

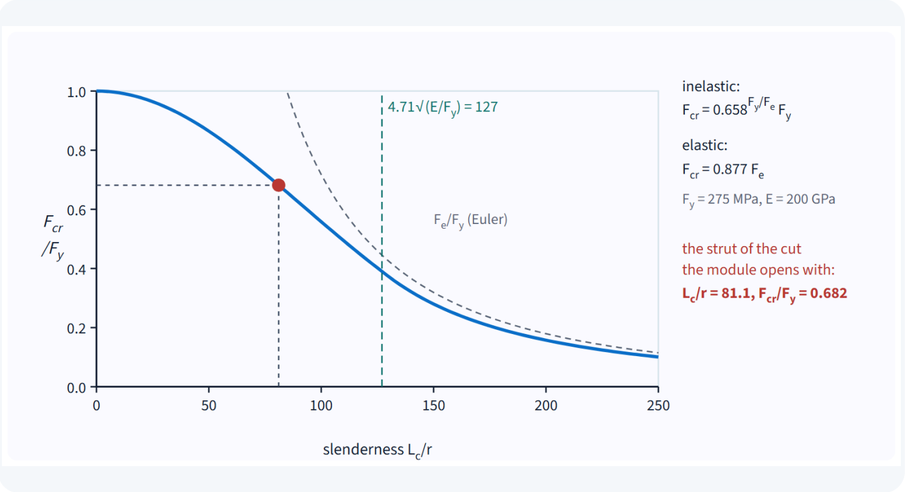
Each member is verified with the **nominal strength $R_n$** found from the compression, bending and shear provisions. In LRFD the load is factored and the strength reduced by φ; in ASD the working load is compared with the strength divided by the safety factor Ω:

$$R_u \le \phi R_n \ \text{(LRFD)} \qquad R_a \le \frac{R_n}{\Omega} \ \text{(ASD)} \qquad P_u = \gamma_H P,\ M_u = \gamma_H M,\ \gamma_H = 1.6$$

The loads coming out of the envelope are the **lateral earth pressure H** of ASCE 7-22: in LRFD combinations, it enters with 1.6 wherever it is added to the load (always in excavation bracing), and with 1.0 in ASD. The sheet pile and the wale given with the module are taken with the strength claimed by the module: $F_yZ$ at compact, $F_yS$ at first yield. Since the wale is laterally restrained by the wall, $L_b = 0$ and lateral-torsional buckling is not a limit state.

$$F_e = \frac{\pi^2 E}{(L_c/r)^2} \qquad F_{cr} = 0.658^{F_y/F_e} F_y \ \left(\frac{L_c}{r} \le 4.71\sqrt{E/F_y}\right) \ \text{otherwise } 0.877F_e \qquad P_n = F_{cr} A_g$$

**A single column curve** serves every section (no curve to be selected as in EN 1993-1-1); r is the small radius of gyration, $L_c = K\,L_b$ the effective length; $L_c/r \le 200$ is recommended. AISC, unlike EN 1993-1-1, does not reduce the flexural strength due to shear force.



# Problem statement

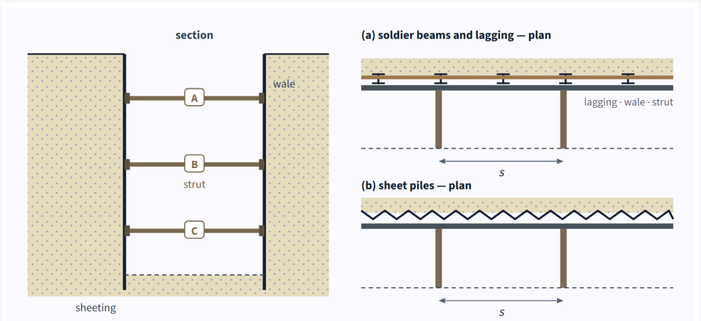
EXAMPLE

## Given

| | |
|---|---|
| Excavation and loads | **The excavation in Example 1 and the loads in Example 4: $P_{max} = 277.47$ kN** |
| Elements | **Elements from Example 6 (S275, $F_y = 275$ MPa); $E = 200$ GPa; LRFD $\gamma_H = 1.6$, $\phi_c = \phi_b = 0.90$; ASD $\Omega_c = 1.67$, $\Omega_b = 1.67$ ( $M_n = F_y S$)** |

## REQUIRED

For the most heavily loaded strut: **$L_c/r$, $F_e$, $F_{cr}$, $P_n$**; LRFD and ASD ratios; LRFD ratio for the sheet pile and the wale.



