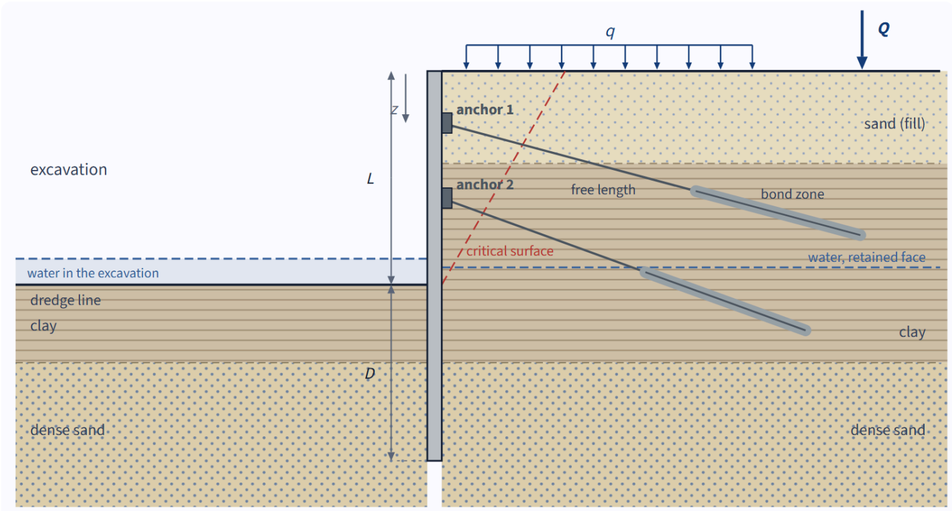
GeoTerva

Geotechnical lecture presentation · Sheet piles and braced cuts

Anchored sheet pile wall (layered soil)

Lecture: cantilever and anchored sheet pile wall in layered sand and clay; stresses on both faces, short and long term, embedment depth, anchor forces, moment, section, earthquake; worked example on each topic.



Layered soil

Short- and long-term

Free-earth support · hinge

Anchor · EN 1993 · AISC 360

Earthquake · basal heave

What will we learn in this lecture?

In a profile of up to six layers of **sand (drained: c′, φ′) and clay (short term c_u, long term c′, φ′)**, the stresses and pressures on both faces of the wall

In undrained clay $\sigma_v \mp 2c_u$, the **tension crack** (water-filled) and the **short/long term** scenarios, the rule of “the larger of each result”

Net pressure diagram and the margin of safety (f with depth, FS_p for passive, given depth)

Statics of the **cantilever**, **single-row anchored (free-earth support)** and **multi-row anchored (hinged method)** walls; moment, anchor force and grout bond

Verifying the sheet pile with **EN 1993-1-1 / AISC 360-22**; **earthquake**, **base heave** and **displacement** checks

LECTURE FLOW

Each topic is covered in **three steps**:

1 · **Theory**: what is the rule, why is it so?

2 · **Problem statement**: given and required

3 · **Solution**: step by step, with numbers

Example wall: 7.5 m excavation height; 3 m sand at the top, 6 m clay, dense sand below; water at 8.5 m depth on both sides; q = 10 kPa surcharge; two rows of anchors (1.5 m and 4.5 m, s = 2.5 m, 15° and 20°). The forces are per metre of wall.

How is the wall retained?

THEORY

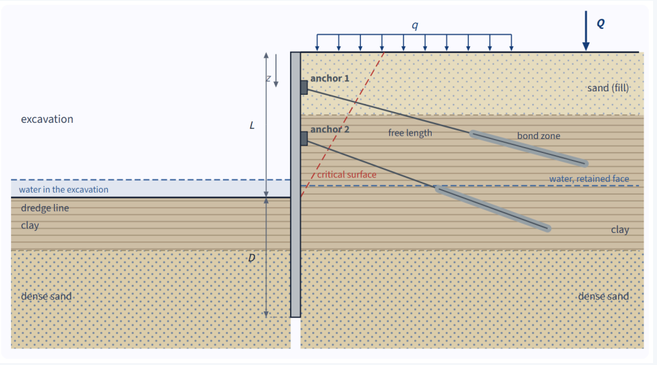
The sheet pile stands by its embedment below the excavation base and, when the retained height is too great for that, by **ground anchors**. Real ground is rarely a single layer: sand pushes and resists by friction, clay by its strength; the clay's strength is not the same immediately after excavation as it is years later. The module takes the profile as it is: **up to six layers**, continuing below the excavation base at both percentages.

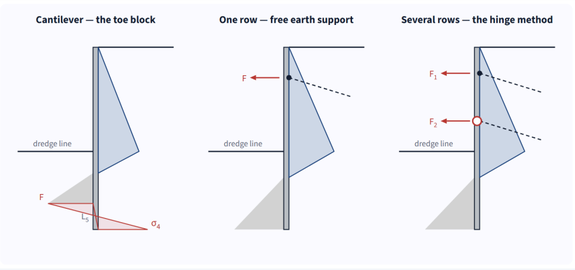
Cantilever: supported by the soil alone; it rotates about a point near the tip (active above the excavation base, the sides swap below). The retained height is economical up to about 5–6 m.

Single row of anchors: the wall is a beam on two supports (the anchor and the passive resistance of the embedment); the tip is taken as free: the **free earth support** is obtained from a single moment equation about the anchor.

Multi-row anchors: the wall is statically indeterminate; the **hinge method** inserts a hinge at every anchor except the topmost one, the spans become simply supported beams, and the part below the lowest anchor is designed with a free earth support.

z is measured downward from the top of the wall; L is the depth of the excavation base, D the embedment. Net pressure is positive toward the excavation; the anchor pulls the wall back with F (per metre). The method is **limit equilibrium**: the soil is taken in its limiting active and passive states.





σ_v, u, σ'_v; Rankine c′–φ′

THEORY

Three stress fields are maintained at each depth: total vertical stress σ_v, pore water pressure u and effective stress $\sigma'_v = \sigma_v - u$. Undrained clay is defined by the first, the drained layer by the second and third. The retained column carries its entire overburden also below the excavation base; the excavated column restarts at the excavation base: this is the difference that the embedment must balance.

$$\sigma_{v,b}(z) = q + \sum \gamma_i \Delta z_i \quad u_b = \gamma_w \max(0, z - D_w) \quad \sigma'_{v,b} = \sigma_{v,b} - u_b$$

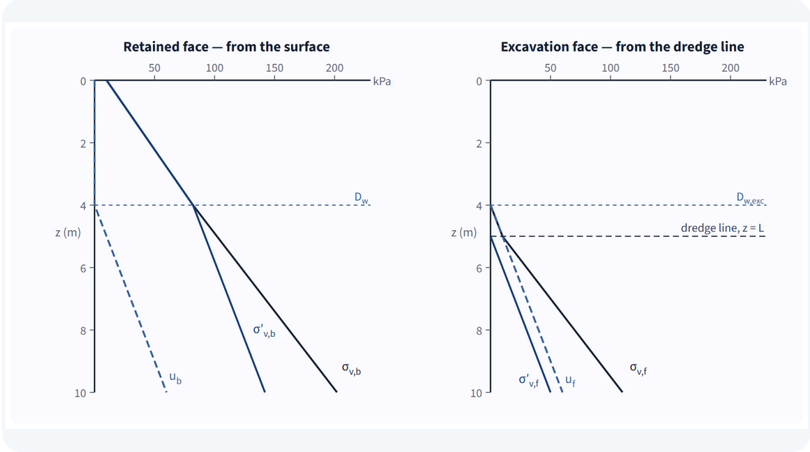
Drained layer (sand, or clay in the long term): Rankine, vertical wall, level surface, no wall friction:

$$K_a = \tan^2\left(45^\circ - \tfrac{\varphi'}{2}\right) \quad K_p = \tan^2\left(45^\circ + \tfrac{\varphi'}{2}\right) \qquad p_a = \max\left(0,\ K_a\sigma'_v - 2c'\sqrt{K_a}\right) + u \quad p_p = \eta_p\left(K_p\sigma'_v + 2c'\sqrt{K_p}\right) + u$$

Each layer has its own K_a, K_p value: σ'_v is continuous but **the pressure jumps at the layer boundary**; the jumps are a genuine feature of the diagram and are not smoothed.

When c′ > 0, the active expression becomes negative above the layer: soil cannot pull the wall, so the soil part is **cut off** at zero (the water part is not cut off).

Uniform surcharge q is added to $\sigma_{v,b}$ at every depth; the tension cut-off accounts for it.



Problem statement

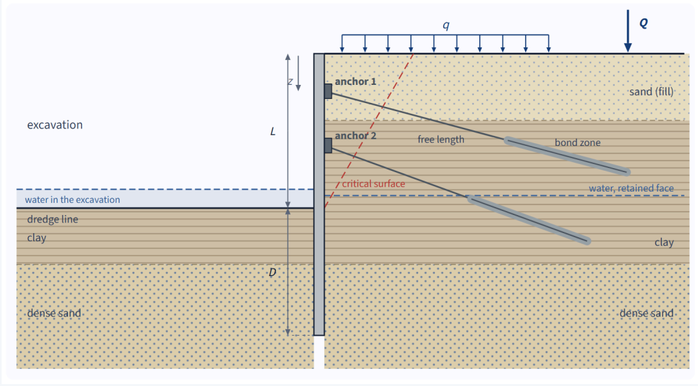
EXAMPLE

Given

Profile	**at the top 3.0 m sand ($\gamma = 18$, $\varphi' = 30^\circ$), 6.0 m clay ($\gamma = 19$, $c' = 5$ kPa, $\varphi' = 24^\circ$, $c_u = 45$ kPa), below it dense sand ($\gamma = 19$, $\varphi' = 36^\circ$); $L = 7.5$ m; water at 8.5 m on both sides; $q = 10$ kPa**
Problem statement	**the retained side $z = 3.0$ m (sand–clay boundary) and the excavation base $z = L = 7.5$ m**

REQUIRED

σ'_v; $\mathbf{p_a}$ for sand, clay (drained, long term); the jump at the sand–clay boundary.



Solution

SOLUTION

1 At the sand–clay boundary (z = 3.0 m; water at 8.5 m, u = 0)

$$\sigma'_v(3.0) = q + \gamma z = 10 + 18(3.0) = 64.0 \text{ kPa}$$

$$\text{sand: } p_a = K_a \sigma'_v = 0.3333(64.0) = \mathbf{21.33} \text{ kPa}$$

$$\text{clay } (\varphi' = 24^\circ)\text{: } p_a = K_a \sigma'_v - 2c'\sqrt{K_a} = 0.4217(64.0) - 2(5)\sqrt{0.4217} = \mathbf{20.50} \text{ kPa}$$

2 At the excavation base (z = L = 7.5 m): the retained column advanced 4.5 m further into the clay

$$\sigma'_v(L) = 64.0 + 19(4.5) = 149.5 \text{ kPa}$$

$$p_a = 0.4217(149.5) - 2(5)\sqrt{0.4217} = \mathbf{56.55} \text{ kPa}$$

σ′$_V$ is continuous (64 kPa), but the pressure jumps at the boundary from **21.33 to 20.50 kPa** (sand K_a = 1/3, clay K_a = 0.4217 and the c′ contribution −6.49 kPa). The active pressure at the excavation base is 56.55 kPa.

Check the solution in GeoTerva ↗

$\sigma_v \mp 2c_u$; “the larger of each result”

THEORY

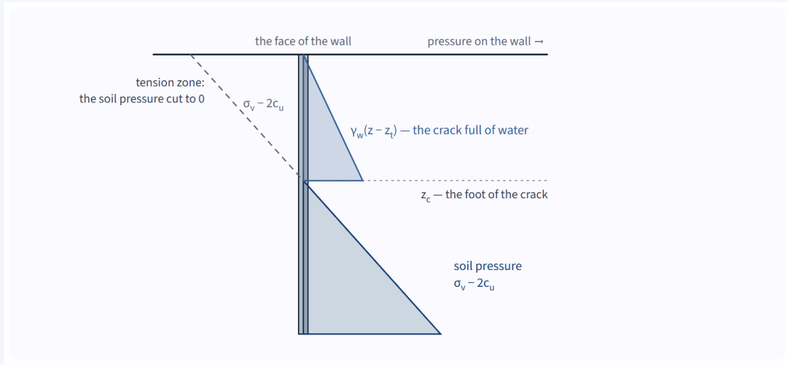
Rapidly loaded saturated clay (the excavation is made in front of it) cannot drain: the strength is c_u, $\varphi = 0$, defined in terms of **total stress** (the water is inside σ_v):

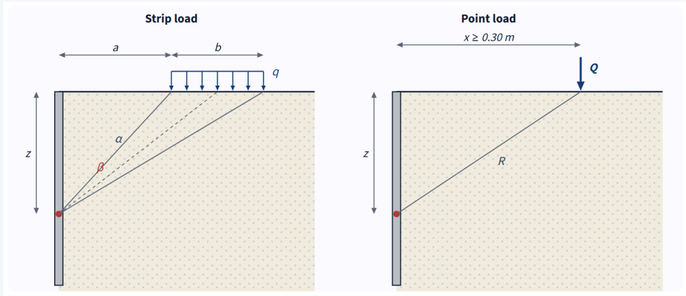
$$p_a = \max(\sigma_v - 2c_u,\ u) \qquad p_p = \eta_p(\sigma_v + 2c_u) \qquad p_{crack} = \gamma_w(z - z_t)\ \ (z_t \le z \le z_c)$$

Above the retained clay, $\sigma_v - 2c_u$ is negative: the clay cracks; z_t is the top of the tension zone and $\mathbf{z_c}$ is its lower end, where $\sigma_v = 2c_u$. An open crack at the surface fills with rain or groundwater and pushes with its own hydrostatic pressure: **assumed water-filled** (safe); it may be declared dry (with a warning), and the consequence of the other case is always reported.

Short term clay undrained; **long term** each layer drained. Which one is critical is not known in advance; the module **runs both** and takes the embedment, each anchor force and the moment as the **larger of the two scenarios** (they do not have to come from the same scenario). A scenario in which the anchor carries no tension does not govern the wall; it is noted.

Surcharge: uniform q is added to $\sigma_{v,b}$; the strip load (a, b, q) and point load (Q, x) are the elastic solution for a rigid wall and are **not multiplied** by K_a; in an earthquake it enters with Ψ_2.





Problem statement

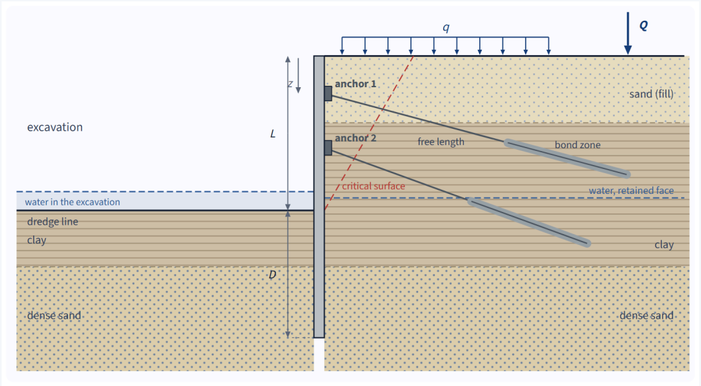
EXAMPLE

Given

Profile	**at the top 3.0 m sand ($\gamma = 18$, $\varphi' = 30°$), 6.0 m clay ($\gamma = 19$, $c' = 5$ kPa, $\varphi' = 24°$, $c_u = 45$ kPa), below it dense sand ($\gamma = 19$, $\varphi' = 36°$); $L = 7.5$ m; water at 8.5 m on both sides; $q = 10$ kPa**
Problem statement	**in the short term $\sigma_v - 2c_u$ above the clay and z_c at the lower end of the crack; results of the two scenarios**

REQUIRED

$\sigma_v - 2c_u$ (above the clay), **z_c** and the water pressure in the crack; the D, F, M values of the short and long term and which one governs.