# Solution (1/2)

SOLUTION

1 Support: column curve ($A_g$ = 65.7 cm², r = 7.40 cm, $L_c$ = 6.00 m)

$$\frac{L_c}{r} = \frac{600}{7.40} = 81.1 \;\le 4.71\sqrt{E/F_y} = 127$$

$$F_e = \frac{\pi^2 E}{(L_c/r)^2} = \frac{\pi^2(200\,000)}{(81.1)^2} = 300.3\ \text{MPa}$$

$$F_{cr} = 0.658^{275/300.3}(275) = 187.4\ \text{MPa}$$

$$P_n = F_{cr}A_g = 187.4 \times 6\,570\ \text{mm}^2 = \mathbf{1{,}231.5}\ \text{kN}$$

At the support **nominal strength $P_n$ = 1,231.5 kN**; slenderness $L_c$/r = 81.1, in the inelastic buckling range (limit 127).

# Solution (2/2)

SOLUTION

2 LRFD and ASD ratios (P = 277.47 kN)

$$\text{LRFD: } P_u = 1.6(277.47) = 444.0\ \text{kN} \leq \phi_c P_n = 0.90(1{,}231.5) = 1{,}108.3\ \text{kN} \quad \eta = \mathbf{0.401}$$

$$\text{ASD: } P_a = 277.47\ \text{kN} \leq \frac{P_n}{\Omega_c} = \frac{1{,}231.5}{1.67} = 737.4\ \text{kN} \quad \eta = \mathbf{0.376}$$

Bracing: LRFD η = **0.401**, ASD η = **0.376**. Sheet pile (LRFD) 0.291, waler 0.673; in EN 1993-1-1 they were 0.221, 0.511 and 0.358.

Check the solution in GeoTerva ↗

# The clay below carries, the layers beside it induce shear and load

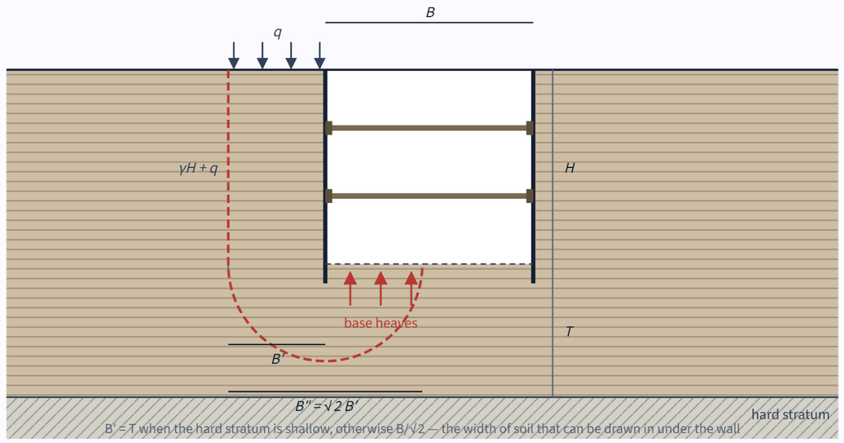
THEORY

The excavation base in clay may heave upward under the weight of the adjacent soil (Terzaghi column, Chang's correction). The formula is derived for homogeneous clay; the module uses it **in layered profiles via the GeoTerva extension**:

$$FS = \frac{5.14\,c\left(1 + 0.2\frac{B''}{L}\right) + \frac{c_{side} H}{B'}}{\sigma_v(H) + q} \qquad B' = \min\left(T, \frac{B}{\sqrt{2}}\right),\ B'' = \sqrt{2}\,B'$$

**c**: the clay below the base (bearing term); **$c_{side}$**: average cohesion over the retained height (sand is taken as zero; side shear term); **$\sigma_v(H)$**: total overburden stress at base level at the excavation edge ($\Sigma\gamma_i H_i$).