Solution

SOLUTION

1 Short term: above the clay (z = 3.0 m, σ_v = 64.0 kPa, c_u = 45 kPa)

$$\sigma_v - 2c_u = 64.0 - 2(45) = -\mathbf{26.0}\ \text{kPa} < 0 \Rightarrow \text{is cut; the crack opens}$$

$$\sigma_v(z_c) = 2c_u: \quad 64.0 + 19(z_c - 3) = 90 \Rightarrow z_c = \mathbf{4.37}\ \text{m}$$

$$p_{crack}(z_c) = \gamma_w(z_c - z_t) = 9.81(1.37) = 13.4\ \text{kPa}$$

2 Both scenarios are run (the module's engine); the wall is designed for the larger of the two

Scenario	$D_{theoretical}$ (m)	$D_{required}$ (f = 1.3)	F_1, F_2 (kN/m)	M_{max} (kN·m/m)	z_M (m)
Short term (clay undrained)	1.549	2.014	34.30 ; 56.94	59.49	6.55
Long term (all drained)	2.403	3.124	41.69 ; 132.32	121.43	6.82

Water-filled crack gives hydrostatic pressure over **1.37 m** above the clay (3.0 → 4.37 m). The long term is **decisive in every result** (D, F_1, F_2, M): the wall is designed for the long-term scenario; the short-term moment is less than half.

Check the solution in GeoTerva ↗

Difference diagram; f versus depth or FS_p for passive

THEORY

The wall feels the **difference** between the two sides: $p_{net} = p_b - p_f$ (retained – excavation). Equilibrium is written on this diagram: above the excavation base, the active pressure and water of the retained soil; below it, the passive of the embedment grows faster than the active, and the net pressure becomes zero at point E and reverses toward the retained side. The top, the anchors, the excavation base, each layer boundary, the water levels, the crack tips, the load edges and the zeros of the truncated pressures divide the wall into **segments**; within each segment the resultant is linear, and force and moment are integrated exactly:

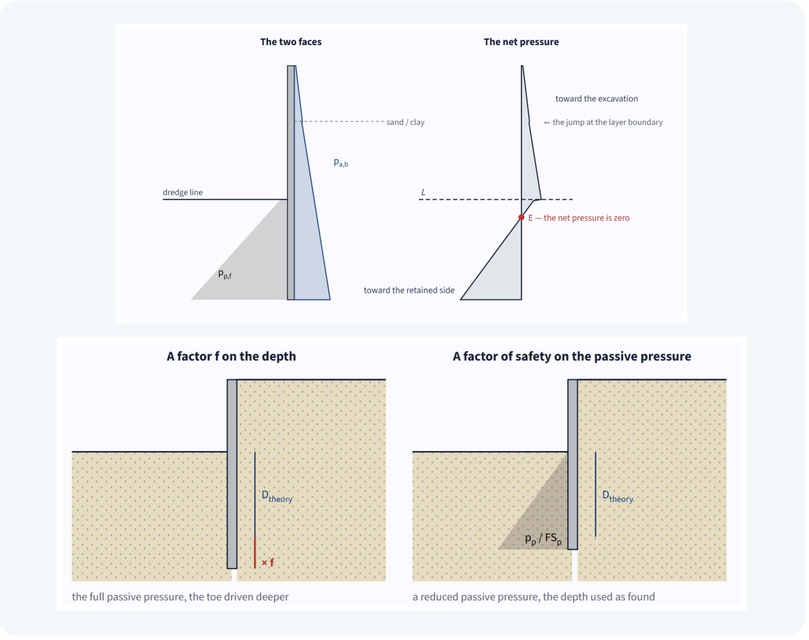
$$F_{seg} = \tfrac{1}{2}(p_0 + p_1)h \qquad M_{seg}(z_r) = \frac{h\,[p_0(2z_0 + z_1) + p_1(z_0 + 2z_1)]}{6} - z_r F_{seg}$$

No allowance is made in the theoretical depth $D_{theoretical}$; two approaches (figure):

Depth factor f: $D_{required} = f \cdot D_{theoretical}$ (f = 1.2–1.3 sand, 1.4–1.6 clay); pressures and moment are those of the theoretical wall.

FS_p on passive side: $\eta_p = 1/FS_p$ multiplies the passive **soil** pressure (never the water pressure); the depth of the reduced diagram becomes $D_{required}$. The $D_{required}$ value of the other approach is always written next to it.

Given depth: verified by $UR_D = D_{required}/D_{given} \le 1$; $FS_{p,mob}$ at which the passive resistance may be divided is found. Moments and anchor forces are taken according to the **larger** of the design and given-depth cases.



Problem statement

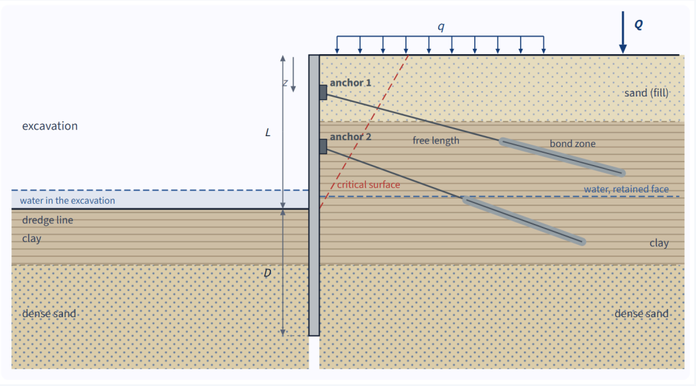
EXAMPLE

Given

Wall	**at the top 3.0 m sand ($\gamma = 18$, $\varphi' = 30°$), 6.0 m clay ($\gamma = 19$, $c' = 5$ kPa, $\varphi' = 24°$, $c_u = 45$ kPa), below it dense sand ($\gamma = 19$, $\varphi' = 36°$); $L = 7.5$ m; water at 8.5 m on both sides; $q = 10$ kPa; two rows of anchors: $z = 1.5$ m ($15°$) and 4.5 m ($20°$), $s = 2.5$ m; $f = 1.3$; S275, $W_{el} = 1\,400$ cm³/m**
Roads	$f = 1.3$; $FS_p = 1.5$; given $D = 3.5$ m and 6.0 m

REQUIRED

For each path, $D_{required}$, M_{max}, T_2 and $FS_{bond,2}$; at the given depth, UR_D, $FS_{p,mob}$.



Solution

SOLUTION

1 At depth f: $D_{required} = 1.3 \times D_{theoretical} = 1.3 \times 2.403 = 3.124$ m (moments and forces are those of the theoretical wall)

Road	$D_{required}$ / given (m)	UR_D	$FS_{p,mob}$	M_{max} (kN·m/m)	T_2 (kN)	$FS_{bond,2}$
f = 1.3 (excavation)	3.124	—	—	121.4	352.0	2.14
FS_p = 1.5 (passive / 1.5)	3.273	—	—	153.3	386.6	**1.95**
Given D = 3.5 m	3.500	0.893	1.63	161.6	395.1	**1.91**
Given D = 6.0 m	6.000	0.521	2.94	255.5	480.4	**1.57**

The passive route with $FS_p = 1.5$ gives a deeper wall (3.27 m) and a **larger moment** (153.3). **If the provided depth is longer than required, the moment and anchor force increase**: for D = 6.0 m, M = 255.5 kN·m/m (2.1 times the excavation) and $T_2 = 480$ kN, $FS_{bond,2} = 1.57 < 2$: the bond is insufficient. Making the wall longer than required does not bring safety.

Check the solution in GeoTerva ↗

Toe block; free-earth support

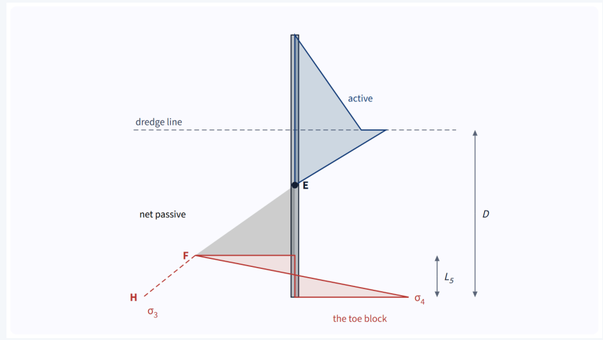
THEORY

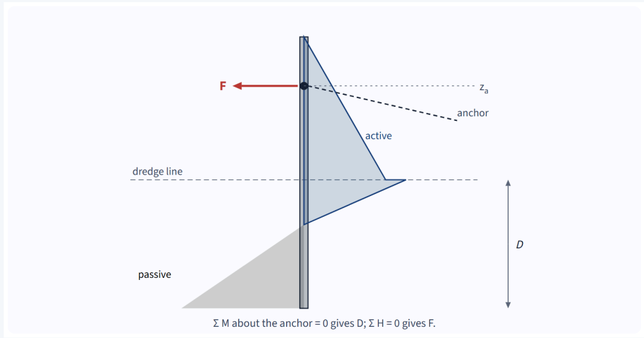
Cantilever: from point E, the net reaction below the excavation base, to F, and from F to the toe block (L_5) with the reversed ordinate σ_4: $\Sigma H = 0$ and $\Sigma M = 0$ about the toe. In a single sand it reduces to a fourth-degree equation for L_4; in a layered profile it is solved numerically on the same diagram. The block must lie below E ($0 < L_5 < L_4$).

Single row of anchors: the free end is assumed to translate; the depth is found **from the moment about the anchor**, the anchor force from horizontal equilibrium:

$$\int_0^{L+D} p_{net}(z)(z - z_a)\,dz = 0 \qquad F = \int_0^{L+D} p_{net}(z)\,dz$$

The free-earth support gives **the shortest wall and the largest moment of the classical methods**; Rowe's moment reduction for flexible walls is not applied.





Hinge method; shear and moment

THEORY

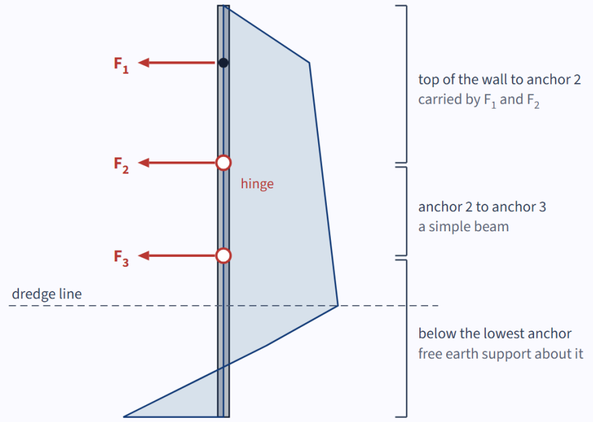
With three or more supports the wall is a continuous beam and is statically indeterminate. The **hinge method** makes it determinate by placing a hinge (zero moment) at every anchor except the topmost: the top span is a beam over the first two anchors, the intermediate ones are simply supported beams, and below the lowest anchor the soil provides a free support relative to that anchor.

$$\int_{z_n}^{L+D} p_{net}(z)(z - z_n)\, dz = 0$$

The force in one row is the sum of the reactions of the spans on either side; the bottom row also takes the horizontal force of the section below it. The moment at each hinge is zero (the module checks this as closure); a row remaining in compression is flagged with a warning.

$$V(z) = \int_0^z p_{net} dz - \sum_{z_a < z} F_i \qquad M(z) = \int_0^z V\, dz$$

V and M are integrated over the elements; the **maximum moment is found where V = 0** (within an element), not selected from the nodes.



Problem statement

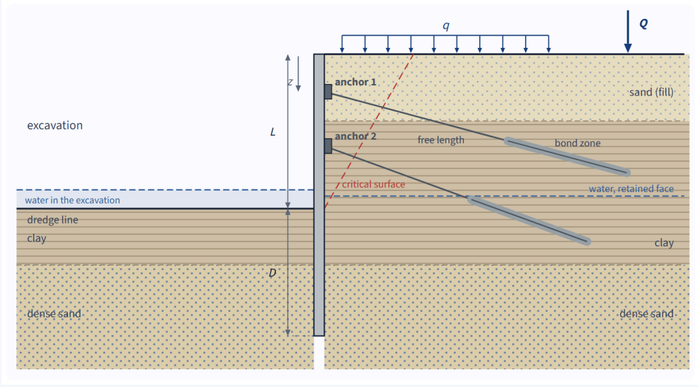
EXAMPLE

Given

Profile and loads	**at the top 3.0 m sand ($\gamma = 18$, $\varphi' = 30^\circ$), 6.0 m clay ($\gamma = 19$, $c' = 5$ kPa, $\varphi' = 24^\circ$, $c_u = 45$ kPa), below it dense sand ($\gamma = 19$, $\varphi' = 36^\circ$); $L = 7.5$ m; water at 8.5 m on both sides; $q = 10$ kPa**
Systems	**(a) cantilever without anchor; (b) single row ($z = 1.5$ m, 15°); (c) two rows (opening); in all, $f = 1.3$, S275 $W_{el} = 1\,400$ cm³/m**

REQUIRED

For each system, **$D_{theoretical}$**, **$D_{required}$**, **M_{max}**, the anchor force and the sheet pile **η**; which one is suitable?



Solution

SOLUTION

1 Same profile, three systems (the module's engine; the larger of the two scenarios in all)

System	Method	$D_{theoretical}$ (m)	$D_{required}$ (m)	M_{max} (kN·m/m)	Anchor (kN/m)	η (EN 1993)
(a) Cantilever	end block	8.033	10.443	1,210.1	—	**4.243**
(b) Single row	free-earth support	2.896	3.765	295.9	128.2	**1.037**
(c) Two rows	hinge	2.403	3.124	121.4	41.7 ; 132.3	0.426

For the 7.5 m retained height the **cantilever** requires 10.4 m of embedment and 1,210 kN·m/m: the sheet pile is insufficient (η = 4.24). **A single row** reduces the embedment to 3.76 m but keeps η = 1.037 > 1 with M = 296 kN·m/m. **Two rows** reduce the moment to 121.4 kN·m/m and the sheet pile is sufficient (η = 0.426): the number of rows governs the moment and the embedment.