B excavation width, L plan length, T the stiff layer below the base; the embedment of the wall is not counted (conservative). Required FS ≈ **1.5**.



# Problem statement

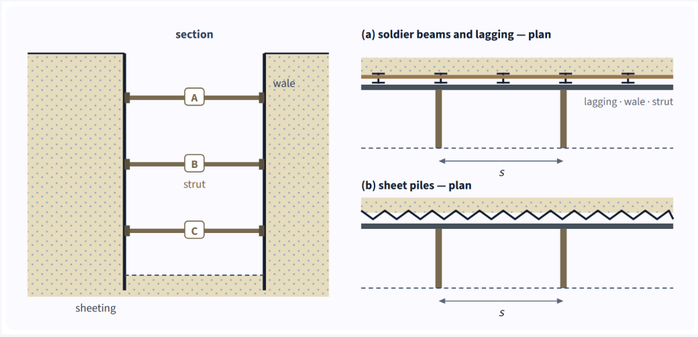
EXAMPLE

## Given

| | |
|---|---|
| Open excavation | $H = 7$ m, $B = 6$ m, $L = 20$ m, clay $T = 4$ m below the base; 3 m of sand on top ($\gamma = 17$), clay below ($\gamma = 18$, $c = 35$ kPa) |
| Comparison | if the sand were 5 m thick (clay 2 m) |

## REQUIRED

$c_{side}$, $\sigma_v(H)$, $B'$ and **FS** (≥ 1.5?); FS as the sand thickness increases?



# Solution

SOLUTION

**1** From the adjacent layers: average cohesion and overburden stress

$$c_{side} = \frac{0\,(3) + 35\,(4)}{7} = \mathbf{20.0}\ \text{kPa}\ \ (\text{sand zero}) \qquad \sigma_v(H) = 17(3) + 18(4) = \mathbf{123}\ \text{kPa}$$

$$B/\sqrt{2} = 4.24\ \text{m} > T = 4\ \text{m} \ \Rightarrow\ B' = 4.00\ \text{m}, \quad B'' = \sqrt{2}B' = 5.66\ \text{m}$$

**2** Factor of safety (bearing term: clay below the base c = 35 kPa)

$$FS = \frac{5.14(35)\left(1 + 0.2\frac{5.66}{20}\right) + \frac{20(7)}{4.00}}{123} = \frac{190.1 + 35.0}{123} = \mathbf{1.83}\ \geq 1.5\ \checkmark$$

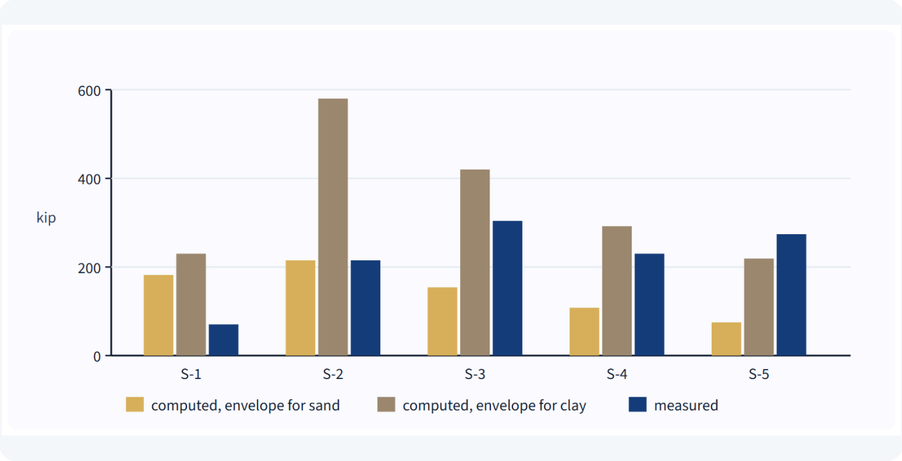
Base heave safety **1.83** (≥ 1.5). When the sand becomes 5 m, the side cohesion drops to 10 kPa, the overburden stress falls to 121 kPa and FS drops to **1.72**: as the sand layer thickens, the side shear contribution decreases.