Check the solution in GeoTerva ↗

Problem statement

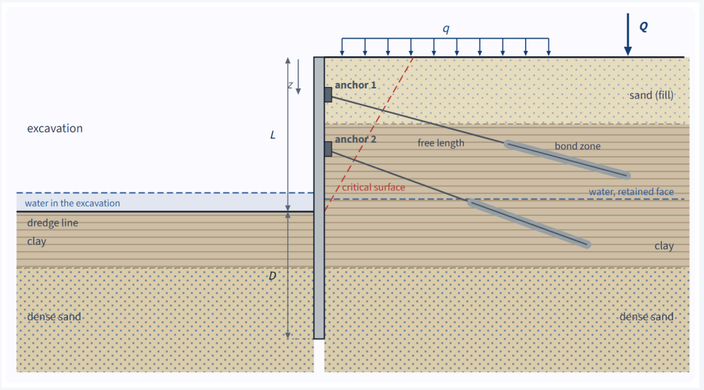
EXAMPLE

Given

Wall	**at the top 3.0 m sand ($\gamma = 18$, $\varphi' = 30^\circ$), 6.0 m clay ($\gamma = 19$, $c' = 5$ kPa, $\varphi' = 24^\circ$, $c_u = 45$ kPa), below it dense sand ($\gamma = 19$, $\varphi' = 36^\circ$); $L = 7.5$ m; water at 8.5 m on both sides; $q = 10$ kPa; two rows of anchors: $z = 1.5$ m (15°) and 4.5 m (20°), $s = 2.5$ m; $f = 1.3$; S275, $W_{el} = 1\,400$ cm³/m**
Result type	**long-term scenario (governing); hinge at $z = 4.5$ m**

REQUIRED

Anchor forces F_1, F_2 and their sums; closure; **M_{max}** and its location; points where V = 0.



Solution (1/2)

SOLUTION

1 Anchor rows (per metre) and horizontal equilibrium

$$F_1 = 41.69\ \text{kN/m}\ \ (z = 1.5\ \text{m}) \qquad F_2 = 132.32\ \text{kN/m}\ \ (z = 4.5\ \text{m})$$

$$F_1 + F_2 = 41.69 + 132.32 = 174.00\ \text{kN/m} = \Sigma H_{net}\ \ (\text{total horizontal force})$$

2 Moment is zero at the hinge; maximum moment where V = 0

$$M(z = 4.5) = 0 \approx 0\ \checkmark\ \ (\text{closure})$$

$$V = 0: \quad z = 3.22\ \text{m}\ \ \text{and}\ \ z_M = 6.82\ \text{m} \qquad M_{max} = \mathbf{121.43}\ \text{kN}\cdot\text{m/m}$$

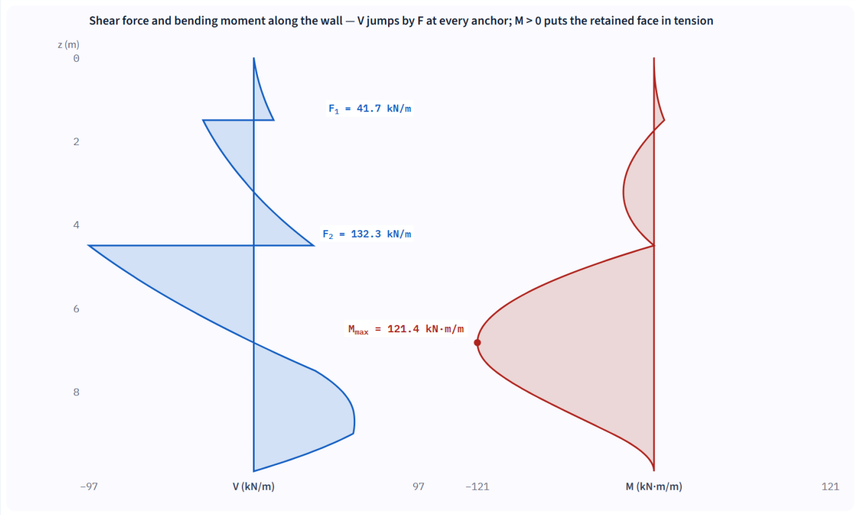
Anchor forces **F_1 = 41.69** and **F_2 = 132.32 kN/m**; their sum is the horizontal resultant of the net diagram (free end). **M_{max} = 121.43 kN·m/m**, below the excavation base at z = 6.82 m (V = 0).

Solution (2/2)

SOLUTION

3 Shear force and moment diagram (V jumps by F at each anchor)
$V(z) = \int p_{net} dz - \Sigma F_i$; $M(z) = \int V\, dz$. The maximum moment is 121.4 kN·m/m at a depth of 6.82 m; the two points where V = 0 (3.22 m and 6.82 m) are local moment peaks.

Closure checks are satisfied: hinge moment ≈ 0, tip shear and moment zero, the segment resultant equals the refined integral (174.0 kN/m). The moment diagram gives a local peak in the upper cantilever part and its largest peak below the excavation base.



Check the solution in GeoTerva ↗

Force, free length, bond and lock-off load

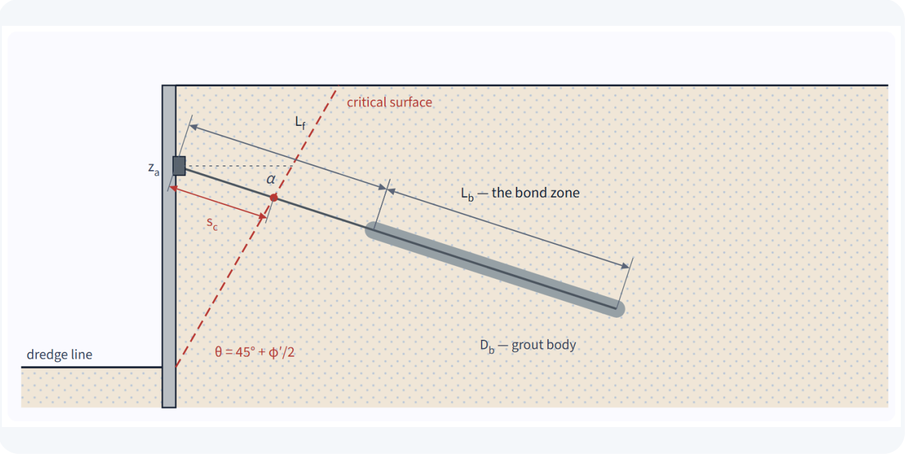
THEORY

The reaction of one row is per metre of wall. Anchors are spaced at s and inclined at α from the horizontal:

$$T = \frac{F\,s}{\cos\alpha} \qquad T_v = T\sin\alpha$$

The injection (bond) zone must remain **behind** the wedge that moves with the wall. The critical surface starts at the excavation corner and rises at θ = 45° + φ′/2; it is cut along the tendon at:

$$s_c = \frac{L - z_a}{\cos\alpha\tan\theta + \sin\alpha} \qquad L_f \geq s_c + \max(1.5\ \text{m},\ L/5)$$



Automatic if the surface is a single sand forming the entire retained height; in a layered or clayey profile a single plane is not a sound construction: the surface is entered from a separate analysis, otherwise the check reports "not performed".

Bond: $T_{ult} = \pi\, D_b\, L_b\, \tau_{ult}$; $FS_{bond} = T_{ult}/\max(T, P_0) \geq FS_{required}$. The anchor is locked off at P_0 and carries that load from the start: the demand is the larger of T and P_0. $P_0/T_{ult} \leq 1$ is also checked separately as an input verification.

Problem statement

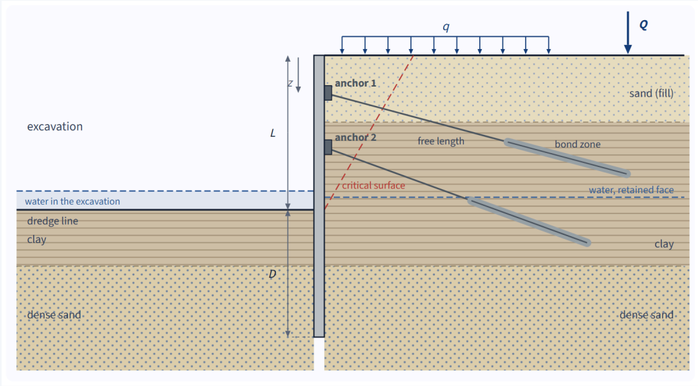
EXAMPLE

Given

Anchors	**row 1:** $z = 1.5$ **m,** $s = 2.5$ **m,** $\alpha = 15°$**,** $F_1 = 41.69$ **kN/m; row 2:** $z = 4.5$ **m,** $\alpha = 20°$**,** $F_2 = 132.32$ **kN/m**
Tie	$D_b = 0.20$ **m,** $L_b = 8$ **m,** $\tau_{ult} = 150$ **kPa; anchor force** $P_0 = 250$ **kN;** $FS_{bond,required} = 2.0$

REQUIRED

For each anchor: **T**, T_V; $\mathbf{T_{ult}}$, $\mathbf{FS_{bond}}$ and P_0/T_{ult}.



Solution (1/2)

SOLUTION

1 Anchor forces (each along the tendon axis)

$$T_1 = \frac{F_1 s}{\cos 15^\circ} = \frac{41.69(2.5)}{0.9659} = \mathbf{107.9}\ \text{kN}$$

$$T_2 = \frac{F_2 s}{\cos 20^\circ} = \frac{132.32(2.5)}{0.9397} = \mathbf{352.0}\ \text{kN}$$

The anchor forces are along the tendon axis; the vertical components $T_{v,1}$ = 27.9 and $T_{v,2}$ = 120.4 kN are reported (not carried into the sheet pile verification).

Solution (2/2)

SOLUTION

2 Bond: $T_{ult} = \pi D_b L_b \tau_{ult} = \pi(0.20)(8)(150) = 754.0$ kN

$$FS_{bond,1} = \frac{T_{ult}}{\max(T_1, P_0)} = \frac{754.0}{\max(107.9, 250)} = \mathbf{3.016} \geq 2 \checkmark$$

$$FS_{bond,2} = \frac{754.0}{\max(352.0, 250)} = \mathbf{2.142} \geq 2 \checkmark$$

At the first anchor the tie is determined by the **lock-off load** ($P_0 > T_1$), at the second by the **service force** (FS 3.02 and 2.14); $P_0/T_{ult} = 0.332 \leq 1$.

Check the solution in GeoTerva ↗

EN 1993-1-1 or AISC 360-22

THEORY

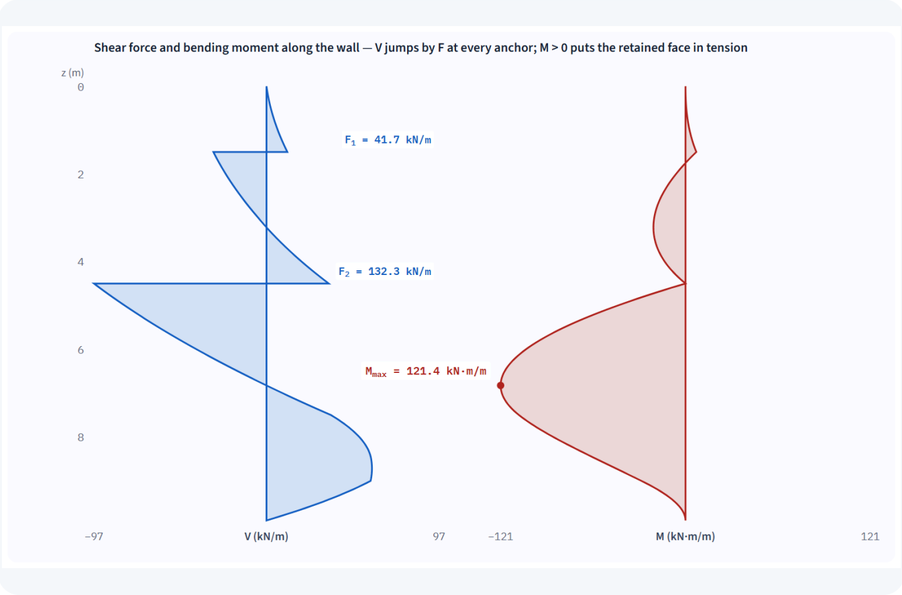
The steel section is verified in bending at the maximum design moment. The section modulus W per metre of wall (from the sheet pile tables; elastic or plastic, depending on the class) is used; the required W_{req} is always written so that the section can be selected from the catalogue:

$$M_{Ed} = \gamma_F M_{max} \le M_{c,Rd} = \frac{W f_y}{\gamma_{M0}} \qquad W_{req} = \frac{M_{Ed}\gamma_{M0}}{f_y}$$

$$\text{LRFD: } \gamma_H M_{max} \le \phi_b M_n \ (\phi_b = 0.90) \qquad \text{ASD: } M_{max} \le \frac{M_n}{\Omega_b} \ (\Omega_b = 1.67)$$

EN 1993-1-1: γ_F = 1.35, γ_{M0} = 1.00. AISC: γ_H = 1.6 (lateral earth pressure H), M_n = $F_y Z$ (compact) or $F_y S$ (first yield).

The shear of the wall, local buckling of its members, and corrosion loss are outside the scope of verification and are stated in the report. Anchor forces and moment are taken as the larger of the design and the given-depth conditions.



Problem statement

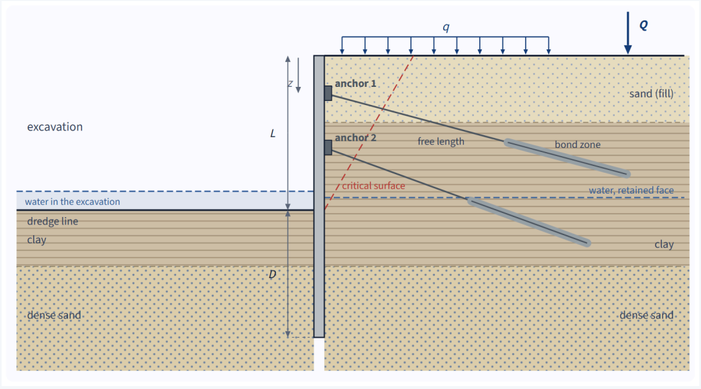
EXAMPLE

Given

Moment	**Example 5:** $M_{max} = 121.43$ **kN·m/m**
Section	**S275 (**$f_y = 275$ **MPa),** $W_{el} = 1\,400$ **cm³/m; EN:** $\gamma_F = 1.35$**,** $\gamma_{M0} = 1.0$**; AISC:** $\gamma_H = 1.6$**,** $\Omega_b = 1.67$ **(**$M_n = F_y S$**)**

REQUIRED

M_{Ed}, **W_{req}**, $M_{c,Rd}$, **η** (EN); η for LRFD and ASD.



Solution

SOLUTION

1 EN 1993-1-1

$$M_{Ed} = 1.35(121.43) = \mathbf{163.92}\ \text{kN·m/m} \qquad W_{req} = \frac{M_{Ed}\gamma_{M0}}{f_y} = \frac{163.92}{275\,000} = \mathbf{596}\ \text{cm}^3\text{/m}$$

$$M_{c,Rd} = 1\,400 \cdot 10^{-6}(275\,000) = 385.0\ \text{kN·m/m} \qquad \eta = \frac{163.92}{385.0} = \mathbf{0.426} \le 1\ \checkmark$$

2 AISC 360-22 ($M_n = F_yS$ = 385.0 kN·m/m)

$$\text{LRFD: } 1.6(121.43) = 194.28 \le 0.90(385.0) = 346.5 \quad \eta = \mathbf{0.561}$$

$$\text{ASD: } 121.43 \le \frac{385.0}{1.67} = 230.5 \quad \eta = \mathbf{0.527}$$

The sheet pile is adequate: EN 1993-1-1 η = **0.426** (required W = 596 cm³/m < 1 400), AISC LRFD **0.561**, ASD **0.527**.