Check the solution in GeoTerva ↗

# Boston and Chicago case studies

THEORY

**Boston** (test section B): 17.7 m deep, five supports; the profile was bounded by two envelopes for sand and clay (sand envelope 53.5 kPa, soft–medium clay envelope 146.1 kPa); the measured loads turned out very different from both estimates, and the supports had been designed for loads far above the measured ones. **Chicago, National Plaza**: 21.3 m deep, six supports; the profile was converted to an equivalent clay ($c_{av}$ = 34.91 kPa, $\gamma_{av}$ = 19.76 kN/m³, γH/c = 12.08, $\sigma_a$ = 282.0 kPa). The Peck envelope gave about **1.8 times** the observed pressure: an envelope drawn around many excavations is conservative for a single excavation; on the same soil, field experience can substantially reduce overdesign.



# What the method assumes

THEORY

The envelopes have been compiled from excavations deeper than about 6 m; **the envelope is not the actual pressure on the wall at any instant**: it gives a safe support load, not wall deflection. Clay is undrained, sand is drained.

The sand + clay profile is converted into **a single equivalent clay** ($c_{av}$, $\gamma_a$); the envelope is selected by $\gamma_a H/c_{av}$. Several clays are first averaged by thickness. $K_s \approx 1$ and $n' = 0.75$ are average values; a variation of $n' = 0.5$–$1.0$ has large effects on the result.

The surcharge is added separately ($\Delta p = q$); water only changes the unit weight. The **sheet pile** is a chain of hinged determinate beams; the **wales** are simple spans between supports. Steel is verified with EN 1993-1-1 (class 4 not designed) or AISC 360-22.

Base heave is checked with the layered GeoTerva extension of the formula derived for homogeneous clay; the embedment of the sheet pile is not counted.

Lateral movement and ground loss around the excavation are expected; if a hard layer exists, the sheet piles are embedded into it.

Summary

# Key points

**Equivalent clay**: $c_{av} = [\gamma_s K_s H_s^2 \tan \varphi'_s + (H - H_s) n' q_u]/(2H)$, $\gamma_a = [\gamma_s H_s + (H - H_s)\gamma_c]/H$; in the example 21.31 kPa and 17.57 kN/m³

**Envelope**: $\gamma_a H/c_{av} = 5.77 > 4 \rightarrow$ soft–medium clay envelope $\sigma_a = \gamma_a H[1 - 4c_{av}/\gamma_a H] = 37.75$ kPa; $n' = 0.5 \rightarrow 57.75$ kPa

**National Plaza**: $c_{av,clay} = 51.16$, $c_{av} = 34.91$ kPa, $\gamma_{av} = 19.76$ kN/m³, $\gamma H/c = 12.08 \rightarrow \sigma_a = 282.0$ kPa (≈ 1.8 times the observed value)

**Support load and sizing**: $P_{max} = 277.47$ kN; sheet pile $M_{max} = 26.99$ kN·m/m; wale M = 104.05 kN·m

**Steel**: EN 1993-1-1 η = 0.221 / 0.511 / 0.358; AISC LRFD strut 0.401, ASD 0.376

**Base heave** (layered): $c_{side} = 20$ kPa, $\sigma_v(H) = 123$ kPa, FS = 1.83 ≥ 1.5

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**NOTE**

All numerical examples have been produced with the module's own calculation engine; the module gives the same values. The worked examples on the module page are the National Plaza (Chicago) case and the module's opening excavation (H = 7 m, sand 3 m + clay).

GeoTerva

Online geotechnical analysis

# From theory to practice

Enter your own excavation's sand and clay layers, support levels, surcharge and your elements in GeoTerva; the module gives the equivalent clay, the envelope, the support loads, the sheet-pile and wale moments, the steel verification and the base heave together.

**Open the module →**

**Theory page →**

**Open on your phone**
The example modules need no registration.

If you would like to use them in your lectures: info@geoterva.com