Check the solution in GeoTerva ↗

Layered pseudo-static: Mononobe–Okabe in sand, undrained wedge in clay

THEORY

Each static coefficient is replaced by the pseudo-static counterpart of its own layer. The seismic angle comes from the horizontal and vertical coefficients:

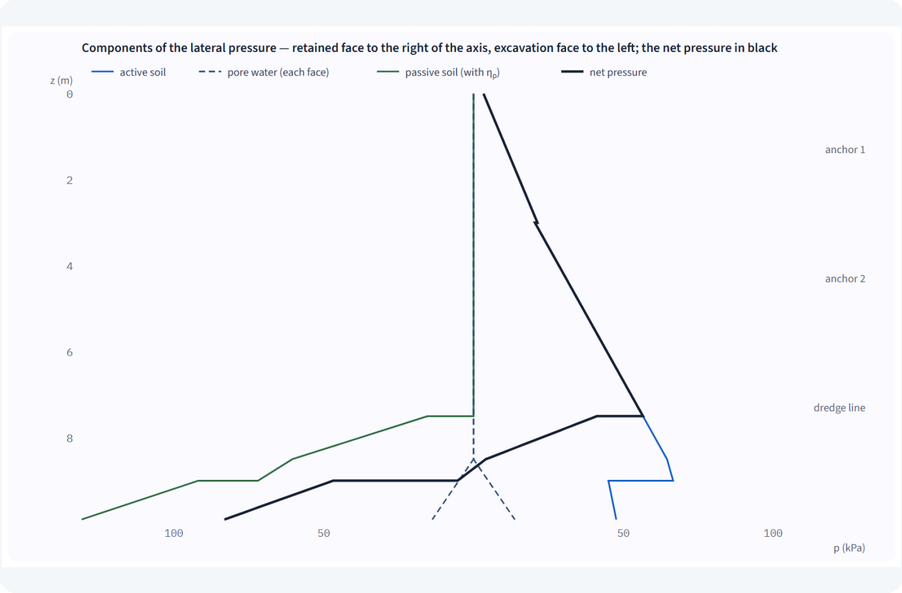
$$\tan\theta = \frac{k_h}{1 \mp k_v} \qquad \sigma_{AE} = (1 \mp k_v) K_{AE} \sigma'_v \qquad \sigma_{PE} = (1 \mp k_v) K_{PE} \sigma'_v$$

$$\text{clay:}\ \ \sigma_{AE} = (1 \mp k_v)(1 + \tan\theta)\sigma_v - 2c_u \qquad \sigma_{PE} = (1 \mp k_v)(1 - \tan\theta)\sigma_v + 2c_u$$

Both scenarios of the static design are run with the two directions of k_V; the largest embedment, moment and anchor forces govern.

An anchored wall cannot rotate about its toe: the dynamic increase in active thrust is **distributed uniformly over the retained height**: $\Delta P_{AE} = \int (p_{AE} - p_A) dz$. Free water in the excavation adds the Westergaard pressure ($7/8\ k_h \gamma_w \sqrt{(hz')}$).

The earthquake is tied to the excavation: **no factor is applied** to the passive soil; D_E, which balances the seismic diagram, must not exceed that of the wall (D_{design}). If the wall is longer than D_E, equilibrium is also satisfied at the given depth in either case, and larger moments and forces are taken. The anchor and tie are also checked for T_E ($FS_{required,E} \approx 1.5$).



Problem statement

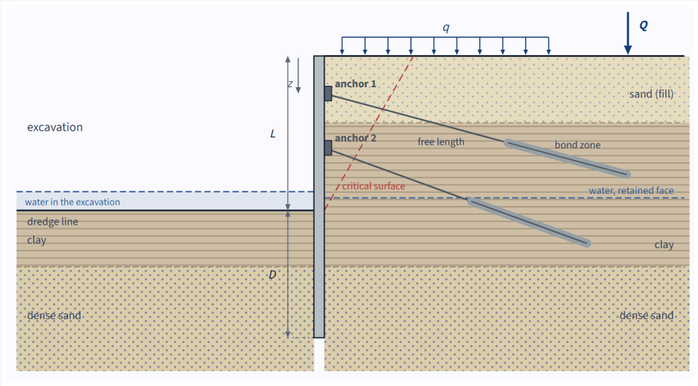
EXAMPLE

Given

Wall	**at the top 3.0 m sand ($\gamma = 18$, $\varphi' = 30^\circ$), 6.0 m clay ($\gamma = 19$, $c' = 5$ kPa, $\varphi' = 24^\circ$, $c_u = 45$ kPa), below it dense sand ($\gamma = 19$, $\varphi' = 36^\circ$); $L = 7.5$ m; water at 8.5 m on both sides; $q = 10$ kPa; two rows of anchors: $z = 1.5$ m (15°) and 4.5 m (20°), $s = 2.5$ m; $f = 1.3$; S275, $W_{el} = 1\,400$ cm³/m**
Earthquake	**$k_h = 0.15$, $k_v = \pm 0.075$; pore water is confined; tie: $T_{ult} = 754$ kN, required $FS_{bond,E} \approx 1.5$**

REQUIRED

The D, M, F values of the four cases (two scenarios × two k_v); **D_E**, **$M_{max,E}$**, anchor forces and bond; can the wall withstand this earthquake?



Solution (1/2)

SOLUTION

1 Four cases (two scenarios of static design, k_v in two directions)

Scenario	k_v	D (m)	M (kN·m/m)	F (kN/m)	F_1 ; F_2
Short	+0.075	2.900	87.4	132.3	51.2 ; 81.0
Short	−0.075	3.110	134.9	205.0	82.4 ; 122.6
Long	+0.075	4.348	277.0	288.5	66.1 ; 222.4
Long	−0.075	3.958	277.2	315.8	81.5 ; 234.3

Governing embedment **D_E = 4.35 m** (long term, k_v = +0.075); maximum moment **277.2 kN·m/m** (long term, k_v = −0.075). ΔP_{AE} = 48.9 kN/m.

Solution (2/2)

SOLUTION

2 Checks

$$D_E = 4.35\ \text{m} \ > D_{design} = 3.12\ \text{m} \ \text{✗} \qquad \eta_E = \frac{M_{Ed,E}}{M_{c,Rd}} = \frac{277.2}{385.0} = 0.720 \ \leq 1 \ \text{✓}$$

$$T_{E,1} = 213.3\ \text{kN}: \quad FS_{bond,E} = \frac{753.98}{\max(213.3, 250)} = 3.02 \ \geq 1.50 \ \text{✓}$$

$$T_{E,2} = 623.3\ \text{kN}: \quad FS_{bond,E} = \frac{753.98}{623.3} = \mathbf{1.21} \ < 1.50 \ \text{✗}$$

The sheet-pile wall **does not withstand** this earthquake: seismic equilibrium requires 4.35 m of embedment (design 3.12 m) and the bond of the second anchor is insufficient ($FS_{bond,E}$ = 1.21 < 1.50, $T_{E,2}$ = 623 kN). The sheet-pile section is adequate (η_E = 0.72): the embedment length and the grout bond (length or diameter) must be increased.

Check the solution in GeoTerva ↗

Heave block at the base in clay; beam on spring foundation

THEORY

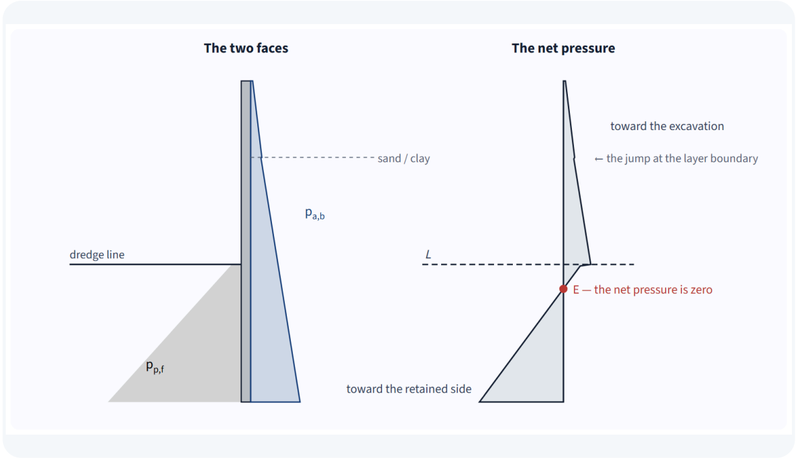
Base heave: if the excavation base is soft clay, the weight of the retained soil may squeeze the clay toward the excavation. As in the clay module, it is checked with a bearing capacity block at the wall toe (B′ = B/√2 or T to the hard layer, whichever is smaller; B″ = √2 B′):

$$FS = \frac{5.14\, c_u \left(1 + 0.2\frac{B''}{L_{exc}}\right) + \frac{c_{side} H}{B'}}{\sigma_v(H) + q}$$

Strip and isolated loads are converted into an equivalent surcharge within B′; if the soil at the base is not clay, no check is applied. **Displacement**: the wall is an elastic beam (EI), elastoplastic springs on each face (modulus of subgrade reaction k_h); each anchor is a prestressed spring ($k = E\, A_t \cos^2\alpha/(L_f s)$, starting with $P_0 \cos\alpha/s$), carrying tension only:

> The springs start from the at-rest pressure ($K_0\sigma'_v$ in a drained layer, σ'_v in undrained clay) and do not exceed the limit. k_h is a model parameter; the result is **characteristic and indicative**, given without limits; it depends strongly on k_h when the springs are soft, and very little once the limits are reached.

> The moment from the displacement model may come out close to the limit-equilibrium moment, but the design moment is always the limit equilibrium one.



Problem statement

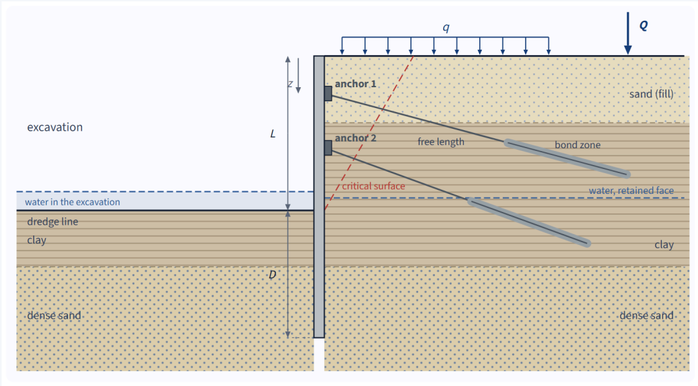
EXAMPLE

Given

Wall	**at the top 3.0 m sand ($\gamma = 18$, $\varphi' = 30^\circ$), 6.0 m clay ($\gamma = 19$, $c' = 5$ kPa, $\varphi' = 24^\circ$, $c_u = 45$ kPa), below it dense sand ($\gamma = 19$, $\varphi' = 36^\circ$); $L = 7.5$ m; water at 8.5 m on both sides; $q = 10$ kPa; two rows of anchors: $z = 1.5$ m (15°) and 4.5 m (20°), $s = 2.5$ m; $f = 1.3$; S275, $W_{el} = 1\,400$ cm³/m**
Excavation and wall	**excavation width $B = 10$ m (long, no stiff layer); clay below the base ($c_u = 45$ kPa); $EI = 44\,100$ kN·m²/m; $k_h = 20/15/40$ MN/m³ (sand/clay/sand); $P_0 = 250$ kN**

REQUIRED

FS_{heave} (≥ 1.5?); **displacement** of the wall top, excavation base and toe; maximum moment of the displacement model.



Solution

SOLUTION

1 Base heave (B′ = B/√2 = 7.07 m; L_{exc} not entered: no shape term)

$$c_{side} = \frac{45(4.5)}{7.5} = 27.0\ \text{kPa}\ \ (\text{sand zero}) \qquad \sigma_v(H) = 18(3) + 19(4.5) = 139.5\ \text{kPa}$$

$$FS = \frac{5.14(45) + 27.0(7.5)/7.07}{139.5 + 10} = \frac{231.3 + 28.6}{149.5} = \mathbf{1.739}\ \geq 1.5\ \checkmark$$

2 Displacement (modulus of subgrade reaction method; prestressed anchors)

$$y_{crest} = -2.6\ \text{mm}\ \ (\text{back, to the retained side}) \qquad y_{excavation\ base} = 11.2\ \text{mm} \qquad y_{tip} = -0.5\ \text{mm}$$

$$M_{max}^{model} = 108.6\ \text{kN·m/m}\ \ (z = 6.86\ \text{m}) \qquad M_{max}^{limit\ equilibrium} = 121.4$$

Base heave safety is **1.74** (≥ 1.5). The top of the wall is pulled **2.6 mm backward** by the prestressed anchors; at the excavation base it moves **11.2 mm** toward the excavation (indicative, not compared with a limit). The model moment (108.6) is close to the limit-equilibrium moment (121.4).

Check the solution in GeoTerva ↗

Assumptions of the module and what it does not do

THEORY

Stated assumptions: no wall friction (δ = 0); the free length is measured along the tendon; the point load is taken at the most loaded section; seismic coefficients layer by layer. **Those shown in two ways**: short and long term (both work), the crack water-filled or dry (the other is noted), the allowance in depth or in the passive (the other is noted).

Limiters: an anchor outside the wall; a layer lacking the strength your scenario requires; failure to find an acceptable embedment; anchors remaining in compression in every scenario; loss of closure; the bond zone lying within the critical wedge.

Out of scope: Rowe moment reduction (flexible wall), deep stability and overall stability (stated as “not checked” in every anchored run), seepage (different water levels are hydrostatic), inclined retained surface, line load and construction stages.

The calculation is **limit equilibrium, preliminary design**; the screen can suggest what to do, the report only states what was assumed and what was found.

Summary

Key points

Stress: σ_v, u, σ'_v on both faces; drained layer Rankine c′–φ′ (**pressure jump** at the layer boundary), undrained clay $\sigma_v \mp 2c_u$; at the surface a **water-filled tension crack** (in the example z_c = 4.37 m)

Both **short and long term** are analysed; embedment, anchor forces and moment use **the larger of the two scenarios** (in the example, long term throughout)

Net pressure = retained − excavation face; point E; allowance: $D = f \cdot D_{theoretical}$ (2.403 → 3.124 m) or FS_p on the passive side; at the given depth the **moment and anchor force increase** (D = 6 m: M = 255 kN·m/m)

System: cantilever (D = 10.4 m, η = 4.2), single row of free-earth support (M = 296, η = 1.04), two-row hinge method (F_1 = 41.7, F_2 = 132.3 kN/m; M = 121.4; η = 0.43)

Anchor: T = Fs/cos α (107.9 and 352.0 kN); bond FS = $T_{ult}/\max(T, P_0)$ = 3.02 and 2.14; **seismic**: D_E = 4.35 > 3.12 m and $FS_{bond,E}$ = 1.21 < 1.5: the sheet-pile wall does not withstand the earthquake

Section: EN 1993-1-1 η = 0.426; AISC LRFD 0.561, ASD 0.527. **Base heave** FS = 1.74; **displacement** crest −2.6 mm, excavation base 11.2 mm (indicator)

References

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Westergaard, H.M. (1933). Water pressures on dams during earthquakes. Transactions of the American Society of Civil Engineers, 98, 418–433.

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NOTE

All numerical examples have been generated with the module's own calculation engine; the module yields the same values. The opening model on the module page is also this wall (7.5 m sand–clay–sand, water at 8.5 m, q = 10 kPa, two rows of anchors).

GeoTerva

Online geotechnical analysis

From theory to practice

Enter the layered profile of your own wall (sand and clay), the water, the surcharge, the anchor rows and their elements in GeoTerva; the module gives together the pressures on both faces, the short- and long-term conditions, the embedment, the anchor forces, the moment, and the section, connection, earthquake, base heave and displacement checks.

Open on your phone
The example modules need no registration.

Open the module →

Theory page →

If you would like to use them in your lectures: info@geoterva.com